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Design of supersonic wind tunnel using method of characteristics

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Abstract: The future of aerospace is supersonic and a supersonic wind tunnel is the fundamental requirement of the day. This design will help build small scale and large scale supersonic wind tunnels for testing objects such as the spikes used in the fore section of supersonic aircrafts, and shock absorbing objects etc. Shocks are evident in a supersonic flow and when air passes through a shock, the velocity reduces and thus not allowing the speed to be maintained constant throughout the section and also reducing the efficiency of the wind tunnel. By using the Method of characteristics, we assume characteristic points and see to it that the shocks, if formed will be in contact with the surface and does not affect the uniformity of the flow through the duct and the velocity of the flow. A thorough understanding of the method of characteristics and its application to the design of supersonic wind tunnel nozzle is required. We make use of the Area-Mach relation, Prandtl Mayer spread sheet solver and other tools to define 64 coordinates in the supersonic section of the nozzle, including the internal points. The Wind Tunnel has been designed in CATIA v5 and the coordinates of the supersonic section of the wind tunnel have been obtained from MS Excel by doing the calculations and using its Goal Seek feature to simplify very complicated equation. The contour of the supersonic section of the wind tunnel is so designed that the shocks cancel each other, the shocks do not affect the velocity of the flow in order to obtain supersonic flow.

Keywords: *Design, Supersonic, Wind Tunnel, Method of Characteristics*

1. Introduction

A supersonic wind tunnel can be simply said as a convergent-divergent duct (fig 1.1) where the velocity increases from subsonic to supersonic speeds. The subsonic flow increases its velocity in a convergent duct and decreases in velocity as the cross sectional area increases in the divergent duct. This is not the case with the supersonic flow regime. In supersonic flow regime, the velocity increases further in a divergent duct and decreases in the convergent duct where the cross-sectional area decreases. The velocity reduces to sonic speed in the convergent duct. At that point, if the cross-sectional area again increases, then the velocity further decreases to subsonic speeds. Generally, a convergent-divergent duct (fig 1.1) can be used for both increasing the velocity from subsonic to supersonic speeds and also for reducing the velocity from supersonic to subsonic speeds.

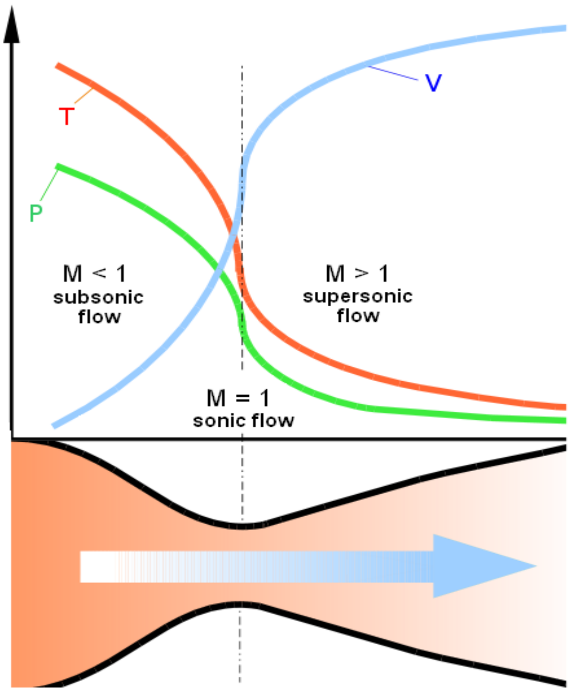


Figure: 1.1 Convergent-Divergent Duct

In the supersonic speed regime, shock waves are involved. Shocks are thin layer of disturbances across which the flow properties change significantly. There are mainly two types of shocks viz. normal shock and oblique shocks. The normal shocks are the stronger ones that stand on the surface, oblique shocks are the weaker ones which make an angle to the surface of the body. In supersonic aircrafts, spikes are used in the fore section of the nose cone for this reason only. They convert the strong normal shocks into weaker oblique shocks. These oblique shocks can also be advantageous. In engines such as the ram jet engines, supersonic air must be brought down to subsonic speeds before combustion. We know that the flow properties change significantly across a shock. As the velocity of air is reduced when passed through a shock, the inlet is designed such that there will be a lot of shock interactions which reduce the flow velocity. In designing the supersonic section of the wind tunnel, we have to make sure that these shock interactions do not have a negative effect on the flow velocity and the speed should further increase downstream with the increase in the cross sectional area. For designing a supersonic wind tunnel, a sound knowledge on all these aspects should be known. We have to design the supersonic area of the nozzle such that the shocks reflect in a direction that is parallel to the flow so that the flow does not pass through the shock which may reduce the velocity. The shock must be either in the direction of the flow or it has to be in contact with the contour of the duct. In this design, the nozzle is designed in such a way that the shocks reflect in the direction of the boundary making it optimum for the expansion of supersonic flow and attaining the desired speed.

The supersonic wind tunnel can be divided into different sections and each section has to be designed separately. These sections are the convergent section having the sub sonic region where the velocity of the flow increases as the cross-section of the duct decreases. Next is the sonic throat, which has to be designed with the help of Area Mach relations to get the cross-sectional area of the throat. In the supersonic region, the duct must be sub divided into two regions, in the first section, the cross-sectional area increases and the slope of the contour increases downstream. In the latter stages of the supersonic flow regime, although the cross sectional area increases, the slop of the contour decreases making the curve parallel to the flow and uniform direction of flow is obtained at the end of this section so that the models that are tested in the wind tunnel experience a uniform flow in the test section which follow the supersonic expansion region.

2. Problem

The main purpose of a wind tunnel is to simulate the in-flight conditions to test various shapes and to check the efficiency of the shapes that are being tested. The primary difference between a supersonic wind tunnel and a sub sonic wind tunnel is the shock waves. The Supersonic flight regime involves a lot of shocks and these make it very difficult for the uniform flow of air which in turn do not give the correct simulation of actual in-flight conditions. To make sure that a uniform flow is present at the test section, we have to design the nozzle such that the characteristics cancel each other or all characteristics have the same angle of attack forming a uniform flow.

The most part of the supersonic wind tunnels being designed these days have their origin from the initial designs in the 1960s. Ongoing developments in this field are connected with aspects of fine tuning details and improving the efficiency and test accuracy. The only area where significant improvement can be achieved is to increase the efficiency of the supersonic wind tunnel.

Supersonic flow can be achieved by maintaining very high pressure difference between the inlet and exit of the nozzle and creating the optimum contour for the nozzle. A convergent divergent nozzle should be designed to achieve the same. The nozzle will accelerate flow from a subsonic speed to a supersonic speed. D

The project utilizes the area Mach relations, in the widely cited book “fundamentals of aerodynamics” by Anderson. The area component is expressed as non-dimensional ratio of the local duct cross-sectional area to the sonic throat cross-sectional area. Test sections usually have simple geometry involving constant cross-section parallel walls and thus a constant cross-sectional area in order to maintain constant velocity throughout the test section.

Blow-down tunnel can be used for designing the supersonic wind tunnel. The pressure ratio comes from a high-pressure tank, upstream of the wind tunnel. This is relatively safe mechanism and very economical when compared to a vacuum chamber type supersonic wind tunnel. The only safety measure that one has to take is that the test section axes doors must be interlocked with the tunnel control valves so that the operator is not harmed by an accidental slow start-up. Also the exit of the wind tunnel must be situated in a position such that the high speed flow coming from the wind tunnel does not hit/harm any individual standing at that place.

The added advantage of the blow-down type supersonic wind tunnel is that, it has a variable runtime. i.e., if we need a higher runtime, then a larger tank can be employed. In the normal course of testing, the runtimes are of the order of a few seconds only. This is a limited for vacuum type supersonic wind tunnels.

The central object of most research and development efforts with respect to testing in supersonic regime has been greatest accuracy and control over test conditions. Critical characteristics of wind tunnel flows are Mach number, Reynolds number, pressure, and temperatures. Precise knowledge and control of these variables in the test section allows for testing that better reflects actual flight conditions.

It is very simple from the area Mach relations to find out the area of the throat and exit area of the nozzle if we know the inlet area and inlet conditions. But, to design the supersonic wind tunnel is not that simple. The various contours that have been adapted and tested in the past have revealed that the design of the nozzle from the inlet section to the throat is relatively very simple when compared to the design of the nozzle from the throat section to the test section as we deal with supersonic flows. All the information regarding the fictional affects, the other factors are very critical in designing the supersonic tunnel.

3. Solution

As stated above, the design of the supersonic wind tunnel from the inlet section to the throat area is relatively simple when compared to the designing of the rest of the nozzle from the throat section till the test section. A simple splined multi section solid and multi section pocket is being used for the design of the nozzle in the convergent section and the throat section. The very famous and reliable “method of characteristics” was used to design the nozzle from the throat section to the test section, which is the most crucial part of the project.

The method of characteristics is a numerical procedure appropriate for solving among other things two-dimensional compressible flow problems. By using this technique flow properties such as direction and velocity can be calibrated at distant points throughout a flow field. The method of characteristics implemented in a computer algorithm is an important element of supersonic competition fluid dynamics software. But we make use of Microsoft Excel, CATIA V5 software to design our supersonic wind tunnel.

3.1 Method of Characteristics

The physical conditions of a two-dimensional, steady, isentropic, irrotational flow can be expressed mathematically by the nonlinear differential equation of the velocity potential. The method of characteristics is a mathematical formulation that can be used to find solutions to the aforementioned velocity potential, satisfying given boundary conditions for which the governing partial differential equations (PDEs) become ordinary differential equations (ODEs). The latter only holds true along a special set of curves known as characteristic curves. As a consequence of the special properties of the characteristic curves, the original problem of finding a solution to the velocity potential is replaced by the problem of constructing these characteristic curves in the physical plane.

The method is founded on the fact that changes in fluid properties in supersonic flows occur across these characteristics, and are brought about by pressure waves propagating along the Mach lines of the flow, which are inclined at the Mach angle to the local velocity vector.

The method of characteristics was first applied to supersonic flows by Prandtl and Busemann in 1929 and has been much used since. This method supersonic nozzle design made the technique more accessible to engineers. In supersonic nozzle design the conventional two-dimensional nozzle is usually considered to consist of several regions such as,

1. Contraction part, where the flow is entirely subsonic
2. The throat region, where the flow accelerates from high subsonic to low subsonic speeds.
3. The initial expansion region, where the slope of the counter increases up to its maximum value
4. The straightening, or “Busemann” region in which the processor area increases but the wall slope decreases to zero.
5. The test section where the flow is uniform and parallel to the axis.

3.2 Characteristics

Characteristics are unique in that the derivatives of the flow properties become unbounded along them. On all other curves, the derivatives are finite. Characteristics are defined by three properties as detailed by John and Keith

1. A characteristic in a two-dimensional supersonic flow is a line along which physical disturbances are propagated at the local speed of sound relative to the gas.
2. A characteristics is a cut across which flow properties are continuous, although they may have discontinuous first derivatives, and along which the derivatives are indeterminate.
3. A characteristic is a cut along which the governing partial differential equations may be manipulated into ordinary differential equations.

“Fluid particles travel along our clients propagating information regarding the condition of the flow. In supersonic flow, the cost equates travel along Mach lines propagating information regarding flow disturbances”. This is described in the first property.
The second property says that Mach line can be considered as an infinitesimally thin interface between two smooth and uniform, but different regions. The line is a boundary between continuous flows along the streamline passing through a field of these Mach waves, the derivative of the velocity and other properties may be discontinuous.
The third property speaks for itself. Ordinary differential equations are often easier to solve than partial differential equations. That is why this property is considered very important.

While the ratios of the areas are relatively straightforward to determine based on desired test section Mach numbers and tunnel runtimes, determining and optimum channel counter is slightly more complicated. As said earlier, it is easy to determine the contoured between the inlet section and the throat section. The region immediately after the sonic throat where the flow is turned away from itself, the air expands into supersonic velocity. This expansion happens gradually over the initial expansion region. In the Prandtl-Meyer expansion scenario, it is assumed that expansion takes place across the fan of originating from an abrupt corner. This phenomena is typically modeled as a continuous series of expansion waves, each turning the airflow and infinitesimal amount, along with the contour of the channel wall. These expansion waves can be thought of as the opposite of stock compression waves, which slow airflow. This is governed by a Prandtl Meyer function.

For the purposes of notation, if one is considering a point P, the point which connects to P by a right-running characteristic1 line is considered ‘A’, and the point connecting with a left-running line is considered point B, as shown in Figure 2.1. Right-running characteristics are considered to be type I, or CI lines. Similarly, left-running characteristics are considered to be type II, or CII lines.

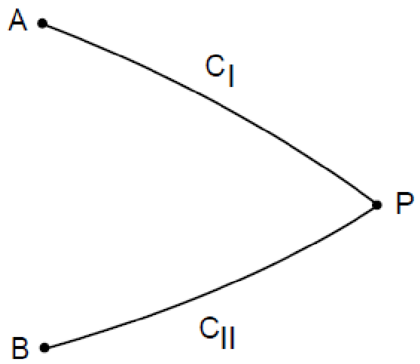


Figure 3-1 Characteristics

4. Design of Supersonic Wind Tunnel Nozzles

It is critical that the stream entering the test section of a wind tunnel be uniform and parallel in order to record valid test data. This requirement becomes more difficult to achieve as the Mach number of the flow increases from the subsonic regime to the supersonic regime where shock waves may form. The design of the divergent portion of the supersonic nozzle contour, in particular the straightening section, is extremely important for this reason. The shape of the expansion contour is largely arbitrary and depends somewhat on the shape of the sonic line2. It has been demonstrated that theoretical results obtained from the method of characteristics, with the assumption of a near linear sonic line, match quite well to experimental values [2]. Also, it is undesirable to have compression shocks in the nozzle, due to boundary layer behavior. Since large pressure gradients arise through these shocks, the shock interaction with the boundary layer can cause irregularities in the flow and even flow separation. Therefore, the Prandtl-Meyer flow in the straightening section should seek to avoid the formation of oblique shock waves.

For this project, the method of characteristics was utilized to design a contour shape that produces test section that is free of shocks. To accomplish this, an initial channel divergence angle is chosen for the expansion region of the contour where the channel simply expands as a linearly diverging section, as pictured in Figure 4-1. Immediately downstream of this section, the channel walls begin to straighten out, gradually becoming horizontal to turn the flow straight and produce uniform streamlines. In normal circumstances, when an incident wave impinges upon a flat wall, that wave is reflected at an angle, as shown in Figure 4-2.

In the case of the straightening section, the wall of the contour is turned exactly through the wave turning angle α at the point at which the wave meets the wall, as shown in Figure 4-2.

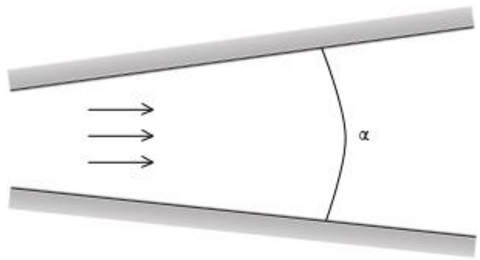


Figure 4-1 Supersonic flow in a two-dimensional diverging channel

Turning the wall in this manner cancels the reflected wave by eliminating the need for it. The angled wall satisfies the boundary condition, as it causes flow to run parallel to the wall.

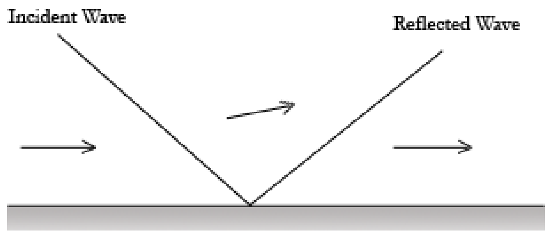


Figure 4-2 Incident and reflected waves

The characteristic net employed in the calculations for this project finds numerous points at which to turn the wall contour to create a continuous smooth curve of wave cancellations. Calculations of the characteristic “net" started with a sample spreadsheet recreating an example method of characteristics calculation presented in John and Keith's Gas Dynamics. The example

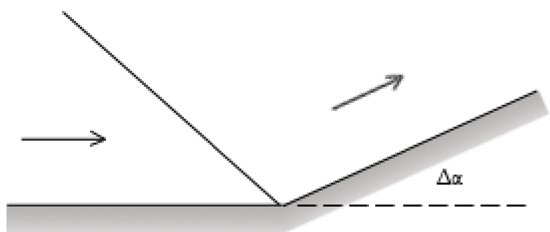


Figure 4-3 Wall parallel to the reflected wave

consisted of a 12^o diverging channel with an initial Mach number of 2 at the inlet. Because the channel was symmetrical, only the top half was considered (for a half-angle divergence of 6^o). The arced initial value line (or “sonic line”) from which the rest of the flow field calculations are carried out was divided into four points having divergence increments of 2^o between 0^o and 6^o. The spreadsheet was designed to match the initial 18 point example in the book, then further expanded to calculate all 32 points in the example expansion region shown in Figure 3-4

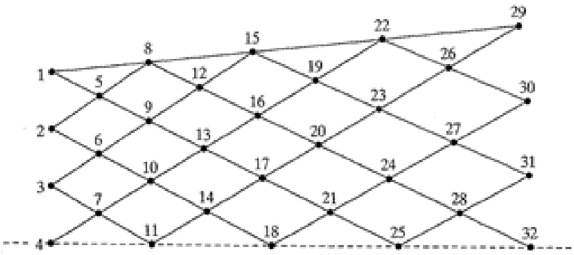


Figure 4-4 Characteristic points

After this was complete, the example mesh was then extended to create the straightening section, which was not present in the example. In this section, each local angle of each wall point was chosen to coincide with the local flow angle in order to cancel out the reflected Mach wave. Knowing the local angle of the wall as a function of axial position along the tunnel, the contour is fully defined. This region-the straightening section-ensures that test section flow is free of shocks.

4.1 Working of Method of characteristics

Calculations begin by dividing the initial value line into four increments to represent increasing angles of divergence. Points 1 through 4 were assigned α value of 6^o, 4^o, 2 ^{o}, and 0 ^{o} respectively. The Prandtl-Meyer angle ν was then calculated using the Prandtl-Meyer function.

$$d\theta = \sqrt{M^2 - 1}\frac{dV}{V} \qquad \textit{Eq.4.1}$$

Where the change in flow angle (relative to its original direction) is represented by θ

$$v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \qquad \text{Eq. 4.2}$$

4.1.1 To find the properties of the points on the initial value line (1, 2, 3, 4)

To begin with, we first calculate the maximum angle that the nozzle can have in order to have optimum expansion. There should not be a case of over expansion or under expansion in the supersonic nozzle. This is found using the Prandtl Meyer spreadsheet solver which is also created as a part of this project using an excel sheet, which gives the maximum angle of expansion. This will be for the minimum length nozzle. The minimum length nozzles are used in rockets and areas where space is a very important issue. We take a convenient angle of expansion lesser than the maximum angle of the minimum length nozzle.

Now, we fix the number of points to be considered in the initial value line. We fix these points such that each point will have equal difference in the angle, making sure that the center line has α = 0o and the point on the wall of the nozzle is the maximum angle we are fixing.

We then fix the y coordinate of the nozzle. Generally, we take y = 1 for ease of scaling.

For the known value of y, we get the x coordinate of the point using the formula

$$x = \frac{y}{\tan\alpha} \qquad \text{Eq.4.3}$$

For the rest of the points, the procedure is different. We first find out the radius of curvature of the initial value line using the Eq.3.4 and then find out the other coordinates of the points on the initial value line using the Eq.3.5 and Eq.3.6

$$R_{IVL} = \sqrt{x^2+y^2} \qquad \text{Eq.4.4}$$

$$x_n = \mathrm{R_{IVL}}.\cos\alpha \qquad \text{Eq.4.5}$$

$$y_n = \mathrm{R_{IVL}}.\sin\alpha \qquad \text{Eq.4.6}$$

4.1.2 To find the properties of points of secondary value line (5, 6, 7…)

The above is the expression for Prandtl - Meyer angle. CI CII

After this, CI and CII were calculated using Eq.4.7 and Eq.4.8

$$\mathrm{C_I} = \nu + \alpha \qquad \text{Eq.4.7}$$

$$C_{II} = \nu - \alpha \qquad \text{Eq.4.8}$$

Then, we find out the Prandtl Meyer angle ν from the Eq.3.9 using the values obtained in the above two equations.

$$\vartheta = \frac{\mathrm{C_I} + C_{II}}{2} \qquad \text{Eq. 4.9}$$

Now, we have the value of the Prandtl Meyer angle. So, by using the “goal seek” feature in MS Excel, we can find out the value of the Mach number at that particular point, using the Eq.4.2

Then, we find out the Mach angle μ from the simple formula as in Eq.3.10

$$\mu = \sin^{-1}\frac{1}{M} \qquad \text{Eq.4.10}$$

Then, we find out the slopes of the two characteristics using the formula given below.

$$m_I = \tan\left(\frac{(\alpha-\mu)_A + (\alpha-\mu)}{2}\right) \qquad \text{Eq.4.11}$$

$$m_{II} = \tan\left(\frac{(\alpha+\mu)_B + (\alpha+\mu)}{2}\right) \qquad \text{Eq.4.12}$$

Here, the subscript “A” relates to the point that connects the first characteristic and “B” relates to the second characteristic.
Now that we found out the slopes of the characteristics also, we can proceed for the calculation of the coordinates of the point. The x-coordinate of the point is first found using the Eq.4.13

$$x = \frac{y_A - y_B + m_{II}.x_B - m_I.x_A}{m_{II} - m_I} \qquad \text{Eq.4.13}$$

After we find the x-coordinate, we can now find out the y-coordinate of the current point. This is done later because, we will be using the x coordinate to find out the y coordinate of the point using any of the two Eq.4.1.4 or Eq.4.1.5 as shown below according to the connivance.

$$y = y_A + m_I(x - x_A) \qquad \text{Eq.4.14}$$

$$y = y_B + m_{II}(x - x_B) \qquad \text{Eq.4.15}$$

This way, we can find out the coordinates of the points on the second value line.

4.1.3 To find the properties of points on contour

The procedure to be followed during the calculation of points on the contour is different from the initial value line or the secondary value line. This is because: the boundary points will not be having the first characteristic. Only the second characteristic is available with the point. But, it has an advantage that it has the known value of α, i.e. the flow angle.

During the expansion region, the flow angle is taken common, i.e. the maximum angle that is taken at all points on the contour. But, in the straightening section, the flow angle at the contour is taken same as the flow angle at its corresponding B point. This way, the flow straightens itself.
As this does not have the first characteristic, the Prandtl Meyer angle is not found using the Eq.3.9. Instead, it is found out using the Eq.4.16

$$C_{II} = \vartheta_B - \alpha_B = \vartheta_P - \alpha_P \qquad \text{Eq.4.16}$$

Then, the first characteristic is found using the Eq.3.9
The slope of the first characteristic is found using the Eq.4.17

$$m_I = \tan\alpha \qquad \text{Eq.4.17}$$

In this case, the y-coordinate is first found out and then, the x-coordinate is found using the y-coordinate. The x & y-coordinate is found using the Eq.4.18 and Eq.4.19

$$y_p = \frac{y_B - m_{II}.x_B}{\left(1 - \frac{m_{II}}{\tan\alpha}\right)} \qquad \text{Eq.4.18}$$

$$x_P = \frac{y_P}{\tan\alpha} \qquad \text{Eq.4.19}$$

4.1.4 To find the properties of points on center line

The procedure to be followed during the calculation of points on the center line is different from the initial value line or the secondary value line. This is because: the boundary points will not be having the second characteristic. Only the first characteristic is available with the point. But, it has an advantage that it has the known value of α, i.e. the flow angle (α=0).

During the expansion region, the flow angle is taken common, i.e. the maximum angle that is taken at all points on the contour. But, in the straightening section, the flow angle at the contour is taken same as the flow angle at its corresponding B point. This way, the flow straightens itself.
As this does not have the second characteristic, the Prandtl Meyer angle is not found using theEq.4.9. Instead, it is found out using the Eq.4.20

$$C_I = \vartheta_A + \alpha_A = \vartheta_P + \alpha_P \qquad \text{Eq.4.20}$$

Then, the second characteristic is found using the Eq.3.9
The slope of the second characteristic is found using the Eq.3.21

$$m_{II} = \tan\alpha \qquad \text{Eq.4.21}$$

In this case, the y-coordinate is first found out and then, the x-coordinate is found using the y-coordinate. The x & y-coordinate is found using the Eq.4.22 and Eq.4.23

$$y_p = \frac{y_B - m_{II}.x_B}{\left(1 - \frac{m_{II}}{\tan\alpha}\right)} \qquad \text{Eq.4.22}$$

$$x_P = \frac{y_P}{\tan\alpha} \qquad \text{Eq.4.23}$$

5. Algorithm

The curve used for the subsonic region is not very important when compared with the supersonic region. A simple spline can be used for the curve for the subsonic region. The use method of characteristics and point-to-point evaluation for generating the contour of the supersonic region which is explained below:

1. Fix on desired Mach number at the exit/at the test section.
2. Formulate the area Mach relations using Excel sheets.
3. Formulate Prandtl Mayer spreadsheet solver using Excel sheets.
4. Prepare a table with area ratio, pressure ratio, and temperatures ratio using the area Mach relations, pressure relations and temperature relations (see table 1).
5. Fixed inlet pressure and area.
6. Fix the height of the throat.

7. Design the nozzle in excel sheet and get the coordinates.
   a. Find out αmax using the Prandtl Mayer spreadsheet solver created in step three.
   b. Fix the number of characteristics that has to be used, i.e. the number of points on the initial line. The more the points, the better the accuracy rate.
   c. Assume each point will have a flow angle at equal intervals so that, the difference between the flow angles of two adjacent points in the initial Mach line is equal.
   d. Assume Mach number to start with. We cannot start with Mach number 1 because they will not be able to calculate properly. So we start with an initial Mach number that is just greater than one.
   e. Assume the height of the throat, i.e. the y-coordinate of the first point on the initial line as 1.
   f. Find out the x-coordinate of the first point of the initial line using the flow angle and the y-coordinate at that point.
   g. Using the first point of the initial line and flow angle at each point on the initial line, find out the coordinates of each point on the initial value line using the radius of curvature of the initial value line.
   h. To get the coordinates of the secondary value line.
      i. Calculate the Rayman invariants
      ii. Using the Rayman invariants, calculate the Prandtl Mayer angle.
      iii. Using the goal seek feature, calculate the Mach number
      iv. Calculate the slope of each characteristic.
      v. Calculate the x-coordinate using the variables that we obtain in the previous steps using the appropriate formula given in the theory.
      vi. Calculate the y-coordinate using the slope of any characteristic, the x-coordinate, and the x-coordinate of the previous value line.
   i. Now find out the coordinates of the points on the contour that does not have the first characteristic. The method is explained in the theory.
   j. And of the coordinates of the points centerline. This method is also explained very clearly in the theory.
8. Repeat steps h, i, j until the required Mach number is reached. From this point, we have to straighten the nozzle. I.e. the nozzle still expands but the slope decreases and finally becoming a straight horizontal line. This is done by using the flow angle on the contour as the flow angle in the previous characteristic's B point.
9. Design using the coordinates obtained in step 7 in CATIA figure (5.1&5.2).

6. Wind tunnel diverging and straightening contour analysis

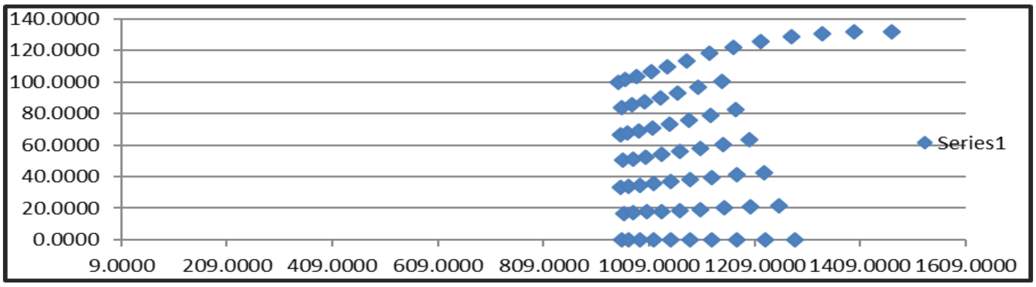
In this case the straightening wall is assumed to be 6° from the horizontal. The initial Mach number at the assumed "throat" is assumed to be 1.02

Table 1 Results of Prandtl Mayer spread sheet solver

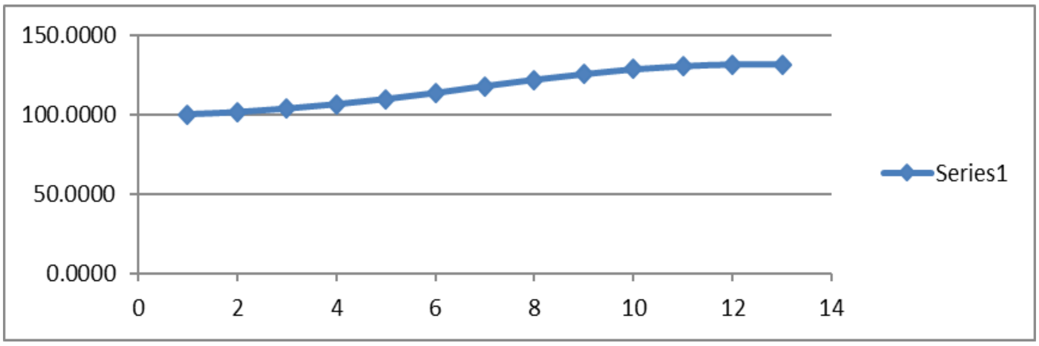
Point	α	ϒ	M	μ	CI	CII	ν	ν	α+μ	α-μ	mI	mII	x	y
1	6	1.4	1.0200	78.6351	6.1257	-5.8743	0.1257		84.6351	-72.6351			951.4364	100.0000
2	4	1.4	1.0200	78.6351	4.1257	-3.8743	0.1257		82.6351	-74.6351			954.3468	66.6667
3	2	1.4	1.0200	78.6351	2.1257	-1.8743	0.1257		80.6351	-76.6351			956.0944	33.3333
4	0	1.4	1.0200	78.6351	0.1257	0.1257	0.1257		78.6351	-78.6351			956.6772	0.0000
5	5	1.4	1.0888	66.7005	6.1257	-3.8743	1.1257	1.1257	71.7005	-61.7005	-2.3752	4.3901	958.2522	83.8115
6	3	1.4	1.0888	66.7005	4.1257	-1.8743	1.1257	1.1257	69.7005	-63.7005	-2.6281	3.7763	960.5821	50.2799
7	1	1.4	1.0888	66.7005	2.1257	0.1257	1.1257	1.1257	67.7005	-65.7005	-2.9321	3.3054	961.7473	16.7588
8	6	1.4	1.1384	61.4505	6.0000	-3.8743	2.1257	2.1257	67.4505	-55.4505	0.1051	2.6854	964.8037	101.4050
9	4	1.4	1.1384	61.4544	6.1257	-1.8743	2.1248	2.1257	65.4544	-57.4544	-1.7029	2.4235	967.7467	67.6431
10	2	1.4	1.1384	61.4544	4.1257	0.1257	2.1248	2.1257	63.4544	-59.4544	-1.8477	2.2022	969.4927	33.8156
11	0	1.4	1.1384	61.4544	2.1257	2.1238	2.1248	2.5257	61.4544	-61.4544	-2.0125	2.0921	970.0746	0.0000
12	5	1.4	1.1821	57.7723	8.1257	-1.8752	3.1249	3.1252	62.7723	-52.7723	-1.3820	2.0606	976.3721	85.4170
13	3	1.4	1.1821	57.7736	6.1248	0.1248	3.1245	3.1248	60.7736	-54.7736	-1.4889	1.8898	978.7351	51.2819
14	1	1.4	1.1821	57.7736	4.1248	2.1248	3.1245	3.1248	58.7736	-56.7736	-1.6074	1.7400	979.8970	17.0913
15	6	1.4	1.2227	54.8733	6.0000	-1.8751	4.1249	4.1249	60.8733	-48.8733	0.1051	1.8668	986.1378	103.6473
16	4	1.4	1.2227	54.8738	8.1249	0.1245	4.1247	4.1247	58.8738	-50.8738	-1.2718	1.7198	989.1408	69.1776
17	2	1.4	1.2227	54.8744	6.1245	2.1245	4.1245	4.1245	56.8744	-52.8744	-1.3675	1.5894	990.9224	34.6155

18	0	1.4	1.2227	54.8736	4.1245	4.1251	4.1248	4.1248	54.8736	-54.8736	-1.4728	1.5925	991.5019	0.0000
19	5	1.4	1.2612	52.4570	10.1249	0.1247	5.1249	5.1248	57.4570	-47.4570	-1.1171	1.6107	1000.5477	87.5503
20	3	1.4	1.2612	52.4572	8.1247	2.1245	5.1248	5.1246	55.4572	-49.4572	-1.1988	1.4919	1002.9740	52.5948
21	1	1.4	1.2612	52.4572	6.1245	4.1248	5.1248	5.1246	53.4572	-51.4572	-1.2876	1.3848	1004.1758	17.5504
22	6	1.4	1.2983	50.3731	6.0000	0.1249	6.1249	6.1249	56.3731	-44.3731	0.1051	1.5349	1012.8654	106.4564
23	4	1.4	1.2983	50.3731	10.1249	2.1248	6.1249	6.1248	54.3731	-46.3731	-1.0692	1.4237	1015.9557	71.0763
24	2	1.4	1.2983	50.3731	8.1248	4.1248	6.1249	6.1248	52.3731	-48.3731	-1.1469	1.3230	1017.8063	35.5831
25	0	1.4	1.2983	50.3731	6.1248	6.1250	6.1249	6.1246	50.3731	-50.3731	-1.2312	1.3452	1018.4309	0.0000
26	5	1.4	1.3345	48.5360	12.1249	2.1249	7.1239	7.1249	53.5360	-43.5360	-0.9642	1.3741	1029.8125	90.1168
27	3	1.4	1.3345	48.5360	10.1249	4.1249	7.1239	7.1249	51.5360	-45.5360	-1.0339	1.2779	1032.3321	54.1449
28	1	1.4	1.3345	48.5360	8.1249	6.1249	7.1239	7.1249	49.5360	-47.5360	-1.1088	1.1898	1033.6094	18.0599
29	6	1.4	1.3699	46.8870	6.0000	2.1234	8.1234	8.1239	52.8870	-40.8870	0.1051	1.3373	1044.5189	109.7834
30	4	1.4	1.3699	46.8870	12.1239	4.1239	8.1234	8.1239	50.8870	-42.8870	-0.9394	1.2443	1047.7211	73.2928
31	2	1.4	1.3699	46.8870	10.1239	6.1239	8.1234	8.1239	48.8870	-44.8870	-1.0074	1.1590	1049.6722	36.6764
32	0	1.4	1.3699	46.8870	8.1239	8.1228	8.1234	8.1249	46.8870	-46.8870	-1.0803	1.1870	1050.3263	0.0000
33	5	1.4	1.4047	45.3885	14.1234	4.1234	9.1231	9.1234	50.3885	-40.3885	-0.8582	1.2191	1063.9644	93.0943
34	3	1.4	1.4047	45.3885	12.1234	6.1234	9.1231	9.1234	48.3885	-42.3885	-0.9208	1.1358	1066.6034	55.9066
35	1	1.4	1.4047	45.3885	10.1234	8.1234	9.1231	9.1234	46.3885	-44.3885	-0.9874	1.0589	1067.9339	18.6441
36	6	1.4	1.4392	44.0139	6.0000	4.1229	10.1229	10.1231	50.0139	-38.0139	0.1051	1.2003	1081.0692	113.6250
37	4	1.4	1.4392	44.0139	14.1231	6.1231	10.1229	10.1231	48.0139	-40.0139	-0.8451	1.1185	1084.4063	75.8188
38	2	1.4	1.4392	44.0139	12.1231	8.1231	10.1229	10.1231	46.0139	-42.0139	-0.9068	1.0428	1086.4278	37.9302
39	0	1.4	1.4392	44.0139	10.1231	10.1228	10.1229	10.1234	44.0139	-44.0139	-0.9725	1.0729	1087.1053	0.0000
40	5	1.4	1.4734	42.7428	16.1229	6.1229	11.1228	11.1229	47.7428	-37.7428	-0.7779	1.1059	1103.0978	96.4895
41	3	1.4	1.4734	42.7428	14.1229	8.1229	11.1228	11.1229	45.7428	-39.7428	-0.8355	1.0311	1105.8209	57.9271
42	1	1.4	1.4734	42.7428	12.1229	10.1229	11.1228	11.1229	43.7428	-41.7428	-0.8966	0.9616	1107.1912	19.3144
43	6	1.4	1.5074	41.5598	6.0000	6.1228	12.1228	12.1228	47.5598	-35.5598	0.1051	1.0971	1122.7054	118.0011
44	4	1.4	1.5074	41.5598	16.1228	8.1228	12.1228	12.1228	45.5598	-37.5598	-0.7715	1.0230	1126.1391	78.7125
45	2	1.4	1.5074	41.5598	14.1228	10.1228	12.1228	12.1228	43.5598	-39.5598	-0.8288	0.9540	1128.2129	39.3691
46	0	1.4	1.5074	41.5598	12.1228	12.1227	12.1228	12.1229	41.5598	-41.5598	-0.8894	0.9848	1128.9064	0.0000
47	5	1.4	1.5413	40.4527	18.1228	8.1228	13.1228	13.1228	45.4527	-35.4527	-0.7135	1.0178	1147.4173	100.3701
48	3	1.4	1.5413	40.4527	16.1228	10.1228	13.1228	13.1228	43.4527	-37.4527	-0.7675	0.9492	1150.2041	60.2425
49	1	1.4	1.5413	40.4527	14.1228	12.1228	13.1228	13.1228	41.4527	-39.4527	-0.8245	0.8849	1151.6023	20.0841
50	5	1.4	1.5751	39.4117	18.1228	10.1228	14.1231	14.1228	44.4117	-34.4117	0.0875	0.9976	1169.1643	122.0657
51	4	1.4	1.5751	39.4123	18.1228	10.1228	14.1225	14.1228	43.4123	-35.4123	-0.7115	0.9811	1172.7404	82.3523
52	2	1.4	1.5751	39.4123	16.1228	12.1228	14.1225	14.1228	41.4123	-37.4123	-0.7655	0.9148	1174.8653	41.3654
53	0	1.4	1.5751	39.4123	14.1228	14.1222	14.1225	14.1228	39.4123	-39.4123	-0.8224	0.8520	1176.0248	0.0000
54	3	1.4	1.6089	38.4299	18.1225	12.1225	15.1225	15.1225	41.4299	-35.4299	-0.7112	0.8823	1199.6384	63.2220
55	1	1.4	1.6089	38.4299	16.1225	14.1225	15.1225	15.1225	39.4299	-37.4299	-0.7651	0.8220	1201.5283	20.9645
56	4	1.4	1.5751	39.4109	4.0014	10.1225	14.1239	14.1225	43.4109	-35.4109	0.0699	0.9138	1220.0988	125.6274
57	2	1.4	1.6427	37.4994	18.1225	14.1225	16.1225	16.1225	39.4994	-35.4994	-0.7661	0.8530	1226.7332	42.4646
58	0	1.4	1.6427	37.4994	16.1225	16.1225	16.1225	16.1225	37.4994	-37.4994	-0.7390	0.7944	1229.8968	0.0000
59	1	1.4	1.6766	36.6152	18.1225	16.1225	17.1225	17.1225	37.6152	-35.6152	-0.7406	0.8545	1255.0508	21.4934
60	3	1.4	1.6089	38.4300	-1.0001	16.1225	15.1224	15.1225	41.4300	-35.4300	0.0524	0.8250	1279.0245	128.7156
61	0	1.4	1.7107	35.7728	18.1225	18.1224	18.1224	18.1225	35.7728	-35.7728	-0.7169	0.7436	1285.9682	0.0000

62	2	1.4	1.6427	37.4995	-2.0000	18.1224	16.1224	16.1224	39.4995	-35.4995	0.0349	0.7971	1337.5051	130.7578
63	1	1.4	1.6766	36.6153	3.0000	14.1224	17.1224	17.1224	37.6153	-35.6153	0.0175	0.7705	1398.2308	131.8177
64	0	1.4	1.7106	35.7737	1.9990	16.1224	18.1214	18.1224	35.7737	-35.7737	0.0000	0.7205	1468.9180	131.8177



Graph 1- Graph *showing the points that we obtained with the calculation*



Graph 2 -*Plotting a curve with the outermost points forming the contour of the tunnel*

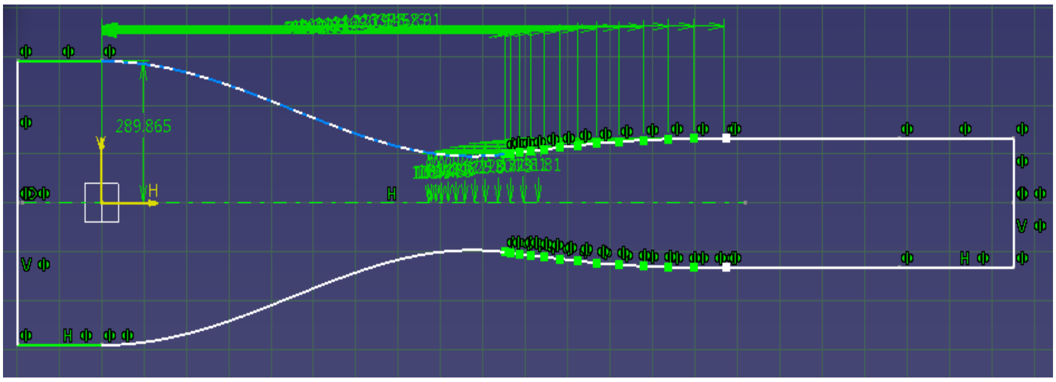


Figure 5.1 – 2D CATIA Model of Convergent –divergent Duct

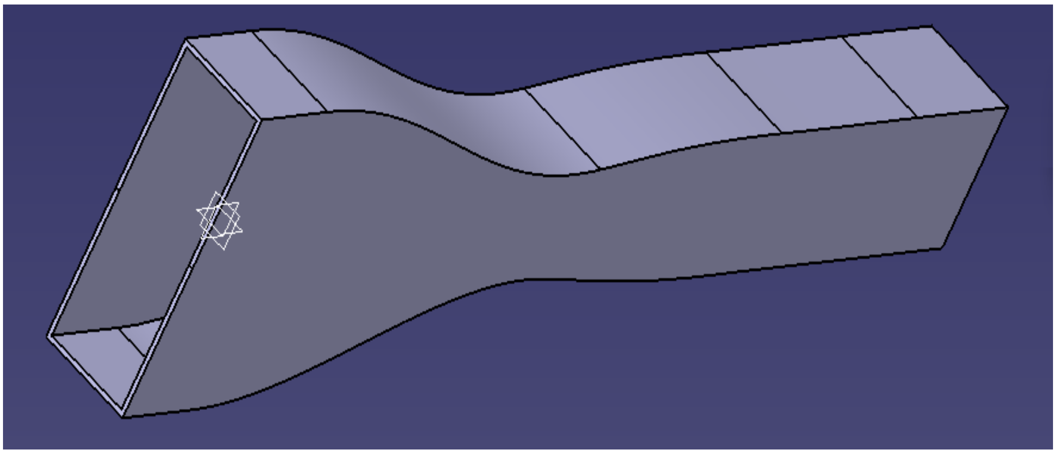


Figure 5.2 -3D CATIA Model of Designed Convergent Divergent Duct

7. Conclusion

The paper presents a good insight about the usefulness of Prandtl Mayer Equations and also shows a way to design the super-sonic wind tunnel which minimizes the loss of shocks and increases the efficiency of the wind tunnel. This makes is possible to construct a wind tunnel of small size which is almost impossible otherwise. The complex calculations have been simplified as far as possible and a clear description has been given for easy understanding.

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