

28)

$$Y_1 = 2X_1 - 3X_2$$

$$Y_2 = 4X_1 + 2X_2$$

$$X = (X_1, X_2)^T$$

$$Y = AX$$

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 2 \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$A \begin{bmatrix} 38 & -5 \\ -5 & 4 \end{bmatrix} A^T = \begin{bmatrix} 81 & -22 \\ 142 & -12 \end{bmatrix} A^T = \begin{bmatrix} 248 & 320 \\ 320 & 544 \end{bmatrix}$$

$$X \sim N \left( \begin{bmatrix} -10 \\ 12 \end{bmatrix}, \begin{bmatrix} 248 & 320 \\ 320 & 544 \end{bmatrix} \right)$$

$\uparrow$   
 variances  
 covariances

$$\rho_{Y_1, Y_2} = \frac{\text{cov}(Y_1, Y_2)}{\sqrt{V(Y_1) V(Y_2)}} = \frac{320}{\sqrt{248 \cdot 544}} = \frac{320}{16 \sqrt{527}} = \frac{20 \sqrt{527}}{527}$$