ChatGPT

7-Day Study Plan: TDEs and Accretion Discs in Schwarzschild Spacetime

Day 1: Introduction to Tidal Disruption Events and Accretion Discs

Readings:
- **Frank, King & Raine (2002)** – *Accretion Power in Astrophysics*, Chapter 1 (pp. 1–5). This introduction explains how gravitational accretion can outshine nuclear power in astrophysical systems [1] and why compact objects like neutron stars and black holes (radius ~ $2GM/c^2$) provide extremely efficient energy release [2].
- **Stone et al. (2019)** – *Stellar Tidal Disruption Events in General Relativity*, Introduction (Section 1, pp. 1–3). Gives a brief overview of TDEs: when a star passes too close to a SMBH and is torn apart, the bound debris can accrete and power a luminous flare [3] (dozens of such events have been observed so far).

Guiding Questions:
- Why is accretion onto compact objects such a powerful energy source compared to nuclear fusion (e.g. how many erg per gram are released)? [1] [4]
- What types of astrophysical systems are powered by accretion onto compact objects (name examples in binaries and galactic nuclei) [1]?
- What is a tidal disruption event (TDE) in basic terms, and what conditions (stellar orbit and tidal radius) lead to a star being disrupted by a black hole [3]?
- Above what black hole mass scale do stellar tidal disruptions require full general relativity to describe, and why is relativistic gravity important in those cases [5]? (Think about how close to the Schwarzschild radius the disruption happens for very massive SMBHs.)

Day 2: Accretion Disc Basics – The Shakura–Sunyaev Thin Disc Model

Readings:
- **Shakura & Sunyaev (1973)** – Original thin-disk paper (A&A **24**, 337). Focus on Sections 1–2, which lay out the assumptions of a **geometrically thin, optically thick** disk and introduce the α-viscosity prescription. This "α-model" parameterizes turbulent viscosity as $\nu = \alpha c_s H$ (with $\alpha \lesssim 1$) [6] – a now-famous approach to handle angular momentum transport in disks.
- **Frank, King & Raine (2002)** – *Accretion Power in Astrophysics*, Sections 5.3 and 5.5. These cover the steady-state thin disk structure and its emitted spectrum. You'll see how mass accretion rate, surface density, and temperature vary with radius, and that a thin disk radiates as a multi-color blackbody with temperature increasing inward (peaking in the innermost regions at $\sim 10^5$ K for an AGN disk, i.e. in the UV/soft X-ray) [7].

Guiding Questions:
- What key assumptions make a **"thin" accretion disk** (geometrically thin, optically thick) and why do these assumptions allow the disk to radiate efficiently (cooling away the accretion heat)?
- What is the **α-viscosity** prescription introduced by Shakura & Sunyaev (1973), and why was this

parametrization needed for disk theory [6]? How does the α-parameter relate to turbulent motion in the disk (e.g. eddy size and speed relative to sound speed)?
- How does an accretion disk transport angular momentum outward and mass inward? (Qualitatively, what role does viscous stress play in allowing matter to spiral in and release gravitational energy as heat?)
- **Disk emission:** Why does a steady thin disk radiate a **multi-color blackbody** spectrum? How does the effective temperature scale with radius, and in what wavelength range does the inner disk emit the bulk of its light [7]? *For example: is the peak emission UV, X-ray, optical?*
- *Extension:* In a black-hole accretion disk, what sets the **inner edge** of the luminous disk? (Foreshadowing Day 3: consider what happens at the innermost stable orbit in Schwarzschild spacetime.)

Day 3: Black Hole Gravity and Schwarzschild Spacetime Effects

Readings:
- **Paczyński & Wiita (1980)** – Introduction of the pseudo-Newtonian potential (A&A **88**, 23–31). Read the summary of the *Paczyński–Wiita potential*, $\Phi_{\rm PW}(r) = -\frac{GM}{r - r_S}$, which is an approximation to the Schwarzschild black hole's gravitational field [8]. Notably, this potential **exactly reproduces** important general relativistic radii: the innermost stable circular orbit (ISCO) and marginally bound orbit, as well as the correct form of orbital angular momentum and epicyclic frequency [9]. This makes it a useful tool for disk simulations.
- **Frank, King & Raine (2002)** – *Accretion Power in Astrophysics*, Section 7.7–7.8 (pp. 234–239). These pages discuss accretion onto black holes in contrast to Newtonian potentials. Key points include: the inner boundary of a thin disk around a Schwarzschild BH is at $R_{\text{ISCO}} = 3R_S$ (three Schwarzschild radii), where matter abruptly plunges inward [10]; the **accretion efficiency** (fraction of rest-mass converted to radiation) is about 5.7% for a non-rotating BH [11] – lower than the ~8% you'd get if matter could orbit stably at $3R_S$ in a Newtonian potential (because in GR the last orbits are less bound) [11]. The text also notes that viscous torques in the disk must vanish at the ISCO boundary [12].

Guiding Questions:
- What is the **Paczyński–Wiita** pseudo-potential and how does it mimic a Schwarzschild black hole's gravity? Which relativistic orbital characteristics (e.g. ISCO location, angular momentum distribution) does it successfully reproduce [9]?
- In Schwarzschild spacetime, what is the radius of the **innermost stable circular orbit** for a test particle? How does the existence of this ISCO affect an accretion disk's inner edge and the fate of gas that reaches that radius [10]?
- What is the **radiative efficiency** of accretion onto a non-rotating (Schwarzschild) black hole, and how does this compare to a Newtonian prediction at the same radius or to accretion onto a neutron star? (Why is a hard surface more efficient than a black hole horizon?) [11] [13]
- In a thin disk around a BH, why must the **viscous torque** go to zero at the ISCO? [12] What physical condition does this reflect at the transition from the disk to the plunging region inside $3R_S$?
- *Advanced:* How would a Kerr (spinning) black hole change the story? (Consider how ISCO radius and efficiency differ for a maximally rotating Kerr BH versus Schwarzschild – e.g. efficiency up to ~42% for prograde orbits [14].)

Day 4: Stellar Tidal Disruption and Debris Fallback (Newtonian Perspective)

Readings:
- **Stone et al. (2019)** – Section 2 *“Tidal Disruption Events in Newtonian Gravity.”* Focus on §2.1 (Stellar Tidal Disruption) for the basic mechanics of a TDE. This will cover defining the **tidal radius** (where the BH’s tidal gravity equals the star’s self-gravity) and what happens when a star is disrupted. Classic results reviewed here: roughly **half of the star’s mass becomes unbound** and flies off, while the other half remains bound to the BH and will return (fall back) on eccentric orbits [15]. The most tightly bound debris returns first, leading to a mass fallback rate that initially rises and then decays roughly as $t^{-5/3}$ [16]. (This $t^{-5/3}$ law for fallback accretion power is a famous result by Rees 1988 and Phinney 1989.) Section 2.2 describes tidal compression/shock of the star, which is interesting but less crucial for our focus.
- *(Optional background:* **Rees (1988)** – Nature **333**, 523 – proposed the TDE flare model. If available, skim to see how he envisioned the stellar debris returning and powering a luminous burst.)*

Guiding Questions:
- **Tidal radius:** What is the tidal disruption radius R_t and how is it related to the black hole’s mass and the star’s density? (Explain qualitatively why a star is disrupted when R_t is of order the distance where tidal force ~ star’s self-gravity.)
- In a full tidal disruption, roughly what fraction of the star’s mass becomes bound to the black hole vs. ejected unbound? What *happens* to the bound debris in the weeks to months after the disruption [15]?
- What is the expected time-dependence of the **fallback accretion rate** of the bound debris? Why do we expect a power-law decay around $t^{-5/3}$ for the fallback of debris onto the BH [16]?
- How does the **energy distribution** of the torn-off stellar debris lead to some material being unbound and some bound? (Hint: material just outside the star’s center of mass gains orbital energy, just inside loses energy.)
- **GR or not?:** For *which black hole masses* are tidal disruptions observable? Why is there an upper limit (on the order of $M_{\rm BH} \sim 10^8 M_\odot$) beyond which an approaching star is swallowed whole rather than tidally disrupted? [5] (*This is known as the Hills mass – above it, the tidal radius lies inside the event horizon.*)

Day 5: Relativistic Effects on Debris Circularization and Disk Formation

Readings:
- **Stone et al. (2019)** – Sections 5.2 *“Capture by the Event Horizon”* and 6.1 *“General Relativistic Circularization.”* Section 5.2 explains that general relativity introduces an event horizon — if a star’s periapse lies within the black hole’s **capture radius**, the star (or its debris) will disappear into the BH instead of producing a flare. This leads to a **suppression of TDEs for very massive BHs** (above the “relativistic Hills mass” ~ few 10^7–$10^8 M_\odot$) [17]. Section 6.1 then examines how relativistic effects influence the **circularization** of the returning debris streams. In Newtonian simulations, the streams often do **not collide** efficiently (miss each other), delaying disk formation [18]. But in GR, **apsidal precession** shifts the orbit of the inbound stream so that it **self-intersects** with the outbound stream, producing strong shocks that dissipate energy and help circularize the debris into a disk [19]. (However, note that GR **nodal** precession due to black hole spin can twist streams out of the plane, sometimes *hindering* intersection [20] – a complication for misaligned disks.)
- ***(Recap Note:*** By this point, you’ve seen three major relativistic effects on TDEs: (1) Very high-mass BHs

directly swallow stars (no TDE) [17]. (2) Apsidal precession in Schwarzschild spacetime causes debris streams to collide and form a disk (whereas in Newtonian gravity, the streams would orbit without intersecting) [19]. (3) If the BH is spinning (Kerr), Lense–Thirring precession can cause the disk to **globally precess** – more on that in Day 7.)*

Guiding Questions:
- **Horizon capture:** Why does general relativity predict a cutoff in observable TDEs for SMBH masses above ~$10^8 M_\odot$? What happens to a star that approaches a black hole above this mass threshold, and why would it not produce a luminous flare [17]?
- **Circularization problem:** In Newtonian TDE models, what is the "circularization problem"? Why might the stellar debris *fail* to quickly form a circular accretion disk in the absence of relativistic effects?
- How does **relativistic perihelion (apsidal) precession** facilitate disk formation in TDEs? Explain how GR causes the tidal stream to self-intersect and shock. What is the role of those shocks in converting the debris's eccentric orbits into a roughly circular disk [19]?
- What observable impact might a delayed versus efficient circularization have on the early light curve of a TDE? [21] (For instance, if disk formation is delayed, how would the early UV/X-ray emission behave?)
- *Advanced:* How could Lense–Thirring **nodal precession** (for a spinning BH) complicate the circularization process [20]? (Think about streams being pulled into different orbital planes – what might that do to stream collisions?)

Day 6: Powering TDE Emission – Stream–Disk Shocks vs. Accretion Power

Readings:
- **Steinberg & Stone (2024)** – *"Stream–disk shocks as the origins of peak light in TDEs,"* Nature **625**, 463–467. **Read the entire article.** This paper presents a 3D radiation-hydrodynamic simulation of a TDE from stellar disruption through peak emission. It addresses the debate: is the luminous optical/UV flare in TDEs powered by **accretion** energy (disk radiation, possibly reprocessed by winds) or by **shock heating** from the debris self-intersections? The authors find that **at early times, "nozzle" shocks near periapse produce some light, but the debris largely does *not* form a disk immediately.** However, **as the fallback rate peaks, the returning stream collides with the nascent disk ("stream–disk" shocks), efficiently circularizing the debris and powering a bright optical/UV peak** [22]. In their simulation, the **peak luminosity is primarily shock-powered** (with accretion power becoming significant only near peak, as the disk starts to form) [23]. They also note that they used a pseudo-Newtonian Paczyński–Wiita potential to approximate the BH's gravity [24], and included radiative transfer to capture how emission emerges. Focus on the Introduction and Results/Discussion sections which describe these findings in prose.

Guiding Questions:
- Why do **simple theoretical models** of TDE emission struggle to explain the high optical/UV luminosities of many events? (What is the "optical excess" problem, and why did it lead to two competing hypotheses for the source of the light? [25])
- What are the two main scenarios for powering TDE light curves that have been debated? Describe **"shock-powered"** emission vs. **"reprocessed accretion-powered"** emission in TDEs [25].
- According to Steinberg & Stone's simulation, what is the dominant mechanism powering the peak of the TDE flare? What do **stream–disk collision shocks** accomplish in terms of debris circularization and energy release, and how do they reproduce the observed peak optical/UV luminosities [22]?

- How does **accretion-driven** power factor in: is accretion (disk) luminosity completely negligible, or when does it begin to contribute according to the simulation? [23]
- What gravitational potential is used in this simulation, and why is that a reasonable choice for capturing Schwarzschild-spacetime effects without full GR? [24] (Connect this to Day 3: what key relativistic feature does the Paczyński–Wiita potential ensure in the simulation?)
- *Big picture:* Given these results, how might a real TDE's light curve evolve from disruption to peak? (Consider an early "flash" from initial shocks, then a plateau/peak as stream–disk shocks power the flare, with disk accretion light possibly rising in X-rays later on.)

Day 7: Disk Oscillations and Schwarzschild Spacetime Signatures

Readings:
- **Nowak & Wagoner (1991)** – *"Diskoseismology: Trapped Adiabatic Oscillations,"* ApJ **378**, 656–664. Read the Introduction and Conclusions (pp. 656–658, 663–664). This seminal work explores how **global oscillation modes** can exist in the inner regions of accretion disks around compact objects – essentially **"sound wave" normal modes in disks (p-modes, g-modes, etc.)**. A crucial insight is that in a relativistic disk (around a BH), certain modes can be **trapped** in the inner disk: General relativity causes the radial **epicyclic frequency** (the natural frequency of radial oscillation in a nearly circular orbit) to drop to zero at the ISCO, creating an inner "reflective" boundary for wave modes. In addition, the disk's outer regions act as a barrier for certain frequencies. Together this means some oscillation modes are confined to a cavity in the inner disk [26]. Nowak & Wagoner approximate these relativistic effects with a modified Newtonian potential and find discrete modes. *You do not need to follow the mathematical derivations; focus on the physical description of why modes are trapped and the implications.*

Guiding Questions:
- What is **diskoseismology**? Why might analyzing oscillations ("seismic" modes) of accretion disks help us learn about the disk's structure and the spacetime around the black hole [27]? (Analogy: like helioseismology probes the Sun's interior.)
- Why can certain oscillation modes be **"trapped" in the inner region** of a black hole's accretion disk? In particular, what effect does Schwarzschild spacetime have on the radial epicyclic frequency profile, and how does this create an inner trapping boundary for oscillations?
- In simple terms, what sort of disk oscillation modes did Nowak & Wagoner find (e.g. vertical p-mode, radial g-mode), and in what part of the disk are these oscillations concentrated? (Inner few R_S.) What provides the restoring forces for these modes?
- **Observational connection:** What are quasi-periodic oscillations (QPOs) as seen in X-ray binaries or active galactic nuclei? How might the trapped inner-disk modes or a precessing disk manifest as QPOs in a TDE or BH accretion scenario [28]?
- Have any TDEs shown evidence of QPOs or periodic modulation? (*Hint:* One TDE candidate, ASASSN-14li, exhibited an X-ray QPO ~ 5 mHz, suggesting possible disk precession or oscillation.) How could detecting such oscillations provide evidence of relativistic disk dynamics in TDEs?

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