Intro To ML – HW2

students:

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**Theory Questions**

Note:  
We denote in this exercise vectors in **bold** font, e.g. and

**Question 1:**

1. Consider the following problems:

We will show that It doesn’t matter whether the following constrain is included or not:

.

We will show that problem (1)’s parameters are equivalent to problem (2)’s:

* : Proof by contradiction: Let be the parameters that minimize the function in problem (1), and let be the parameters that minimize the function in problem (2), so that (2)’s target function gives a smaller solution than (1)’s. We conclude that that minimizes the function in problem (**2**)- so the constraint is satisfied (otherwise, if there wasn’t like this, the problems (1) and (2) would be equivalent). Because of , we ge that meaning also satisfies the constrain and would be even better solution: in contradiction that (2) gives the minimalist value which can be achieved.
* : Proof by contradiction: Let be the parameters that minimize the function in problem (2), and let be the parameters that minimize the function in problem (1), so that (1)’s target function gives a smaller solution than (2)’s. It’s obvious that (1)’s parameters satisfy the (2)’s constrains, so again, we get the same minimal value for both (1) and (2) target functions, in contradiction.



* By comparing (1) to 0 we get the relation: .
* By comparing (2) to 0 we get the new constrain: .
* By comparing (3) to 0 we get the relation: or .

So now our Lagrangian is:

=

1. Our dual problem is:

.

**Question 2:**

1. By placing the star notation (\*) for the optimal solution, we don’t need to write the constrains, because we assume that they are already satisfied. We get:

On the right member, we didn't use to ease notation.



So as required.

**Question 3:**

1. Not submitting the full answer, just note that we try to optimize:

1. Firstly, we evaluate the expression:

For not fixed, and fixed we will use the partial derivative compared to 0. First, the problem in section a, for un-fixed is equivalent to:

We can see the objective function is concave.

* If or so any , fits. We’ll choose arbitrary value,
* If and then

In total, with respect to the constraint we get:

1. The expression under the takes and we it . In total, time complexity is . We should keep in memory all our samples, s and labels, so space complexity is

If we assume that distance calculations were done in advance, and we keep the same matrix at all the iterations (which is true), for a single iteration, (now assuming all are pre-estimated), the time and space complexity are just .

**Question 4:**

1. This statement is false. Counter-example:

Given and , and are valid positive definite kernels. So, which is negative definite, because

.

1. This statement is correct. Proof:

Assumption- and are valid positive definite kernels. In this section we assume that is not 0 otherwise is not even well defined so setting will count as a valid counter-example.

Now for the proof:

Positive:

. is positive from the definition of the norm of un-zeroed vector, and gives a positive value because of the assumption.

In total, we get that hence .

Symmetry:

.

1. This statement is correct. Proof:

Assumption- is a valid positive definite kernel and .

Positive:

. gives a positive value, and because of the assumption. In total, we get that hence.

Symmetry:

.

1. This statement is false. Counter-example:

Given and , is a valid positive definite kernel. So, which is negative definite, because .

1. In this section, we assume that the kernels definition stands for semi-positive matrixes as well as positive ones. Otherwise, we can take the constant function as a counter example.

Now for the proof (this statement is correct):

Assign is clearly a PDK.

Symmetry:

.

From lecture 7 (slide 28) we get: . is positive because is a PDK, so in total we get that .

1. This statement is correct. Proof:

Assumption- is a valid positive definite kernel and is a polynomial with positive coefficients- .

Positive:

First, we will prove by induction that for any valid positive definite kernel , is valid positive definite kernel too:

Base:

For : is valid positive definite kernel.

For : is a valid positive definite kernel from the multiplication property proved in class.

Step:

Assuming is a valid positive definite kernel, is a valid positive definite kernel as well from the multiplication property proved in class: .

We proved in class the addition property.

We proved in section c the multiplication by a positive factor property.

In total, we get that is a valid positive definite kernel.

Symmetry:

**Question 5:**

1. Proof by induction on the Tth iteration:

Base:

for we had no iterations so the claim is clearly satisfied.

Step:

Assume the claim is true for , and we will prove for :

* Prediction is correct:
* Prediction is mistake: **.** By the induction assumption we directly get our proof by construction.

1. The weight vectorcan be expressed by where is the number of times that was misclassified (now we denote instead of ). Therefore, which would be the kernel version of the perceptron.

We get the algorithm:

1. Initialize the weight vector , and initialize t: .
2. Given a sample , predict positive iff
   1. If the prediction was correct, set .
   2. If the prediction was mistaken, set .
3. Go over 1.

For demonstration, let's assume that and the previous predictions were:

in that case, we get:

|  |  |  |
| --- | --- | --- |
| t |  |  |
|  | 1 | -1 |
|  | 0 | 1 |
|  | 1 | 1 |

Given the sample the algorithm will predict positive iff

And act accordingly. it is easy to see from this example that the samples which were correctly predicted are not “taken into account”, and the others influence the weights similarly as the original.