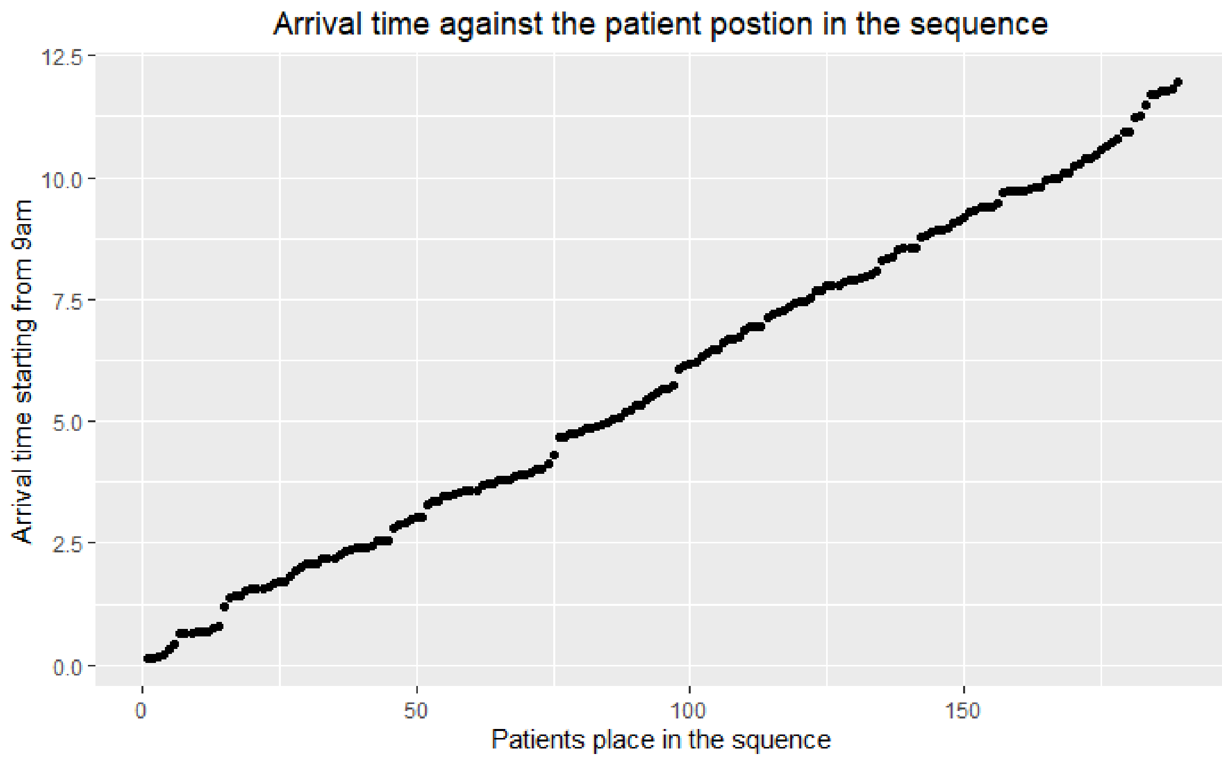
Summative Assignment

The Poisson Process

Q1)

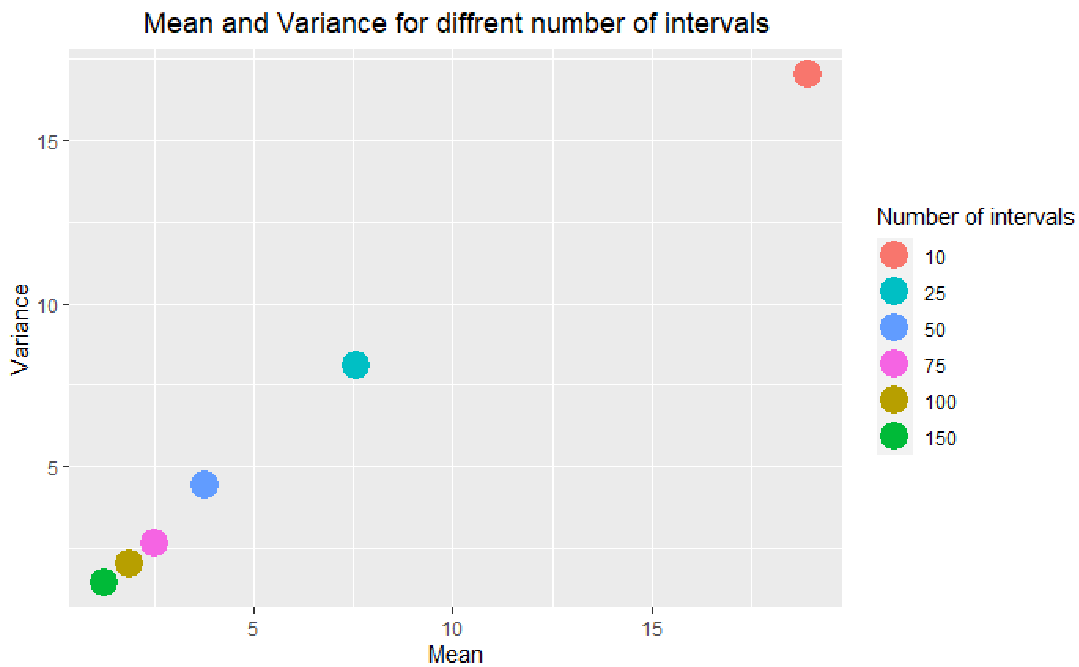
Using the data provided this the plot I produced by plotting each arrival time against its position in the sequence.



From this graph I believe that the data is following a Poisson process. In a Poisson process the events occur randomly in time at a constant rate which suggest that at every constant unit in time there is we should except roughly the same number of new patients. This would roughly give us a somewhat linearish graph, like what I was able to produce by plotting the data. If I where to draw a straight line on my graph of in increasing function of a constant per time interval like what the Poisson process the two would roughly overlap.

Q2)

When splitting the data into intervals of equal length and then recording the mean and variance I was able to produce the following.



As we can see the higher the number of intervals, we split the data into then the lower the mean and variance. What should be noted is that for every point the mean is roughly equivalent to the variance. This supports the idea the arrivals follow a Poisson process as in a Poisson distribution the mean and the variance are roughly equivalent

$$mean\ \{x1, \ldots, xk\} \approx variance\ \{x1, \ldots, xk\}$$

This because in a of Poisson distribution the mean and variance are both equal to μ (the average occurrences in each interval).

$$E(X) = \mu \text{ and } V(X) = \sigma^2 = \mu$$

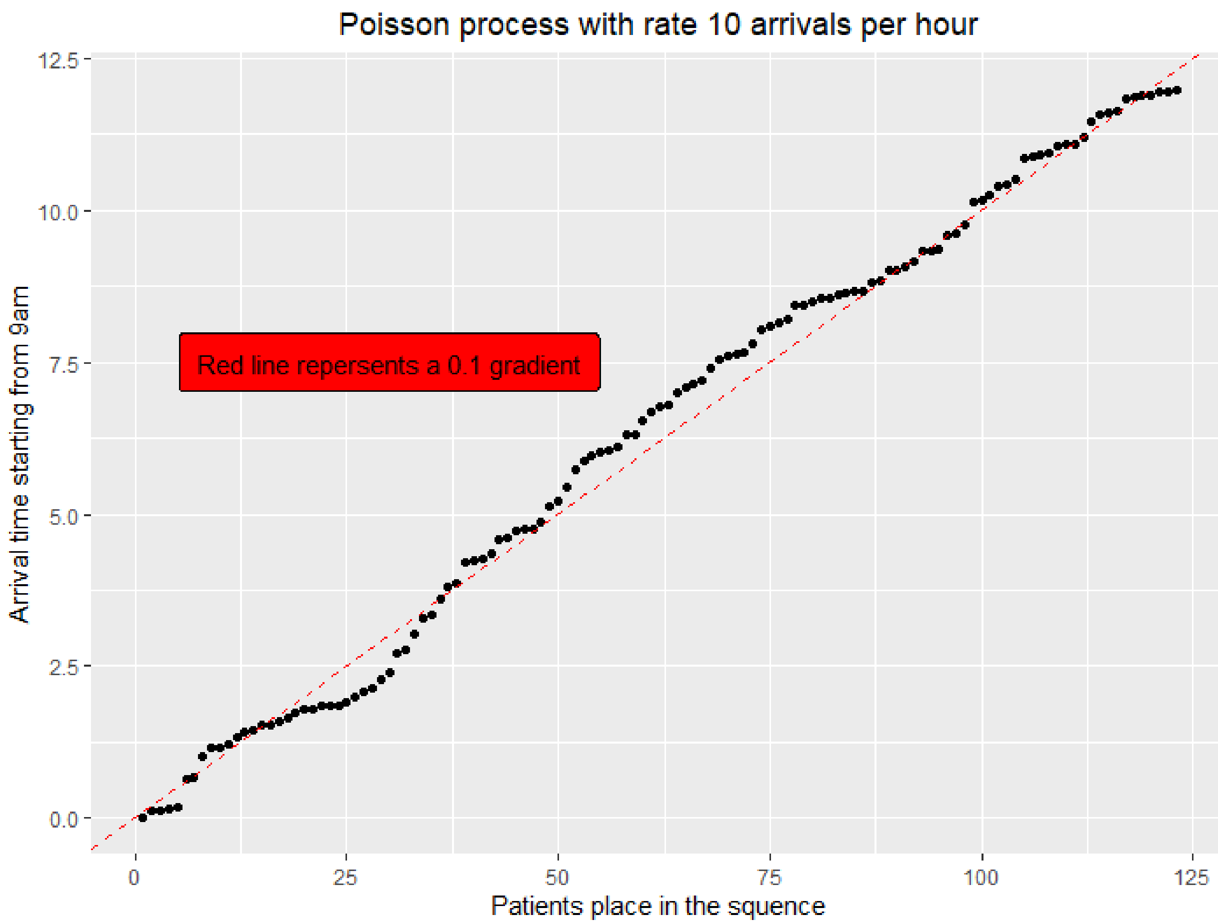
Using the fact *mean {x1, . . . , xk}* $\approx \lambda L/k$ I rearranged it to find the rate parameter by doing

$$(mean\ \{x1, \ldots, xk\} * k\ ) / L \approx \lambda$$

Using the different number of intervals and the means they produced I was able to consistently get an estimate of the rate parameter λ being **15.75.**

Q3)

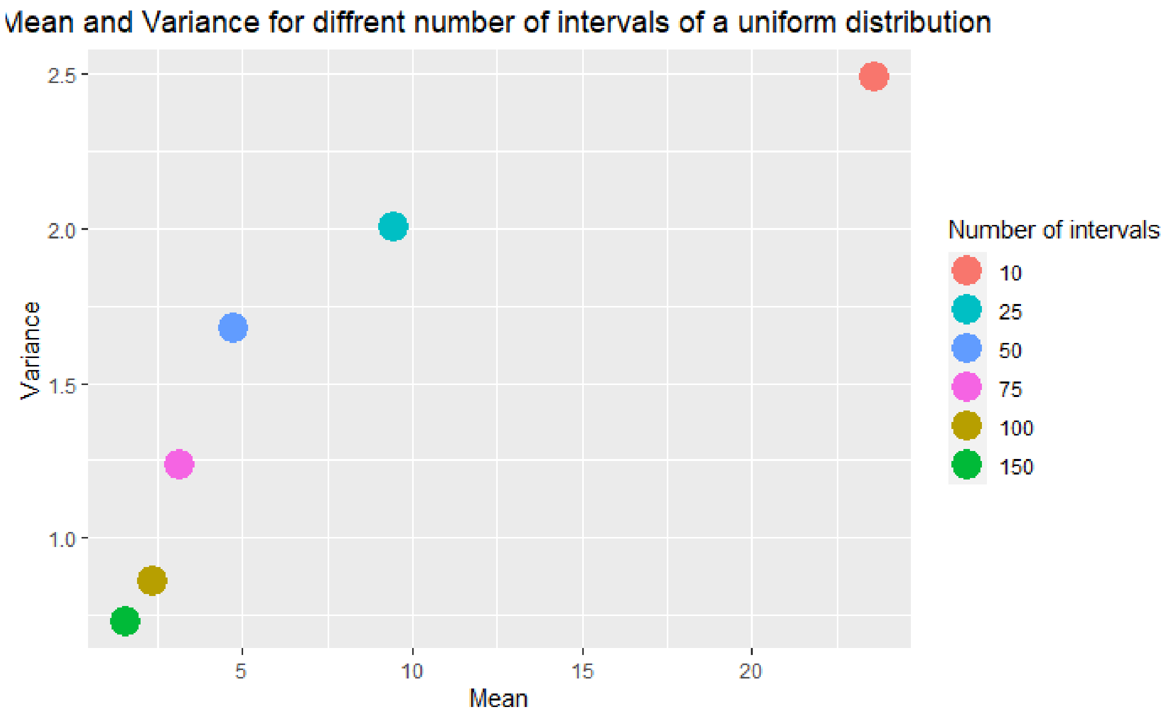
Using the cumsum(rexp(500, 10)) and filtering results that went over 12 hours I was able to produce the following.



The red line represents a constant growth rate at 10 patients per hour. The results gotten from a Poisson process gets a result that "sits" along that line showing that the Poisson is consistent with arrivals at a rate of lambda = 10.

Note as Poisson process is a sequence of random variables and it being a stochastic process, Each time you run the Poisson process, it will produce a different sequence of random outcomes so the graph will look different each time but should roughly match this line.

Q4)

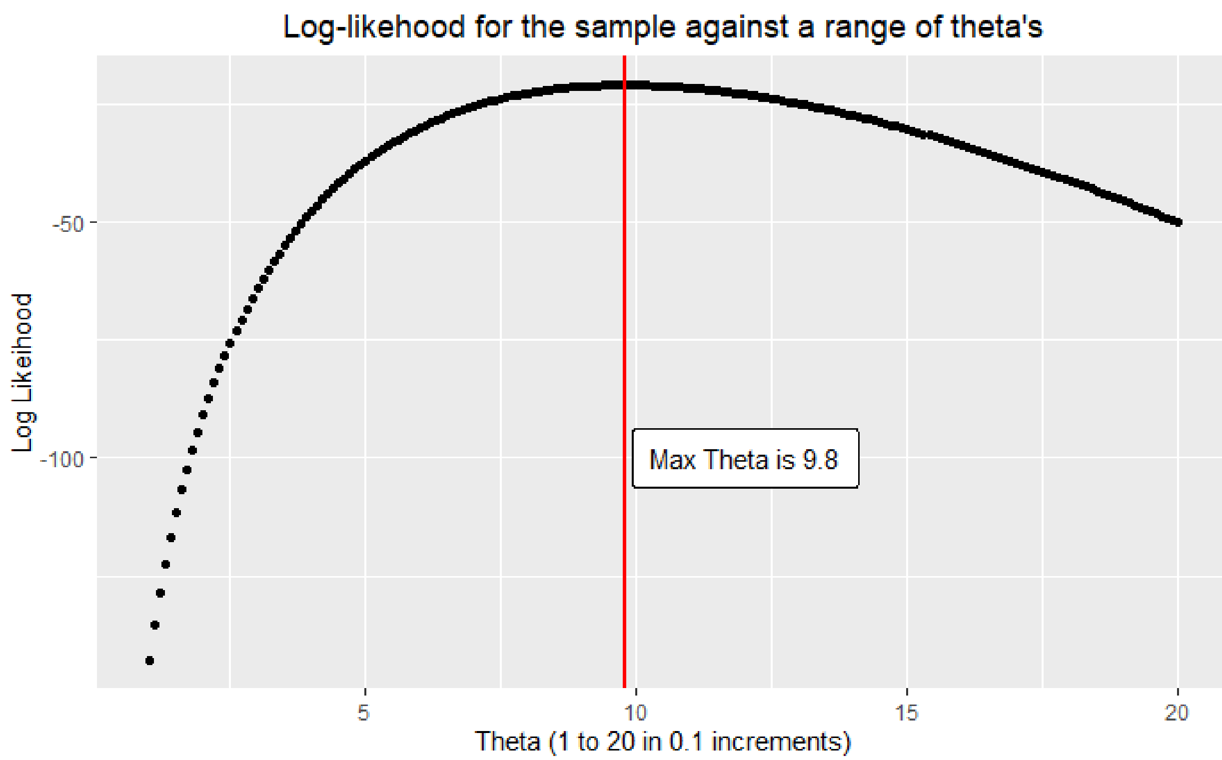


I would say that this uniform distribution is not following a Poisson process as there is large difference between each "number of intervals" gotten mean and variance. So, the mean is not equivalent to variance (which is a key way of telling if something is Poisson).

Maximum Likelihood

Q5) See code

Q6)



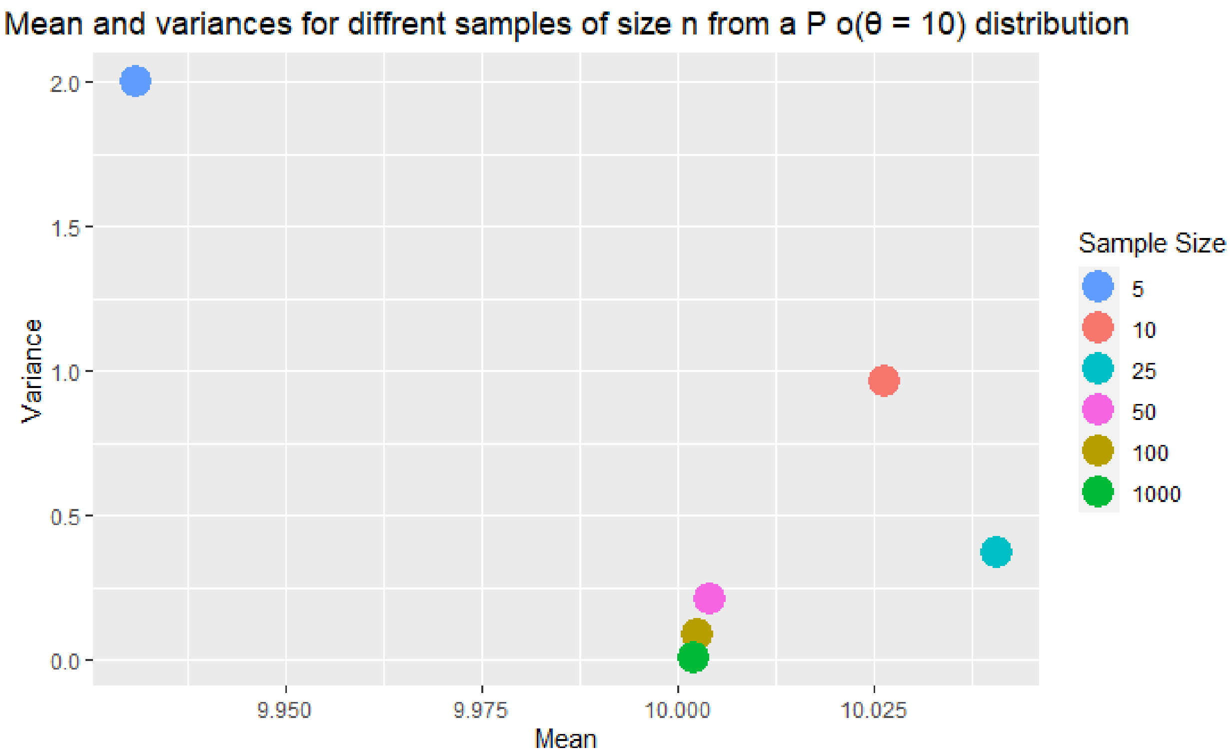
This is the graph of log-likelihood for the given sample against theta (range 1 to 20 in 0.1 increments). The red line is the point which gets the maximum theta which is 9.8 (Log likehood –21.09).

Q7)

By modifying the function from Q5 to find the θ which maximizes the log-likelihood of a sample. By using the sample given in question 6 It returned a maximum of 9.777779 which rounded to 1.dp is 9.8 which is what I got as the estimated max from my q6 plot. This means that this function agrees with my graph

Q8) See code

Q9)



This is the graph I made for this question.

I believe that these results for the maximum likelihood is consistent but not fully unbiased, instead I believe its asymptotically unbiased.

This is because as the sample size increases the Mean tends towards the lambda used to generate the results (10)

E[Estimate theta] -> theta and n -> infinity

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Also, when the sample size increases the Variance tends towards 0.

Var(Theta estimate) -> 0 as n -> infinity

Both of these factors make me believe that this is asymptotically unbiased.