211

ipynb

212)

$p \cdot V^{k} = C$

$p = C \cdot V^{-k}$

$\log(P) = -k \cdot \log(V) + \log(C)$

$y = a \cdot x + b$

$$a = \frac{\operatorname{cov}(\log(V), \log(P))}{V(\log(P))} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i}^{n} (x_i - \bar{x})} \overset{\text{ipynb}}{=} -1{,}40391000\ldots$$

$k = -a = 1{,}40391000$

$b = \bar{y} - a \cdot \bar{x} = 9.6774580600\ldots$

$C = e^{b} = 15953.8915067428 35\ldots$

$$P = \frac{C}{V^{k}} = 24.8339967 55\ldots$$

213)

$$M_X(t) = \frac{e^{t} - e^{-t} + 4}{6} \qquad M_X(t) = 1 + t \cdot E(X) + \frac{t^2 \cdot E(X^2)}{2!} + \frac{t^3 \cdot E(X^3)}{3!} + \ldots$$

$$E(X^k) = \frac{d^k}{dt^k} M_X(0)$$

$$\frac{d^k}{dt^k} M_X(t) = \begin{cases} \frac{e^{t} - e^{-t}}{6}, & \text{gdy } 2 \nmid k \\ \frac{e^{t} + e^{-t}}{6}, & \text{gdy } 2 \mid k \end{cases} \Big|_{t=0} = \begin{cases} 0, & \text{gdy } 2 \nmid k \\ \frac{1}{3}, & \text{gdy } 2 \mid k \end{cases}$$

Z14

$X \sim N(1,2)$, $Y \sim N(4,7)$ $f(x) = \frac{1}{\delta\sqrt{2\pi}} \cdot \exp\left(\frac{(x-\mu)^2}{2\delta^2}\right)$

Niech $A = X+Y$. Wiemy, że

$$A \sim N(1+4, 2+7) = N(5, 9)$$

Niech $B = -Y$.

$$M_B(t) = M_{-Y}(t) = M_Y(-t) = \exp(-\mu t + \frac{\delta^2 \cdot t^2}{2})$$

$C = X + B = X - Y$, więc

$$C \sim N(1-4, 2+7) \sim N(-3, 9)$$

Niech $D = 3X$ $D \sim N(3 \cdot 1, 3^2 \cdot 2) = N(3, 18)$

$E = 4Y$ $E \sim N(4 \cdot 4, 16 \cdot 7) = N(16, 112)$

$F = D + E = N(19, 130)$

$$P(X+Y>0) = P(A>0) = \int_0^\infty \frac{1}{3\sqrt{2\pi}} \cdot \exp\left(-\frac{(x-5)^2}{2 \cdot 9}\right) dx =$$

$\approx 0,95$

Wolfram

$$P(X-Y<2) = P(C<2) = \int_{-\infty}^2 \frac{1}{3\sqrt{2\pi}} \cdot \exp\left(-\frac{(x+3)^2}{2 \cdot 9}\right) dx \approx$$

≈ 0.95221

$$P(3X+4Y>20) = P(F>20) = \int_{20}^\infty \frac{1}{\sqrt{260\pi}} \cdot \exp\left(-\frac{(x-19)^2}{2 \cdot 130}\right) dx =$$

≈ 0.465