(Only one correct option)

41. Let $f: R \to R$ be defined as $f(x) = 3^{-|x|} - 3^{x} + \text{sgn}(e^{-x}) + 2$ (where sgn x denotes signum function of x). Then which one of the following is correct ?
(A) if is injective but not surjective
(B) f is surjective but not injective
(C) f is injective as well as surjective
(D) f is neither injective nor surjective

42. Let $f: D \to R$ be defined as $f(x) = \frac{x^2+2x+a}{x^2+4x+3a}$ where D and R denote the domain of f and the set of all real numbers respectively. If f is surjective mapping then the range of a is:
(A) $0 \le a \le 1$ (B) $0 < a \le 1$
(C) $0 \le a < 1$ (D) $0 < a < 1$

43. If $f(x) = x^2 + bx + c$ and $f(2 + t) = f(2 - t)$ for all real numbers t, then which of the following is true ?
(A) $f(1) < f(2) < f(4)$ (B) $f(2) < f(1) < f(4)$
(C) $f(2) < f(4) < f(1)$ (D) $f(4) < f(2) < f(1)$

44. If $f(x) = \pi\left(\frac{\sqrt{x+7}-4}{x-9}\right)$, then the range of function $y = \sin(2f(x))$ is
(A) $[0, 1]$ (B) $\left(0, \frac{1}{\sqrt{2}}\right]$
(C) $\left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$ (D) $(0, 1)$

45. Let $f(x) = \max.\{\sin t : 0 \le t \le x\}$, $g(x) = \min.\{\sin t : 0 \le t \le x\}$ and $h(x) = [f(x) - g(x)]$ where [ ] denotes greatest integer functon, then the range of $h(x)$ is:
(A) $\{0, 1\}$ (B) $\{1, 2\}$
(C) $\{0, 1, 2\}$ (D) $\{-3, -2, -1, 0, 1, 2, 3\}$

46. The sum of all possible values of n where $n \in N$, $x > 0$ and $10 < n \le 100$ such that the equation $[2x^2] + x - n = 0$ has a solution, is equal to:
(A) 150 (B) 175
(C) 190 (D) 210

47. If the range of the funciton $f(x) = \frac{x-1}{p-x^2+1}$ does not contain any values belonging to the interval $\left[-1, \frac{-1}{3}\right]$ then the true set of values of p, is :
(A) $(-\infty, -1)$ (B) $\left(-\infty, \frac{-1}{4}\right)$
(C) $(0, \infty)$ (D) $(-\infty, 0)$

48. The fundamental period of the function $f(x) = 4\cos^4\left(\frac{x-\pi}{4\pi^2}\right) - 2\cos\left(\frac{x-\pi}{2\pi^2}\right)$ is equal to
(A) π^3 (B) $4\pi^2$
(C) $3\pi^2$ (D) $2\pi^2$

49. Let f be a function defined as $f : \left(0, e^{\frac{-3}{2}}\right] \to \left[\frac{-1}{4}, \infty\right)$, $f(x) = (\ln x)^2 + 3\ln x + 2$, then $f^{-1}(x)$ equals:
(A) $\log\left(\frac{-3+\sqrt{4x+1}}{2}\right)$ (B) $\log\left(\frac{-3-\sqrt{4x+1}}{2}\right)$
(C) $e^{\frac{-3+\sqrt{4x+1}}{2}}$ (D) $e^{\frac{-3-\sqrt{4x+1}}{2}}$

50. Let $f : X \to Y$ be defined as $f(x) = \sin x + \cos x + 2\sqrt{2}$. If f is invertible, then $X \to Y$, is:
(A) $\left[\frac{-3\pi}{4}, \frac{-\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$
(B) $\left[\frac{-\pi}{4}, \frac{-3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$
(C) $\left[\frac{-3\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$
(D) $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

51. If the range of function $f(x) = \frac{x^2+x+c}{x^2+2x+c}, x \in R$ is $\left[\frac{5}{6}, \frac{3}{2}\right]$, then c is equal to
(A) -4 (B) 3
(C) 4 (D) 5

52. Let $f: R \to [1, \infty)$ be defined as $f(x) = \log_{10}(\sqrt{3x^2-4x+k+1}+10)$. If $f(x)$ is surjective, then:
(A) $k = \frac{1}{3}$ (B) $k < \frac{1}{3}$
(C) $k < \frac{1}{3}$ (D) $k = 1$

53. If the domain of function $f(x) = \sqrt{\log_2\left(\frac{x-2}{3-x}\right)}$ is [a, b), then the value of (2a – b) is:
(A) 2 (B) 3
(C) 4 (D) 5

54. The range of function $f(x) = \text{sgn}(\sin x) + \text{sgn}(\cos x) + \text{sgn}(\tan x) + \text{sgn}(\cot x)$, $x \neq \frac{n\pi}{2} (n \in I)$ is :

Note: sign k denotes signum function of k]

(A) $\{-2, 4\}$ (B) $\{-2, 0, 4\}$

(C) $\{-4, -2, 0, 4\}$ (D) $\{0, 2, 4\}$

55. If a plynomial function 'f'satisfies the relation $\log_2[f(x)] = \log_2\left(2+\frac{2}{3}+\frac{2}{9}+.....+\infty\right) \cdot \log_3\left(1+\frac{f(x)}{f\left(\frac{1}{x}\right)}\right)$ and f(10) = 1001, then the value of $f(20)$ is:

(A) 2002 (B) 7999

(C) 8001 (D) 16001

56. If $x = \frac{4l}{1+l^2}$ and $y = \frac{2-2l^2}{1+l^2}$ where 'l' is a parameter and range of $f(x, y) = x^2 - xy + y^2$ is $[a, b]$ then $(a + b)$ is equal to:

(A) 4 (B) 6

(C) 8 (D) 12

57. If the equation $|x - 2| - |x + 1| = p$ has exactly one solution, then number of integral values of p, is

(A) 3 (B) 4

(C) 5 (D) 7

58. Let a, b are positive real numbers such that a – b = 10, then the smalest value of the constant K for which $\sqrt{(x^2+ax)} - \sqrt{(x^2+bx)} < K$ for all x > 0, is:

(A) 2 (B) 3

(C) 4 (D) 5

59. The sum of al possible solution (s) of the equation $||x+2|-3| = \text{sgn}\left(1-\left|\frac{(x-2)(x^2+10x+24)}{(x^2+1)(x+4)(x^2+4x-12)}\right|\right)$ is

(A) 0 (B) – 8

(C) –10 (D) Not applicable

Not: sgn (y) denotes the signum function of y]

60. If the equation $(p^2-4)(p^2-9)x^3 + \left[\frac{p-2}{2}\right]x^2 + (p-4)(p^2-5p+6)x + \{2p-1\} = 0$ is satisfied by all values of x in (0, 3] then sum of all possible integral values of 'p' is

(A) 0 (B) 5

(C) 9 (D) 10

Note: {y} and [y] denotes fractional part function and greatest integer function of y respectively.]