$\boxed{z1}$

$X_1, X_2, \dots, X_n$ - niezależne i $X_i \sim \exp(\lambda)$, czyli

$$f_{X_1,X_2,X_3,\dots,X_n}(x_1,\dots,x_n) = \lambda \cdot e^{-\lambda \cdot x_1} \cdot \lambda \cdot e^{-\lambda \cdot x_2} \cdot \dots \cdot \lambda \cdot e^{-\lambda x_n} =$$

$$= \lambda \cdot e^{-\lambda \cdot \sum_{i=1}^{n} x_i}$$

$$Y_i = \sum_{k=1}^{i} X_k$$

$Y_1 = X_1$

$Y_2 = X_1 + X_2 \iff X_2 = Y_2 - X_1 = Y_2 - Y_1$

$Y_3 = X_1 + X_2 + X_3 \iff X_3 = Y_3 - X_1 - X_2 = Y_3 - Y_2$

$\vdots$

$X_n = Y_n - Y_{n-1}$

$$f_{Y_1,Y_2,\dots,Y_n}(y_1,y_2,\dots,y_n) = f_{X_1,X_2,X_3,\dots,X_n}(x_1,x_2,\dots,x_n) \cdot |\mathcal{J}| = \lambda^n \cdot e^{-\lambda \sum_{i=1}^{n} x_i} \cdot |\mathcal{J}|$$

$$|\mathcal{J}| = \begin{vmatrix} \frac{dx_1}{dy_1} & \frac{dx_1}{dy_2} & \dots & \frac{dx_1}{dy_n} \\ \frac{dx_2}{dy_1} & & & \\ \vdots & & & \\ \frac{dx_n}{dy_n} & & & \frac{dx_n}{dy_n} \end{vmatrix} = \begin{vmatrix} 1 & & & \\ -1 & 1 & & \\ & -1 & 1 & \\ & & \ddots & \\ & & & 1 \end{vmatrix} = 1$$

$Y_n = \sum_{i=1}^{n} X_i$, więc

$$f_{Y_1,Y_2,\dots,Y_n}(y_1,y_2,\dots,y_n) = \lambda^n \cdot e^{-\lambda \cdot \sum_{i=1}^{n} x_i} \cdot 1 = \lambda^n \cdot e^{-\lambda \cdot Y_n}$$

$\square$