Limites Trigonométricos Resolvidos

Sete páginas e 34 limites resolvidos

Usar o limite fundamental e alguns artifícios : $\lim\limits_{x\to 0}\frac{senx}{x}=1$

1. $\lim\limits_{x\to 0}\frac{x}{\operatorname{sen} x}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{x}{\operatorname{sen} x}=\frac{0}{0}$, é uma indeterminação.

$\lim\limits_{x\to 0}\frac{x}{\operatorname{sen} x}=\lim\limits_{x\to 0}\frac{1}{\frac{\operatorname{sen} x}{x}}=\frac{1}{\lim\limits_{x\to 0}\frac{\operatorname{sen} x}{x}}=1$ logo $\lim\limits_{x\to 0}\frac{x}{\operatorname{sen} x}=1$

2. $\lim\limits_{x\to 0}\frac{\operatorname{sen} 4x}{x}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{\operatorname{sen} 4x}{x}=\frac{0}{0}\ \rightarrow\ \lim\limits_{x\to 0}4.\frac{\operatorname{sen} 4x}{4x}=4.\lim\limits_{y\to 0}\frac{\operatorname{sen} y}{y}=4.1=4$ logo

$\lim\limits_{x\to 0}\frac{\operatorname{sen} 4x}{x}=4$

3. $\lim\limits_{x\to 0}\frac{\operatorname{sen} 5x}{2x}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{5}{2}.\frac{\operatorname{sen} 5x}{5x}=\lim\limits_{y\to 0}\frac{5}{2}.\frac{\operatorname{sen} y}{y}=\frac{5}{2}$ logo $\lim\limits_{x\to 0}\frac{\operatorname{sen} 5x}{2x}=\frac{5}{2}$

4. $\lim\limits_{x\to 0}\frac{\operatorname{sen} mx}{nx}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{\operatorname{sen} mx}{nx}=\lim\limits_{x\to 0}\frac{m}{n}.\frac{\operatorname{sen} mx}{mx}=\frac{m}{n}.\lim\limits_{y\to 0}\frac{\operatorname{sen} y}{y}=\frac{m}{n}.1=\frac{m}{n}$ logo $\lim\limits_{x\to 0}\frac{\operatorname{sen} mx}{nx}=\frac{m}{n}$

5. $\lim\limits_{x\to 0}\frac{\operatorname{sen} 3x}{\operatorname{sen} 2x}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{\operatorname{sen} 3x}{\operatorname{sen} 2x}=\lim\limits_{x\to 0}\frac{\frac{\operatorname{sen} 3x}{x}}{\frac{\operatorname{sen} 2x}{x}}=\lim\limits_{x\to 0}\frac{3.\frac{\operatorname{sen} 3x}{3x}}{2.\frac{\operatorname{sen} 2x}{2x}}=\frac{3}{2}.\frac{\lim\limits_{x\to 0}\frac{\operatorname{sen} 3x}{3x}}{\lim\limits_{x\to 0}\frac{\operatorname{sen} 2x}{2x}}=\frac{3}{2}.\frac{\lim\limits_{y\to 0}\frac{\operatorname{sen} y}{y}}{\lim\limits_{t\to 0}\frac{\operatorname{sen} t}{t}}=\frac{3}{2}.1=$

$\frac{3}{2}$ logo $\lim\limits_{x\to 0}\frac{\operatorname{sen} 3x}{\operatorname{sen} 2x}=\frac{3}{2}$

6. $\lim\limits_{x\to 0}\frac{senmx}{sennx}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{\operatorname{sen} mx}{\operatorname{sen} nx}=\lim\limits_{x\to 0}\frac{\frac{\operatorname{sen} mx}{x}}{\frac{\operatorname{sen} nx}{x}}=\lim\limits_{x\to 0}\frac{m.\frac{\operatorname{sen} mx}{mx}}{n.\frac{\operatorname{sen} nx}{nx}}=\lim\limits_{x\to 0}\frac{m}{n}.\frac{\frac{\operatorname{sen} mx}{mx}}{\frac{\operatorname{sen} nx}{nx}}=\frac{m}{n}$ Logo

$\lim\limits_{x\to 0}\frac{senmx}{sennx}=\frac{m}{n}$

7. $\lim\limits_{x\to 0}\frac{tgx}{x}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{tgx}{x}=\frac{0}{0}\ \rightarrow \lim\limits_{x\to 0}\frac{tgx}{x}=\lim\limits_{x\to 0}\frac{\frac{\operatorname{sen} x}{\cos x}}{x}=\lim\limits_{x\to 0}\frac{\operatorname{sen} x}{\cos x}.\frac{1}{x}=$

$\lim\limits_{x\to 0}\frac{\operatorname{sen} x}{x}.\frac{1}{\cos x}=\lim\limits_{x\to 0}\frac{\operatorname{sen} x}{x}.\lim\limits_{x\to 0}\frac{1}{\cos x}=1$ Logo $\lim\limits_{x\to 0}\frac{tgx}{x}=1$

8. $\lim\limits_{a\to 1}\frac{tg(a^2-1)}{a^2-1}=?\ \rightarrow\ \lim\limits_{a\to 1}\frac{tg(a^2-1)}{a^2-1}=\frac{0}{0}$ $\rightarrow$Fazendo $t=a^2-1,\begin{cases}x\to 1\\ t\to 0\end{cases}$ $\rightarrow$ $\lim\limits_{t\to 0}\frac{tg(t)}{t}=1$

logo $\lim\limits_{a\to 1}\frac{tg(a^2-1)}{a^2-1}=1$

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9. $\lim\limits_{x\to 0}\frac{x-\operatorname{sen} 3x}{x+\operatorname{sen} 2x}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{x-\operatorname{sen} 3x}{x+\operatorname{sen} 2x}=\frac{0}{0}\ \rightarrow\ f(x)=\frac{x-\operatorname{sen} 3x}{x+\operatorname{sen} 2x}=\frac{x.\left(1-\frac{\operatorname{sen} 3x}{x}\right)}{x.\left(1+\frac{\operatorname{sen} 5x}{x}\right)}=$

$\frac{x.\left(1-3.\frac{\operatorname{sen} 3x}{3.x}\right)}{x.\left(1+5.\frac{\operatorname{sen} 5x}{5.x}\right)}=\frac{1-3.\frac{\operatorname{sen} 3x}{3.x}}{1+5.\frac{\operatorname{sen} 5x}{5.x}}\ \rightarrow\ \lim\limits_{x\to 0}\frac{1-3.\frac{\operatorname{sen} 3x}{3.x}}{1+5.\frac{\operatorname{sen} 5x}{5.x}}=\frac{1-3}{1+5}=\frac{-2}{6}=-\frac{1}{3}$ logo

$\lim\limits_{x\to 0}\frac{x-\operatorname{sen} 3x}{x+\operatorname{sen} 2x}=-\frac{1}{3}$

10. $\lim\limits_{x\to 0}\frac{tgx-\operatorname{sen} x}{x^3}=?\ \rightarrow\ \lim\limits_{x\to 0}\frac{tgx-\operatorname{sen} x}{x^3}=\lim\limits_{x\to 0}\frac{\operatorname{sen} x}{x}.\frac{1}{\cos x}.\frac{\operatorname{sen}^2 x}{x^2}.\frac{1}{1+\cos x}=\frac{1}{2}$

$f(x)=\frac{tgx-\operatorname{sen} x}{x^3}=\frac{\frac{\operatorname{sen} x}{\cos x}-\operatorname{sen} x}{x^3}=\frac{\frac{\operatorname{sen} x-\operatorname{sen} x.\cos x}{\cos x}}{x^3}=\frac{\operatorname{sen} x.(1-\cos x)}{x^3.\cos x}=\frac{\operatorname{sen} x}{x}.\frac{1}{x^2}.\frac{1-\cos x}{\cos x}=$

$\frac{\operatorname{sen} x}{x}.\frac{1}{x^2}.\frac{1-\cos x}{\cos x}.\frac{1+\cos x}{1+\cos x}=\frac{\operatorname{sen} x}{x}.\frac{1}{\cos x}.\frac{1-\cos^2 x}{x^2}.\frac{1}{1+\cos x}=\frac{\operatorname{sen} x}{x}.\frac{1}{\cos x}.\frac{\operatorname{sen}^2 x}{x^2}.\frac{1}{1+\cos x}$

Logo $\lim\limits_{x\to 0}\frac{tgx-\operatorname{sen} x}{x^3}=\frac{1}{2}$

11. $\lim\limits_{x\to 0}\frac{\sqrt{1+tgx}-\sqrt{1+\operatorname{sen} x}}{x^3}=?\rightarrow\ \lim\limits_{x\to 0}\frac{tgx-\operatorname{sen} x}{x^3}.\frac{1}{\sqrt{1+tgx}+\sqrt{1+\operatorname{sen} x}}=$

$\lim\limits_{x\to 0}\frac{\operatorname{sen} x}{x}.\frac{1}{\cos x}.\frac{\operatorname{sen}^2 x}{x^2}.\frac{1}{1+\cos x}.\frac{1}{\sqrt{1+tgx}+\sqrt{1+\operatorname{sen} x}}=1.\frac{1}{1}.\frac{1}{1}.\frac{1}{2}.\frac{1}{2}=\frac{1}{4}$

$f(x)=\frac{\sqrt{1+tgx}-\sqrt{1+senx}}{x^3}=\frac{1+tgx-1-\operatorname{sen} x}{x^3}.\frac{1}{\sqrt{1+tgx}+\sqrt{1+\operatorname{sen} x}}=\frac{tgx-\operatorname{sen} x}{x^3}.\frac{1}{\sqrt{1+tgx}+\sqrt{1+\operatorname{sen} x}}$

$\lim\limits_{x\to 0}\frac{\sqrt{1+tgx}-\sqrt{1+\operatorname{sen} x}}{x^3}=\frac{1}{4}$

12. $\lim\limits_{x\to a}\frac{\operatorname{sen} x-\operatorname{sen} a}{x-a}=?\ \rightarrow\ \lim\limits_{x\to a}\frac{\operatorname{sen} x-\operatorname{sen} a}{x-a}=\lim\limits_{x\to a}\frac{2\operatorname{sen}\left(\frac{x-a}{2}\right).\cos\left(\frac{x+a}{2}\right)}{2.\left(\frac{x-a}{2}\right)}=$

$\lim\limits_{x\to a}\frac{2\operatorname{sen}(\frac{x-a}{2})}{2.\left(\frac{x-a}{2}\right)}.\frac{\cos\left(\frac{x+a}{2}\right)}{1}=\cos a$ Logo $\lim\limits_{x\to a}\frac{\operatorname{sen} x-\operatorname{sen} a}{x-a}=\operatorname{cosa}$

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13. $\lim\limits_{a\to 0}\dfrac{\text{sen}(x+a)-\text{sen}\,x}{a}=\,?$ → $\lim\limits_{a\to 0}\dfrac{\text{sen}(x+a)-\text{sen}\,x}{a}=\lim\limits_{a\to a}\dfrac{2\,\text{sen}\left(\dfrac{x+a-x}{2}\right)}{2.\left(\dfrac{x-a}{2}\right)}.\dfrac{.\cos\left(\dfrac{x+a+x}{2}\right)}{1}=$

$\lim\limits_{a\to a}\dfrac{2\,\text{sen}\left(\dfrac{a}{2}\right)}{2.\left(\dfrac{a}{2}\right)}.\dfrac{.\cos\left(\dfrac{2x+a}{2}\right)}{1}=\cos x$ Logo $\lim\limits_{a\to 0}\dfrac{\text{sen}(x+a)-\text{sen}\,x}{a}=\text{cosx}$

14. $\lim\limits_{a\to 0}\dfrac{\cos(x+a)-\cos x}{a}=\,?$ → $\lim\limits_{a\to 0}\dfrac{\cos(x+a)-\cos x}{a}=\lim\limits_{a\to 0}\dfrac{-2\,\text{sen}\left(\dfrac{x+a+x}{2}\right).\text{sen}\left(\dfrac{x-a-x}{2}\right)}{a}=$

$\lim\limits_{a\to 0}\dfrac{-2.\,\text{sen}\left(\dfrac{2x+a}{2}\right).\text{sen}\left(\dfrac{-a}{2}\right)}{2.\left(\dfrac{-a}{2}\right)}=\lim\limits_{a\to 0}-\text{sen}\left(\dfrac{2x+a}{2}\right).\dfrac{\text{sen}\left(\dfrac{-a}{2}\right)}{\left(\dfrac{-a}{2}\right)}=-\text{sen}\,x$ Logo

$\lim\limits_{a\to 0}\dfrac{\cos(x+a)-\cos x}{a}=\text{-senx}$

15. $\lim\limits_{x\to a}\dfrac{\sec x-\sec a}{x-a}=\,?$ → $\lim\limits_{x\to a}\dfrac{\sec x-\sec a}{x-a}=\lim\limits_{x\to a}\dfrac{\dfrac{1}{\cos x}-\dfrac{1}{\cos a}}{x-a}=\lim\limits_{x\to a}\dfrac{\dfrac{\cos a-\cos x}{\cos x.\cos a}}{x-a}=$

$\lim\limits_{x\to a}\dfrac{\cos a-\cos x}{(x-a).\cos x.\cos a}=\lim\limits_{x\to a}\dfrac{-2.\text{sen}\left(\dfrac{a+x}{2}\right).\text{sen}\left(\dfrac{a-x}{2}\right)}{(x-a).\cos x.\cos a}=$

$\lim\limits_{x\to a}\dfrac{-2.\text{sen}\left(\dfrac{a+x}{2}\right)}{1}.\dfrac{\text{sen}\left(\dfrac{a-x}{2}\right)}{-2.\left(\dfrac{a-x}{2}\right)}.\dfrac{1}{\cos x.\cos a}=\lim\limits_{x\to a}\dfrac{\text{sen}\left(\dfrac{a+x}{2}\right)}{1}.\dfrac{\text{sen}\left(\dfrac{a-x}{2}\right)}{\left(\dfrac{a-x}{2}\right)}.\dfrac{1}{\cos x.\cos a}=$

$\dfrac{\text{sen}\,a}{1}.1.\dfrac{1}{\cos a.\cos a}=\dfrac{\text{sen}\,a}{\cos a}.\dfrac{1}{\cos a}=tga.\sec a$ Logo $\lim\limits_{x\to a}\dfrac{\sec x-\sec a}{x-a}=tga.\sec a$

16. $\lim\limits_{x\to 0}\dfrac{x^2}{1-\sec x}=\,?$ → $\lim\limits_{x\to 0}\dfrac{x^2}{1-\sec x}=\lim\limits_{x\to 0}\dfrac{1}{-\dfrac{\text{sen}^2 x}{x^2}.\dfrac{1}{\cos x}.\dfrac{1}{(1+\cos x)}}=-2$

$f(x)=\dfrac{x^2}{1-\dfrac{1}{\cos x}}=\dfrac{x^2}{\dfrac{\cos x-1}{\cos x}}=\dfrac{x^2.\cos x}{-1.(1-\cos x)}=\dfrac{1}{-\dfrac{(1-\cos x)}{x^2}.\dfrac{1}{\cos x}.\dfrac{(1+\cos x)}{(1+\cos x)}}=$

$\dfrac{1}{-\dfrac{1-\cos^2 x}{x^2}.\dfrac{1}{\cos x}.\dfrac{1}{(1+\cos x)}}=\dfrac{1}{-\dfrac{\text{sen}^2 x}{x^2}.\dfrac{1}{\cos x}.\dfrac{1}{(1+\cos x)}}$

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17. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\cot gx}{1-tgx}=\,?$ → $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\cot gx}{1-tgx}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\dfrac{1}{tgx}}{1-tgx}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\dfrac{tgx-1}{tgx}}{1-tgx}=$

$\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\dfrac{-1.(1-tgx)}{tgx}}{1-tgx}=\lim\limits_{x\to\frac{\pi}{4}}-\dfrac{1}{tgx}=-1$ Logo $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\cot gx}{1-tgx}=\text{-}1$

18. $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{\text{sen}^2 x}=\,?$ → $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{\text{sen}^2 x}=\lim\limits_{x\to 0}\dfrac{(1-\cos x).(1+\cos x+\cos^2 x)}{1-\cos^2 x}=$

$\lim\limits_{x\to 0}\dfrac{(1-\cos x).(1+\cos x+\cos^2 x)}{(1-\cos x).(1+\cos x)}=\lim\limits_{x\to 0}\dfrac{1+\cos x+\cos^2 x}{1+\cos x}=\dfrac{3}{2}$ Logo $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{\text{sen}^2 x}=\dfrac{3}{2}$

19. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\text{sen}\,3x}{1-2.\cos x}=\,?$ → $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\text{sen}\,3x}{1-2.\cos x}=\lim\limits_{x\to\frac{\pi}{3}}-\dfrac{\text{sen}\,x.(1+2.\cos x)}{1}=-\sqrt{3}$

$f(x)=\dfrac{\text{sen}\,3x}{1-2.\cos x}=\dfrac{\text{sen}(x+2x)}{1-2.\cos x}=\dfrac{\text{sen}\,x.\cos 2x+\text{sen}\,2x.\cos x}{1-2.\cos x}=\dfrac{\text{sen}\,x.(2\cos^2 x-1)+2.\text{sen}\,x.\cos x.\cos x}{1-2.\cos x}=$

$\dfrac{\text{sen}\,x.\left[(2\cos^2 x-1)+2\cos^2 x\right]}{1-2.\cos x}=\dfrac{\text{sen}\,x.\left[4\cos^2 x-1\right]}{1-2.\cos x}=-\dfrac{\text{sen}\,x.(1-2.cox).(1+2.cox)}{1-2.\cos x}=-\dfrac{\text{sen}\,x.(1+2.\cos x)}{1}$

20. $\lim\limits_{x\to\pi/4}\dfrac{\text{sen}\,x-\cos x}{1-tgx}=\,?$ → $\lim\limits_{x\to\pi/4}\dfrac{\text{sen}\,x-\cos x}{1-tgx}=\lim\limits_{x\to\pi/4}(-\cos x)=-\dfrac{\sqrt{2}}{2}$

$f(x)=\dfrac{\text{sen}\,x-\cos x}{1-tgx}=\dfrac{\text{sen}\,x-\cos x}{1-\dfrac{\text{sen}\,x}{\cos x}}=\dfrac{\text{sen}\,x-\cos x}{1-\dfrac{\text{sen}\,x}{\cos x}}=\dfrac{\text{sen}\,x-\cos x}{\dfrac{\cos x-\text{sen}\,x}{\cos x}}=\dfrac{\text{sen}\,x-\cos x}{\dfrac{-1.(\text{sen}\,x-\cos x)}{\cos x}}=$

$-\dfrac{\text{sen}\,x-\cos x}{1}.\dfrac{\cos x}{\cos x-\text{sen}\,x}=-\cos x$

21. $\lim\limits_{x\to 3}(3-x).\cos\sec(\pi x)=\,?$ → $\lim\limits_{x\to 3}(3-x).\cos\sec(\pi x)=0.\infty$

$f(x)=(3-x).\cos\sec(\pi x)=(3-x).\dfrac{1}{\text{sen}(\pi x)}=\dfrac{3-x}{\text{sen}(\pi-\pi x)}=\dfrac{3-x}{\text{sen}(3\pi-\pi x)}=\dfrac{1}{\dfrac{\pi.\text{sen}(3\pi-\pi x)}{\pi.(3-x)}}=$

$\dfrac{1}{\dfrac{\pi.\text{sen}(3\pi-\pi x)}{(3\pi-\pi x)}}$ → $\lim\limits_{x\to 3}(3-x).\cos\sec(\pi x)=\lim\limits_{x\to 3}\dfrac{1}{\dfrac{\pi.\text{sen}(3\pi-\pi x)}{(3\pi-\pi x)}}=\dfrac{1}{\pi}$

22. $\lim\limits_{x\to\infty}x.\text{sen}(\dfrac{1}{x})=\,?$ → $\lim\limits_{x\to\infty}x.\text{sen}(\dfrac{1}{x})=\infty.0$

$\lim\limits_{x\to\infty}\dfrac{\text{sen}\left(\dfrac{1}{x}\right)}{\dfrac{1}{x}}=\lim\limits_{t\to 0}\dfrac{\text{sen}\,t}{t}=1$ → Fazendo $t=\dfrac{1}{x}$ $\begin{cases}x\to+\infty\\ t\to 0\end{cases}$

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23. $\lim\limits_{x\to\pi/6}\dfrac{2.\operatorname{sen}^2 x+\operatorname{sen}x-1}{2.\operatorname{sen}^2 x-3.\operatorname{sen}x+1}=\,?\ \rightarrow\ \lim\limits_{x\to\pi/6}\dfrac{2.\operatorname{sen}^2 x+\operatorname{sen}x-1}{2.\operatorname{sen}^2 x-3.\operatorname{sen}x+1}=\lim\limits_{x\to\pi/6}\dfrac{1+\operatorname{sen}x}{-1+\operatorname{sen}x}=\dfrac{1+\operatorname{sen}\frac{\pi}{6}}{-1+\operatorname{sen}\frac{\pi}{6}}=$

$\dfrac{1+\frac{1}{2}}{-1+\frac{1}{2}}=-3\ \rightarrow\ f(x)=\dfrac{2.\operatorname{sen}^2 x+\operatorname{sen}x-1}{2.\operatorname{sen}^2 x-3.\operatorname{sen}x+1}=\dfrac{\left(\operatorname{sen}x-\frac{1}{2}\right).(\operatorname{sen}x+1)}{\left(\operatorname{sen}x-\frac{1}{2}\right).(\operatorname{sen}x-1)}=\dfrac{(\operatorname{sen}x+1)}{(\operatorname{sen}x-1)}=\dfrac{1+\operatorname{sen}x}{-1+\operatorname{sen}x}$

24. $\lim\limits_{x\to1}(1-x).tg\left(\dfrac{\pi x}{2}\right)=\,?\ \rightarrow\ \lim\limits_{x\to1}(1-x).tg\left(\dfrac{\pi x}{2}\right)=0.\infty\ \rightarrow\ f(x)=(1-x).tg\left(\dfrac{\pi x}{2}\right)=$

$(1-x).\cot g\left(\dfrac{\pi}{2}-\dfrac{\pi x}{2}\right)=\dfrac{(1-x)}{tg\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}=\dfrac{\frac{\pi}{2}.(1-x).\frac{2}{\pi}}{tg\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}=\dfrac{\frac{2}{\pi}}{\dfrac{tg\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}{\frac{\pi}{2}.(1-x)}}=\dfrac{\frac{2}{\pi}}{\dfrac{tg\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}{\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}}\ \rightarrow$

$\lim\limits_{x\to1}(1-x).tg\left(\dfrac{\pi x}{2}\right)=\lim\limits_{x\to1}\dfrac{\frac{2}{\pi}}{\dfrac{tg\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}{\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}}=\dfrac{\frac{2}{\pi}}{\lim\limits_{t\to0}\frac{tg(t)}{t}}=\dfrac{2}{\pi}$ Fazendo uma mudança de variável,

temos : $t=\dfrac{\pi}{2}-\dfrac{\pi x}{2}\begin{cases}x\to1\\t\to0\end{cases}$

25. $\lim\limits_{x\to1}\dfrac{1-x^2}{\operatorname{sen}(\pi x)}=\,?\ \rightarrow\ \lim\limits_{x\to1}\dfrac{1-x^2}{\operatorname{sen}(\pi x)}=\lim\limits_{x\to1}\dfrac{1+x}{\dfrac{\pi.\operatorname{sen}(\pi-\pi x)}{(\pi-\pi x)}}=\dfrac{2}{\pi}$

$f(x)=\dfrac{1-x^2}{\operatorname{sen}\pi x}=\dfrac{(1-x)(1+x)}{\operatorname{sen}(\pi-\pi x)}=\dfrac{1+x}{\dfrac{\operatorname{sen}(\pi-\pi x)}{(1-x)}}=\dfrac{1+x}{\dfrac{\pi.\operatorname{sen}(\pi-\pi x)}{\pi.(1-x)}}=\dfrac{1+x}{\dfrac{\pi.\operatorname{sen}(\pi-\pi x)}{(\pi-\pi x)}}$

26. $\lim\limits_{x\to0}\cot g2x.\cot g\left(\dfrac{\pi}{2}-x\right)=\,?\ \rightarrow\ \lim\limits_{x\to0}\cot g2x.\cot g\left(\dfrac{\pi}{2}-x\right)=\infty.0$

$f(x)=\cot g2x.\cot g\left(\dfrac{\pi}{2}-x\right)=\cot g2x.tgx=\dfrac{tgx}{tg2x}=\dfrac{tgx}{\frac{2tgx}{1-tg^2x}}=tgx.\dfrac{1-tg^2x}{2.tgx}=\dfrac{1-tg^2x}{2}$

$\lim\limits_{x\to0}\cot g2x.\cot g\left(\dfrac{\pi}{2}-x\right)=\lim\limits_{x\to0}\dfrac{1-tg^2x}{2}=\dfrac{1}{2}$

27. $\lim\limits_{x\to0}\dfrac{\sqrt{\cos x}-\sqrt[3]{\cos x}}{\operatorname{sen}^2 x}=\lim\limits_{t\to1}\dfrac{-t^2}{1+t+t^2+...+t^{10}+t^{11}}=-\dfrac{1}{12}$

$f(x)=\dfrac{\sqrt{\cos x}-\sqrt[3]{\cos x}}{\operatorname{sen}^2 x}=\dfrac{t^3-t^2}{1-t^{12}}=\dfrac{-t^2.(1-t)}{(1-t)(1+t+t^2+...+t^{10}+t^{11})}=\dfrac{-t^2}{1+t+t^2+...+t^{10}+t^{11}}$

$t=\sqrt[2.3]{\cos x}=\sqrt[6]{\cos x}\quad\begin{cases}x\to0\\t\to1\end{cases}\quad t^6=\cos x,\quad t^{12}=\cos^2 x,\quad \operatorname{sen}^2 x=1-t^{12}$

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BriotxRuffini :

	1	0	0	...	0	-1
1	•	1	1	...	1	1
	1	1	1	...	1	***0***

28. $\lim\limits_{x\to\pi/4}\dfrac{\operatorname{sen}2x-\cos2x-1}{\cos x-\operatorname{sen}x}=\,?\ \rightarrow\ \lim\limits_{x\to\pi/4}\dfrac{\operatorname{sen}2x-\cos2x-1}{\cos x-\operatorname{sen}x}=\lim\limits_{x\to\pi/4}(-2.\cos x)=-2.\cos\dfrac{\pi}{4}=-2.\dfrac{\sqrt{2}}{2}=$

$-\sqrt{2}$

$f(x)=\dfrac{\operatorname{sen}2x-\cos2x-1}{\cos x-\operatorname{sen}x}=\dfrac{2.\operatorname{sen}x\cos x-(2\cos^2 x-1)-1}{\cos x-\operatorname{sen}x}=\dfrac{2.\operatorname{sen}x.\cos x-2\cos^2 x+1-1}{\cos x-\operatorname{sen}x}=$

$\dfrac{2.\operatorname{sen}x.\cos x-2\cos^2 x}{\cos x-\operatorname{sen}x}=\dfrac{-2.\cos x.(\cos x-\operatorname{sen}x)}{\cos x-\operatorname{sen}x}=-2.\cos x$

29. $\lim\limits_{x\to1}\dfrac{\operatorname{sen}(x-1)}{\sqrt{2x-1}-1}=\,?\ \rightarrow\ \lim\limits_{x\to1}\dfrac{\operatorname{sen}(x-1)}{\sqrt{2x-1}-1}=\lim\limits_{x\to1}\dfrac{1}{2}.\dfrac{\operatorname{sen}(x-1)}{(x-1)}.\dfrac{\sqrt{2x-1}+1}{1}=1$

$f(x)=\dfrac{\operatorname{sen}(x-1)}{\sqrt{2x-1}-1}=\dfrac{\operatorname{sen}(x-1)}{\sqrt{2x-1}-1}.\dfrac{\sqrt{2x-1}+1}{\sqrt{2x-1}+1}=\dfrac{\operatorname{sen}(x-1)}{2x-1-1}.\dfrac{\sqrt{2x-1}+1}{1}=\dfrac{\operatorname{sen}(x-1)}{2.(x-1)}.\dfrac{\sqrt{2x-1}+1}{1}=$

$\dfrac{1}{2}.\dfrac{\operatorname{sen}(x-1)}{(x-1)}.\dfrac{\sqrt{2x-1}+1}{1}$

30. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{1-2.\cos x}{x-\frac{\pi}{3}}=\,?\ \rightarrow\ \lim\limits_{x\to\frac{\pi}{3}}\dfrac{1-2.\cos x}{x-\frac{\pi}{3}}=\lim\limits_{x\to\pi/3}2.\operatorname{sen}\left(\dfrac{\pi/3+x}{2}\right).\dfrac{\operatorname{sen}\left(\dfrac{\pi/3-x}{2}\right)}{\left(\dfrac{\pi/3-x}{2}\right)}=$

$2.\operatorname{sen}\left(\dfrac{\pi/3+\pi/3}{2}\right).=2.\operatorname{sen}\left(\dfrac{2\pi/3}{2}\right).=2.\operatorname{sen}\left(\dfrac{\pi}{3}\right).=\ 2.\dfrac{\sqrt{3}}{2}=\sqrt{3}$

$f(x)=\dfrac{1-2.\cos x}{x-\frac{\pi}{3}}=\dfrac{2.\left(\frac{1}{2}-\cos x\right)}{x-\frac{\pi}{3}}=\dfrac{2.\left(\cos\frac{\pi}{3}-\cos x\right)}{x-\frac{\pi}{3}}=\dfrac{2.(-2)\operatorname{sen}\left(\dfrac{\pi/3+x}{2}\right).\operatorname{sen}\left(\dfrac{\pi/3-x}{2}\right)}{-1.2.\left(\dfrac{\pi/3-x}{2}\right)}=$

$\dfrac{2.\operatorname{sen}\left(\dfrac{\pi/3+x}{2}\right).\operatorname{sen}\left(\dfrac{\pi/3-x}{2}\right)}{\left(\dfrac{\pi/3-x}{2}\right)}=2.\operatorname{sen}\left(\dfrac{\pi/3+x}{2}\right).\dfrac{\operatorname{sen}\left(\dfrac{\pi/3-x}{2}\right)}{\left(\dfrac{\pi/3-x}{2}\right)}$

31. $\lim\limits_{x\to0}\dfrac{1-\cos2x}{x.\operatorname{sen}x}=\,?\ \rightarrow\ \lim\limits_{x\to0}\dfrac{1-\cos2x}{x.\operatorname{sen}x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2.\operatorname{sen}x}{x}=2$

Limites Trigonométricos Resolvidos

Sete páginas e 34 limites resolvidos

$$f(x)=\frac{1-\cos 2x}{x.\operatorname{sen} x}=\frac{1-\left(1-2\operatorname{sen}^2 x\right)}{x.\operatorname{sen} x}=\frac{1-1+2\operatorname{sen}^2 x}{x.\operatorname{sen} x}=\frac{2.\operatorname{sen}^2 x}{x.\operatorname{sen} x}=\frac{2.\operatorname{sen} x}{x}$$

32. $\lim\limits_{x\to 0}\frac{x}{\sqrt{1+\operatorname{sen} x}-\sqrt{1-\operatorname{sen} x}}=\ ?\ \rightarrow \lim\limits_{x\to 0}\frac{x}{\sqrt{1+\operatorname{sen} x}-\sqrt{1-\operatorname{sen} x}}=\lim\limits_{x\to 0}\frac{\sqrt{1+\operatorname{sen} x}+\sqrt{1-\operatorname{sen} x}}{\frac{2.\operatorname{sen} x}{x}}=\frac{1+1}{2.1}$

$=1$

$$f(x)=\frac{x}{\sqrt{1+\operatorname{sen} x}-\sqrt{1-\operatorname{sen} x}}=\frac{x.\left(\sqrt{1+\operatorname{sen} x}+\sqrt{1-\operatorname{sen} x}\right)}{1+\operatorname{sen} x-(1-\operatorname{sen} x)}=\frac{x.\left(\sqrt{1+\operatorname{sen} x}+\sqrt{1-\operatorname{sen} x}\right)}{1+\operatorname{sen} x-1+\operatorname{sen} x}=$$

$$\frac{x.\left(\sqrt{1+\operatorname{sen} x}+\sqrt{1-\operatorname{sen} x}\right)}{2.\operatorname{sen} x}=\frac{\sqrt{1+\operatorname{sen} x}+\sqrt{1-\operatorname{sen} x}}{2.\frac{\operatorname{sen} x}{x}}=\frac{1+1}{2.1}=1$$

33. $\lim\limits_{x\to 0}\frac{\cos 2x}{\cos x-\operatorname{sen} x}=\lim\limits_{x\to 0}\frac{\cos x+\operatorname{sen} x}{1}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\sqrt{2}$

$$f(x)=\frac{\cos 2x}{\cos x-\operatorname{sen} x}=\frac{\cos 2x.(\cos x+\operatorname{sen} x)}{(\cos x-\operatorname{sen} x)(\cos x+\operatorname{sen} x)}=\frac{\cos 2x.(\cos x+\operatorname{sen} x)}{\cos^2 x-\operatorname{sen}^2 x}=\frac{\cos 2x.(\cos x+\operatorname{sen} x)}{\cos 2x}=$$

$$\frac{\cos 2x.(\cos x+\operatorname{sen} x)}{\cos 2x}=\frac{\cos x+\operatorname{sen} x}{1}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\sqrt{2}$$

34. $\lim\limits_{x\to\frac{\pi}{3}}\frac{\sqrt{3}-2.\operatorname{sen} x}{x-\frac{\pi}{3}}=\ ?\ \rightarrow \lim\limits_{x\to\frac{\pi}{3}}\frac{\sqrt{3}-2.\operatorname{sen} x}{x-\frac{\pi}{3}}=\lim\limits_{x\to\frac{\pi}{3}}\frac{2.\left(\frac{\sqrt{3}}{2}-\operatorname{sen} x\right)}{x-\frac{\pi}{3}}=\lim\limits_{x\to\frac{\pi}{3}}\frac{2.\left(\operatorname{sen}\frac{\pi}{3}-\operatorname{sen} x\right)}{x-\frac{\pi}{3}}=$

$$\lim_{x\to\frac{\pi}{3}}\frac{2.\left(\operatorname{sen}\left(\frac{\frac{\pi}{3}-x}{2}\right).\cos\left(\frac{\frac{\pi}{3}+x}{2}\right)\right)}{x-\frac{\pi}{3}}=\lim_{x\to\frac{\pi}{3}}\frac{2.\left(\operatorname{sen}\left(\frac{\frac{\pi-3x}{3}}{2}\right).\cos\left(\frac{\frac{\pi+3x}{3}}{2}\right)\right)}{\frac{3x-\pi}{3}}=$$

$$\lim_{x\to\frac{\pi}{3}}\frac{2.\left(\operatorname{sen}\left(\frac{\pi-3x}{6}\right).\cos\left(\frac{\pi+3x}{6}\right)\right)}{\frac{-1.(\pi-3x)}{3}}$$

35. ?