Z12

$$f_x(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$$

$$f_y(y) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{y^2}{2}}$$

$$f_{x,y}(x,y) = \frac{1}{2\pi} \cdot e^{-\frac{x^2+y^2}{2}}$$

$$\begin{cases} X = R \cdot \cos\theta \\ Y = R \cdot \sin\theta \end{cases}$$

$$J = \begin{vmatrix} \frac{dx}{dr} & \frac{dx}{d\theta} \\ \frac{dy}{dr} & \frac{dy}{d\theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\cdot\sin\theta \\ \sin\theta & r\cdot\cos\theta \end{vmatrix} =$$

$$= r\cdot\cos^2\theta + r\cdot\sin^2\theta = r$$

czyli $g(r,\theta) = f(r\cos\theta, r\sin\theta)\cdot r =$

$$= \frac{1}{2\pi}\cdot r \cdot \exp\left\{\frac{-r^2\cdot\cos^2\theta + r^2\cdot\sin^2\theta}{2}\right\} =$$

$$= \frac{1}{2\pi}\cdot r \cdot \exp\left\{-\frac{r^2}{2}\right\} \square$$

Z13)

$$f_{XY}(x,y) = \frac{1}{2\pi} \cdot e^{-\frac{x^2+y^2}{2}}$$

$D = R^2 = X^2 + Y^2$, $\Theta = \tan^{-1} \frac{Y}{X}$

$R = \sqrt{D}$

$\Downarrow$

$X = \sqrt{D} \cdot \cos\Theta$

$Y = \sqrt{D} \cdot \sin\Theta$

$$J = \begin{vmatrix} \frac{dx}{dD} & \frac{dx}{d\Theta} \\ \frac{dy}{dD} & \frac{dy}{d\Theta} \end{vmatrix} = \begin{vmatrix} \frac{\cos\Theta}{2\sqrt{D}} & -\sqrt{D}\cdot\sin\Theta \\ \frac{\sin\Theta}{2\sqrt{D}} & \sqrt{D}\cdot\cos\Theta \end{vmatrix} =$$

$$= \frac{\cos^2\Theta + \sin^2\Theta}{2} = \frac{1}{2}$$

d)

$$g(d,\Theta) = f(\sqrt{d}\cdot\cos\Theta, \sqrt{d}\cdot\sin\Theta) =$$

$$= \frac{1}{2\pi}\cdot\frac{1}{2}\cdot\exp\left(-\frac{d(\sin^2\Theta+\cos^2\Theta)}{2}\right) = \frac{1}{4\pi}\, e^{-\frac{d}{2}}$$

c)

$$g_D(d) = \int_0^{2\pi} \frac{1}{4\pi}\cdot e^{-\frac{d}{2}}\, d\Theta = \frac{1}{4\pi} e^{-\frac{d}{2}} \cdot \int_0^{2\pi} 1\, d\Theta = \frac{1}{2}\cdot e^{-\frac{d}{2}}$$

$$g_\Theta(\Theta) = \int_0^{\infty} \frac{1}{4\pi}\cdot e^{-\frac{d}{2}}\, dd = \frac{1}{4\pi}\cdot \int e^{-\frac{d}{2}}\, dd = \frac{1}{4\pi}\cdot\left(-2e^{-\frac{x}{2}}\Big|_0^{\infty}\right) =$$

$$= \frac{1}{2\pi}$$

b) $\frac{1}{2\pi}\cdot\frac{1}{2}\cdot e^{-\frac{d}{2}} = \frac{1}{4\pi}\cdot e^{-\frac{d}{2}}$ czyli niezależne