NaturalLI: Natural Logic Inference for Common Sense Reasoning

Abstract

Common-sense reasoning is important for a range of AI applications, both in NLP and many vision or robotics tasks. We propose NaturalLI: a Natural Logic inference system for inferring common sense facts – for instance, *cats have tails* or *tomatoes are fragile* – from a very large database of known facts. The system provides logically valid derivations, while also being flexible to backing off to derivations which are only likely valid, accompanied by an associated confidence. We show that our system is able to capture strict Natural Logic inferences on the FraCaS test suite, and demonstrate the system's ability to infer previously unseen facts with 50% recall and 91% precision.

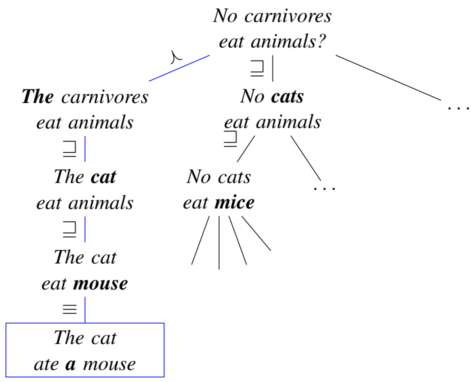


Figure 1: A Natural Logic inference cast as search, proving that the query *no carnivores eat animals* is false given the known fact *the cat ate a mouse*. The valid path is one of many candidates taken; the known fact found is one of many known facts in the database. The edge labels denote Natural Logic inference steps, as described in Section 2.1.

1 Introduction

Natural Logic allows us to reason about language without an intermediate logical form. Although projects like the Abstract Meaning Representation (Banarescu et al., 2013) have made headway providing a broad-coverage logical representation for language, it remains appealing to use the unstructured language itself as the vessel for inference. This is particularly true for common sense types of facts, where the meaning representation is often less crisp than in Freebase-style factoids.

A key application for natural language inference is, in turn, database completion: given a set of facts, predict whether an unseen fact should belong in the database. Many such databases – e.g., output from the OpenIE project (Banko et al., 2007) – are in reality semi-structured or unstructured text. This makes Natural Logic an appealing tool for the task.

However, prior work on Natural Logic has a few shortcomings: (i) it assumes we know the premise for our inference, (ii) it requires explicit alignment between this premise and the query, and (iii) it enforces strict entailment, whereas many applications could make use of probabilistic entailment accompanied by a confidence. We address the first two shortcomings by formulating Natural Logic inference as a search problem from a query to any supporting premise. The third point is addressed by allowing imprecise inference steps, at the cost of an associated learned penalty.

This then allows us to efficiently infer whether an unseen common sense fact is *true* or *false* on the basis of a very large, noisy, unstructured collection of known facts. An example inference is shown in Figure 1; Section 2 explains the inference steps in the search, Sections 3 and 4 describe the search problem in detail.

2 MacCartney's Natural Logic

Broadly, Natural Logic aims to capture common logical inferences by appealing directly to the structure of language, as opposed to running deduction on an abstract logical form. That is to say, Natural Logic takes the meaning representation of a sentence to be the surface form of the sentence itself. In part, this lends itself to computationally efficient inference; in another part, it frees the system from parsing to an abstract logic – this is particularly relevant in our case, where the set of potential antecedents is very large.

We build off of the variant of Natural Logic introduced by the NatLog inference system (MacCartney and Manning, 2007; MacCartney and Manning, 2008), based off of earlier theoretical work on Natural Logic and Monotonicity Calculus (Van Benthem, 1986; Valencia, 1991). Later work has formalized many aspects of the logic (Icard III, 2012; Djalali, 2013), which we will appeal to in future sections, but are not necessary for an understanding of the contributions of this work. An elegant introduction to Natural Logic can be found in Icard III and Moss (2014); a thorough treatment of MacCartney's Natural Logic can be found in MacCartney and Manning (2009).

At a high level, Natural Logic proofs operate by mutating spans of text in precise ways to ensure that mutated sentence follows from the original. We therefore must introduce three components for a complete proof system: we define the valid atomic mutations over lexical entries (Section 2.1), define the effect these mutations have on the validity of the inference (Section 2.2), and define a practical system for executing these proofs. We introduce MacCartney's alignment-based approach in Section 2.3, and show that we can generalize and simplify this system in Section 3.

2.1 Lexical Relations

MacCartney and Manning (2007) introduces seven set-theoretic relations between the denotations of any two lexical items. The denotation of a lexical item is the set of objects in the domain of discourse $\mathcal{D}$ which that lexical item refers to. For instance, the denotation of *cat* would be the set of all cats. We can then compare the denotation of two lexical items in terms of set algebra: if we define the set of cats to be φ and the set of animals to be ψ, we can state that $\varphi \subseteq \psi$. This example should evoke a taste of Natural Logic: e.g., if $\varphi \subseteq \psi$,

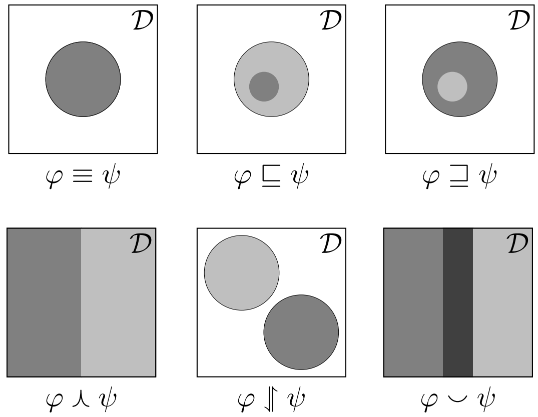


Figure 2: The model-theoretic interpretation of the MacCartney relations. The figure shows the relation between the denotation of φ, in dark gray, compared to the denotation of of ψ, in light gray. The universe is denoted by $\mathcal{D}$.

then anything which holds for all elements in ψ (all animals) must hold for all elements of φ (all cats).

The six informative relations are summarized in Figure 2; the last relation (#) corresponds to to the completely uninformative relation. For instance, the example search path in Figure 1 makes use of the following relations:

No x y	$\curlywedge$	*The x y*
cat	$\sqsupseteq$	*carnivore*
animals	$\sqsupseteq$	*mouse*
mouse	$\equiv$	*a mouse*

The middle two middle two entries have straightforward interpretations: the set of cats is a subset of the set of carnivores; the set of mice is a subset of animals. The last entry states that the denotation of *mouse* and *a mouse* is close enough that they can be considered equivalent.

The first entry is more subtle in that the denotation of quantifiers is not over the universe of entities e, but rather over functions from entities to truth values t: $e \rightarrow (e \rightarrow t)$. Our claim is really the conjunction of two claims: $\forall x \forall y \quad \neg\,(\textit{no } x\ y \land \textit{the } x\ y)$ and $\forall x \forall y \quad (\textit{no } x\ y \lor \textit{the } x\ y)$. This is directly analogous to the claims used to construct the set-theoretic definition of $\curlywedge$ in Figure 2: $\varphi \cap \psi = \varnothing$ and $\varphi \cup \psi = \mathcal{D}$. A more thorough treatment can be found in Icard III and Moss (2014).

Note that $\sqsubseteq$ and $\sqsupseteq$ are inverses. Examples of the last two relations ($\mid$ and $\smile$) and the complete independence relation (#) are given below:

cat	$\upharpoonleft\downharpoonright$	*dog*
animal	$\smile$	*nonhuman*
cat	$\#$	*friendly*

We proceed to describe the relationship between these lexical mutations, and the validity of executing the mutation in a derivation.

2.2 Monotonicity and Polarity

We introduce two important concepts: *monotonicity* as a property of arguments to natural language quantifiers, and *polarity* as a property of lexical items in a sentence. Much like monotone functions in calculus, a monotone quantifier has an output truth value which is non-decreasing as the input "increases"

To offer a concrete example, *some* is a monotone quantifier taking two entities as input (i.e., *some* x y – *some cats eat mice*), and returning a truth value. Therefore, we can replace an argument to *some* with a superset of that argument, and be guaranteed that the truth value of the new predicate will be "at least" the truth value of the original statement. The "at least" relation on truth values is trivially defined as $F \leq T$; therefore, for two truth values t_1 and t_2, $t_1 \leq t_2$ is precisely material implication $t_1 \Rightarrow t_2$. With this we can now show that, e.g., *some cats eat mice* $\Rightarrow$ *some animals eat mice*.

Analogous to monotone quantifiers, we can define *antitone*[1] quantifiers as quantifiers which has an output truth value which is non-increasing as the input increases. For instance, *no* is an antitone quantifier. Note that quantifiers can have different monotonicity for each argument: *all* is monotone in its first and antitone in its second argument. Furthermore, not all quantifiers are monotone or antitone – *most* is the classic example of a quantifier which is *nonmonotone* in its first argument.

So far we have looked only at sentences with a single quantifier; moreover, monotonicity itself is not always sufficient to warrant mutations on lexical items. For this, we introduce *polarity*, a property of lexical items in a sentence determined by the quantifiers acting on it. All lexical items begin are *upward* polarity by default; monotone quantifiers preserve polarity, and antitone quantifiers reverse polarity. For example, *cats* in *all cats eat mice* has downward polarity; or, *mice* in *no cats don't eat mice* has upward polarity (in the scope of two antitone quantifiers).

[1] Monotone and antitone are often referred to as *monotone up* and *monotone down*.

Relation	**Polarity of Context**	
	Upward	Downward
$e_1 \sqsubseteq e_2$	$s_1 \sqsubseteq s_2$	$s_1 \sqsupseteq s_2$
$e_1 \sqsupseteq e_2$	$s_1 \sqsupseteq s_2$	$s_1 \sqsubseteq s_2$
$e_1 \upharpoonleft\downharpoonright e_2$	$s_1 \upharpoonleft\downharpoonright s_2$	$s_1 \smile s_2$
$e_1 \smile e_2$	$s_1 \smile s_2$	$s_1 \upharpoonleft\downharpoonright s_2$
$e_1 \equiv e_2$	$s_1 \equiv s_2$	$s_1 \equiv s_2$
$e_1 \curlywedge e_2$	$s_1 \curlywedge s_2$	$s_1 \curlywedge s_2$
$e_1 \# e_2$	$s_1 \# s_2$	$s_1 \# s_2$

Table 1: The projection table used for a relation between a phrase e_1 and its candidate mutation e_2, in terms of the produced relation between sentence s_1 and the new sentence s_2. Values are given for when the entities are in a monotone and antitone context.

$\bowtie$	$\sqsubseteq$	$\sqsupseteq$	$\curlywedge$	$\upharpoonleft\downharpoonright$	$\smile$
$\sqsubseteq$	$\sqsubseteq$	$\#$	$\upharpoonleft\downharpoonright$	$\upharpoonleft\downharpoonright$	$\#$
$\sqsupseteq$	$\#$	$\sqsupseteq$	$\smile$	$\#$	$\smile$
$\curlywedge$	$\smile$	$\upharpoonleft\downharpoonright$	$\equiv$	$\sqsupseteq$	$\sqsubseteq$
$\upharpoonleft\downharpoonright$	$\#$	$\upharpoonleft\downharpoonright$	$\sqsubseteq$	$\#$	$\sqsubseteq$
$\smile$	$\smile$	$\#$	$\sqsupseteq$	$\sqsupseteq$	$\#$

Table 2: The join table, as copied from Icard III (2012). Note that the $\#$ always joins to yield $\#$, and $\equiv$ always joins to yield the input relation.

We now have a repertoire or lexical mutations, and a polarity marking on each lexical item. We have two remaining elements to define for a full proof theory: how do lexical mutation of a word with a certain polarity *trickle up* the sentence to affect the relation between entire sentences, and how do we chain individual lexical mutations together for a full derivation. The first is studied in depth by Icard III (2012);[2] we axiomatically define the relation between two sentences differing by a mutation in Table 1. The second is discussed in the next section.

2.3 Proof By Alignment

In the framework of MacCartney and Manning (2007), the task is to infer whether a single logical antecedent entails a query consequent. The natural approach is to generate an alignment between the antecedent and the consequent, and classify each aligned segment into one of the relations in Fig-

[2] Formally, we optimistically assume every quantifier is *additive* and *multiplicative*.

ure 2. Inference then reduces to projecting each of these relations according to Table 1 and iteratively *joining* pairs of projected relations together to get the final entailment relation. This *join table* is given in Table 2.

To illustrate, we can consider the inference from *Stimpy is a cat* to *Stimpy is not a poodle*. An alignment of the two statements would provide three lexical mutations: *cat*→*dog*, $\cdot$ →*not*, and *dog*→*poodle*. MacCartney and Manning (2007) then constructs a proof as follows:

Mutation	$\beta(e_i)$	$\beta(x_{i-1}, e_i)$	$\beta(x_0, x_i)$
cat→*dog*	$\downharpoonleft\!\upharpoonright$	$\downharpoonleft\!\upharpoonright$	$\downharpoonleft\!\upharpoonright$
$\cdot$ →*not*	$\curlywedge$	$\curlywedge$	$\sqsubseteq$
dog→*poodle*	$\sqsupseteq$	$\sqsubseteq$	$\sqsubseteq$

where $\beta(e_i)$ is the lexical relation, as per Figure 2, $\beta(x_{i-1}, e_i)$ is the relation projected according to Table 1, and $\beta(x_0, x_i)$ is the relation between the antecedent and the current candidate fact. $\beta(x_0, x_i)$ is obtained from repeatedly applying the join table in Table 2 to $\beta(x_0, x_{i-1})$ and $\beta(x_{i-1}, e_i)$. For instance, joining $\beta(x_0, x_0)$ with $\beta(x_0, e_1)$ ($\downharpoonleft\!\upharpoonright \bowtie \curlywedge$) yields $\beta(x_0, x_1)$ ($\sqsubseteq$).

The final relation between the antecedent and the consequent is taken to be the last $\beta(x_0, x_i)$ entry. In our example above, we would conclude that *Stimpy is a cat* $\sqsubseteq$ *Stimpy is not a poodle*, and therefore the inference is valid.

3 Inference as a Finite State Machine

The proof system from Section 2.3 can be naturally cast as a finite state automata. This is appealing for at least two reasons: first, it is an efficient means of keeping track of only the relevant information when running Natural Logic inference as a search (see Section 4). Second, this formulation makes clear a theoretical contribution of this work: in the case where relevant output of the system is only whether the derivation is *valid*, *invalid*, or *unknown*, we can collapse the automata losslessly into only these three states. This is both computationally convenient, and conceptually elegant, in that it makes many of the opaque patterns in the join table (Table 2) more clear.

In more detail, taking notation from Section 2.3, we can take the states of our FSA to be the $\beta(x_0, x_i)$ values – the relation between the current fact and the first antecedent. The transitions become the projected lexical relations $\beta(x_{i-1}, e_i)$; note that these are deterministic from the mutation and an analysis of the polarity of the lexical item. The FSA is then described in Figure 3a.

We can cluster each of the states in the FSA – that is, each of the relations between the first and current fact – into whether this model-theoretic relation corresponds to a valid, invalid, or unknown inference. Following MacCartney and Manning (2007), we can cluster $\equiv$ and $\sqsubseteq$ as valid inferences. Provably invalid inferences correspond to $\downharpoonleft\!\upharpoonright$ and $\curlywedge$. All other relations ($\sqsupseteq$, $\smile$) correspond to inferences of unknown validity.

We note that there is never a case where two states in a cluster have the same outgoing transition go to states in two different clusters. That is, for states x and y in the same cluster (i.e., *valid*, *invalid*, *unknown*), if for relation r, x transitions to x' and y transitions to y', it is always the case that x' and y' are also in the same cluster.

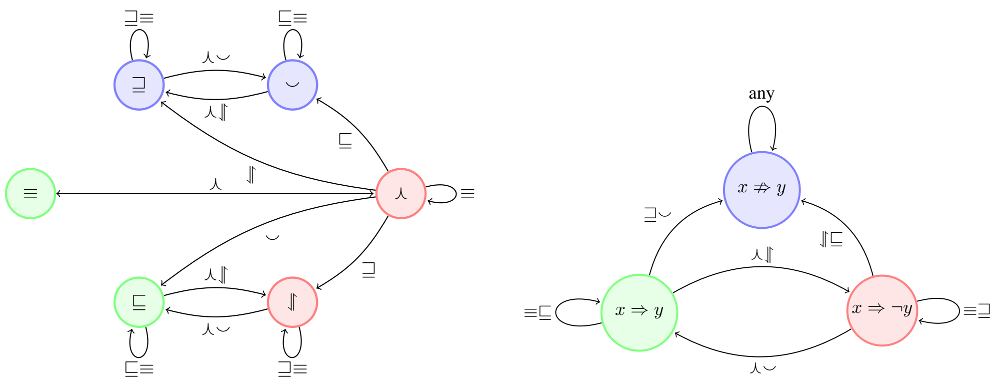
We can therefore collapse the automata into just three states, as shown in Figure 3b. A few observations deserve passing remark. First, it becomes obvious from the collapsed FSA that even though the states $\sqsupseteq$ and $\smile$ appear to track, in fact there is no "escaping" these states back into either a valid or invalid inference. Second, the hierarchy over relations presented in Icard III (2012) becomes apparent – in particular, $\curlywedge$ always behaves as negation, whereas its two "weaker" versions ($\downharpoonleft\!\upharpoonright$ and $\smile$) still behave as negation, but in fewer contexts.

We have not presented both prerequisites for our inference engine formulated as a search problem. The lexical relations from Section 2 will define the transitions in our search. The reformulation of Natural Logic inference as traversing an FSA will allow our search to efficiently track our inference state. We proceed to describe the detail of our search problem.

4 Inference As Search

For common-sense reasoning, we are not given a well-defined antecedent, but rather have at our disposal a large database of true facts. This makes the align-and-classify approach of Section 2.3 substantially less appealing, as candidate antecedents are not readily available.

We therefore approach the problem as a search task: given the consequent (the query), we search over the space of possible facts for a valid antecedent in our database. The nodes in our search problem correspond to candidate facts; the edges are mutations of these facts; the costs over these



(a) (b)

Figure 3: (a) The join table in Table 2 expressed as a finite state automata. Omitted edges go to the unknown state (#), with the exception of omitted edges from $\equiv$, which go to the state of the edge type. Green states ($\equiv$, $\sqsubseteq$) denote valid inferences; red states ($|$, $\curlywedge$) denote invalid inferences; blue states ($\sqsupseteq$, $\smile$) denote inferences of unknown validity. (b) The join table collapsed into the three meaningful states over truth values.

edges encode the confidence (or likelihood) that this edge maintains maintains an informative inference.

We define the problem by specifying the state space, the valid transitions, along with the weights of the transitions. We then describe how these weights translate to the confidence of truth of a fact, and show that our search is at worst a generalization of JC distance (Jiang and Conrath, 1997) – a common WordNet similarity metric.

4.1 Nodes

The space of possible nodes in our search is the space of possible states in a derivation. To a first approximation, a node is a pair (w, s) of a surface form w tagged with word sense and polarity, and an inference state s in our collapsed FSA (Figure 3b). In addition to this minimal information, a node in our search keeps track of some additional information to handle various details.

Lexical mutations As per our definition, transitions between nodes are transitions between facts. However, the lexical resources for the mutations are over lexical items (e.g., *feline* $\sqsupseteq$ *cat*) rather than entire facts. This motivates the inclusion of an index i in our node, denoting the index of the item which may be mutated. Importantly, this also imposes a natural ordering of items to mutate, making search more efficient at the expense of rare search errors.

Predicting deletions Although inserting lexical items in a derivation (deleting words from the reversed derivation) is trivial, the other direction is not. For brevity, we refer to a deletion in the derivation as an insertion, as from the perspective of the search algorithm constructing a reverse derivation, we are inserting lexical items. naïvely, at every node in our search, we must consider every item in the vocabulary as a possible insertion.

This is largely handled by storing the database of known facts as a trie. Since search mutates the fact left-to-right (as per above), we can look up completions in the trie to use as candidate insertions. To illustrate, given a search state with fact $w_0, w_1, \ldots, w_n$ and mutation index i, we would look up completions w_{i+1} for $w_0, w_1, \ldots, w_i$ in our trie of known facts.

Although this approach works well when i is relatively large, there are likely many candidate insertions for small i. We special case the most extreme case for this, where $i = 0$; that is, when we are inserting into the beginning of the fact. In this case, rather than taking all possible lexical items that start any fact, we take all items which are followed by the first word of our current fact. For instance, given a search state with fact $w_0, w_1, \ldots, w_n$, we would propose candidate insertions w_{-1} such that $w_{-1}, w_0, w'_1, \ldots, w'_k$ is a known fact, for some $w'_1, \ldots, w'_k$.

Polarity tracking Mutating quantifiers can change the polarity information on a span in the fact. Since we do not have the full parse tree at our disposal at search, we track a small amount of metadata to guess the scope of the mutated quantifier.

4.2 Transitions

We begin by introducing some terminology. A *transition template* is a broad class of transitions; for instance WordNet hypernymy. A *transition* or *transition instance* is a particular instantiation of a transition template. For example, the transition from *cat* to *feline*. Lastly, an *edge* in the search space connects two facts, which are separated by a single transition instance. For example, an edge exists between *some cats have tails* and *some felines have tails*.

Note that the edges in the search are not constructed *a priori*. It is sufficient to store the transitions, and construct particular edges on demand. At a high level, we include most relations in WordNet as transitions, and parametrize insertions and deletions by the part of speech of the token being inserted/deleted. The full table of transitions is given in Table 3, along with the relation that transition introduces, and an example edge in our derivation corresponding to that transition.

It should be noted that the mapping from transitions to relation types is intentionally imprecise. For instance, clearly nearest neighbors do not preserve equivalence ($\equiv$); more subtly, while *all cats like milk* $\rightthreetimes$ *all cats hate milk*, it is not the case that *some cats like milk* $\rightthreetimes$ *some cats hate milk*.[3] We mitigate this imprecision by introducing a cost for each transition, and learning the appropriate value for this cost (see Section 5).

The cost of an edge is parametrized by two values; the cost of an edge from fact (f, v, p) with surface form f, validity v and polarity p to a new fact (f', v', p') using a transition instance t_i of type t is given by $f_{t_i} \cdot \theta_{t,v,p}$, where:

> f_{t_i}: A value associated with every transition instance t_i, intuitively corresponding to how "far" the endpoints of a mutation are.
>
> $\theta_{t,v,p}$: A learned cost for taking a transition of type t, if the source of the edge is in a validity state of v and the word being mutated has polarity p.

The notation for f_{t_i} is chosen to evoke an analogy to features, and we will refer to this quantity as the feature value. We set f_{t_i} to be 1 in most cases; the exception are the edges over the WordNet hypernym tree, and the nearest neighbors edges. In the first case, we take $\uparrow_{w \to w'}$ as transitioning from word w to its hypernym w' (and visa versa for $\downarrow_{w \to w'}$), and set:

$$f_{\uparrow_{w \to w'}} = \log \frac{p(w')}{p(w)} \quad = \log p(w') - \log p(w)$$
$$f_{\downarrow_{w \to w'}} = \log \frac{p(w)}{p(w')} \quad = \log p(w) - \log p(w')$$

We define $p(w)$ to be the probability (normalized frequency) of a word or any of its hyponyms in the Google N-Grams corpus (Brants and Franz, 2006). Intuitively, this ensures that relatively long paths through fine-grained sections of WordNet are not unduly penalized. For instance, the path from *cat* to *animal* traverses six intermediate nodes, naïvely yielding a prohibitive search depth of 6.

For nearest neighbors edges, we take Neural Network embeddings learned in Huang et al. (2012) corresponding to each vocabulary entry. We then define $f_{NN_{w \to w'}}$ to be the arc cosine of the cosine similarity between word vectors associated with lexical items w and w':

$$f_{NN_{w \to w'}} = \arccos\left(\frac{w \cdot w'}{\|w\| \|w'\|}\right)$$

The set of features which fire along a path can be expressed as a vector $\mathbf{f}$. Each element of $\mathbf{f}$ corresponds to sum of the values of that feature along the path. Correspondingly, we define the weight vector $\boldsymbol{\theta}$ as the weight for every element of $\mathbf{f}$. The cost of a path can then be expressed as the dot product: $\theta^{\mathrm{T}}\mathbf{f}$.

4.3 Generalizing Similarities

An elegant property of our definitions of f_{t_i} is its ability to generalize JC distance, and upper-bound distributional similarity. Let us assume we have words w_1 and w_2, with a least common subsumer lcs. The JC distance $\text{dist}_{\text{jc}}(w_1, w_2)$ is:

$$\text{dist}_{\text{jc}}(w_1, w_2) = \log \frac{p(\text{lcs})^2}{p(w_1)p(w_2)} \qquad (1)$$

For simplicity, we simplify $\theta_{\uparrow,v,p}$ and $\theta_{\uparrow,v,p}$ as simply $\theta_{\uparrow}$ and $\theta_{\downarrow}$. The derivation generalizes trivially to the the case where weights are further parametrized. Without loss of generality, we also assume that a path in our search is only modifying

[3]The latter example is a consequence of the projection table in Table 1 being overly optimistic.

Template	Relation	Example edge
WordNet hypernym	$\sqsubseteq$	*some cats like milk* $\sqsubseteq$ *some felines like milk*
WordNet hyponym	$\sqsupseteq$	*some felines like milk* $\sqsupseteq$ *some cats like milk*
WordNet antonym†	$\curlywedge$	*all cats like milk* $\curlywedge$ *all cats hate milk*
WordNet synonym/pertainym†	$\equiv$	*some cats like milk* $\equiv$ *some cats enjoy milk*
Distributional nearest neighbor	$\equiv$	*some cats like milk* $\equiv$ *some cats like dairy*
Delete word†	$\sqsubseteq$	*some tabby cats like milk* $\sqsubseteq$ *some cats like milk*
Add word†	$\sqsupseteq$	*some cats like milk* $\sqsupseteq$ *some tabby cats like milk*
Quantifier weaken	$\sqsubseteq$	*all cats like milk* $\sqsubseteq$ *some cats like milk*
Quantifier strengthen	$\sqsupseteq$	*some cats like milk* $\sqsupseteq$ *all felines like milk*
Quantifier negate	$\curlywedge$	*some cats like milk* $\curlywedge$ *no felines like milk*
Quantifier synonym	$\equiv$	*some cats like milk* $\equiv$ *a few cats like milk*
Change word sense	$\equiv$	

Table 3: The edges allowed during inference. Entries with a dagger are parametrized by their part-of-speech tag, from the restricted list of {noun,adjective,verb,other}. The first column describes the type of the transition. The set-theoretic relation introduced by each relation is given in the second column. The third column gives an example of the transition in practice, as an edge in the search graph.

a single word w_1, ending at a mutation of the word w_2.

We can factorize the cost of a path, $\boldsymbol{\theta}^T\mathbf{f}$, along the path from w_1 to w_2 through its lowest common subsumer (lcs), $[w_1, w_1^{(1)}, \ldots, \text{lcs}, \ldots, w_2^{(1)}, w_2]$, as follows:

$$\begin{aligned}\theta^{\mathrm{T}}\phi &= \theta_{\uparrow}\left(\left[\log p(w_1^{(1)}) - \log p(w_1)\right] + \ldots\right) + \\ &\quad \theta_{\downarrow}\left(\left[\log p(\text{lcs}) - \log p(w_1^{(n)})\right] + \ldots\right) \\ &= \theta_{\uparrow}\left(\log\frac{p(\text{lcs})}{p(w_1)}\right) + \theta_{\downarrow}\left(\log\frac{p(\text{lcs})}{p(w_2)}\right) \\ &= \log\frac{p(\text{lcs})^{\theta_{\uparrow}+\theta_{\downarrow}}}{p(w_1)^{\theta_{\uparrow}} + p(w_2)^{\theta_{\downarrow}}}\end{aligned}$$

Note that setting both $\theta_{\uparrow}$ and $\theta_{\downarrow}$ to 1 exactly yield the Formula (1) for JC distance.

It is also worth noting that the nearest neighbors path provides an upper bound on the true similarity between the start and end words in the path. In this way, the search based approach presented here can be thought of as generalizing and formalizing the intuition that similar objects have similar properties (e.g., as presented in Angeli and Manning (2013)) using both common classes of similarity metrics.

4.4 Confidence Estimation

The last component in inference is translating a search path into a probability of truth. We notice from Section 4.2 that the *cost* of a path can be represented as $\boldsymbol{\theta}^T\mathbf{f}$. We can normalize this value by negating every element of the weight vector and passing it through a sigmoid:

$$\text{confidence} = \frac{1}{1 + e^{\boldsymbol{\theta}^T\mathbf{f}}}$$

Importantly, note that the cost vector must be non-negative for the search to be well-defined, and therefore the confidence value will be constrained to be between 0 and $\frac{1}{2}$.

At this point, we have a confidence that the given path has not violated strict Natural Logic. However, to translate this value into a probability we need to incorporate whether the inference path is confidently valid, or confidently invalid. To illustrate, a fact with a low confidence should translate to a probability of $\frac{1}{2}$, rather than a probability of 0.

We therefore define the probability of validity as follows. We take v to be 1 if the final fact of our derivation is in the *valid* state with respect to the antecedent, and -1 if this fact is in the *invalid* state. In practice, as our search is reversed, we take the state of the antecedent in our database when compared to the consequent, rather than visa versa. For completeness, if no path is given we can set $v = 0$. The probability of validity then becomes:

$$p(\text{valid}) = \frac{v}{2} + \frac{1}{1 + e^{v\boldsymbol{\theta}^T\mathbf{f}}} \tag{2}$$

Note that in the case where $v = -1$, the above expression reduces to $\frac{1}{2} -$ confidence; in the case where $v = 0$ it reduces to simply $\frac{1}{2}$. Furthermore, note that the probability of truth makes use of the same parameters as the cost in the search. Thus, as better weights are learned, the search is likewise more likely to produce derivations which would confidently support or disprove the query. We proceed to describe how these weights are learned.

5 Learning Transition Weights

The learning task can be viewed as a constrained optimization problem. Subject to the constraint that all elements of the cost vector $\boldsymbol{\theta}$ must be nonnegative, we optimize the probability from Equation (2) compared against the gold annotation. As training data, we are given a number of facts, annotated with a truth value of *true* or *false*. We assume that all facts in our database are true; therefore, $p(\text{valid})$ corresponds directly to $p(\text{true})$.

We learn costs using an iterative algorithm. At each iteration, we take the costs from the previous iteration and run the derivation search over every example, providing a predicted probability of truth for each query fact. Optimizing the likelihood of the training data according to Equation (2) is impractical as the objective is nonconvex; the rest of today is intended to be spent on implementing a piecewise optimization to mitigate this. A simple heuristic (weight = 1 - # times this feature fired in good examples / # of times feature fired total) does ok, but is a little braindead.

6 Experiments

We evaluate our system on two tasks: the FraCaS test suite used by MacCartney and Manning (2007) and MacCartney and Manning (2008), evaluate the systems ability to capture Natural Logic inferences even without the explicit alignments of these previous systems. In addition, we evaluate the system's ability to predict common-sense facts from a large corpus of OpenIE extractions.

6.1 FraCaS Entailment Corpus

The FraCaS textual entailment corpus (Cooper et al., 1996) is a small corpus of entailment problems, aimed at providing a comprehensive test of an entailment system's handling of various entailment patterns.

We process the corpus following MacCartney and Manning (2007). 12 problems were discarded as degenerate – lacking either an antecedent or a consequent. 151 problems were discarded as involving multiple antecedents to justify the inference. Lastly, it should be noted that many of the sections of the corpus are not directly applicable to Natural Logic inferences; MacCartney and Manning (2007) identify three sections which are in the scope of their system.

§	Category	Count	Precision		Recall		Accuracy		
			N	M08	N	M08	N	M07	M08
1	Quantifiers	44	91	95	100	100	95	84	97
2	Plurals	24	80	90	29	64	38	42	75
3	Anaphora	6	100	100	20	60	33	50	50
4	Ellipses	25	100	100	5	5	28	28	24
5	Adjectives	15	80	71	66	83	73	60	80
6	Comparatives	16	90	88	100	89	87	69	81
7	Temporal	36	75	86	53	71	52	61	58
8	Verbs	8	–	80	0	66	25	63	62
9	Attitudes	9	–	100	0	83	22	55	89
Applicable (1,5,6)		75	89	89	94	94	89	76	90

Table 4: Results on the FraCaS textual entailment suite. N is this work; M07 refers to MacCartney and Manning (2007); M08 refers to MacCartney and Manning (2008). The relevant sections of the corpus intended to be handled by this system are sections 1, 5, and 6 (not 2 and 9, which are also included in M08).

Results on the corpus are given in Table 4. Since the corpus is not a blind test set, the results are presented less as a comparison of performance, but rather as a comparison of the expressive power of our search-based approach compared with MacCartney's align-and-classify approach. For the experiments, costs were hard-coded to represent a strict logical entailment system – costs corresponding to valid Natural Logic mutations were set to a small constant cost; other costs were set to infinity.

The results validate the system's ability to capture valid Natural Logic inferences as well as the state-of-the-art system of MacCartney and Manning (2008). Note that our system is comparatively crippled in this framework along at least two dimensions: It cannot appeal to the antecedent when constructing the search, leading to the introduction of search errors which are entirely absent from prior work. Second the derivation process itself does not have access to the full parse tree of the candidate fact.

Although precision is fairly high even on the non-applicable sections of FraCaS, recall is significantly lower than prior work. This is a direct consequence of not having alignments to appeal to. For instance, we can consider two inferences:

System	Precision	Recall	Accuracy
Lookup	100.0	12.0	56.0
NaturalLI Only	89.8	41.0	66.6
NaturalLI + Lookup	91.5	49.9	72.6

Table 5: Accuracy inferring common-sense facts on a balanced test set. Lookup queries the lemmatized lower-case fact directly in the 300M fact database. NaturalLI Only disallows such lookups, and infers every query from only unseen facts. NaturalLI + Lookup takes the union of the two systems.

$$\textit{Jack saw Jill is playing} \overset{?}{\Rightarrow} \textit{Jill is playing}$$

$$\textit{Jill saw Jack is playing} \overset{?}{\Rightarrow} \textit{Jill is playing}$$

It is clear from the parse of the sentence that the first is valid and the second is not; however, from the perspective of the search algorithm both make the same two edits: inserting *Jack* and *saw*.

6.2 Common Sense Reasoning

We validate our system's ability to validate unseen common sense facts from a large database of such facts. Whereas evaluation of FraCaS shows that our search formulation captures applicable inferences as well as prior work, this evaluation presents a new use-case for Natural Logic which is not trivial with prior work.

For our database of facts, we run the Ollie OpenIE system (Mausam et al., 2012) over Wikipedia, Simple Wikipedia,[4] and a small subset of CommonCrawl. Extractions with confidence below 0.25 or which contained pronouns were discarded. This yielded in a total of 305 million unique extractions composed entirely of lexical items which mapped into our vocabulary (186 707 words). Each of these extracted triples (e_1, r, e_2) was then flattened into a plain-text fact $e_1\ r\ e_2$. In general, each fact in the database could be arbitrary unstructured text; our use of Ollie extractions is motivated by a desire to extract and query concise facts.

For our evaluation, we infer the top 689 most confident facts from the ConceptNet project (Tandon et al., 2011). To avoid redundancy with WordNet, we take all facts from eight ConceptNet relations: MemberOf, HasA, UsedFor, CapableOf, Causes, HasProperty, Desires, and CreatedBy. We then treat the *surface text* field of these facts as our candidate query. This yields facts like the following:

not all birds can fly
noses are used to smell
nobody wants to die
music is used for pleasure

For negative examples, we take the 689 ReVerb extractions (Fader et al., 2011) judged as false by Mechanical Turk workers (**?**). This provides a set of *plausible* queries, similar in many ways to the database of Ollie extractions, and ensures that our recall is not due to an over-zealous search. The search costs are tuned from a balanced set of true ConceptNet and 540 false ReVerb extractions.

Results are shown in Table 5. We compare against the baseline of looking up each fact verbatim in the fact database. Note that both the query and the facts in the database are lemmatized and lower-cased; therefore, it is not in principle unreasonable to expect at database of 300 million extractions to contain these facts. Nonetheless, only 12% of facts were found via a lookup to the database. We show that NaturalLI improves this recall four-fold, at only an 8.5% drop in precision. Furthermore, if we prohibit matching the query fact verbatim, we still recover all but 8% of our recall.

7 Related Work

A large body of work is devoted to compiling open-domain knowledge bases. For instance, OpenIE systems (Yates et al., 2007; Fader et al., 2011; Mausam et al., 2012) extract concise facts via surface or dependency patterns. In a similar vein, NELL (Carlson et al., 2010; Gardner et al., 2013) continuously learns new high-precision facts from the internet. The MIT Media Lab's CONCEPTNET project (Liu and Singh, 2004) has been working on creating a large knowledge base emphasizing common sense facts.

A natural alternative to the approach taken in this paper is to extend knowledge bases by inferring and adding new facts directly. For instance, Snow et al. (2006) present an approach to enriching the WORDNET taxonomy; Tandon et al. (2011) extend CONCEPTNET with new facts; Soderland et al. (2010) use REVERB extractions to enrich a domain-specific ontology. Chen et al. (2013) and Socher et al. (2013) use Neural Tensor Networks to predict unseen relation triples in

[4] http://simple.wikipedia.org/

WordNet and Freebase. This work runs inference over arbitrary text, without restricting itself to a particular set of relations, or even entities. Yao et al. (2012) and Riedel et al. (2013) present a related line of work, inferring new relations between Freebase entities by appealing to inferences over both Freebase and OpenIE relations. This work, however, focuses primarily on common-sense reasoning rather than inferring relations between named entities.

The goal of tacking common-sense reasoning is by no means novel in itself. Work such as Reiter (1980; McCarthy (1980) attempt to reason about the truth of a consequent in the absence of strict logical entailment. Similarly, Pearl (1989) presents a framework for assigning confidences to inferences which can be reasonably assumed. Our approach differs from these attempts in part in its use of Natural Logic as the underlying inference engine, and more substantially in its attempt at creating a truly broad-coverage system dealing with millions of candidate antecedents.

Many NLP applications query large knowledge bases. Prominent examples include question answering (Voorhees, 2001), semantic parsing (Zelle and Mooney, 1996; Zettlemoyer and Collins, 2007; Kwiatkowski et al., 2013; Berant and Liang, 2014), information extraction (Hoffmann et al., 2011; Surdeanu et al., 2012), and recognizing textual entailment (Schoenmackers et al., 2010; Berant et al., 2011). A long-term goal of this work is to improve accuracy on these downstream tasks by providing a *probabilistic* knowledge base over both explicitly known and likely true facts.

Fader et al. (2014) propose a system for question answering based on a sequence of paraphrase rewrites followed by a fuzzy query to a structured knowledge base. This work can be thought of as an elegant framework for unifying this two-stage process, while explicitly tracking the “risk” taken with each paraphrase step.

8 Conclusion

We have presented NaturalLI, an inference system over *unstructured text* intended to infer common sense facts. We have shown that we can run inference over a large set of antecedents while maintaining valid Natural Logic semantics, and have shown that we can learn how to infer unseen common sense facts.

Future work will focus on enriching the class of inferences we can make with Natural Logic – in particular, reasoning with meronymy and relational entailment. Furthermore, in the future we hope to learn with lexicalized parameters, and making use of the syntactic structure of a fact during search.

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