Intro To ML – HW4

students:

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**Theory Questions**

**Question 1:**

**Question 2:**

**Question 3:**

1. Denote and *.*

Denote

Hence: *.*

Denote the matrix which all its elements are equal to as . Thus, generalizing the discussion to matrices, we get: and since and are symmetrical (by definition of ), then and we get: **.** Assuming we are given and that matrix multiplication takes for matrixes dimension’s , we conclude that the time complexity to compute in total is .

1. We will show that any where can be expressed by a linear combination of :

Denote as the covariance matrix formed of , meaning: . We note that every is an eigen-vector of , with eigen-value (by the definition of principal component). Thus:

.

Hence:

(note that we didn’t prove that , which is easy to show). Denote and we get: as we expected.

1. . Calculating depends on the number of features, meaning it takes and it we repeat this times (as the number of samples). In total, this phase’s time complexity is .

**Question 4:**

Consider the following optimization problem:

Where is a matrix of the variables, is a column vector of size , and is a column vector of size of coefficients.

Consider the SVD of , we get the constraint:

We want now to recover , so from the recitation: . By multiplying both members with from left, we get: . Denote this solution by .

Assume is singular. Denote by a vector in the null space of . Namely a vector such that . Suppose other solution to the optimization problem: , so fulfills . Thus, we get: .

Hence any solution of the form contains all possible solutions to .

From the recitation, we know that the projection matrix, can be calculated by . We would like to project to the null space of so we can get the minimum distance vector from . Then the null space projection would be: , and for we get: .

Then, from the definition of projection, the closest vector to in this null space is **.**

We know that the null space is linear, hence as well.

Now we can solve the optimization problem:

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**Programming Questions**

**The code source files stay under /specific/a/home/cc/students/cs/avicaciularu/ML/HW4/:**

**Q5- Q5.py**

**Q6- Q6.py**

**Question 5:**

**Question 6:**