Lista de exercícios 01

01) $\underline{5} \cdot \underline{5} \cdot \underline{5} \cdot \underline{5} \cdot \underline{5} \cdot \underline{5} = 5^6 = 15.625$

02) a) $\underline{100} \cdot \underline{99} \cdot \underline{98} \cdot \underline{97} \cdot \underline{96} = 9.034.502.400$ cartelas

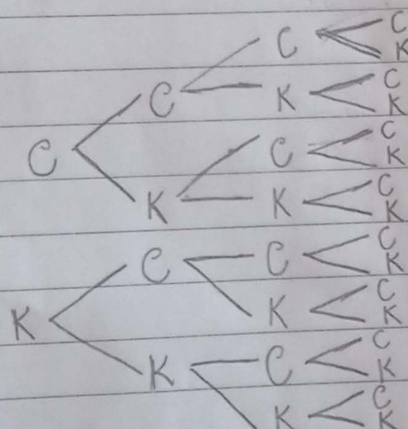
b) $P(A) = \frac{5}{100} = \frac{1}{20}$

03)

$RJ \xrightarrow{10} SP \xrightarrow{5} BSB \rightarrow 10 \cdot 5 = 50$

04) COMB TOTAIS $\rightarrow 2^4 = 16$

* árvore de possibilidades



a) $P(0C) = P(4K) = \frac{1}{16}$

b) $P(1C) = \frac{4}{16} = \frac{1}{4}$

c) $P(2C) = \frac{6}{16} = \frac{3}{8}$

d) $P(3C) = \frac{4}{16} = \frac{1}{4}$

e) $P(4C) = \frac{1}{16}$

f) $P(C) = P(1C) + P(2C) + P(3C) + P(4C)$

$= \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{15}{16}$

05) $\underline{15} \cdot \underline{14} = 210$

06) $\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 10^5 - 1$ registros

07)

p/ uma central.

$\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 10000$ telefones

· nº de centrais

$\underline{9} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 9000$ centrais

08) $\underline{26} \cdot \underline{26} \cdot \underline{26} - \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 175.760.000$ placas

09) $\underline{5} \cdot \underline{3} \cdot \underline{6} \cdot \underline{1} = 90$ maneiras

10) $\underline{20} \cdot \underline{20} \cdot \underline{20} = 8000$

11) $\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 10000 - 10 = 9990$

12) · 1ª marca: $\underline{3} \cdot \underline{5} = 15$
· 2ª marca: $\underline{5} \cdot \underline{8} = 40$
· $\underline{15} + \underline{40} = 55$ possibilidades

13) · p/ 2 resultados
$\underline{13} \cdot \underline{3} = 39$ TOTAL $= 39 \cdot 3^{12}$ possibilidades
· p/ 3 resultados
3^{12} possibilidades

- Fatorial e Permutação Simples

01) a) $3! = 3 \cdot 2 \cdot 1 = 6$
b) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
c) $\frac{10!}{8!} = \frac{10 \cdot 9 \cdot \cancel{8!}}{\cancel{8!}} = 90$
d) $\frac{12!}{10!\,2!} = \frac{\overset{6}{\cancel{12}} \cdot 11 \cdot \cancel{10!}}{\cancel{10!} \cdot \cancel{2} \cdot 1} = 66$

02) $12! = 479001600$
$13! = 13 \cdot 12! = 6227020800$

03) $P(n) = A(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$

· Em uma competição, haviam 6 competidores. De quantas maneiras pode ser a classificação? $P(6) = 6! = 720$

04) $P(6) = 6! = 720$

05) $P(5) = 5! = 120$

06) $P(8) = 8! = 40.320$

07) $P(6) = 6! = 720$

08) $P(5) = 5! = 120$

09) a) $P(6) = 6! = 720$

b) $P(5) = 5! = 120$

c)

1º 2º 3º 4º 5º 6º

$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

7º 8º 9º 10º 11º 12º

$\underline{5} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

$T - 6! \cdot 5! \cdot 5$

10) $P(3) = 3! = 6$

- Arranjos simples

01) a) $A(5,3) = \frac{5!}{2!} = 5 \cdot 4 \cdot 3 = 60$

b) $A(2,1) = \frac{2!}{1!} = 2$

c) $A(5,5) = \frac{5!}{0!} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

d) $A(20,18) = \frac{20!}{2!} = 20 \cdot 19 \cong 1,22 \cdot 10^{18}$

02) $A(n,p) = \frac{n!}{(n-p)!}$

No arranjo, determinamos o número de maneiras de selecionar p objetos em uma certa ordem dentro de um conjunto de n objetos distintos.

Na permutação, determinamos o número de agrupamentos que podemos formar com certo número de elementos.

03) $A(10,4) = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$ maneiras

04) $A(10,2) = 10 \cdot 9 = 90$ maneiras

05) a) $A(8,2) = 8 \cdot 7 = 56$

b) $56 \cdot 56 = 3136$

06) $A(10,5) = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30240$

07) $A(15,10) = \frac{15!}{5!}$

08) $A \rightarrow A(6,2) = 6 \cdot 5 = 30$

$B \rightarrow A(4,2) = 4 \cdot 3 = 12$

multi-$A \rightarrow A(10,2) = 10 \cdot 9 = 90$ rotas

Permutações com elementos repetidos e permutações circulares

01) $P_9^{(2,2)} = \dfrac{9!}{2!2!} = \dfrac{9\cdot8\cdot7\cdot6\cdot5\cdot\cancel{4}^{2}\cdot3}{\cancel{2}\cdot1} = 90720$

02) $A(10,3) = 10\cdot9\cdot8 = 720$

03) a) $A(6,5) = 6\cdot5\cdot4\cdot3\cdot2 = 720$

b) $A(6,3)\cdot A(8,2) = 6\cdot5\cdot4\cdot8\cdot7 = 6720$

c) $A(8,2)\cdot A(7,3) = 8\cdot7\cdot7\cdot6\cdot5 = 11.760$

d) $A(21,5) = 21\cdot20\cdot19\cdot18\cdot17 = 2.441.880$

04) a) $\underline{2}\cdot\underline{4!} = 2\cdot24 = 48$

b) $6! = 720$

c) $\underline{2}\cdot A(4,2)\cdot\underline{1}\cdot A(2,2) = 2\cdot4\cdot3\cdot1\cdot2\cdot1 = 48$

05) $\underbrace{4!\cdot4!\cdot4!\cdot4!\cdot4!}_{\text{permutando as cidades em cada país}}\cdot\underbrace{5!}_{\text{permutando os países}} = 955.514.880$

06) a) 50 - 65

$P_{11}^{(5,6)} = \dfrac{11!}{5!6!} = \dfrac{11\cdot\cancel{10}\cdot\cancel{9}^{3}\cdot\cancel{8}^{2}\cdot7}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1} = 462$

b) $C \xrightarrow{30-35} BJ \xrightarrow{20-35} T$

$P_6^{(3,3)}\cdot P_5^{(2,3)}$

$\dfrac{6!}{3!3!}\cdot\dfrac{5!}{2!\cdot3!} = \dfrac{\cancel{6}\cdot5\cdot4\cdot5\cdot\cancel{4}^{2}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{2}} = 200$

07) $P_{C(10)} = \dfrac{10!}{10} = 9! = 362.880$

08) ?

1) $\dfrac{5}{H/M}\cdot\dfrac{5}{M/H}\cdot\dfrac{4}{H/M}\cdot\dfrac{4}{M/H}\cdot\dfrac{3}{H/M}\cdot\dfrac{3}{M/H}\cdot\dfrac{2}{H/M}\cdot\dfrac{2}{M/H}\cdot\dfrac{1}{H/M}\cdot\dfrac{1}{M/H} =$

$= 2\cdot5!\cdot5! = 28800 = P_{L(n)}$

2) $P_{C(n)} = \dfrac{P_{L(n)}}{n} = \dfrac{28800}{10} = 2880$

09) 1) $P_{C(5)} = \dfrac{5!}{5} = 24$

2) $P_{C(4)} = \dfrac{4!}{4} = 6\cdot2 = 12$

3) $24 - 12 = 12$ possibilidades

10) $P_{C(6)} = \dfrac{6!}{6} = 5! = 120$

- Combinação com ou sem repetições

01) $C(7,2) = \frac{7!}{5!2!} = \frac{7\cdot \cancel{6}^{3}}{\cancel{2!}} = 21$

02) $C(14,5) = \frac{14\cdot 13\cdot \cancel{12}\cdot 11\cdot \cancel{10}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 2002$

03) $C(5,2)\cdot C(6,2) = \frac{5\cdot \cancel{4}}{\cancel{2}}\cdot\frac{6\cdot 5}{2} = 150$

04) $2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2 = 2^6 = 64$

$P(^{3,3}_{6}) = \frac{6!}{3!3!} = \frac{\cancel{6}\cdot 5\cdot 4}{\cancel{3}\cancel{2}} = 20$

05) a) $C(n,n) = \frac{\cancel{n!}}{\cancel{n!}(n-n)!} = \frac{1}{0!} = \frac{1}{1} = 1$

$C(n,0) = \frac{\cancel{n!}}{\cancel{n!}0!} = \frac{1}{1} = 1$

b) $C(n,r) = C(n,n-r)$

$C(n,n-r) = \frac{n!}{(n-r)![n-(n-r)]!} = \frac{n!}{(n-r)!(\cancel{n}-\cancel{n}+r)!} =$

$= \frac{n!}{(n-r)!r!} = C(n,r)$

06) $C(6,4) = \frac{\cancel{6}^{3}\cdot 5\cdot \cancel{4}\cdot \cancel{3}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 15$

07) $A(5,3) = 5\cdot 4\cdot 3 = 60$

08) $C(12,2) = \frac{\cancel{12}^{6}\cdot 11}{\cancel{2}} = 66$

09)

a) c c c c c

$P(^{5}_{5}) = C(5,5) = 1$

b) c c k k k

$P(^{2,3}_{5}) = \frac{5!}{2!3!} = \frac{5\cdot \cancel{4}^{2}}{\cancel{2}} = 10$

c) c k k k k

$P(^{4}_{5}) = \frac{5!}{4!} = 5$

10) 7 sabores

4 sorvetes

$C_R(7,4) = P(^{6,4}_{10}) = \frac{10!}{6!4!} = \frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}\cdot 7}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 210$

11)

1. $C(12,4) = \frac{\cancel{12}\cdot 11\cdot \cancel{10}^{5}\cdot 9}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 495$

2. 2R: $\frac{\cancel{2}\cdot 1}{\cancel{2}}$ | 2B - $C(10,2)$: $\frac{\cancel{10}^{5}\cdot 9}{\cancel{2}} = 45$ $P(2R) = \frac{45}{495}$

3. 1R: 2 | 3R - $C(10,3)$: $\frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}}{\cancel{3}\cdot \cancel{2}\cdot 1} = 240$ $P(3R) = \frac{240}{495}$

4. $C(10,4) = \frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}\cdot 7}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 210$ $P(4B) = \frac{210}{495}$

ou

$495 - 45 - 240 = 210$

12) $C(10,4) = \dfrac{10\cdot\overset{3}{\cancel{9}}\cdot\cancel{8}\cdot 7}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 210$

a) $C(6,4) = \dfrac{\overset{3}{\cancel{6}}\cdot 5\cdot\cancel{4}\cdot\cancel{3}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 15 \quad P(4B) = \dfrac{15}{210}$

b) $C(4,2)\cdot C(6,2) = \dfrac{\cancel{4}\cdot 3}{\cancel{2}}\cdot\dfrac{6\cdot 5}{\cancel{2}} = 90 \quad P(2B\ e\ 2A) = \dfrac{90}{210}$

c) $C(6,3)\cdot C(4,1) = \dfrac{\cancel{6}\cdot 5\cdot 4}{\cancel{3}\cdot\cancel{2}}\cdot 4 = 80 \quad P(3B\ e\ 1A) = \dfrac{80}{210}$

d) $C(4,4) = 1 \quad P(4A) = \dfrac{1}{210}$

13) a) $C(10,6) = \dfrac{10!}{6!\,4!} = \dfrac{10\cdot\overset{3}{\cancel{9}}\cdot\cancel{8}\cdot 7}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 210$

b) $C(5,3)\cdot C(5,3) = \dfrac{5\cdot\cancel{4}\cdot\cancel{3}}{\cancel{3}\cdot\cancel{2}\cdot 1}\cdot\dfrac{5\cdot 4\cdot\cancel{3}}{\cancel{3}\cdot\cancel{2}\cdot 1} = 100$