

2/4/

$x_1, \dots, x_n$   
 $y_1, \dots, y_n$

$$\sim N(\mu, \sigma^2)$$

$$Z = \frac{\bar{X} - \bar{Y}}{\sigma \sqrt{\frac{n+k}{nk}}}$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right), \quad \bar{Y} \sim N\left(\mu, \frac{\sigma^2}{k}\right)$$

$$\bar{X} - \bar{Y} \sim N\left(0, \frac{\sigma^2 k - \sigma^2 n}{nk}\right)$$

$$\frac{1}{\sigma \sqrt{\frac{n+k}{nk}}} \cdot (\bar{X} - \bar{Y}) \sim N\left(0, \frac{nk}{\sigma^2 \cdot (n+k)}\right) \cdot \frac{\sigma^2 (k-n)}{nk} = N\left(0, \frac{k-n}{n+k}\right)$$