

78) $f(x, y) = xy$ $[0, 2] \times [0, 1]$, $z = x + y$

$$\begin{cases} z = x + y \\ y = T \end{cases} \rightarrow \begin{cases} x = z - T \\ y = T \end{cases} \quad \begin{vmatrix} \frac{\partial x}{\partial z} & \frac{\partial x}{\partial T} \\ \frac{\partial y}{\partial z} & \frac{\partial y}{\partial T} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1$$

$$g(z, T) = f(x(z-T), y(T)) = (z-T)T = zT - T^2$$

$$\begin{cases} 0 < x < 2 \\ 0 < y < 1 \end{cases} \quad \begin{cases} 0 < z - t < 2 \\ 0 < t < 1 \end{cases} \rightarrow -2 < t - z < 0$$

$$\begin{cases} z - 2 < t < z \\ 0 < t < 1 \end{cases}$$

$$t \in (z-2, z) \cap (0, 1)$$

- $z \in [0, 1]$, $t \in [0, z]$
- $z \in [1, 2]$, $t \in [0, 1]$
- $z \in [2, 3]$, $t \in [z-2, 1]$

$$\int zt - t^2 dt = \frac{zt^2}{2} - \frac{t^3}{3}$$

$$g_1(z) = \begin{cases} \left. \frac{zt^2}{2} - \frac{t^3}{3} \right|_0^z, & z \in [0, 1] \\ \left. \frac{zt^2}{2} - \frac{t^3}{3} \right|_0^1, & z \in [1, 2] \\ \left. \frac{zt^2}{2} - \frac{t^3}{3} \right|_{z-2}^1, & z \in [2, 3] \end{cases} = \begin{cases} \frac{z^3}{6}, & z \in [0, 1] \\ \frac{3z-2}{6}, & z \in [1, 2] \\ \frac{1}{6}(-z^3 + 15z - 18), & z \in [2, 3] \end{cases}$$

$$g_1(z) \geq 0 \text{ alle } z \in [0, 3] \quad \checkmark$$