Indian Institute of Technology, Bombay — Chemical Engineering

CL686: Advanced Process Control — **Assignment 5, 2025**

Date Due: 13 April 2025, 11.59 PM (Note that time is not 12 PM midnight since that is confusing to set.)

Mode of Submission: Moodle submission

Late Submission Policy: Submissions late by 0-24 hrs will incur a 25% penalty, and submissions late by 24-48 hrs will incur a 50% penalty. Submissions later than 48 hrs will not be considered.

Topic: Offset Free Tracking and Unmeasured Disturbance Rejection

AIM: Implement both estimator and controller. In particular, consider problem of tracking time-varying setpoints in presence of disturbances/model-plant mismatch. Estimate states and use **Innovation Bias**, and **State Augmentation** approaches to capture effect of unmeasured disturbance, with LQOC controller design for Quadruple Tank System.

Task at Hand

1. Implement Innovation Bias approach along with Linear Quadratic Optimal Controller (LQOC) design.
2. Implement State Augmentation approach along with Linear Quadratic Optimal Controller (LQOC) design.
3. Compare the performance of the above controllers in terms of Sum of squared error (SSE) value.

NOTE: All estimators must be implemented considering nonlinear ODEs of Quadruple Tank System as the plant. The estimators will be based on linear-model though. The same Assignment 4 Mat file (file: Continuous_time_linear_perturbation_model_without_disturbance.mat), which contains the (linearized) matrices for the continuous-time model, must be used for this assignment.

Quadruple Tank System Description

Consider a nonlinear system governed by the following set of coupled nonlinear ODEs,

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \tag{1}$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \tag{2}$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 + \frac{\gamma_3}{A_3}F_d \tag{3}$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 + \frac{(1-\gamma_3)}{A_4}F_d \tag{4}$$

The initial steady-state operating conditions of the system are listed in the table below:

- **System State:** h_1, h_2, h_3, h_4 are the liquid levels in the quadruple tank system.
- **Process inputs:** v_1 and v_2 are the voltage applied to the pump.
- **Disturbance inputs:** F_d acts as the disturbance input to the process. For this assignment, it will not be considered (i.e. its value will be kept at 0). [However, enterprising students can investigate its effect as an additional not-to-be-submitted exercise. In the current assignment, disturbance is a proxy for model-plant mismatch.]

Variable	Optimum operating point
A_1, A_3 $[cm^2]$	28
A_2, A_4 $[cm^2]$	32
a_1, a_3 $[cm^2]$	0.071
a_2, a_4 $[cm^2]$	0.057
g $[cm/s^2]$	981
k_1, k_2 $[cm^3/Vs]$	3.33, 3.35
$\gamma_1, \gamma_2, \gamma_3$	0.7, 0.6, 0.4

Condition for Simulation:

- **Steady state inputs:** $\mathbf{U}_s = [3 \quad 3]^T$; $D_s = 0$
- **Equilibrium/Steady State Operating Point:**
  $\mathbf{X}_s = [12.263 \quad 12.7831 \quad 1.6339 \quad 1.409]^T$
- **Sampling interval**, $T = 4$ sec
- **Number of samples**, $N_s = 200$
- **Initial state**, $\mathbf{X}(0) = \mathbf{X}_s + [1 \quad 1 \quad 1 \quad 1]^T$.
- **Setpoint**, $\mathbf{R}(k) = \mathbf{y}_{sp}(k) + \mathbf{Y}_s$, where

$$\mathbf{y}_{sp}(k) = \begin{cases} 1 & \text{if } k \leq \frac{N_s}{2} \\ 10 & \text{otherwise} \end{cases}$$

LQOC controller design implementation

Steady State Controller Gain

In this implementation, use $\mathbf{G}_{\infty,c}$ as the controller gain where $\mathbf{G}_{\infty,c}$ is the steady state linear quadratic optimal controller matrix.

- **Weighting matrices:**
  $\mathbf{W}_x = 10 \times I_{4\times4}$
  $\mathbf{W}_u = I_{2\times2}$
  where $\mathbf{I}$ is identity matrix of appropriate dimension
- Solution for Algebraic Riccati Equation (ARE) needed for LQOC can be found using MATLAB Control System Toolbox function *dlqr* as follows
  $[\mathbf{G}_{\infty,c}, \mathbf{S}_{\infty,c}, \mathbf{P}_c] = dlqr(\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{W}_x, \mathbf{W}_u)$

Estimator design with innovation bias approach implementation

Steady State Estimator Gain

In this implementation, use $\mathbf{L}_{\infty,inno}$ as the estimator gain where $\mathbf{L}_{\infty,inno}$ is obtained by solving the LQOC problem for the dual system.

- **Weighting matrices:**
  $\mathbf{W}_x = I_{4\times4}$
  $\mathbf{W}_u = 10 \times I_{2\times2}$

- Solution for Algebraic Riccati Equation (ARE) can be found using MATLAB Control System Toolbox function *dlqr* as follows
  $[\mathbf{L}^T_{\infty,inno}, \mathbf{S}_{\infty,innov}, P_{innov}] = dlqr(\mathbf{\Phi}^T, \mathbf{C}^T, \mathbf{W}_x, \mathbf{W}_u)$

  NOTE: Please refer to Prof. Sachin's slides (Part5_StateEstimation_SetPtTracking_DistRejection) for the detailed algorithm (mainly slides 71-72), and MATLAB help for above function. Note that you can skip step 3 (innovation filtering) and step 6 (input constraints) of slide 72.

Estimator with state augmentation (input bias) approach implementation

In this approach, disturbance ($\boldsymbol{\beta}(k)$) is assumed only in the plant model, while the measurement model is assumed to be disturbance free. Thus, $\mathbf{\Gamma}_{\boldsymbol{\beta}} = \mathbf{\Gamma}$, $\mathbf{C}_{\boldsymbol{\eta}} = [0]$, $\eta(k) = 0$. Disturbance $\boldsymbol{\beta}(k)$ has to be estimated along with states $\boldsymbol{x}(k)$.

Steady State Estimator Gain

In this implementation, use $\mathbf{L}_{\infty,sa}$ as the estimator gain where $\mathbf{L}_{\infty,sa}$ is the steady state solution of the Riccati equation.

- **Weighting matrices:**
  $\mathbf{W}_x = 0.2 \times I_{6\times 6}$
  $\mathbf{W}_u = 5 \times I_{2\times 2}$

- Estimator is designed by solving the LQ-OC problem (steady state controller) in Matlab for the dual system as follows
  $[\mathbf{L}^T_{\infty,sa}, \mathbf{S}_{\infty,sa}, P_{sa}] = dlqr(\mathbf{\Phi}^T_{sa}, \mathbf{C}^T_{sa}, \mathbf{W}_x, \mathbf{W}_u)$
  where, $\mathbf{\Phi}_{sa}$ and $\mathbf{C}_{sa}$ are the augmented process and measurement matrices.
  NOTE: Please refer to Prof. Sachin's slides (Part5_StateEstimation_SetPtTracking_DistRejection) for the detailed algorithm (mainly slides 80-81), and MATLAB help for above function. Note that you can skip step 3 of slide 81 since there are no input constraints in the current problem.

Performance index calculation

- **Sum Squared Error for set point tracking**
  $SSEST_i = \sum_{k=1}^{N_s}(Y_i(k) - R_i(k))^2$, for $i = 1, 2$.
  Calculate the SSE values for both innovation bias and state augmentation based LQOC controller design.

- **Sum Squared Error for estimation errors**
  $SSESE_i = \sum_{k=1}^{N_s}(\widehat{X}_i(k) - X_i(k))^2$, for $i = 1, 2, 3, 4$. Here, $\widehat{X}_i(k)$ and $X_i(k)$ are the estimated and true i^{th} state
  Calculate the SSE values for both innovation bias and state augmentation based LQOC controller design.

- **Sum Squared Deviations for Manipulated variables**
  $SSEMV_i = \sum_{k=1}^{N_s}(U_i(k) - U_{s,i})^2$, for $i = 1, 2$.
  Here $U_{s,i}$ is the linearization point for the i^{th} input.

  This is a measure of perturbations in the manipulated variables (a proxy for control effort).

Plots to generate

1. Plot states (original units and not in deviation form) of Nonlinear plant obtained using both innovation bias and state augmentation based LQOC controller design. You can have one figure per state. Don't forget to insert a legend.

2. Plot the innovation for both approaches.

3. Plot the input trajectory (original units and not in deviation form) along with the steady state values (i.e. linearization point which is 3 for both the inputs) for both approaches.
4. Plot the estimation error for all four states for both approaches.
5. Plot the measurement output (original units and not in deviation form) for both approaches along with the setpoint for each state in the same plot using 'hold on'.

MATLAB Code To Submit

- Submit all files in a zipped folder. The submitted folder should have the following MATLAB files:
  - File for LQOC controller with innovation bias approach implementation.
  - File for LQOC controller with state augmentation approach implementation.
  - Function file of system dynamics.
  - MAT file for continuous-time linear state space model. (Use the MAT file provided in Assignment 4).
  - File, which on execution, will generate the various plots and return the value of performance indices. This is the main file which will be executed by the TAs.

  Keep the main file name as *Assn_05_ROLLNO.m* where ROLLNO is your roll number. [**Get in touch with TA for any issues.**]
- Make sure that the plots are labelled properly with axes names, titles, and legends.

Learning is fun. Best of Luck!