1ª lista de exercícios 02

01) $P(A) = \dfrac{1}{3}$ e $P(B) = \dfrac{1}{2}$ $\quad\to P(A\cap B) = P(A)\cdot P(B)$

$= \dfrac{1}{3}\cdot\dfrac{1}{2} = \dfrac{1}{6}$

a) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\cup B) = \dfrac{1}{3} + \dfrac{1}{2} - \dfrac{1}{6} = \dfrac{3+2-1}{6} = \dfrac{4}{6} = \dfrac{2}{3}$

b) $P(A^c \cup B^c) = P(A\cap B)^c = 1 - P(A\cap B)$

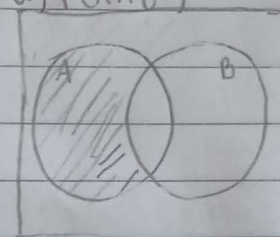$= 1 - \dfrac{1}{6} = \dfrac{5}{6}$

$P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$

$= \dfrac{2}{3} + \dfrac{1}{2} - \dfrac{1}{3}\cdot\dfrac{1}{2} = \dfrac{7}{6} - \dfrac{1}{3} = \dfrac{15}{18} = \dfrac{5}{6}$

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02) A e B independentes
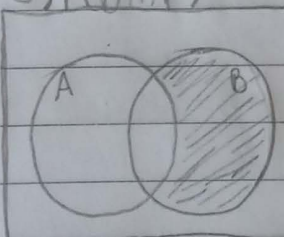
a) $P(A\cap B^c)$

$P(A\cap B^c) = P(A) - P(A\cap B)$

$P(A\cap B^c) = P(A) - \big[P(A)\cdot P(B)\big]$

$P(A\cap B^c) = P(A)\big[1 - P(B)\big]$

$P(A\cap B^c) = P(A)\cdot P(B^c)$

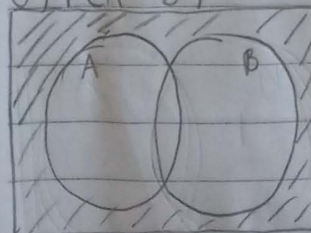**b)** $P(B\cap A^c)$

$P(B\cap A^c) = P(B) - P(A\cap B)$

$P(B\cap A^c) = P(B) - \big[P(A)\cdot P(B)\big]$

$P(B\cap A^c) = P(B)\big[1 - P(A)\big]$

$P(B\cap A^c) = P(B)\cdot P(A)$

c) $P(A^c \cap B^c)$

$P(A^c \cap B^c) = P(A^c) - P(B\cap A^c)$

$P(A^c \cap B^c) = P(A^c) - \big[P(B)\cdot P(A^c)\big]$

$P(A^c \cap B^c) = P(A^c)\big[1 - P(B)\big]$

$P(A^c \cap B^c) = P(A^c)\cdot P(B^c)$

03) $S = \{1,2,3,4,5,6\} \rightarrow P = \{2,3,5\}$; $I = \{1,3,5\}$

$P(P/I) = \dfrac{P(P \cap I)}{P(I)} = \dfrac{2/6}{3/6} = \dfrac{2}{3}$

04) $S' = \{(1,1); (1,3); (1,5); (3,1); (3,3); (3,5); (5,1); (5,3); (5,5)\}$

$P(8/2I) = \dfrac{2}{9}$

05) N° diferentes: $6 \cdot 5 = 30$

Soma par:

$\underbrace{3 \cdot 2}_{(1,3,5)} + \underbrace{3 \cdot 2}_{(2,4,6)} = 12$

$P(\text{SOMA PAR}/N = DIF) = \dfrac{12}{30} = \dfrac{2}{5}$

06) a) Sim
b) Sim

07) $N(S) = 2^3 = 8$

$\underline{R \, K \, K} + \underline{K \, C \, K} + \underline{K \, K \, C} = 3$

$P(C) = \dfrac{3}{8}$

08) a) SOMA IMPAR:

$\underbrace{4}_{(P)} \cdot \underbrace{5}_{(I)} + \underbrace{5}_{(I)} \cdot \underbrace{4}_{(P)} = 40$

· OCORRER 2, SOMA IMPAR:

$2 \cdot 5 + 5 \cdot 2 = 10$

$P(2/\text{IMPAR}) = \dfrac{10}{40} = \dfrac{1}{4}$

b) OCORRER 2:

$2 \cdot 8 + 8 \cdot 2 = 16$

SOMA IMPAR, OCORRENDO 2:

$2 \cdot 5 + 5 \cdot 2 = 10$

$P(\text{IMPAR}/2) = \dfrac{10}{16} = \dfrac{5}{4}$

09)

$$P(A/V) = \frac{P(A \cap V)}{P(V)} = \frac{P(A) \cdot P(V)}{P(V)}$$

$$P(A/V) = \frac{4/9 \cdot 3/10}{3/10} = \frac{4}{9}$$

10)

	VIVER +10	Ñ VIVER +10
H	$\frac{1}{4}$	$\frac{3}{4}$
M	$\frac{1}{5}$	$\frac{4}{5}$

a) $P(Hv) = \frac{1}{4} \cdot \frac{4}{5} = \frac{1}{5}$

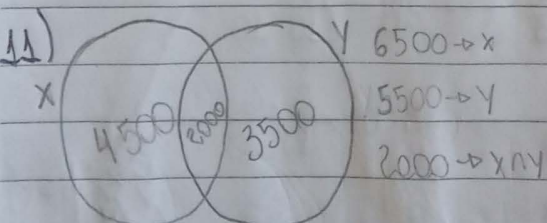b) $P(Mv) = \frac{1}{5} \cdot \frac{3}{4} = \frac{3}{20}$

c) $P(Hv \cap Mv) = \frac{1}{4} \cdot \frac{1}{5} = \frac{1}{20}$

d) $P(v) = 1 - P(M) = 1 - \frac{3}{4} \cdot \frac{4}{5}$

$$P(v) = 1 - \frac{3}{5} = \frac{2}{5}$$

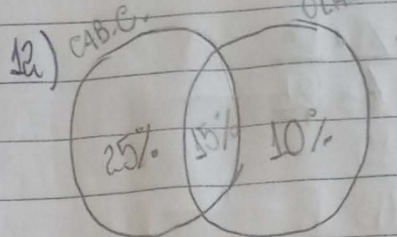e) $P(Hm \cap Mm) = \frac{3}{4} \cdot \frac{4}{5} = \frac{3}{5}$

11)

Y $6500 \to x$

X $5500 \to y$

$4500 \quad 2000 \quad 3500$ $2000 \to x \cap y$

$$P(y/x) = \frac{2000}{6500} = \frac{4}{13}$$

12)

CAB.C.   OLHOS.C.

$25\% \quad 15\% \quad 10\%$

$40\% \to CAB.C.$

$25\% \to OLHOS C.$

$15\% \to CAB.C. \cap OLHO.C$

$20\% \to$ Ñ POSSUI

a) $P(OLHOS C / CAB.C.) = \frac{15\%}{40\%} = \frac{3}{8}$

b) $P(Ñ CAB.C / OLHOS C.) = \frac{10\%}{25\%} = \frac{2}{5}$

c) $P(Ñ CABC \cap Ñ OLHOS C.) = 100\% - 25\% - 15\% - 10\%$

$$= 20\%$$

13)

MASTERCARD   VISA

$450 \quad 200 \quad 350$
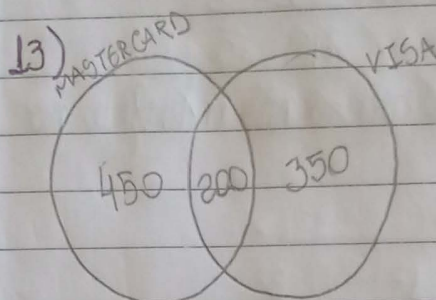
$650 \to MASTERCARD$

$550 \to VISA$

$200 \to AMBAS$

$$P(MC/V) = \frac{200}{550} = \frac{4}{11}$$

14)

	A	B	AB	O
∂	0,25	0,30	0,20	0,25
$\emptyset$	0,75	0,70	0,80	0,75

a) $P(0) = 0,25$

b) $P(\bar{0}) = 0,75$

c) $P(A \cap B) = P(A) \cdot P(B) = 0,25 \cdot 0,3 = 7,5\%$

d) $P(\bar{B} \cup \overline{AB}) = P(\bar{B}) + P(\overline{AB}) - P(\bar{B} \cap \overline{AB})$

$$= 0,7 + 0,8 - (0,25 + 0,25)$$

$$= 0,7 + 0,8 - 0,5$$

$$P(\bar{B} \cup \overline{AB}) = 1,0$$

15) a) $P(V \cap B) = \dfrac{5}{7} \cdot \dfrac{\cancel{6}}{63} = \dfrac{15}{21}$

b) $P(V \cap B) = \dfrac{5}{7} \cdot \dfrac{2}{7} = \dfrac{10}{49}$

16) $P(\tilde{N} \text{ TROCAR}) = \dfrac{100 + 50 + 150}{1050} = \dfrac{300}{1050} = \dfrac{2}{7}$

17) a) $P(\text{TOT. CURADO}/A) = \dfrac{24}{60} = \dfrac{2}{5}$

b) $P(A / \text{TOT. CURADO}) = \dfrac{24}{40} = \dfrac{3}{5}$

c) Não, pois um evento não interfere no outro.

18) a) ATUAL $\quad 1= \quad\quad 2:$
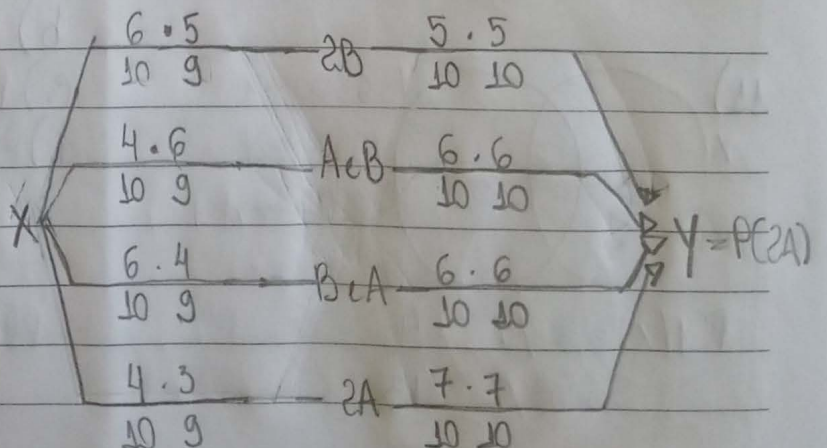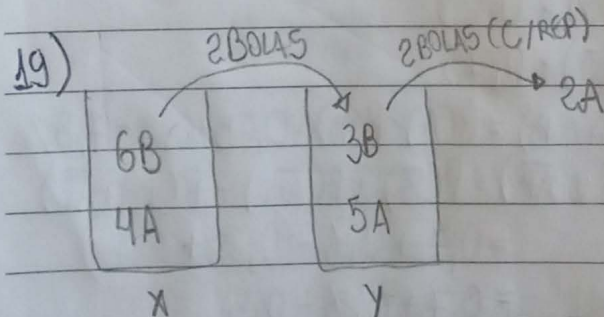
$B \xrightarrow{0,3} A \xrightarrow{0,2} C = 0,06$

$B \xrightarrow{0,1} D \xrightarrow{0,2} C = 0,02$

$B \xrightarrow{0,1} E \xrightarrow{0,1} C = 0,01$

$\underline{ 0,09}$

b) $P(E_2 / B \to C) = \dfrac{0,01}{0,09} = \dfrac{1}{9}$

19)

6B	3B
4A	5A
x	y

2 BOLAS $\quad$ 2 BOLAS (C/REP) $\to$ 2A

$\dfrac{6 \cdot 5}{10 \quad 9} \quad\quad 2B \quad \dfrac{5 \cdot 5}{10 \quad 10}$

$\dfrac{4 \cdot 6}{10 \quad 9} \quad\quad A e B \quad \dfrac{6 \cdot 6}{10 \quad 10}$

$\dfrac{6 \cdot 4}{10 \quad 9} \quad\quad B e A \quad \dfrac{6 \cdot 6}{10 \quad 10}$

$\dfrac{4 \cdot 3}{10 \quad 9} \quad\quad 2A \quad \dfrac{7 \cdot 7}{10 \quad 10}$

$X \quad\quad\quad\quad Y = P(2A)$

$P(2A) = \cancel{8} \cdot \cancel{8} \cdot \cancel{8} \cdot \cancel{8} + \cancel{4} \cdot \cancel{8} \cdot \cancel{8} \cdot \cancel{8} + \cancel{8} \cdot \cancel{4} \cdot \cancel{8} \cdot 6 + \cancel{4} \cdot \cancel{8} \cdot 7 \cdot 7$

$\cancel{10} \; \cancel{8} \; \cancel{10} \; \cancel{10} \quad \cancel{8} \; \cancel{8} \; \cancel{10} \; \cancel{10} \quad \cancel{10} \; \cancel{8} \; \cancel{10} \; \cancel{10} \quad \cancel{10} \; \cancel{8} \; \cancel{10} \; 10$

$= \dfrac{1}{12} + \dfrac{12}{125} + \dfrac{12}{125} + \dfrac{49}{750} = \dfrac{511}{1500} = 34,06\%$

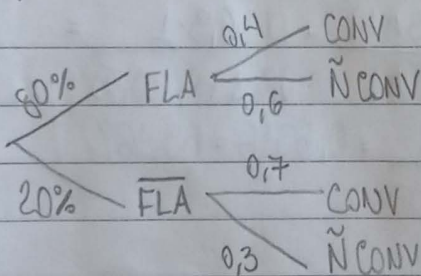$P(2A \times / 2A) = \dfrac{P(2A \times \cap 2A)}{P(2A)} = \dfrac{0,0653}{0,3406} = 0,1918 = 19,18\%$

20)
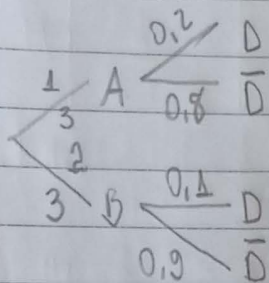
$P(D) = P(D/A) + P(D/B)$

$= 0,2 \cdot \dfrac{1}{3} + 0,1 \cdot \dfrac{2}{3} = \dfrac{0,2}{3} + \dfrac{0,2}{3} = \dfrac{0,4}{3} = 0,13$

22)

a) $P(FLA \cap CONV) = 0,8 \cdot 0,4 = 0,32 = 32\%$

b) $P(CONV) = 0,8 \cdot 0,4 + 0,2 \cdot 0,7 = 0,32 + 0,14 = 0,46 = 46\%$

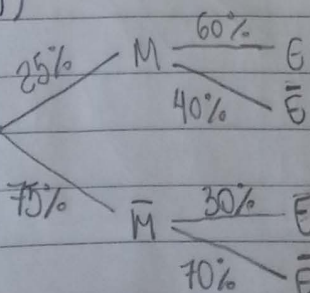c) $P(\tilde{N}CONV) = 1 - P(CONV) = 1 - 0,46 = 0,54 = 54\%$

21) EFICIÊNCIA = 99%

FALHA = 1%

$P(D) = 1 - P(2F)$

$P(D) = 1 - 0,1 \cdot 0,1 = 1 - 0,01$
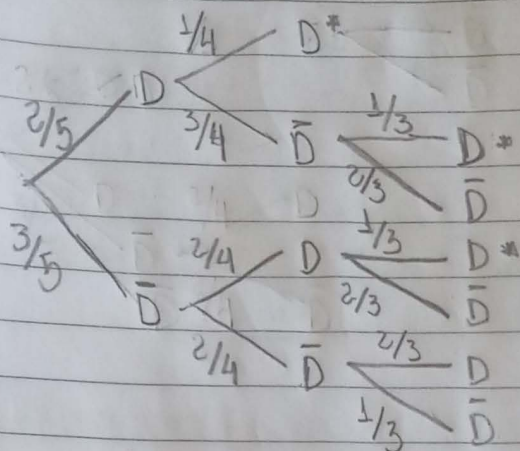
$P(D) = 99,99\%$

23)

a) $P(E) = 0,25 \cdot 0,6 + 0,3 \cdot 0,75 = 0,35 = 35\%$

b) $P(M/E) = \dfrac{P(M \cap E)}{P(E)} = \dfrac{0,25 \cdot 0,6}{0,35} = \dfrac{0,125}{0,35} = \dfrac{5}{14} = 35,71\%$
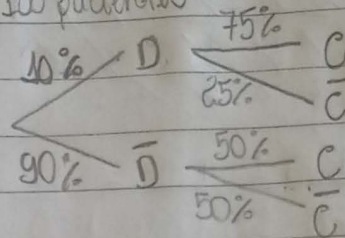
24)
     1ª T   2ª T   3ª T

Tree diagram:
- $2/5$ → D
  - $1/4$ → D^*
  - $3/4$ → $\bar{D}$
    - $1/3$ → D^*
    - $2/3$ → $\bar{D}$
- $3/5$ → $\bar{D}$
  - $2/4$ → D
    - $1/3$ → D^*
    - $2/3$ → $\bar{D}$
  - $2/4$ → $\bar{D}$
    - $2/3$ → D
    - $1/3$ → $\bar{D}$

a) $P_{(2D)} = \dfrac{2}{5} \cdot \dfrac{1}{4_2} = \dfrac{1}{10}$

b) $P_{(2D)} = \dfrac{2}{5} \cdot \dfrac{3}{4} \cdot \dfrac{1}{3} + \dfrac{3}{5} \cdot \dfrac{2}{4} \cdot \dfrac{1}{3} = \dfrac{1}{10} + \dfrac{1}{10} = \dfrac{2}{10} = \dfrac{1}{5}$

25) $P_{(1=\bar{D}/3T)} = \dfrac{P_{(1=\bar{D} \wedge 3T)}}{P_{(3T)}} =$

$= \dfrac{\dfrac{3}{5} \cdot \dfrac{2}{4_2} \cdot \dfrac{1}{3}}{\dfrac{2}{10}} = \dfrac{\dfrac{1}{10}}{\dfrac{2}{10}} = \dfrac{1}{2}$

26)

Tree diagram:
- $1/2$ → A
  - $4/9$ → PAR
  - $5/9$ → ÍMPAR
- $1/2$ → B
  - $2/5$ → PAR
  - $3/5$ → ÍMPAR

$P_{(A/PAR)} = \dfrac{P_{(A \cap PAR)}}{P_{(PAR)}} = \dfrac{\dfrac{1}{2} \cdot \dfrac{4}{9}}{\dfrac{1}{2} \cdot \dfrac{4}{9} + \dfrac{1}{2} \cdot \dfrac{2}{5}} =$

$= \dfrac{\dfrac{2}{9}}{\dfrac{2}{9} + \dfrac{1}{5}} = \dfrac{\dfrac{2}{9}}{\dfrac{19}{45}} = \dfrac{2 \cdot \overset{5}{45}}{9 \cdot 19} = \dfrac{10}{19}$

27) 100 pacientes

Tree diagram:
- 10% → D
  - 75% → C
  - 25% → $\bar{C}$
- 90% → $\bar{D}$
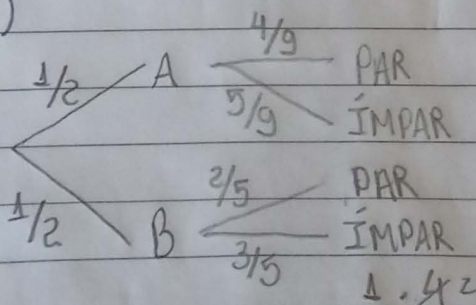 - 50% → C
  - 50% → $\bar{C}$

$P_{(D/C)} = \dfrac{P_{(D \cap C)}}{P_{(C)}} = \dfrac{0,1 \cdot 0,75}{0,9 \cdot 0,5 + 0,1 \cdot 0,75} =$

$= \dfrac{0,075}{0,525} = 0,1429 = 14,29\%$

28)

Tree diagram:
- 30% → S
  - 100% → A
  - 0% → $\bar{A}$
- 70% → $\bar{S}$
  - 25% → A
  - 75% → $\bar{A}$
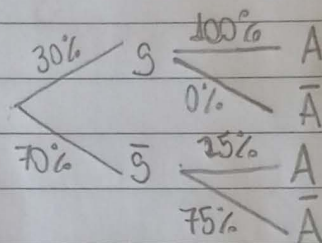
$P_{(S/A)} = \dfrac{P_{(S \cap A)}}{P_{(A)}} = \dfrac{0,3 \cdot 1}{0,3 \cdot 1 + 0,7 \cdot 0,25} =$

$= \dfrac{0,3}{0,4750} = 0,6316 = 63,16\%$