

**READ THESE INSTRUCTIONS**

- Use pencil only
- Initial top right corner of all pages (except the first one).
- Do NOT write on the exam.
- Do not remove the staple from your exam.
- Do not crumple or fold your exam.
- **Answering Questions:**
  - Handwriting that is illegible (messy, small, not straight) will lose points.
  - Indentation matters. Keep code aligned correctly for any answers that require code.
  - Answer all questions on the answer sheet(s) provided, not on the exam.
  - Make sure you write your answers on the answer sheets IN ORDER.
  - Do not cram answers together. Leave space.
- Help me ... help you!

**Failure to comply will result in loss of letter grade**

Grade Table (don't write on it)

Question	Points	Score
1	21	
2	20	
3	15	
4	19	
5	15	
6	10	
7	15	
8	10	
9	10	
10	10	
11	24	
Total:	169	

1. Remember to answer all questions on your answer sheet, not directly on the test (including a-g below).

(a) (3 points) Linked lists are always allocated in the:

(a) heap

(b) (3 points) Linked lists use what type of memory allocation?

(b) Dynamic

(c) (3 points) Arrays provide us with what type of access?

(c) direct or random

(d) (3 points) Accessing an item in a linked list costs?

(d)  $O(n)$

(e) (3 points) Insertion and deletion operations are faster in?

(e) linked lists

(f) (3 points) You receive values in some random order, but you want to store them in sequential order. Which is better, array or linked list?

(f) linked lists

(g) (3 points) You have a set of ordered values. Now you want to search for a value in that set. Which is better, array or linked list?

(g) array

2. (20 points) List the complexities from fastest to slowest.

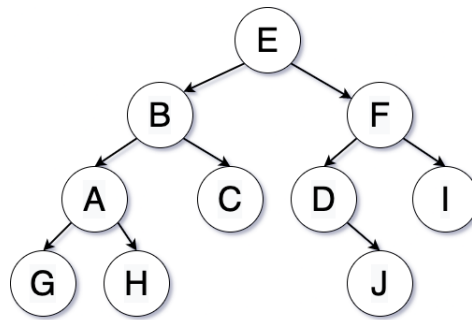
### Complexity Choices

$O(n!)$   $O(2^n)$   $O(1)$   $O(n \lg n)$   $O(n^2)$   $O(n^n)$   $O(n)$   $O(\log n)$   $O(1000)$

**Solution:** The spreadsheet below shows the growth of each choice based on the N value in column 1. The columns go from least cost on the left to greatest cost on the right. And even though it doesn't look correct, the  $O(1000)$  is a constant, therefore it falls right after  $O(1)$ . Again, this is why I dislike asymptotic notation for certain aspects of algorithm analysis.

N	$O(1)$	$O(\lg N)$	$O(N)$	$O(N \lg N)$	$O(N^2)$	$O(2^N)$	$O(N!)$	$O(N^N)$
1	1	0	1	0	1	2	1	1
2	1	1	2	2	4	4	2	4
3	1	2	3	5	9	8	6	27
4	1	2	4	8	16	16	24	256
5	1	2	5	12	25	32	120	3125
6	1	3	6	16	36	64	720	46656
7	1	3	7	20	49	128	5040	823543
8	1	3	8	24	64	256	40320	16777216
9	1	3	9	29	81	512	362880	387420489
10	1	3	10	33	100	1024	3628800	10000000000
11	1	3	11	38	121	2048	39916800	285311670611
12	1	4	12	43	144	4096	479001600	891610044825
13	1	4	13	48	169	8192	6227020800	302875106592

3. (15 points) Give a Pre-order / Post-order / In-order traversal of the following tree:

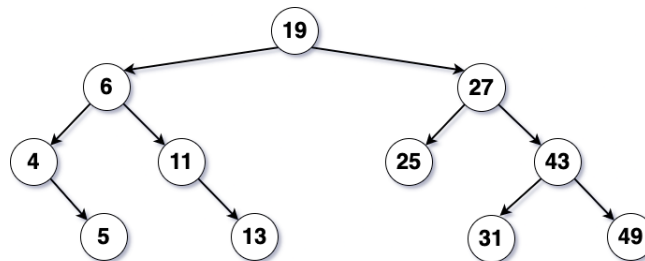


PreOrder	E	B	A	G	H	C	F	D	J	I
InOrder	G	A	H	B	C	E	D	J	F	I
PostOrder	G	H	A	C	B	J	D	I	F	E

4. Given the following list of numbers: **19, 6, 11, 4, 13, 5, 27, 43, 49, 31, 25**

- (a) (10 points) Process them from left to right and draw your resulting Binary Search Tree.

**Solution:**



- (b) (3 points) Is this tree complete?

**Solution:** No

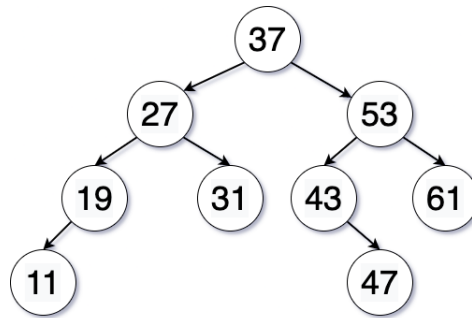
- (c) (3 points) Is this tree full?

**Solution:** No

- (d) (3 points) Is this tree balanced?

**Solution:** Yes

Use the following tree for the following questions dealing with node deletion and AVL trees. For each question, start with the original tree.

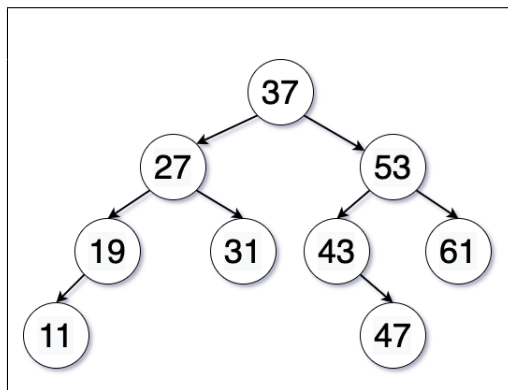


Assuming the tree is a plain Binary Search Tree:

5. (15 points) Delete **37** from the binary search tree above and draw the TWO possible resulting trees.

Predecessor (31)	Successor (43)
<pre>graph TD; 31((31)) --&gt; 27((27)); 31 --&gt; 53((53)); 27 --&gt; 19((19)); 19 --&gt; 11((11)); 53 --&gt; 43((43)); 43 --&gt; 47((47)); 53 --&gt; 61((61));</pre>	<pre>graph TD; 43((43)) --&gt; 27((27)); 43 --&gt; 53((53)); 27 --&gt; 19((19)); 19 --&gt; 11((11)); 53 --&gt; 47((47)); 47 --&gt; 31((31)); 53 --&gt; 61((61));</pre>

6. (10 points) Delete **43** from the binary search tree above and draw the resulting tree.



7. (15 points) Write a pseudo code implementation for deleting a node from a binary search tree.

if node is null:

Just return and stop the recursion (looking for value to delete)

if current node is less than the value we're looking for:

recursively call delete with right child

if current node is great than the value:

recursively call delete with left child

otherwise, we've found the node we want to delete:

- if the node has no children, delete node were done

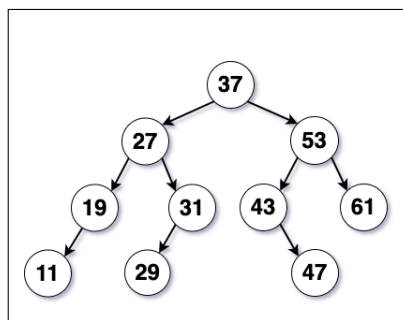
- if the node has one child, replace deleted node with existing child

- node has 2 children:

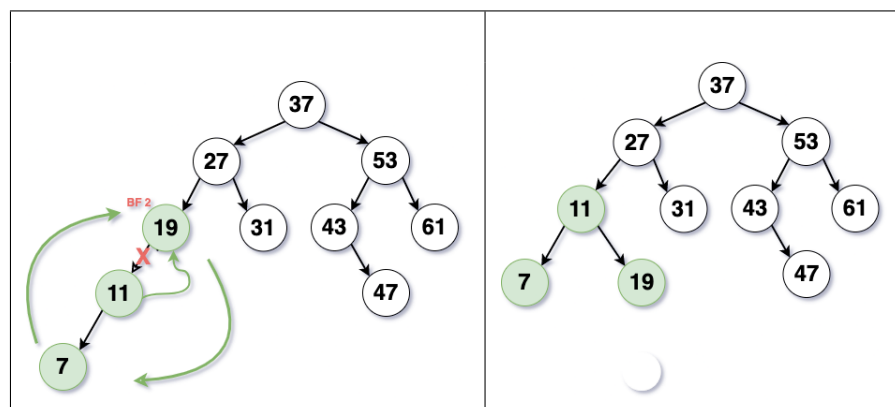
- replace deleted node with inorder successor or inorder predecessor

Now assume the tree is an AVL tree. Remember for each question use original tree.

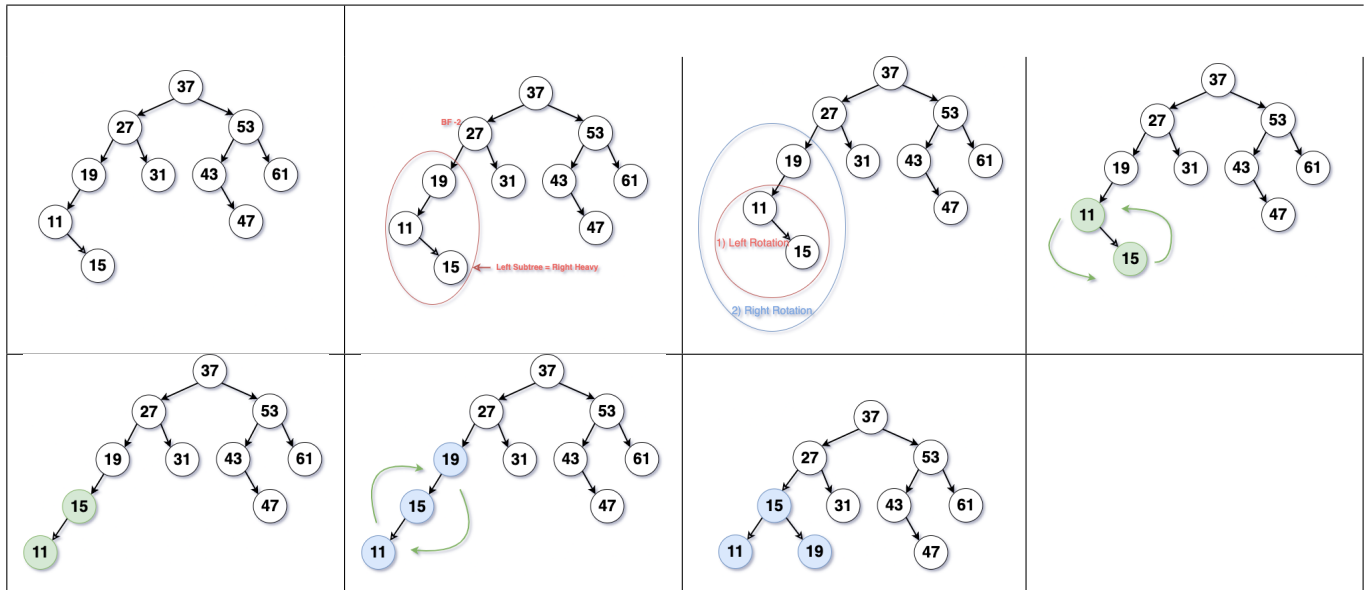
8. (10 points) Insert the value **29**.



9. (10 points) Insert the value **7**.

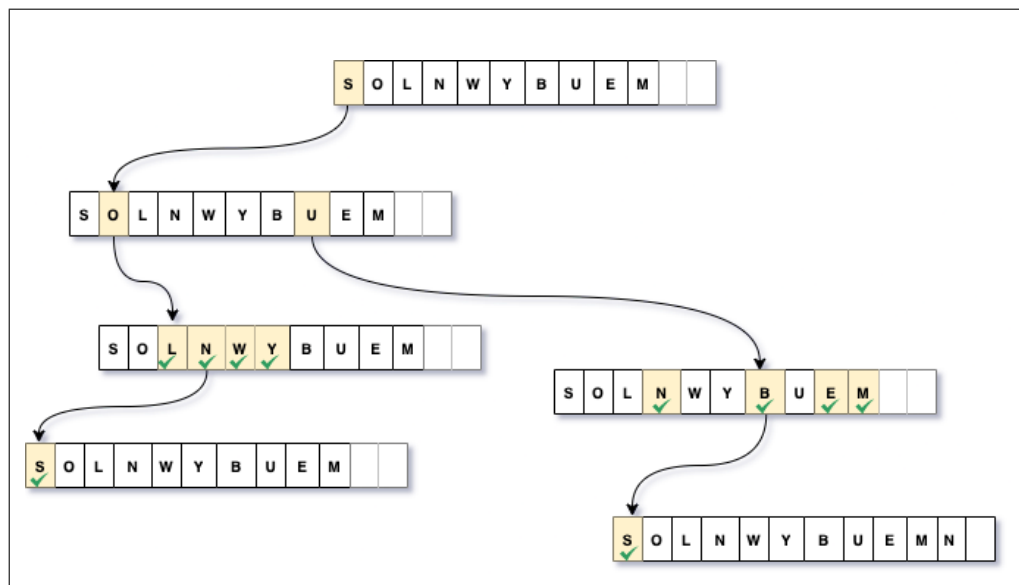


10. (10 points) Insert the value **15**.



11. (a) (15 points) Add the following words to a Trie data structure.

**sol, sons, sow, soy, subs, sue, sum, sun**



(b) (3 points) What is the complexity of creating a Trie?

**Solution:**  $O(N)$

(c) (3 points) What is the complexity of searching for a specific word in a Trie?

**Solution:**  $O(N)$

(d) (3 points) What is the complexity of searching for all words with a given prefix in a Trie?

**Solution:**  $O(N)$