Chapter 1 Solutions

1.1-1. (a) $E = \int_0^2 (1)^2 dt + \int_2^3 (-1)^2 dt = 3$

(b) $E = \int_0^2 (-1)^2 dt + \int_2^3 (1)^2 dt = 3$

(c) $E = \int_0^2 (2)^2 dt + \int_2^3 (-2)^2 dt = 12$

(d) $E = \int_3^5 (1)^2 dt + \int_5^6 (-1)^2 dt = 3$

Comments: Changing the sign of a signal does not change its energy. Doubling a signal quadruples its energy. Shifting a signal does not change its energy. Multiplying a signal by a constant K increases its energy by a factor K^2.

1.1-2.

$$E_x = \int_0^1 t^2 dt = \frac{1}{3}\, t^3\Big|_0^1 = \frac{1}{3}, \qquad E_{x_1} = \int_{-1}^0 (-t)^2 dt = \frac{1}{3}\, t^3\Big|_{-1}^0 = \frac{1}{3},$$

$$E_{x_2} = \int_0^1 (-t)^2 dt = \frac{1}{3}\, t^3\Big|_0^1 = \frac{1}{3}, \qquad E_{x_3} = \int_1^2 (t-1)^2 dt = \int_0^1 x^2 dx = \frac{1}{3},$$

$$E_{x_4} = \int_0^1 (2t)^2 dt = \frac{4}{3}\, t^3\Big|_{-1}^0 = \frac{4}{3}$$

1.1-3. (a)

$$E_x = \int_0^2 (1)^2 dt = 2, \qquad E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2,$$

$$E_{x+y} = \int_0^1 (2)^2 dt = 4, \qquad E_{x-y} = \int_1^2 (2)^2 dt = 4$$

Therefore $E_{x \pm y} = E_x + E_y$.

(b)

$$E_x = \int_0^{2\pi} \sin^2 t\, dt = \frac{1}{2}\int_0^{2\pi} (1) dt - \frac{1}{2}\int_0^{2\pi} \cos(2t) dt = \pi + 0 = \pi$$

$$E_y = \int_0^{2\pi} (1)^2\, dt = 2\pi$$

$$E_{x+y} = \int_0^{2\pi} (\sin t + 1)^2\, dt = \int_0^{2\pi} \sin^2(t) dt + 2\int_0^{2\pi} \sin(t) dt + \int_0^{2\pi} (1)^2 dt = \pi + 0 + 2\pi = 3\pi$$

In both cases (a) and (b), $E_{x+y} = E_x + E_y$. Similarly we can show that for both cases $E_{x-y} = E_x + E_y$.

(c) As seen in part (a),

$$E_x = \int_0^\pi \sin^2 t\, dt = \pi/2$$

Furthermore,

$$E_y = \int_0^\pi (1)^2\, dt = \pi$$

Thus,

$$E_{x+y} = \int_0^\pi (\sin t{+}1)^2\, dt = \int_0^\pi \sin^2(t)dt{+}2\int_0^\pi \sin(t)dt{+}\int_0^\pi (1)^2 dt = \pi/2{+}2(2){+}\pi\frac{3\pi}{2}{+}4$$

Additionally,

$$E_{x-y} = \int_0^\pi (\sin t - 1)^2\, dt = \pi/2 - 4 + \pi = \frac{3\pi}{2} - 4$$

In this case, $E_{x+y} \neq E_{x-y} \neq E_x + E_y$. Hence, we cannot generalize the conclusions observed in parts (a) and (b).

1.1-4. $P_x = \frac{1}{4}\int_{-2}^{2}(t^3)^2 dt = 64/7$

(a) $P_{-x} = \frac{1}{4}\int_{-2}^{2}(-t^3)^2 dt = 64/7$

(b) $P_{2x} = \frac{1}{4}\int_{-2}^{2}(2t^3)^2 dt = 4(64/7) = 256/7$

(c) $P_{cx} = \frac{1}{4}\int_{-2}^{2}(ct^3)^2 dt = 64c^2/7$

Comments: Changing the sign of a signal does not affect its power. Multiplying a signal by a constant c increases the power by a factor c^2.

1.1-5. (a) Power of a sinusoid of amplitude C is $C^2/2$ [Eq. (1.4a)] regardless of its frequency ($\omega \neq 0$) and phase. Therefore, in this case $P = 5^2 + (10)^2/2 = 75$.

(b) Power of a sum of sinusoids is equal to the sum of the powers of the sinusoids [Eq. (1.4b)]. Therefore, in this case $P = \frac{(10)^2}{2} + \frac{(16)^2}{2} = 178$.

(c) $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. Hence from Eq. (1.4b) $P = \frac{(10)^2}{2} + \frac{1}{2} + \frac{1}{2} = 51$.

(d) $10\cos 5t \cos 10t = 5(\cos 5t{+}\cos 15t$. Hence from Eq. (1.4b) $P = \frac{(5)^2}{2} + \frac{(5)^2}{2} = 25$.

(e) $10\sin 5t \cos 10t = 5(\sin 15t{-}\sin 5t$. Hence from Eq. (1.4b) $P = \frac{(5)^2}{2} + \frac{(-5)^2}{2} = 25$.

(f) $e^{j\alpha t}\cos\omega_0 t = \frac{1}{2}\left[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t}\right]$. Using the result in Prob. 1.1-5, we obtain $P = (1/4) + (1/4) = 1/2$.

1.1-6. First, $x(t) = \begin{cases} \frac{2A}{T}t & 0 \le t < \frac{T}{2} \\ 0 & \frac{T}{2} \le t < T \\ x(t+T) & \forall t \end{cases}$. Next, $P_x = \frac{1}{T}\int_0^{T/2}\left(\frac{2A}{T}t\right)^2 dt = \frac{4A^2}{T^2}\int_0^{T/2} t^2 dt = \frac{4A^2}{T^2}\frac{t^3}{3}\Big|_0^{T/2} = \frac{4A^2}{T^2}\frac{T^3}{3(8)} = \frac{A^2}{6}$. Since power is finite, energy must be infinite. Thus,

$$P_x = \frac{A^2}{6} \text{ and } E_x = \infty.$$

1.1-7. (a) i. By definition, $E[Tx_1(t)] = \int_{t=-\infty}^{\infty} (Tx_1(t))^2\,dt = \int_{t=-\infty}^{\infty} T^2 x_1^2(t)dt = T^2 \int_{t=-\infty}^{\infty} x_1^2(t)dt = T^2 E[x_1(t)]$.

$$E[Tx_1(t)] = T^2 E[x_1(t)].$$

ii. By definition, $E[x_1(t-T)] = \int_{t=-\infty}^{\infty} (x_1(t-T))^2\,dt$. Substituting $t' = t - T$ and $dt' = dt$ yields $\int_{t'=\infty}^{\infty} x_1^2(t')dt' = E[x_1(t)]$.

$$E[x_1(t-T)] = E[x_1(t)].$$

iii. By definition, $E[x_1(t) + x_2(t)] = \int_{t=-\infty}^{\infty} (x_1(t) + x_2(t))^2\,dt = \int_{t=-\infty}^{\infty} (x_1^2(t) + 2x_1(t)x_2(t) + x_2^2(t))\,dt$. However, $x_1(t)$ and $x_2(t)$ are non-overlapping so their product $x_1(t)x_2(t)$ must be zero. Thus, $E[x_1(t) + x_2(t)] = \int_{t=-\infty}^{\infty} (x_1^2(t) + x_2^2(t))\,dt = \int_{t=-\infty}^{\infty} x_1^2(t)dt + \int_{t=-\infty}^{\infty} x_2^2(t)dt = E[x_1(t)] + E[x_2(t)]$.

$$\text{If } (x_1(t) \neq 0) \Rightarrow (x_2(t) = 0) \text{ and } (x_2(t) \neq 0) \Rightarrow (x_1(t) = 0),$$

$$\text{Then, } E[x_1(t) + x_2(t)] = E[x_1(t)] + E[x_2(t)].$$

iv. By definition, $E[x_1(Tt)] = \int_{t=-\infty}^{\infty} x_1^2(Tt)dt$.
First, consider the case $T > 0$. Substituting $t' = Tt$ and $dt' = Tdt$ yields $E[x_1(Tt)] = \int_{t'=-\infty}^{\infty} x_1^2(t')\frac{dt'}{T} = \frac{1}{T}\int_{t'=-\infty}^{\infty} x_1^2(t')dt' = \frac{E[x_1(t)]}{T} = \frac{E[x_1(t)]}{|T|}$.
Next, consider the case $T < 0$. Substituting $t' = Tt$ and $dt' = Tdt$ yields $E[x_1(Tt)] = \int_{t'=\infty}^{-\infty} x_1^2(t')\frac{dt'}{T} = \frac{-1}{T}\int_{t'=-\infty}^{\infty} x_1^2(t')dt' = \frac{E[x_1(t)]}{-T}$. For $T < 0$, we know $T = -|T|$. Making this substitution yields $E[x_1(Tt)] = \frac{E[x_1(t)]}{|T|}$.
Since energy is the same whether $T < 0$ or $T > 0$, we know

$$E[x_1(Tt)] = \frac{E[x_1(t)]}{|T|}.$$

(b) To begin, notice that signal $y(t) = t(u(t) - u(t-1))$ has energy equal to $E[y(t)] = \int_0^1 t^2 dt = 1/3$.
To determine $E[x(t)]$, consider dividing $x(t)$ into three non-overlapping pieces: a first piece $x_a(t)$ from $(-2 \le t < -1)$, a second piece $x_b(t)$ from $(-1 \le t < 0)$, and a third piece $x_c(t)$ from $(0 \le t < 3)$. Since the pieces are non-overlapping, the total energy $E[x(t)] = E[x_a(t)] + E[x_a(t)] + E[x_a(t)]$.
Using the properties of energy, we know that shifting or reflecting a signal does not affect its energy. Notice that $y(t/3)$ is the same as a flipped and shifted version of $x_c(t)$. Thus, $E[x_c(t)] = E[y(t/3)] = 3(1/3) = 1$. Also, it is possible to combine $x_a(t)$ with a flipped and shifted version of $x_b(t)$ to equal a flipped and shifted version of $2y(t/2)$. Thus, $E[x_a(t) + x_b(t)] = E[2y(t/2)] = 4(2)(1/3) = 8/3$.
Thus,

$$E[x(t)] = 11/3.$$

1.1-8. (a)

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)x^*(t)\,dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=m}^{n}\sum_{r=m}^{n} D_k D^*{}_r e^{j(\omega_k - \omega_r)t}\,dt$$

The integrals of the cross-product terms (when $k \neq r$) are finite because the integrands are periodic signals (made up of sinusoids). These terms, when divided by $T \to \infty$, yield zero. The remaining terms ($k = r$) yield

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^{n} |D_k|^2\, dt = \sum_{k=m}^{n} |D_k|^2$$

(b) i.

$$\begin{aligned} x(t) &= 5 + 10\cos(100t + \pi/3) \\ &= 5 + 5e^{j(100t+\frac{\pi}{3})} + 5e^{-j(100t+\frac{\pi}{3})} \\ &= 5 + 5e^{j\pi/3}e^{j100t} + 5e^{-j\pi/3}e^{-j100t} \end{aligned}$$

Hence,

$$P_x = 5^2 + |5e^{j\pi/3}|^2 + |5e^{-j\pi/3}|^2 = 25 + 25 + 25 = 75.$$

Thought of another way, note that $D_0 = 5, D_{\pm 1} = 5$ and thus $P_x = 5^2 + 5^2 + 5^2 = 75$.

ii.

$$\begin{aligned} x(t) &= 10\cos(100t + \pi/3) + 16\sin(150t + \pi/5) \\ &= 5e^{j\pi/3}e^{j100t} + 5e^{-j\pi/3}e^{-j100t} - j8e^{j\pi/5}e^{j150t} + j8e^{-j\pi/5}e^{-j150t} \end{aligned}$$

Hence,

$$P_x = |5e^{j\pi/3}|^2 + |5e^{-j\pi/3}|^2 + |-j8e^{j\pi/5}|^2 + |j8e^{-j\pi/5}|^2 = 25+25+64+64 = 178.$$

Thought of another way, note that $D_{\pm 1} = 5$ and $D_{\pm 2} = 8$. Hence, $P_x = 5^2 + 5^2 + 8^2 + 8^2 = 178$.

iii. $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. In this case, $D_{\pm 1} = 5$, $D_{\pm 2} = 0.5$ and $D_{\pm 3} = 0.5$. Hence, $P = 5^2 + 5^2 + (0.5)^2 + (0.5)^2 + (0.5)^2 + (0.5)^2 = 51$

iv. $10\cos 5t \cos 10t = 5(\cos 5t + \cos 15t)$. In this case, $D_{\pm 1} = 2.5$ and $D_{\pm 2} = 2.5$. Hence, $P = (2.5)^2 + (2.5)^2 + (2.5)^2 + (2.5)^2 = 25$

v. $10\sin 5t \cos 10t = 5(\sin 15t - \sin 5t)$. In this case, $D_{\pm 1} = 2.5$ and $D_{\pm 2} = 2.5$. Hence, $P = (2.5)^2 + (2.5)^2 + (2.5)^2 + (2.5)^2 = 25$

vi. $e^{j\alpha t}\cos\omega_0 t = \frac{1}{2}\left[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t}\right]$. In this case, $D_{\pm 1} = 0.5$. Hence, $P = (1/2)^2 + (1/2)^2 = 1/2$.

1.1-9. First, notice that $x(t) = x^2(t)$ and that the area of each pulse is one. Since $x(t)$ has an infinite number of pulses, the corresponding energy must also be infinite. To compute the power, notice that N pulses requires an interval of width $\sum_{i=0}^{N} 2(i+1) = N^2 + 3N$. As $N \to \infty$, power is computed by the ratio of area to width, or $P = \lim_{N\to\infty} \frac{N}{N^2+3N} = 0$. Thus,

$$P = 0 \text{ and } E = \infty.$$

1.2-1. Refer to Figure S1.2-1.

1.2-2. Refer to Figure S1.2-2.

1.2-3. (a) $x_1(t)$ can be formed by shifting $x(t)$ to the left by 1 plus a time-inverted version of $x(t)$ shifted to left by 1. Thus,

$$x_1(t) = x(t+1) + x(-t+1) = x(t+1) + x(1-t).$$

integrands are periodic signals (made up of sinusoids). These terms, when divided by $T \to \infty$, yield zero. The remaining terms ($k = r$) yield

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^{n} |D_k|^2 \, dt = \sum_{k=m}^{n} |D_k|^2$$

(b) i.

$$\begin{aligned} x(t) &= 5 + 10\cos(100t + \pi/3) \\ &= 5 + 5e^{j(100t+\frac{\pi}{3})} + 5e^{-j(100t+\frac{\pi}{3})} \\ &= 5 + 5e^{j\pi/3}e^{j100t} + 5e^{-j\pi/3}e^{-j100t} \end{aligned}$$

Hence,

$$P_x = 5^2 + |5e^{j\pi/3}|^2 + |5e^{-j\pi/3}|^2 = 25 + 25 + 25 = 75.$$

Thought of another way, note that $D_0 = 5, D_{\pm 1} = 5$ and thus $P_x = 5^2 + 5^2 + 5^2 = 75$.

ii.

$$\begin{aligned} x(t) &= 10\cos(100t + \pi/3) + 16\sin(150t + \pi/5) \\ &= 5e^{j\pi/3}e^{j100t} + 5e^{-j\pi/3}e^{-j100t} - j8e^{j\pi/5}e^{j150t} + j8e^{-j\pi/5}e^{-j150t} \end{aligned}$$

Hence,

$$P_x = |5e^{j\pi/3}|^2 + |5e^{-j\pi/3}|^2 + |-j8e^{j\pi/5}|^2 + |j8e^{-j\pi/5}|^2 = 25+25+64+64 = 178.$$

Thought of another way, note that $D_{\pm 1} = 5$ and $D_{\pm 2} = 8$. Hence, $P_x = 5^2 + 5^2 + 8^2 + 8^2 = 178$.

iii. $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. In this case, $D_{\pm 1} = 5$, $D_{\pm 2} = 0.5$ and $D_{\pm 3} = 0.5$. Hence, $P = 5^2 + 5^2 + (0.5)^2 + (0.5)^2 + (0.5)^2 + (0.5)^2 = 51$

iv. $10\cos 5t \cos 10t = 5(\cos 5t + \cos 15t)$. In this case, $D_{\pm 1} = 2.5$ and $D_{\pm 2} = 2.5$. Hence, $P = (2.5)^2 + (2.5)^2 + (2.5)^2 + (2.5)^2 = 25$

v. $10\sin 5t \cos 10t = 5(\sin 15t - \sin 5t)$. In this case, $D_{\pm 1} = 2.5$ and $D_{\pm 2} = 2.5$. Hence, $P = (2.5)^2 + (2.5)^2 + (2.5)^2 + (2.5)^2 = 25$

vi. $e^{j\alpha t}\cos\omega_0 t = \frac{1}{2}\left[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t}\right]$. In this case, $D_{\pm 1} = 0.5$. Hence, $P = (1/2)^2 + (1/2)^2 = 1/2$.

1.1-9. First, notice that $x(t) = x^2(t)$ and that the area of each pulse is one. Since $x(t)$ has an infinite number of pulses, the corresponding energy must also be infinite. To compute the power, notice that N pulses requires an interval of width $\sum_{i=0}^{N} 2(i+1) = N^2 + 3N$. As $N \to \infty$, power is computed by the ratio of area to width, or $P = \lim_{N\to\infty} \frac{N}{N^2+3N} = 0$. Thus,

$$P = 0 \text{ and } E = \infty.$$

1.2-1. Refer to Figure S1.2-1.

1.2-2. Refer to Figure S1.2-2.

1.2-3. (a) $x_1(t)$ can be formed by shifting $x(t)$ to the left by 1 plus a time-inverted version of $x(t)$ shifted to left by 1. Thus,

$$x_1(t) = x(t+1) + x(-t+1) = x(t+1) + x(1-t).$$

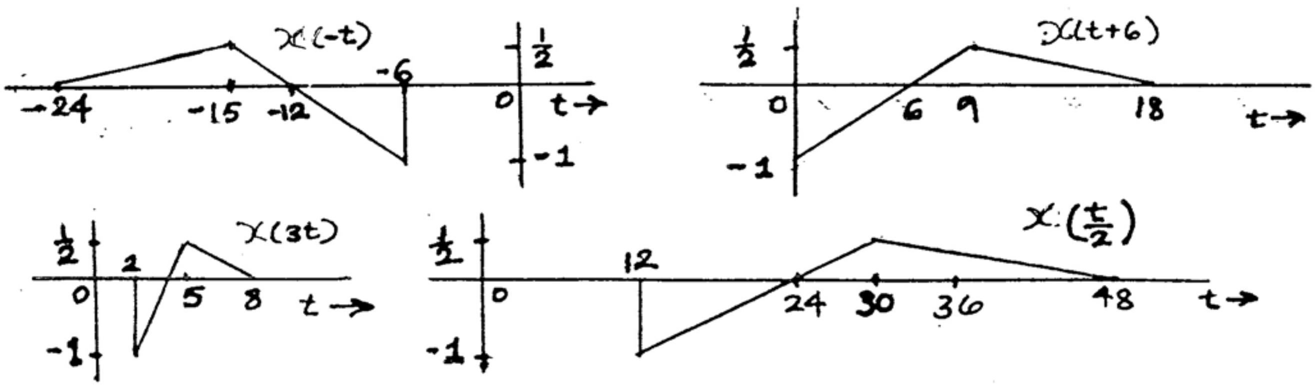

Figure S1.2-1

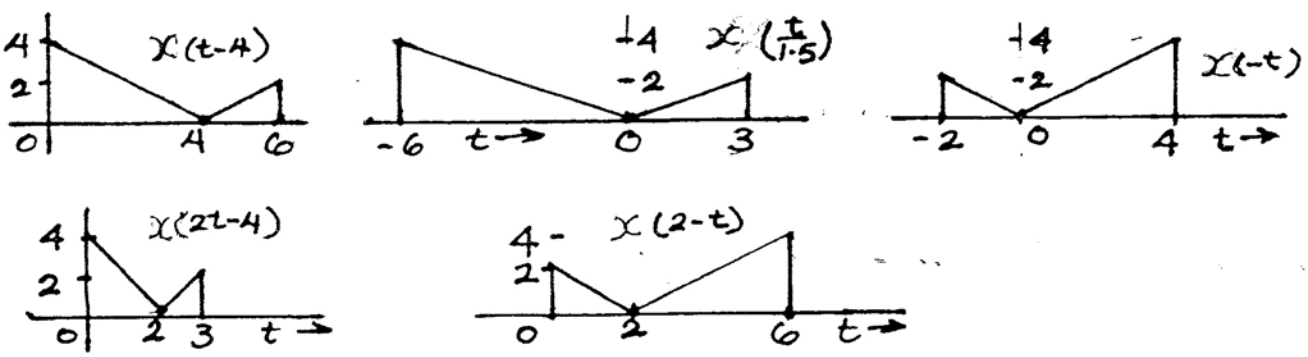

Figure S1.2-2

(b) $x_2(t)$ can be formed by time-expanding $x(t)$ by factor 2 to obtain $x(t/2)$ Now, left-shift $x(t/2)$ by unity to obtain $x(\frac{t+1}{2})$. We now add to this a time-inverted version of $x(\frac{1-t}{2})$ to obtain $x_2(t)$. Thus,

$$x_2(t) = x(\frac{t+1}{2}) + x(\frac{1-t}{2}).$$

(c) Observe that $x_3(t)$ is composed of two parts:
First, a rectangular pulse to form the base is constructed by time-expanding $x_2(t)$ by a factor of 2. This is obtained by replacing t with $t/2$ in $x_2(t)$. Thus, we obtain $x_2(t/2) = x(\frac{t+2}{4}) + x(\frac{2-t}{4})$.
Second, the two triangles on top of the rectangular base are constructed by time-expanded (factor of 2) and shifted versions of $x(t)$ according to $x(t/2) + x(-t/2)$. Thus,

$$x_3(t) = x(\frac{t+2}{4}) + x(\frac{2-t}{4}) + x(t/2) + x(-t/2).$$

(d) $x_4(t)$ can be obtained by time-expanding $x_1(t)$ by a factor 2 and then multiplying it by 4/3 to obtain $\frac{4}{3}x_1(t/2) = \frac{4}{3}\left[x(\frac{t+2}{2}) + x(\frac{2-t}{2})\right]$. From this, we subtract a rectangular pedestal of height 1/3 and width 4. This is obtained by time-expanding $x_2(t)$ by 2 and multiplying it by 1/3 to yield $\frac{1}{3}x_2(t/2) = \frac{1}{3}\left[x(\frac{t+2}{4}) + x(\frac{2-t}{4})\right]$. Hence,

$$x_4(t) = \frac{4}{3}\left[x(\frac{t+2}{2}) + x(\frac{2-t}{2})\right] - \frac{1}{3}\left[x(\frac{t+2}{4}) + x(\frac{2-t}{4})\right].$$

(e) $x_5(t)$ is a sum of three components: (i) $x_2(t)$ time-compressed by a factor 2, (ii) $x(t)$ left-shifted by 1.5, and (iii) $x(t)$ time-inverted and then right shifted by 1.5. Hence,

$$x_5(t) = x(t+0.5) + x(0.5-t) + x(t+1.5) + x(1.5-t).$$

1.2-4.

$$E_{-x} = \int_{-\infty}^{\infty} [-x(t)]^2\,dt = \int_{-\infty}^{\infty} x^2(t)\,dt = E_x$$

$$E_{x(-t)} = \int_{-\infty}^{\infty} [x(-t)]^2\,dt = \int_{-\infty}^{\infty} x^2(x)\,dx = E_x$$

$$E_{x(t-T)} = \int_{-\infty}^{\infty} [x(t-T)]^2\,dt = \int_{-\infty}^{\infty} x^2(x)\,dx = E_x,$$

$$E_{x(at)} = \int_{-\infty}^{\infty} [x(at)]^2\,dt = \frac{1}{a}\int_{-\infty}^{\infty} x^2(x)\,dx = E_x/a$$

$$E_{x(at-b)} = \int_{-\infty}^{\infty} [x(at-b)]^2\,dt = \frac{1}{a}\int_{-\infty}^{\infty} x^2(x)\,dx = E_x/a,$$

$$E_{x(t/a)} = \int_{-\infty}^{\infty} [x(t/a)]^2\,dt = a\int_{-\infty}^{\infty} x^2(x)\,dt = aE_x$$

$$E_{ax(t)} = \int_{-\infty}^{\infty} [ax(t)]^2\,dt = a^2\int_{-\infty}^{\infty} x^2(t)\,dt = a^2E_x$$

Comment: Multiplying a signal by constant a increases the signal energy by a factor a^2.

1.2-5. (a) Calling $y(t) = 2x(-3t+1) = t(u(-t-1) - u(-t+1))$, MATLAB is used to sketch $y(t)$.

```
>> t = [-1.5:.001:1.5]; y = inline('t.*((t<=-1)-(t<=1))');
>> plot(t,y(t),'k-'); axis([-1.5 1.5 -1.1 1.1]);
>> xlabel('t'); ylabel('2x(-3t+1)');
```

(b) Since $y(t) = 2x(-3t+1)$, $0.5*y(-t/3+1/3) = 0.5(2)x(-3(-t/3+1/3)+1) = x(t)$. MATLAB is used to sketch $x(t)$.

```
>> y = inline('t.*((t<=-1)-(t<=1))');
>> t = [-3:.001:5]; x = 0.5*y(-t/3+1/3);
>> plot(t,x,'k-'); axis([-3 5 -0.6 0.6]);
>> xlabel('t'); ylabel('x(t)');
```

1.2-6. MATLAB is used to compute each sketch. Notice that the unit step is in the exponent of the function $x(t)$.

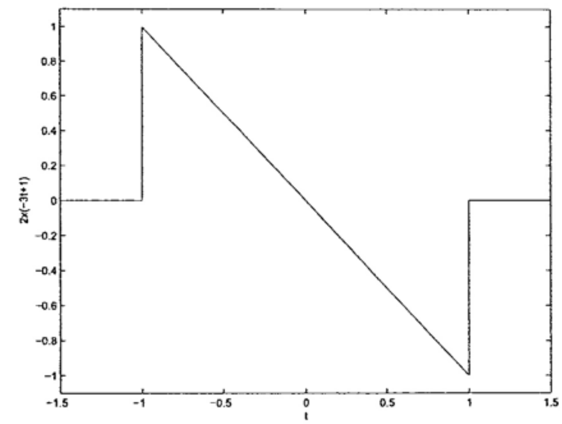

Figure S1.2-5a: Plot of $2x(-3t+1)$.

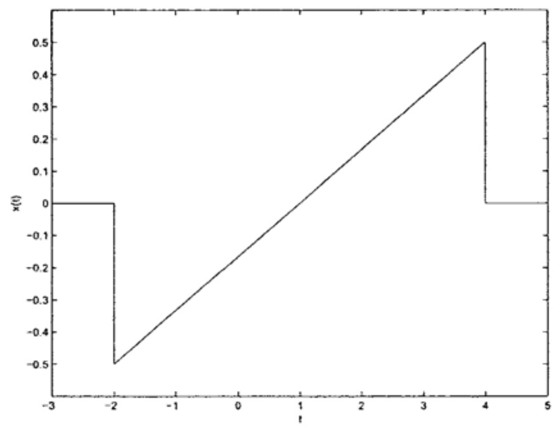

Figure S1.2-5b: Plot of $x(t) = 0.5y(-t/3 + 1/3)$.

(a)
```
>> t = [-1:.001:1];
>> x = inline('2.^(-t.*(t>=0))');
>> plot(t,x(t),'k'); axis([-1 1 0 1.1]);
>> xlabel('t'); ylabel('x(t)');
```

(b)
```
>> plot(t,0.5*x(1-2*t),'k'); axis([-1 1 0 1.1]);
>> xlabel('t'); ylabel('y(t)');
```

1.3-1. (a) False. Figure 1.11b is an example of a signal that is continuous-time but digital.

(b) False. Figure 1.11c is discrete-time but analog.

(c) False. e^{-t} is neither an energy nor a power signal.

(d) False. $e^{-t}u(t)$ has infinite duration but is an energy signal.

(e) False. $u(t)$ is a power signal that is causal.

(f) True. A periodic signal, by definition, exists for all t.

1.3-2. (a) True. Every bounded periodic signal is a power signal.

(b) False. Signals with bounded power are not necessarily periodic. For example, $x(t) = \cos(t)u(t)$ is non-periodic but has a bounded power of $P_x = 0.25$.

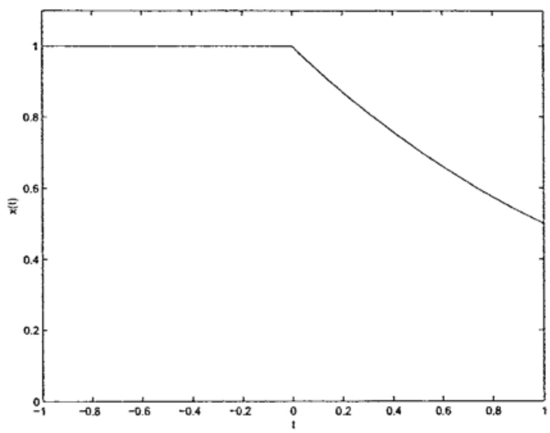

Figure S1.2-6a: Plot of $x(t) = 2^{-tu(t)}$.

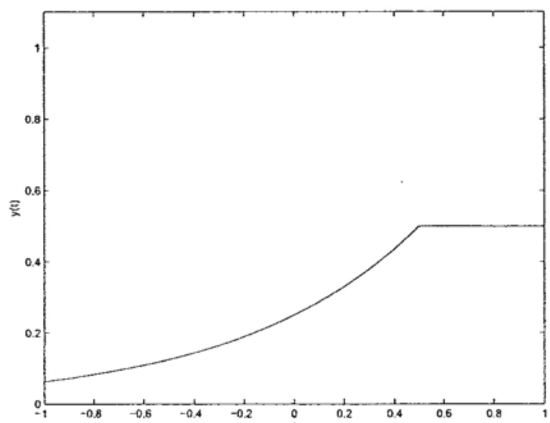

Figure S1.2-6b: Plot of $y(t) = 0.5x(1-2t)$.

(c) True. If an energy signal $x(t)$ has energy E, then the energy of $x(at)$ is $\frac{E}{a}$ (a real and positive).

(d) False. If a power signal $x(t)$ has power P, then the power of $x(at)$ is generally not $\frac{P}{a}$. A counter-example provides a simple proof. Consider the case of $x(t) = u(t)$, which has $P = 0.5$. Letting $a = 2$, $x(at) = x(2t) = u(2t) = u(t)$, which still has power $P = 0.5$ and not $P/a = P/2$.

1.3-3. (a) For periodicity, $x_1(t) = \cos(t) = \cos(t + T_1) = x_1(t + T_1)$. Since cosine is a 2π-periodic function, $T_1 = 2\pi$. Similarly, $x_2(t) = \sin(\pi t) = \sin(\pi t + \pi T_2) = \sin(\pi(t + T_2)) = x_2(t + T_2)$. Thus, $\pi T_2 = 2\pi k$. The smallest possible value is $T_2 = 2$. Thus,

$$T_1 = 2\pi \text{ and } T_2 = 2.$$

(b) Periodicity requires $x_3(t) = x_3(t+T_3)$ or $\cos(t)+\sin(\pi t) = \cos(t+T_3)+\sin(\pi t + \pi T_3)$. This requires $T_3 = 2\pi k_1$ and $\pi T_3 = 2\pi k_2$ for some integers k_1 and k_2. Combining, periodicity thus requires $T_3 = 2\pi k_1 = 2k_2$ or $\pi = k_1/k_2$. However, π is irrational. Thus, no suitable k_1 and k_2 exist, and $x_3(t)$ cannot be periodic.

(c)

$$
\begin{aligned}
P_{x_1} &= \tfrac{1}{2\pi}\int_0^{2\pi}\cos^2(t)dt \\
&= \tfrac{1}{2\pi}\left(0.5(t+\sin(t)\cos(t))\big|_{t=0}^{2\pi}\right) \\
&= \tfrac{1}{2\pi}\tfrac{1}{2}2\pi = \tfrac{1}{2} \\
P_{x_2} &= \tfrac{1}{2}\int_0^2 \sin^2(\pi t)dt \\
&= \tfrac{1}{2}\left(\tfrac{1}{2\pi}(\pi t - \sin(\pi t)\cos(\pi t))\big|_{t=0}^{2}\right) \\
&= \tfrac{1}{2}\tfrac{1}{2\pi}2\pi = \tfrac{1}{2} \\
P_{x_3} &= \lim_{t\to\infty}\tfrac{1}{2T}\int_{-T}^{T}(\cos(t)+\sin(\pi t))^2\,dt \\
&= \lim_{t\to\infty}\tfrac{1}{2T}\int_{-T}^{T}\left(\cos^2(t)+\sin^2(t)+\cos(t)\sin(\pi t)\right)dt \\
&= P_{x_1}+Px_2+\lim_{t\to\infty}\tfrac{1}{2T}\int_{-T}^{T}0.5\left(\sin(\pi t-t)+\sin(\pi t+t)\right)dt \\
&= P_{x_1}+Px_2+0 = 1
\end{aligned}
$$

Thus,

$$P_{x_1} = P_{x_2} = \frac{1}{2} \text{ and } P_{x_3} = 1.$$

1.3-4. No, $f(t) = \sin(\omega t)$ is not guaranteed to be a periodic function for an arbitrary constant ω. Specifically, if ω is purely imaginary then $f(t)$ is in the form of hyperbolic sine, which is not a periodic function. For example, if $\omega = \jmath$ then $f(t) = \jmath\sinh(t)$. Only when ω is constrained to be real will $f(t)$ be periodic.

1.3-5. (a) $E_{y_1} = \int_{-\infty}^{\infty} y_1^2(t)dt = \int_{-\infty}^{\infty}\frac{1}{9}x^2(2t)dt$. Performing the change of variable $t' = 2t$ yields $\int_{-\infty}^{\infty}\frac{1}{9}x^2(t')\frac{dt'}{2} = \frac{E_x}{18}$. Thus,

$$E_{y_1} = \frac{E_x}{18} \approx \frac{1.0417}{18} = 0.0579.$$

(b) Since $y_2(t)$ is just a $(T_{y_2} = 4)$-periodic replication of $x(t)$, the power is easily obtained as

$$P_{y_2} = \frac{E_x}{T_{y_2}} = \frac{E_x}{4} \approx= 0.2604.$$

(c) Notice, $T_{y_3} = T_{y_2}/2 = 2$. Thus, $P_{y_3} = \frac{1}{2}\int_{T_{y_3}} y_3^2(t)dt = \frac{1}{2}\int_{T_{y_3}}\frac{1}{9}y_2(2t)dt$. Performing the change of variable $t' = 2t$ yields $P_{y_3} = \frac{1}{2}\int_{T_{y_2}}\frac{1}{9}y_2(t')\frac{dt'}{2} = \frac{1}{36}\int_0^4 x(t')dt' = \frac{E_x}{36}$. Thus,

$$P_{y_3} = \frac{E_x}{36} \approx 0.0289.$$

1.3-6. For all parts, $y_1(t) = y_2(t) = t^2$ over $0 \le t \le 1$.

(a) To ensure $y_1(t)$ is even, $y_1(t) = t^2$ over $-1 \le t \le 0$. Since $y_1(t)$ is $(T_1 = 2)$-periodic, $y_1(t) = y_1(t+2)$ for all t. Thus, $y_1(t) = \begin{cases} t^2 & -1 \le t \le 1 \\ y_1(t+2) & \forall t \end{cases}$. $P_{y_1} = \frac{1}{T_1}\int_{-1}^{1}(t^2)^2dt = 0.5\frac{t^5}{5}\Big|_{t=-1}^{1} = 1/5$. Thus,

$$P_{y_1} = 1/5.$$

A sketch of $y_1(t)$ over $-3 \le t \le 3$ is created using MATLAB.

```
>> t = [-3:.001:3]; mt = mod(t,2);
>> y_1 = (mt<=1).*(mt.^2) + (mt>1).*((mt-2).^2);
>> plot(t,y_1,'k'); xlabel('t'); ylabel('y_1(t)'); axis tight;
```

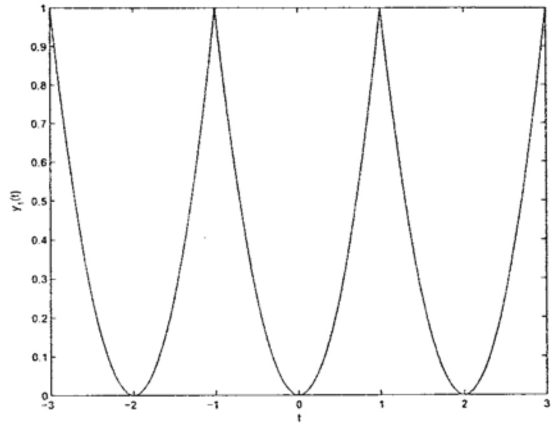

Figure S1.3-6a: Plot of $y_1(t)$

(b) Let

$$y_2(t) = \begin{cases} k & 1 \leq t < 1.5 \\ t^2 & 0 \leq t < 1 \\ -y_2(-t) & \forall t \\ y_2(t+3) & \forall t \end{cases}.$$

With this form, $y_2(t)$ is odd and ($T_2 = 3$)-periodic. The constant k is determined by constraining the power to be unity, $P_{y_2} = 1 = \frac{1}{3}\left(k^2 + \frac{2}{5}t^5\big|_{t=0}^{1}\right)$. Solving for k yields $k^2 = 3 - 2/5 = 13/5$ or $k = \sqrt{13/5}$. Thus,

$$y_2(t) = \begin{cases} \sqrt{13/5} & 1 \leq t < 1.5 \\ t^2 & 0 \leq t < 1 \\ -y_2(-t) & \forall t \\ y_2(t+3) & \forall t \end{cases}.$$

A sketch of $y_2(t)$ over $-3 \leq t \leq 3$ is created using MATLAB.

```
>> t = [-3:.001:3]; mt = mod(t,3);
>> y_2 = (mt<1).*(mt.^2)-(mt>=2).*((mt-3).^2)+...
        ((mt>=1)&(mt<1.5))*sqrt(13/5)-...
        ((mt>=1.5)&(mt<2))*sqrt(13/5);
>> plot(t,y_2,'k'); xlabel('t'); ylabel('y_2(t)'); axis([-3 3 -2 2]);
```

(c) Define $y_3(t) = y_1(t) + jy_2(t)$. To be periodic, $y_3(t)$ must equal $y_3(t+T_3)$ for some value T_3. This implies that $y_1(t) = y_1(t+T_3)$ and $y_2 = y_2(t+T_3)$. Since $y_1(t)$ is ($T_1 = 2$)-periodic, T_3 must be an integer multiple of T_1. Similarly, since $y_2(t)$ is ($T_2 = 3$)-periodic, T_3 must be an integer multiple of T_2. Thus, periodicity of $y_3(t)$ requires $T_3 = T_1k_1 = 2k_1 = T_2k_2 = 3k_2$, which is satisfied letting $k_1 = 3$ and $k_2 = 2$. Thus,

$y_3(t)$ is periodic with $T_3 = 6$.

(d) Noting $y_3(t)y_3^*(t) = y_1^2(t) + y_2^2(t)$, $P_{y_3} = \frac{1}{T_3}\int_{T_3}\left(y_1^2(t) + y_2^2(t)\right)dt = P_{y_1} + P_{y_2}$. Thus,

$$P_{y_3} = 1 + \frac{1}{5} = \frac{6}{5}.$$

1.4-1. Refer to Figure S1.4-1.

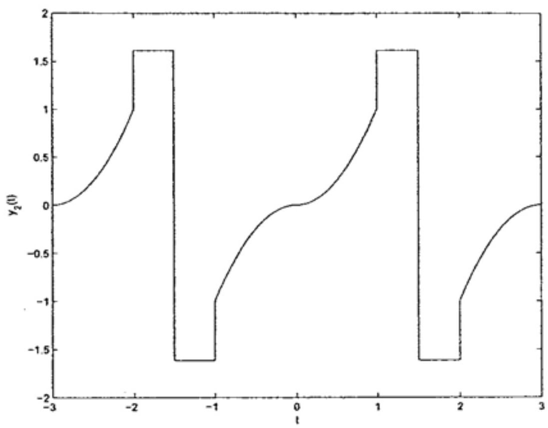

Figure S1.3-6b: Plot of $y_2(t)$

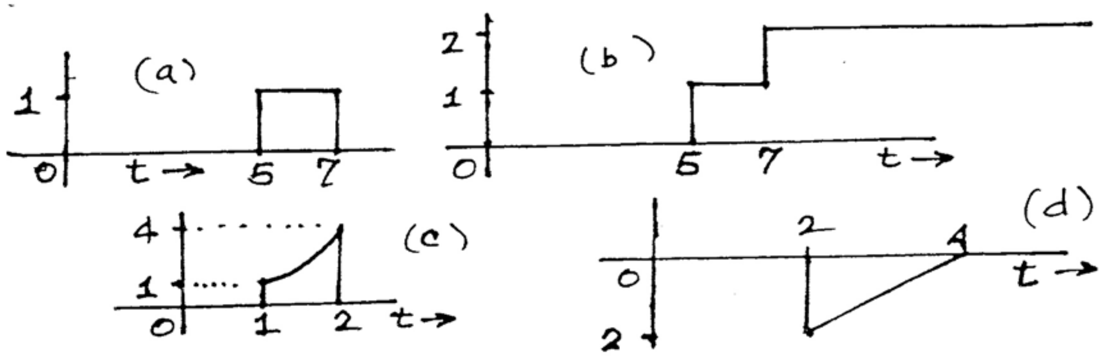

Figure S1.4-1

1.4-2.

$$
\begin{aligned}
x_1(t) &= (4t+1)[u(t+1)-u(t)] + (-2t+4)[u(t)-u(t-2)] \\
&= (4t+1)u(t+1) - 6tu(t) + 3u(t) + (2t-4)u(t-2)
\end{aligned}
$$

$$
\begin{aligned}
x_2(t) &= t^2[u(t)-u(t-2)] + (2t-8)[u(t-2)-u(t-4)] \\
&= t^2u(t) - (t^2-2t+8)u(t-2) - (2t-8)u(t-4)
\end{aligned}
$$

1.4-3. Using the fact that $f(x)\delta(x) = f(0)\delta(x)$, we have

(a) 0
(b) $\frac{2}{9}\delta(\omega)$
(c) $\frac{1}{2}\delta(t)$
(d) $-\frac{1}{5}\delta(t-1)$
(e) $\frac{1}{2-j3}\delta(\omega+3)$
(f) $k\delta(\omega)$ (use L' Hôpital's rule)

1.4-4. In these problems remember that impulse $\delta(x)$ is located at $x = 0$. Thus, an impulse $\delta(t-\tau)$ is located at $\tau = t$, and so on.

(a) The impulse is located at $\tau = t$ and $x(\tau)$ at $\tau = t$ is $x(t)$. Therefore

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau = x(t).$$

(b) The impulse $\delta(\tau)$ is at $\tau = 0$ and $x(t-\tau)$ at $\tau = 0$ is $x(t)$. Therefore

$$\int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)\,d\tau = x(t).$$

Using similar arguments, we obtain

(c) 1
(d) 0
(e) e^3
(f) 5
(g) $x(-1)$
(h) $-e^2$

1.4-5. For sketches, refer to Figure S1.4-5.

(a) Recall that the derivative of a function at the jump discontinuity is equal to an impulse of strength equal to the amount of discontinuity. Hence, dx/dt contains impulses $4\delta(t+4)$ and $2\delta(t-2)$. In addition, the derivative is -1 over the interval $(-4,\ 0)$, and is 1 over the interval $(0,\ 2)$. The derivative is zero for $t < -4$ and $t > 2$. The result is sketched in Figure S1.4-5(a).

(b) Using the procedure in part (a), Figure S1.4-5(b) depicts d^2x/dt^2 for the signal in Figure P1.4-2a.

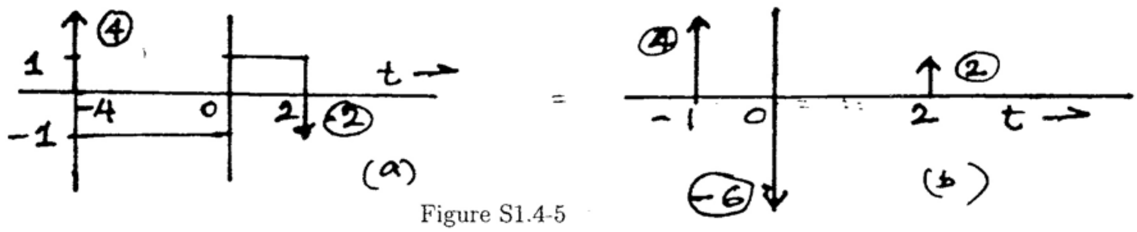

Figure S1.4-5

1.4-6. For sketches, refer to Figure S1.4-6.

(a) Recall that the area under an impulse of strength k is k. Over the interval $0 \le t \le 1$, we have

$$y(t) = \int_0^t 1\,dx = t \qquad 0 \le t \le 1.$$

Over the interval $0 \le t < 3$, we have

$$y(t) = \int_0^1 1\,dx + \int_1^t (-1)\,dx = 2 - t \qquad 1 \le t < 3.$$

At $t = 3$, the impulse (of strength unity) yields an additional term of unity. Thus (assuming $\epsilon \to 0$),

$$y(t) = \int_0^1 1\,dx + \int_1^{3-\epsilon} (-1)\,dx + \int_{3-\epsilon}^{t} \delta(x-3)\,dx = 1 + (-2) + 1 = 0 \qquad t > 3$$

(b)

$$y(t) = \int_0^t [1-\delta(x-1)-\delta(x-2)-\delta(x-3)+\cdots]\,dx = tu(t)-u(t-1)-u(t-2)-u(t-3)-\ldots$$

Figure S1.4-6

1.4-7. Changing the variable t to $-x$, we obtain

$$\int_{-\infty}^{\infty} \phi(t)\delta(-t)\,dt = -\int_{\infty}^{-\infty} \phi(-x)\delta(x)\,dx = \int_{-\infty}^{\infty} \phi(-x)\delta(x)\,dx = \phi(0).$$

This shows that

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt = \int_{-\infty}^{\infty} \phi(t)\delta(-t)\,dt = \phi(0).$$

Therefore

$$\delta(t) = \delta(-t).$$

1.4-8. Letting $at = x$, we obtain (for $a > 0$)

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{a}\int_{-\infty}^{\infty} \phi(\frac{x}{a})\delta(x)\,dx = \frac{1}{a}\phi(0)$$

Similarly for $a < 0$, we show that this integral is $-\frac{1}{a}\phi(0)$. Therefore

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{|a|}\phi(0) = \frac{1}{|a|}\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt$$

Therefore

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

1.4-9.

$$\begin{aligned}\int_{-\infty}^{\infty} \dot\delta(t)\phi(t)\,dt &= \phi(t)\delta(t)|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \dot\phi(t)\delta(t)\,dt \\ &= 0 - \int \dot\phi(t)\delta(t)\,dt = -\dot\phi(0)\end{aligned}$$

1.4-10. For sketches, refer to Figure S1.4-10.

(a) $s_{1,2} = \pm j3$

(b) $e^{-3t}\cos 3t = 0.5[e^{-(3+j3)t} + e^{-(3-j3)t}]$. Therefore the frequencies are $s_{1,2} = -3 \pm j3$.

(c) Using the argument in (b), we find the frequencies $s_{1,2} = 2 \pm j3$

(d) $s = -2$

(e) $s = 2$

(f) $5 = 5e^{0t}$ so that $s = 0$.

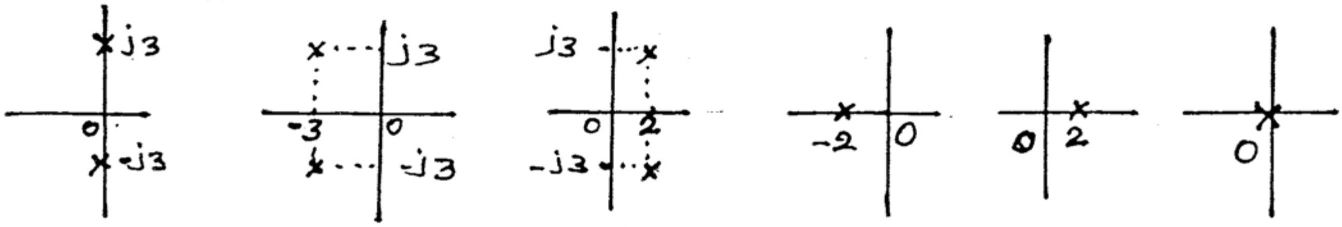

Figure S1.4-10

1.5-1. For sketches, refer to Figure S1.5-1.

(a) $x_e(t) = 0.5[u(t) + u(-t)] = 0.5$ and $x_o(t) = 0.5[u(t) - u(-t)]$.

(b) $x_e(t) = 0.5[tu(t) - tu(-t)] = 0.5|t|$ and $x_o(t) = 0.5[tu(t) + tu(-t)] = 0.5t$.

(c) $x_e(t) = 0.5[\sin\omega_0 t + \sin(-\omega_0 t)] = 0$ and $x_o(t) = 0.5[\sin\omega_0 t - \sin(-\omega_0 t)] = \sin\omega_0 t$.

(d) $x_e(t) = 0.5[\cos\omega_0 t + \cos(-\omega_0 t)] = \cos\omega_0 t$ and $x_o(t) = 0.5[\cos\omega_0 t - \cos(-\omega_0 t)] = 0$.

(e) $\cos(\omega_0 t + \theta) = \cos\omega_0 t\cos\theta - \sin\omega_0 t\sin\theta$. Hence $x_e(t) = \cos\omega_0 t\cos\theta$ and $x_o(t) = -\sin\omega_0 t\sin\theta$

(f) $x_e(t) = 0.5[\sin\omega_0 t\,u(t) + \sin(-\omega_0 t)u(-t)] = 0.5[\sin\omega_0 t\,u(t) - \sin\omega_0 t\,u(-t)]$ and $x_o(t) = 0.5[\sin\omega_0 t\,u(t) - \sin(-\omega_0 t)u(-t)] = 0.5[\sin\omega_0 t\,u(t) + \sin\omega_0 t\,u(-t)] = 0.5\sin\omega_0 t$.

(g) $x_e(t) = 0.5[\cos\omega_0 t\,u(t) + \cos(-\omega_0 t)u(-t)] = 0.5[\cos\omega_0 t\,u(t) + \cos\omega_0 t\,u(-t)] = 0.5\cos\omega_0 t$ and $x_o(t) = 0.5[\cos\omega_0 t\,u(t) - \cos(-\omega_0 t)u(-t)] = 0.5[\cos\omega_0 t\,u(t) - \cos\omega_0 t\,u(-t)]$.

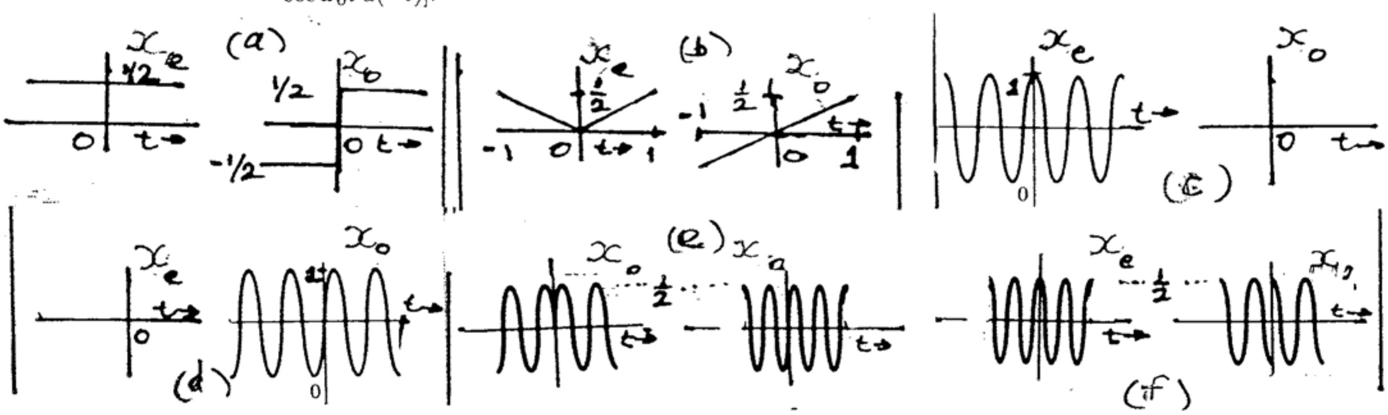

Figure S1.5-1

1.5-2. (a)

$$x_e(t) = \frac{1}{2}[e^{-2t}u(t) + e^{2t}u(-t)]$$

$$x_0(t) = \frac{1}{2}[e^{-2t}u(t) - e^{2t}u(-t)]$$

(b) $E_{x_e} = \int_{-\infty}^{\infty} x_e^2(t)dt$. Because $e^{-2t}u(t)$ and $e^{2t}u(-t)$ are disjoint in time, the cross-product term in $x_e^2(t)$ is zero. Hence,

$$E_{x_e} = \int_{-\infty}^{\infty} x_e^2(t)dt = \frac{1}{4}\left[\int_0^{\infty} e^{-4t}dt + \int_{-\infty}^{0} e^{4t}dt\right] = \frac{1}{8}.$$

Using a similar argument, we have

$$E_{x_o} = \frac{1}{8}.$$

Also,

$$E_x = \int_0^{\infty} e^{-4t}dt = \frac{1}{4}.$$

Hence,

$$E_x = E_{x_e} + E_{x_o}.$$

(c) To generalize this result, we first consider causal $x(t)$. In this case, $x(t)$ and $x(-t)$ are disjoint. Moreover, energy or $x(t)$ is identical to that of $x(-t)$. Hence,

$$E_{x_e} = \frac{1}{4}\left[\int_0^{\infty} |x(t)|^2 dt + \int_{-\infty}^{0} |x(-t)|^2 dt\right] = \frac{1}{2}E_x.$$

Using a similar argument, it follows that $E_{x_o} = \frac{1}{2}E_x$. Hence, for causal signals,

$$E_x = E_{x_e} + E_{x_o}$$

Identical arguments hold for anti-causal signals. Thus, for anti-causal signal $x(t)$

$$E_x = E_{x_e} + E_{x_o}$$

Now, every signal can be expressed as a sum of a causal and an anti-causal signal. Also, the signal energy is equal to the sum of energies of the causal and the anti-causal components. Hence, it follows that for a general case

$$E_x = E_{x_e} + E_{x_o}$$

1.5-3. (a)

$$\begin{aligned} x_e(t)x_o(t) &= \frac{1}{4}[x(t) + x(-t)][x(t) - x(-t)] \\ &= \frac{1}{4}[|x(t)|^2 - |x(-t)|^2] \end{aligned}$$

Since the areas under $|x(t)|^2$ and $|x(-t)|^2$ are identical, it follows that

$$\int_{-\infty}^{\infty} x_e(t)x_o(t)dt = 0$$

(b)

$$\int_{-\infty}^{\infty} x_e(t)dt = \frac{1}{2}\int_{-\infty}^{\infty} x(t)dt + \frac{1}{2}\int_{-\infty}^{\infty} x(-t)dt$$

Because the areas under $x(t)$ and $x(-t)$ are identical, it follows that

$$\int_{-\infty}^{\infty} x_e(t)dt = \int_{-\infty}^{\infty} x(t)dt.$$

1.5-4. $x_o(t) = 0.5(x(t) - x(-t)) = 0.5(\sin(\pi t)u(t) - \sin(-\pi t)u(-t)) = 0.5\sin(\pi t)(u(t) + u(-t))$. Since $\sin(0) = 0$, this reduces to $x_o(t) = 0.5\sin(\pi t)$, which is a $(T = 2)$-periodic signal. Therefore,

$$x_o(t) = 0.5\sin(\pi t) \text{ is a periodic signal.}$$

1.5-5. $x_e(t) = 0.5(x(t) + x(-t)) = 0.5(\cos(\pi t)u(t) + \cos(-\pi t)u(-t)) = 0.5\cos(\pi t)(u(t) + u(-t))$. Written another way, $x_e(t) = \begin{cases} 0.5\cos(\pi t) & t \neq 0 \\ 1 & t = 0 \end{cases}$. Since there exists no $T \neq 0$ such that $x_e(t+T) = x_e(t)$,

$$x_e(t) \text{ is not a periodic function.}$$

It is worth pointing out that sometimes the unit step is defined as $u(t) = \begin{cases} 1 & t > 0 \\ 0.5 & t = 0 \\ 0 & t < 0 \end{cases}$. Using this alternate definition, $x_e(t)$ is periodic.

1.5-6. (a) Using the figure, $x(t) = (t+1)(u(t+1)-u(t)) + (-t+1)(u(t)-u(t-1))$. MATLAB is used to plot $v(t) = 3x\left(-\frac{1}{2}(t+1)\right)$.

```
>> x = inline('(t+1).*((t>=-1)&(t<0))+(-t+1).*((t>=0)&(t<1))');
>> t = [-5:.001:5]; v = 3*x(-0.5*(t+1));
>> plot(t,v,'k-'); xlabel('t'); ylabel('v(t)');
```

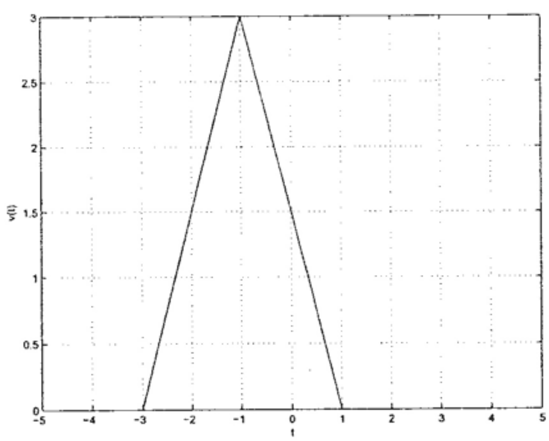

Figure S1.5-6a: Plot of $v(t) = 3x\left(-\frac{1}{2}(t+1)\right)$.

(b) Since $v(t)$ is finite duration, $P_v = 0$. Signal is unaffected by shifting, so $v(t)$ is shifted to start at $t = 0$. By symmetry, the energy of the first half is equal to the energy of the second half. Thus, $E_v = 2\int_0^2 \left(\frac{3}{2}t\right)^2 dt = 2\frac{9}{4}\frac{t^3}{3}\Big|_{t=0}^{t=2} = \frac{24}{2} = 12$. Thus,

$$E_v = 12 \text{ and } P_v = 0.$$

(c) Using 1.5-6a, $v(t) = (1.5t+4.5)(u(t+3)-u(t+1))+(-1.5t+1.5)(u(t+1)-u(t-1))$. MATLAB is used to determine and plot $v_e(t)$.

```
>> v_str = ['(1.5*t+4.5).*((t>=-3)&(t<-1))+',...
            '(-1.5*t+1.5).*((t>=-1)&(t<1))'];
>> v = inline(v_str); t = [-4:.001:4]; v_e = 0.5*(v(t)+v(-t));
>> plot(t,v_e,'k'); xlabel('t'); ylabel('v_e(t)');
>> axis([-4 4 -.25 1.75]);
```

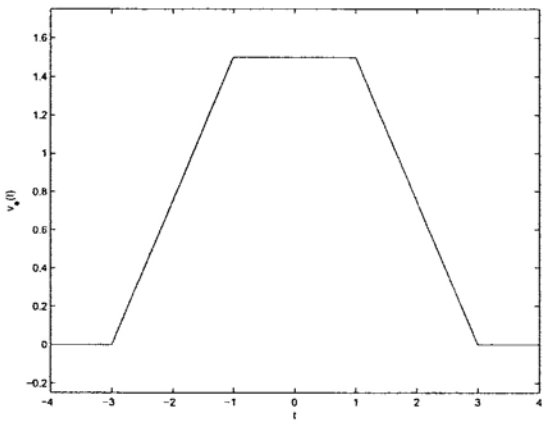

Figure S1.5-6c: Plot of $v_e(t)$.

Thus,

$$v_e(t) = \begin{cases} 3t/4 + 9/4 & -3 \le t < -1 \\ 1.5 & -1 \le t < 1 \\ -3t/4 + 9/4 & 1 \le t < 3 \\ 0 & \text{otherwise} \end{cases}.$$

(d) Using 1.5-6a, $v(t) = (1.5t+4.5)(u(t+3)-u(t+1))+(-1.5t+1.5)(u(t+1)-u(t-1))$. MATLAB is used to create the four desired plots.

```
>> v_str = ['(1.5*t+4.5).*((t>=-3)&(t<-1))+',...
            '(-1.5*t+1.5).*((t>=-1)&(t<1))'];
>> v = inline(v_str);
>> t = [-6:.001:1]; a = 2; b = 3; ax = [-6 1 -.5 9.5];
>> subplot(221); plot(t,v(a*t+b),'k-'); grid;
>> xlabel('t'); ylabel('v(at+b)'); axis(ax);
>> subplot(222); plot(t,v(a*t)+b,'k-'); grid;
>> xlabel('t'); ylabel('v(at)+b'); axis(ax);
>> subplot(223); plot(t,a*v(t+b),'k-'); grid;
>> xlabel('t'); ylabel('av(t+b)'); axis(ax);
>> subplot(224); plot(t,a*v(t)+b,'k-'); grid;
>> xlabel('t'); ylabel('av(t)+b'); axis(ax);
```

(e) Using 1.5-6a, $v(t) = (1.5t+4.5)(u(t+3)-u(t+1))+(-1.5t+1.5)(u(t+1)-u(t-1))$. MATLAB is used to create the four desired plots.

```
>> v_str = ['(1.5*t+4.5).*((t>=-3)&(t<-1))+',...
            '(-1.5*t+1.5).*((t>=-1)&(t<1))'];
>> v = inline(v_str);
>> t = [-3:.001:3]; a = -3; b = -2; ax = [-3 3 -11.5 3.5];
```

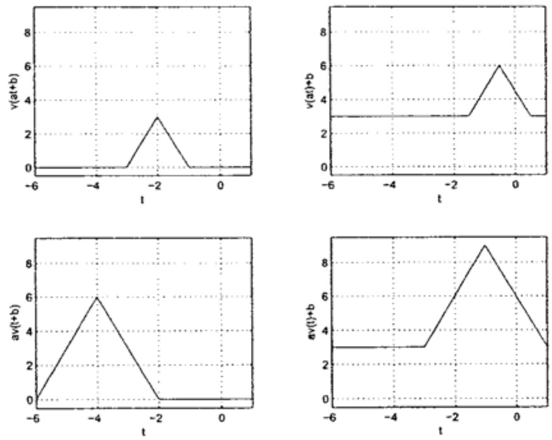

Figure S1.5-6d: Plots of $v(2t+3)$, $v(2t)+3$, $2v(t+3)$, and $2v(t)+3$.

```
>> subplot(221); plot(t,v(a*t+b),'k-'); grid;
>> xlabel('t'); ylabel('v(at+b)'); axis(ax);
>> subplot(222); plot(t,v(a*t)+b,'k-'); grid;
>> xlabel('t'); ylabel('v(at)+b'); axis(ax);
>> subplot(223); plot(t,a*v(t+b),'k-'); grid;
>> xlabel('t'); ylabel('av(t+b)'); axis(ax);
>> subplot(224); plot(t,a*v(t)+b,'k-'); grid;
>> xlabel('t'); ylabel('av(t)+b'); axis(ax);
```

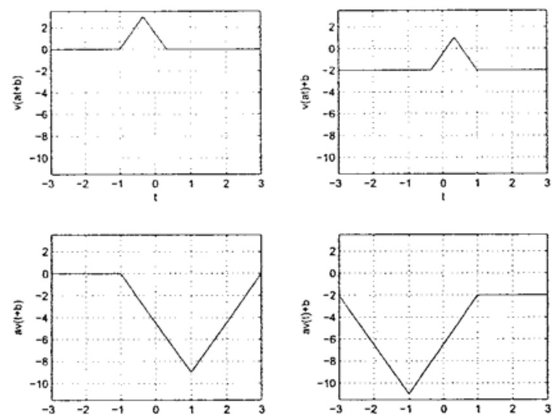

Figure S1.5-6e: Plots of $v(-3t-2)$, $v(-3t)-2$, $-3v(t-2)$, and $-3v(t)-2$.

1.5-7. (a) Using the figure, $y(t) = t(u(t) - u(t-1)) + (u(t-1) - u(t-2))$. MATLAB is used to plot $y_o(t) = \frac{y(t)-y(-t)}{2}$.

```
>> y = inline('t.*((t>=0)&(t<1))+((t>=1)&(t<2))');
>> t = [-5:.001:5]; y_o = (y(t)-y(-t))/2;
>> plot(t,y_o,'k-'); xlabel('t'); ylabel('y_o(t)'); axis([-5 5 -.6 .6]);
```

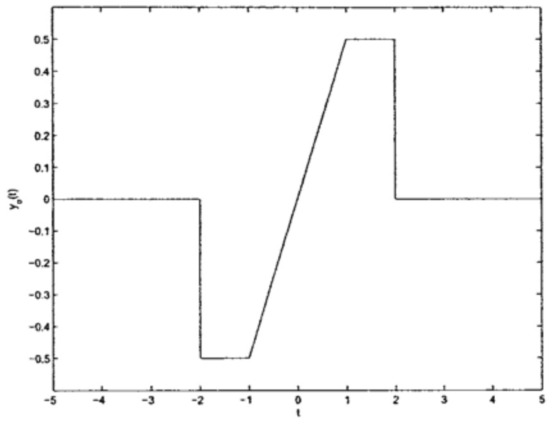

Figure S1.5-7a: Plot of $y_o(t) = \frac{y(t)-y(-t)}{2}$.

Thus,

$$y_o(t) = \begin{cases} -1/2 & -2 \leq t < -1 \\ t/2 & -1 \leq t < 1 \\ 1/2 & 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases}.$$

(b) Since $y(t) = 0.2x(-2t-3)$, $5y(-0.5t-1.5) = 5(0.2)x(-2(-0.5t-1.5)-3) = x(t)$. MATLAB is used to sketch $x(t)$.

```
>> y = inline('t.*((t>=0)&(t<1))+((t>=1)&(t<2))');
>> t = [-8:.001:0]; x = 5*y(-0.5*t-1.5);
>> plot(t,x,'k-'); xlabel('t'); ylabel('x(t)'); axis([-8 0 -.5 5.5]);
```

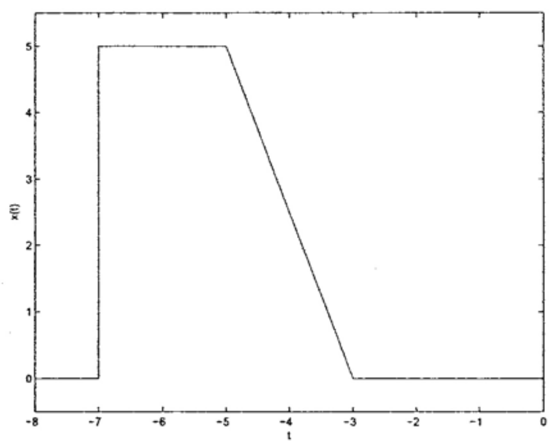

Figure S1.5-7b: Plot of $x(t) = 5y(-0.5t-1.5)$.

Thus,

$$x(t) = \begin{cases} 5 & -7 \leq t < -5 \\ -5(t+3)/2 & -5 \leq t < -3 \\ 0 & \text{otherwise} \end{cases}.$$

1.5-8. Let the graphed signal be named $y(t)$.

(a) Since $y(t) = -0.5x(-3t+2)$, $-2y(-t/3+2/3) = -2(-0.5)x(-3(-t/3+2/3)+2) = x(t)$. MATLAB is used to sketch $x(t)$.

```
>> y = inline('((t>=-1)&(t<0))+(-t+1).*((t>=0)&(t<1))');
>> t = [-2:.001:6]; x = -2*y(-t/3+2/3);
>> plot(t,x,'k-'); xlabel('t'); ylabel('x(t)'); axis([-2 6 -2.5 0.5]);
```

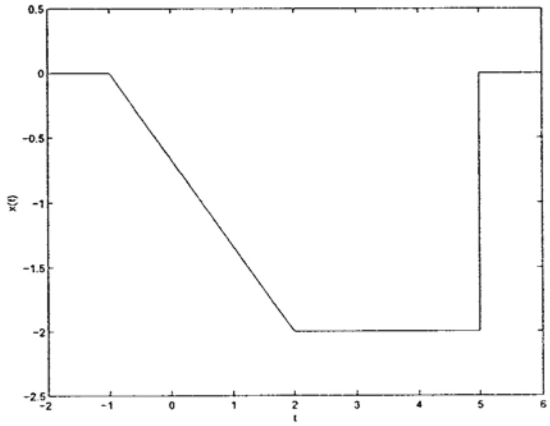

Figure S1.5-8a: Plot of $x(t) = -2y(-t/3 + 2/3)$.

Thus,

$$x(t) = \begin{cases} -2(t+1)/3 & -1 \leq t < 2 \\ -2 & 2 \leq t < 5 \\ 0 & \text{otherwise} \end{cases}.$$

(b) The even portion of $x(t)$ is $x_e(t) = 0.5(x(t) + x(-t))$.

```
>> x = inline('-2*(t+1)/3.*((t>=-1)&(t<2))-2*((t>=2)&(t<5))');
>> t = [-6:.001:6]; x_e = 0.5*(x(t)+x(-t));
>> plot(t,x_e,'k-'); xlabel('t'); ylabel('x_e(t)'); axis([-6 6 -1.5 0.5]);
```

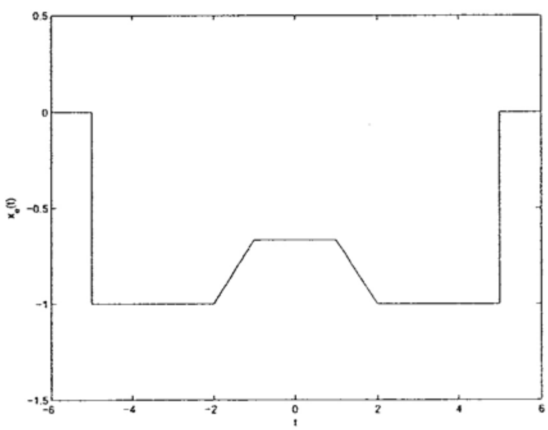

Figure S1.5-8b: Plot of $x_e(t)$.

Thus,

$$x_e(t) = \begin{cases} -1 & 2 \leq |t| < 5 \\ (-|t|-1)/3 & 1 \leq |t| < 2 \\ -2/3 & |t| < 1 \\ 0 & \text{otherwise} \end{cases}.$$

(c) The odd portion of $x(t)$ is $x_o(t) = 0.5(x(t) - x(-t))$.

```
>> x = inline('-2*(t+1)/3.*((t>=-1)&(t<2))-2*((t>=2)&(t<5))');
>> t = [-6:.001:6]; x_o = 0.5*(x(t)-x(-t));
>> plot(t,x_o,'k-'); xlabel('t'); ylabel('x_e(t)'); axis([-6 6 -1.5 1.5]);
```

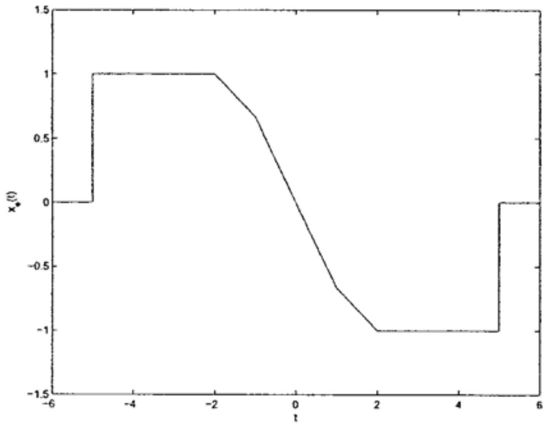

Figure S1.5-8c: Plot of $x_o(t)$.

Thus,

$$x_o(t) = \begin{cases} 1 & -5 \leq t < -2 \\ (-t+1)/3 & -2 \leq t < -1 \\ -2t/3 & -1 \leq t < 1 \\ -(t+1)/3 & 1 \leq t < 2 \\ -1 & 2 \leq t < 5 \\ 0 & \text{otherwise} \end{cases}.$$

1.5-9. Notice, $w_{cs}^*(-t) = 0.5(w(-t) + w^*(t))^* = 0.5(w^*(-t) + w(t)) = w_{cs}(t)$. In Cartesian form, this becomes $w_{cs}^*(-t) = x(-t) - jy(-t) = x(t) + jy(t) = w_{cs}(t)$. Equating the real portions yields $x(-t) = x(t)$, and equating the imaginary portions yields $-y(-t) = y(t)$. Thus, by definition, the real portion of $w_{cs}(t)$ is even and the imaginary portion of $w_{cs}(t)$ is odd.

1.5-10. Notice, $-w_{ca}^*(-t) = -0.5(w(-t) - w^*(t))^* = 0.5(-w^*(-t) + w(t)) = w_{ca}(t)$. In Cartesian form, this becomes $-w_{ca}^*(-t) = -x(-t) + jy(-t) = x(t) + jy(t) = w_{ca}(t)$. Equating the real portions yields $-x(-t) = x(t)$, and equating the imaginary portions yields $y(-t) = y(t)$. Thus, by definition, the real portion of $w_{ca}(t)$ is odd and the imaginary portion of $w_{ca}(t)$ is even.

1.5-11. Complex signal $w(t)$ is defined over $(0 \leq t \leq 1)$.

(a) Assigning certain properties to $w(t)$ allows us to plot $w(t)$ over $(-1 \leq t \leq 1)$.

i. If $w(t)$ is even, $w(t) = w(-t)$, it is even in both the real and imaginary components. Thus, the graph folds back on itself and appears unchanged.

Consider, for example, point (2,1), which now corresponds to both $t = 1$ and $t = -1$.

ii. If $w(t)$ is odd, $w(t) = -w(-t)$, it is odd in both the real and imaginary components. Thus, the graph reflects about both the real and imaginary axes.

iii. If $w(t)$ is conjugate symmetric, $w(t) = w^*(-t)$, it is even in the real component and odd in the imaginary component. Thus, the graph reflects about the real axis.

iv. If $w(t)$ is conjugate antisymmetric, $w(t) = -w^*(-t)$, it is odd in the real component and even in the imaginary component. Thus, the graph reflects about the imaginary axis.

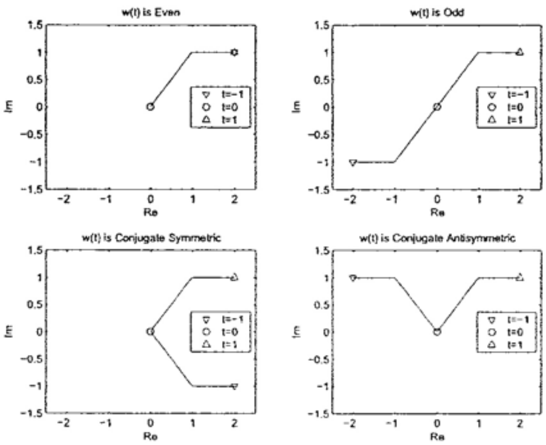

Figure S1.5-11a: Plots of $w(t)$.

(b) Since $w(t)$ is only given over $(0 \leq t \leq 1)$, $w(3t)$ can be determined only for $(0 \leq t \leq 1/3)$. Since the function does not change, only the time at which it occurs, the complex-plane graph of $w(3t)$ looks identical to the original complex-plane graph of $w(t)$ with the exception that the points are assigned different times. For example, point (2,1) occurs now at $t = 1/3$.

1.5-12. (a) We know $x(t) = t^2(1+j)$ over $(1 \leq t \leq 2)$. Since $x(t)$ is skew-Hermitian, $x(t) = -x^*(-t)$ and thus $x(t) = -t^2(1-j)$ over $(-2 \leq t \leq -1)$. To minimize energy, $x(t)$ is set to zero everywhere else. Thus,

$$x(t) = \begin{cases} t^2(1+j) & 1 \leq t \leq 2 \\ -t^2(1-j) & -2 \leq t \leq -1 \\ 0 & \text{otherwise} \end{cases}.$$

(b) MATLAB is used to sketch $y(t) = \text{Real}\{x(t)\}$.

```
>> x_str = ['(t.^2*(1+j)).*((t>=1)&(t<=2))+',...
            '(-t.^2*(1-j)).*((t>=-2)&(t<=-1))'];
>> x = inline(x_str);
>> t = [-2.5:.001:2.5]; plot(t,real(x(t)),'k');
>> xlabel('t'); ylabel('y(t)'); axis([-2.5 2.5 -4.5 4.5]);
```

(c) MATLAB is used to sketch $z(t) = \text{Real}\{jx(-2t+1)\}$.

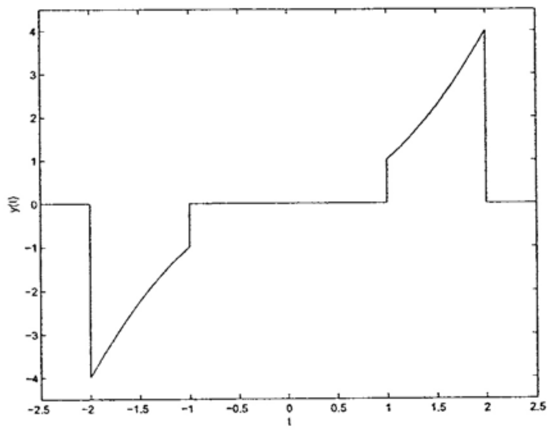

Figure S1.5-12b: Plot of $y(t) = \text{Real}\{x(t)\}$.

```
>> x_str = ['(t.^2*(1+j)).*((t>=1)&(t<=2))+',...
           '(-t.^2*(1-j)).*((t>=-2)&(t<=-1))'];
>> x = inline(x_str);
>> t = [-2.5:.001:2.5]; plot(t,real(j*x(-2*t+1)),'k');
>> xlabel('t'); ylabel('z(t)'); axis([-2.5 2.5 -4.5 4.5]);
```

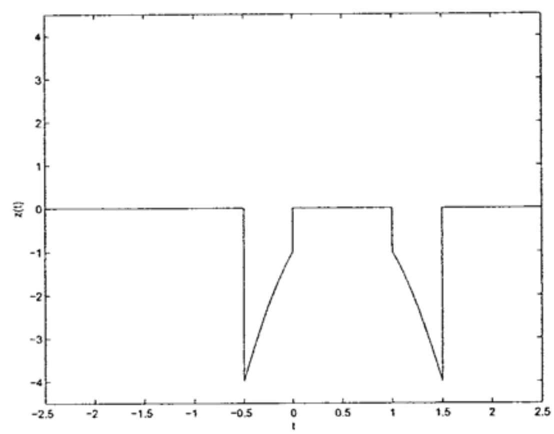

Figure S1.5-12c: Plot of $z(t) = \text{Real}\{jx(-2t+1)\}$.

(d) Since $x(t)$ is finite duration, $P_x = 0$. Using symmetry, $E_x = 2\int_1^2 (t^2(1+j))(t^2(1-j))dt = 2\int_1^2 2t^4 dt = \frac{4t^5}{5}\Big|_{t=1}^{2} = \frac{4}{5}(32-1) = \frac{124}{5}$. Thus,

$$E_x = \frac{124}{5} = 24.8 \text{ and } P_x = 0.$$

1.6-1. If $x(t)$ and $y(t)$ are the input and output, respectively, of an ideal integrator, then

$$\dot{y}(t) = x(t)$$

and

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau = \int_{-\infty}^{0} x(\tau)\,d\tau + \int_{0}^{t} x(\tau)\,d\tau = \underbrace{y(0)}_{\text{zero-input}} + \underbrace{\int_{0}^{t} x(\tau)\,d\tau}_{\text{zero-state}}$$

1.6-2. From Newton's law

$$x(t) = M\frac{dv}{dt}$$

and

$$v(t) = \frac{1}{M}\int_{-\infty}^{t} x(\tau)\,d\tau = \frac{1}{M}\int_{-\infty}^{0} x(\tau)\,d\tau + \frac{1}{M}\int_{0}^{t} x(\tau)\,d\tau = v(0) + \frac{1}{M}\int_{0}^{t} x(\tau)\,d\tau$$

1.7-1. Only (b), (f), and (h) are linear. All the remaining are nonlinear. This can be verified by using the procedure discussed in Example 1.9.

1.7-2. (a) The system is time-invariant because the input $x(t)$ yields the output $y(t) = x(t-2)$. Hence, if the input is $x(t-T)$, the output is $x(t-T-2) = y(t-T)$, which makes the system time-invariant.

(b) The system is time-varying. The input $x(t)$ yields the output $y(t) = x(-t)$. Thus, the output is obtained by changing the sign of t in $x(t)$. Therefore, when the input is $x(t-T)$, the output is $x(-t-T) = x(-[t+T]) = y(t+T)$, which represents the original output advanced by T (not delayed by T).

(c) The system is time-varying. The input $x(t)$ yields the output $y(t) = x(at)$, which is a scaled version of the input. Thus, the output is obtained by replacing t in the input with at. Thus, if the input is $x(t-T)$ ($x(t)$ delayed by T), the output is $x(at-T) = x(a[t-\frac{T}{a}])$, which is $x(at)$ delayed by T/a (not T). Hence the system is time-varying.

(d) The system is time-varying. The input $x(t)$ yields the output $y(t) = tx(t)$. For the input $x(t-T)$, the output is $tx(t-T)$, which is not $tx(t)$ delayed by T. Hence the system is time-varying.

(e) The system is time-varying. The output is a constant, given by the area under $x(t)$ over the interval $|t| \le 5$. Now, if $x(t)$ is delayed by T, the output, which is the area under the delayed $x(t)$, is another constant. But this output is not the same as the original output delayed by T. Hence the system is time-varying.

(f) The system is time-invariant. The input $x(t)$ yields the output $y(t)$, which is the square of the second derivative of $x(t)$. If the input is delayed by T, the output is also delayed by T. Hence the system is time-invariant.

1.7-3. We construct the table below from the first three rows of data. Let r_j denote the jth row.

Row	$x(t)$	$q_1(0)$	$q_2(0)$	$y(t)$
r_1	0	1	-1	$e^{-t}u(t)$
r_2	0	2	1	$e^{-t}(3t+2)u(t)$
r_3	$u(t)$	-1	-1	$2u(t)$
$r_4 = \frac{1}{3}(r_1+r_2)$	0	1	0	$(t+1)e^{-t}u(t)$
$r_5 = \frac{1}{2}(r_1+r_3)$	$\frac{1}{2}u(t)$	0	-1	$(\frac{1}{2}e^{-t}+1)u(t)$
$r_6 = (r_4+r_5)$	$\frac{1}{2}u(t)$	1	-1	$(1.5e^{-t}+te^{-t}+1)u(t)$
$r_7 = 2(r_6+r_1)$	$u(t)$	0	0	$(e^{-t}+2te^{-t}+2)u(t)$

In our case, the input $x(t) = u(t+5) - u(t-5)$. From r_7 and the superposition and time-invariance properties, we have

$$\begin{aligned} y(t) &= r_7(t+5) - r_7(t-5) \\ &= \left[e^{-(t+5)} + 2(t+5)e^{-(t+5)} + 2\right] u(t+5) - \left[e^{-(t-5)} + 2(t-5)e^{-(t-5)} + 2\right] u(t-5) \end{aligned}$$

1.7-4. If the input is $kx(t)$, the new output $y(t)$ is

$$y(t) = k^2x^2(t)/\left(k\frac{dx}{dt}\right) = k[x^2(t)/\left(\frac{dx}{dt}\right)]$$

Hence the homogeneity is satisfied. If the input-output pair is denoted by $x_i \to y_i$, then

$$x_1 \to y_1 = (x_1)^2/(\dot{x}_1) \quad \text{and} \quad x_2 \to y_2 = (x_2)^2/(\dot{x}_2)$$

$$\text{But} \quad x_1 + x_2 \to (x_1+x_2)^2/(\dot{x}_1+\dot{x}_2) \neq y_1 + y_2$$

1.7-5. From the hint it is clear that when $v_c(0) = 0$, the capacitor may be removed, and the circuit behaves as shown in Figure S1.7-5. It is clearly zero-state linear. To show that it is zero-input nonlinear, consider the circuit with $x(t) = 0$ (zero-input). The current $y(t)$ has the same direction (shown by arrow) regardless of the polarity of v_c (because the input branch is a short). Thus the system is zero-input nonlinear.

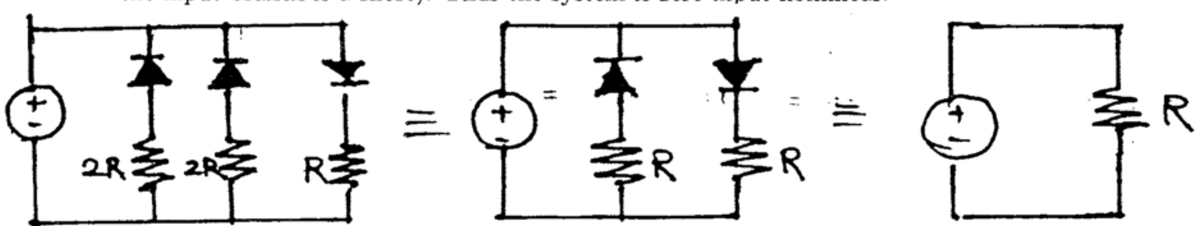

Figure S1.7-5

1.7-6. The solution is trivial. The input is a current source, which has infinite impedance. Hence, as far as the output $y(t)$ is concerned, the circuit behaves as shown in Figure S1.7-6. The nonlinear elements are irrelevant in computing the output $y(t)$, and the output $y(t)$ satisfies the linearity conditions. Yet, the circuit is nonlinear because it contains nonlinear elements.

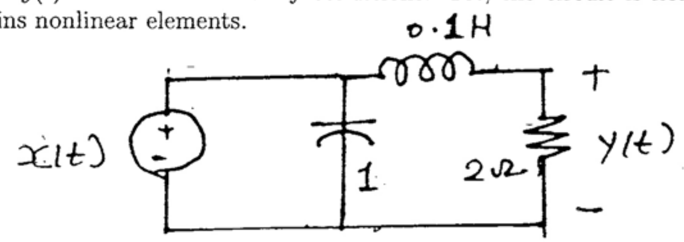

Figure S1.7-6

1.7-7. (a) $y(t) = x(t-2)$. Thus, the output $y(t)$ always starts after the input by 2 seconds (see Figure S1.7-7a). Clearly, the system is causal.

(b) $y(t) = x(-t)$. The output $y(t)$ is obtained by time inversion in the input. Thus, if the input starts at $t = 0$, the output starts before $t = 0$ (see Figure S1.7-7b). Hence, the system is not causal.

(c) $y(t) = x(at)$, $a > 1$. The output $y(t)$ is obtained by time compression of the input by factor a. Hence, the output can start before the input (see Figure S1.7-7c), and the system is not causal.

(d) $y(t) = x(at)$, $a < 1$. The output $y(t)$ is obtained by time expansion of the input by factor $1/a$. Hence, the output can start before the input (see Figure S1.7-7d), and the system is not causal.

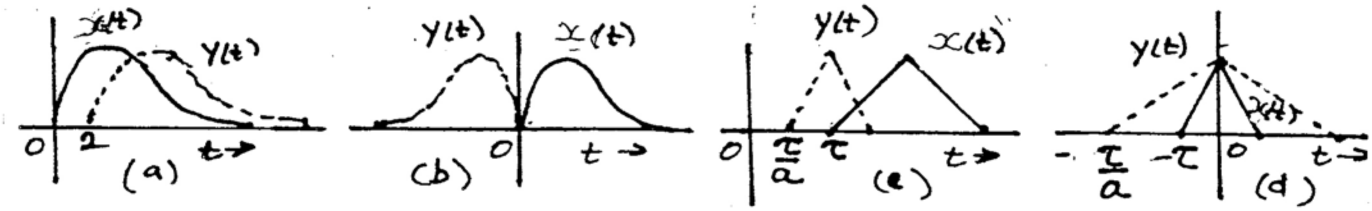

Figure S1.7-7

1.7-8. (a) Invertible because the input can be obtained by taking the derivative of the output. Hence, the inverse system equation is $y(t) = dx/dt$.

(b) Not invertible for even values of n, because the sign information is lost. However, the system is invertible for odd values of n. The inverse system equation is $y(t) = [x(t)]^{1/n}$.

(c) Not invertible because differentiation operation irretrievable loses the constant part of $x(t)$.

(d) The system $y(t) = x(3t - 6) = x(3[t - 2])$ represents an operation of signal compression by factor 3, and then time delay by 2 seconds. Hence, the input can be obtained from the output by first advancing the output by 2 seconds, and then time-expanding by factor 3. Hence, the inverse system equation is $y(t) = x(\frac{t}{3} + 2)$. Although the system is invertible, it is not realizable because it involves the operation of signal compression and signal advancing (which makes it noncausal). However, if we can accept time delay, we can realize a noncausal system.

(e) Not invertible because cosine is a multiple valued function, and $\cos^{-1}[x(t)]$ is not unique.

(f) Invertible. $x(t) = \ln y(t)$.

1.7-9. (a) Yes, the system is linear. Begin assuming $y_1(t) = r(t)x_1(t)$ and $y_2(t) = r(t)x_2(t)$. Applying $ax_1(t) + bx_2(t)$ to the system yields $y(t) = r(t)\,(ax_1(t) + bx_2(t)) = ar(t)x_1(t) + br(t)x_2(t) = ay_1(t) + by_2(t)$.

(b) Yes, the system is memoryless. By inspection, it is clear that the system only depends on the current input.

(c) Yes, the system is causal. Since the system is memoryless, the system cannot depend on future values and must be causal.

(d) No, the system is not time-invariant. Since the system function depends on the independent variable t, it is unlikely that the system is time-invariant. To explicitly verify, let $y(t) = r(t)x(t)$. Next, delay $x(t)$ by τ to obtain a new input $x_2 = x(t-\tau)$. Applying $x_2(t)$ to the system yields $y_2(t) = r(t)x_2(t) = r(t)x(t-\tau) \neq r(t-\tau)x(t-\tau) = y(t-\tau)$. Since, the system operator and the time-shift operator do not commute, the system is not time-invariant.

1.7-10. Using the sifting property, this system operation is rewritten as $y(t) = 0.5\,(x(t) - x(-t))$.

(a) This system extracts the odd portion of the input.

(b) Yes, the system is BIBO stable. If the input is bounded, then the output is necessarily bounded. That is, if $|x(t)| \leq M_x < \infty$, then $|y(t)| = |0.5\,(x(t) - x(-t))\,| \leq 0.5\,(|x(t)| + |-x(-t)|) \leq M_x < \infty$.

(c) Yes, the system is linear. Let $y_1(t) = 0.5(x_1(t) - x_1(-t))$ and $y_2(t) = 0.5(x_2(t) - x_2(-t))$. Applying $ax_1(t) + bx_2(t)$ to the system yields $y(t) = 0.5\,(ax_1(t) + bx_2(t) - (ax_1(-t) + bx_2(-t))) = 0.5a(x_1(t) - x_1(-t)) + 0.5b(x_2(t) - x_2(-t)) = ay_1(t) + by_2(t)$.

(d) No, the system is not memoryless. For example, at time $t = 1$ the output $y(1) = 0.5(x(1) - x(-1))$ depends on a past value of the input, $x(-1)$.

(e) No, the system is not causal. For example, at time $t = -1$ the output $y(-1) = 0.5(x(-1) - x(1))$ depends on a future value of the input, $x(1)$.

(f) No, the system is not time-invariant. For example, let the input be $x(t) = t(u(t+1) - u(t-1))$. Since this input is already odd, the output is just the input, $y(t) = x(t)$. Shifting by a non-zero τ, $x(t-\tau)$ is not odd, and the output is not $y(t-\tau) = x(t-\tau)$. Thus, the system cannot be time-invariant.

1.7-11. (a) No, the system is not BIBO stable. The system returns the time-delayed derivative, or slope, of the input signal. A square-wave is a bounded signal which, due to point discontinuities, has infinite slope at certain instants in time. Thus, a bounded input may not result in a bounded output, and the system cannot be BIBO stable.

(b) Yes, the system is linear. Begin assuming $y_1(t) = \frac{d}{dt}x_1(t-1)$ and $y_2(t) = \frac{d}{dt}x_2(t-1)$. Applying $ax_1(t) + bx_2(t)$ to the system yields $y(t) = \frac{d}{dt}\,(ax_1(t-1) + bx_2(t-1)) = a\frac{d}{dt}x_1(t-1) + b\frac{d}{dt}x_2(t-1) = ay_1(t) + by_2(t)$.

(c) No, the system is not memoryless. By inspection, it is clear that the system depends on a past value of the input. For example, at $t = 0$, the output $y(0)$ depends on the time-derivative of $x(-1)$, a past value.

(d) Yes, the system is causal. By inspection, it is clear that the system does not depend on future values.

(e) Yes, the system is time-invariant. To explicitly verify, let $y(t) = \frac{d}{dt}x(t-1)$. Next, delay $x(t)$ by τ to obtain a new input $x_2 = x(t-\tau)$. Applying $x_2(t)$ to the system yields $y_2(t) = \frac{d}{dt}x_2(t) = \frac{d}{dt}x(t-1-\tau) = y(t-\tau)$. Since, the system operator and the time-shift operator commute, the system is time-invariant. In more loose terms, the derivative operator returns the delayed slope of a signal independent of when that signal is applied.

1.7-12. (a) Yes, the system is BIBO stable. From the definition of the system, we know that $y(t)$ is either $x(t)$ or 0. Correspondingly, if $|x(t)| < \infty$ then $|y(t)| < \infty$, and the system must be BIBO stable.

(b) No, the system is not linear. Consider two signals: $x_1(t) = 1$ and $x_2(t) = \cos(t)$. The corresponding outputs of these individual signals is $y_1(t) = 1$ and $y_2(t) = \begin{cases} \cos(t) & \text{if } \cos(t) > 0 \\ 0 & \text{if } \cos(t) \leq 0 \end{cases}$. However, if we create a third input $x_3(t) = x_1(t) + x_2(t)$, the system output is $y_3(t) = 1 + \cos(t) \neq y_1(t) + y_2(t)$. Since superposition does not apply, the system cannot be linear.

(c) Yes, the system is memoryless. By inspection, it is clear that the system only depends on the current input.

(d) Yes, the system is causal. Since the system is memoryless, the system cannot depend on future values and must be causal.

(e) Yes, the system is time-invariant. Consider delaying $x(t)$ by τ to obtain a new input $x_2 = x(t-\tau)$. Applying $x_2(t)$ to the system yields $y_2(t) = \begin{cases} x(t-\tau) & \text{if } x(t-\tau) > 0 \\ 0 & \text{if } x(t-\tau) \leq 0 \end{cases} = y(t-\tau)$. Since the system operator and the time-shift operator commute, the system is time-invariant.

1.7-13. (a) No, Bill is not correct. The $x_1(3t)$ term represents a compression rather than the necessary dilation. One way to construct $x_2(t)$ is $x_2(t) = 2x_1(t/3) - x_1(t-1)$. However, this form is not unique; $x_2(t) = 2x_1(t) + x_1(t-1) + 2x_1(t-2)$ also works and may be more useful.

(b) The output $y_1(t)$ is given for the input signal $x_1(t)$. The expression $x_2(t) = 2x_1(t)+x_1(t-1)+2x_1(t-2)$ forms $x_2(t)$ from a superposition of scaled and shifted copies of $x_1(t)$. Since the system is linear and time invariant, the operations of scaling, summing, and shifting commute with the system operator. Thus, $y_2(t) = 2y_1(t) + y_1(t-1) + 2y_1(t-2)$. Notice, it is not true that $y_2(t) = 2y_1(t/3) - y_1(t-1)$; linearity and time-invariance cannot help with the time-scaling operation. MATLAB is used to plot $y_2(t)$.

```
>> t = [-1:.001: 4]; y_1 = inline('t.*((t>=0)&(t<1))+(t>=1)');
>> y_2 = 2*y_1(t)+y_1(t-1)+2*y_1(t-2);
>> plot(t,y_2,'k-'); xlabel('t'); ylabel('y_2(t)'); axis([-1 4 -.5 5.5]);
```

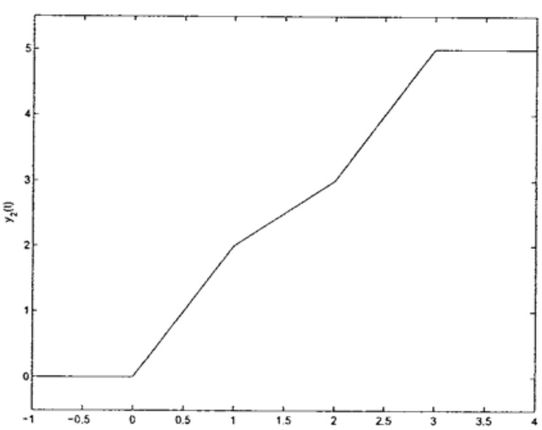

Figure S1.7-13b: Plot of $y_2(t) = 2y_1(t) + y_1(t-1) + 2y_1(t-2)$.

1.8-1. The loop equation for the circuit is

$$3y_1(t) + Dy_1(t) = x(t) \quad \text{or} \quad (D+3)y_1(t) = x(t) \tag{1}$$

Also

$$Dy_1(t) = y_2(t) \Longrightarrow y_1(t) = \frac{1}{D}y_2(t) \tag{2}$$

Substitution of (2) in (1) yields

$$\frac{(D+3)}{D}y_2(t) = x(t) \quad \text{or} \quad (D+3)y_2(t) = Dx(t)$$

1.8-2. The currents in the resistor, capacitor and inductor are $2y_2(t)$, $Dy_2(t)$ and $(2/D)y_2(t)$, respectively. Therefore

$$(D + 2 + \frac{2}{D})y_2(t) = x(t)$$

or

$$(D^2 + 2D + 2)y_2(t) = Dx(t) \tag{1}$$

Also

$$y_1(t) = Dy_2(t) \quad \text{or} \quad y_2(t) = \frac{1}{D}y_1(t) \tag{2}$$

Substituting of (2) in (1) yields

$$\frac{D^2 + 2D + 2}{D}y_1(t) = Dx(t)$$

or

$$(D^2 + 2D + 2)y_1(t) = D^2x(t)$$

1.8-3. The freebody diagram for the mass M is shown in Figure 1.8-3. From this diagram it follows that

$$M\ddot{y} = B(\dot{x} - \dot{y}) + K(x - y)$$

or

$$(MD^2 + BD + K)y(t) = (BD + K)x(t)$$

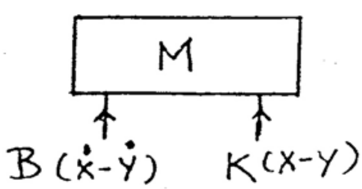

Figure S1.8-3

1.8-4. The loop equation for the field coil is

$$(DL_f + R_f)i_f(t) = x(t) \tag{1}$$

If $T(t)$ is the torque generated, then

$$T(t) = K_f i_f(t) = (JD^2 + BD)\theta(t) \tag{2}$$

Substituting of (1) in (2) yields

$$\frac{K_f}{DL_f + R_f} x(t) = (JD^2 + BD)\theta(t)$$

or

$$(JD^2 + BD)(DL_f + R_f)\theta(t) = K_f x(t)$$

1.8-5.

$$[q_i(t) - q_0(t)]\triangle t = A\triangle h$$

or

$$\dot{h}(t) = \frac{1}{A}[q_i(t) - q_0(t)] \tag{1}$$

But

$$q_0(t) = Rh(t) \tag{2}$$

Differentiation of (2) yields

$$\dot{q}_0(t) = R\dot{h}(t) = \frac{R}{A}[q_i(t) - q_0(t)]$$

and

$$\left(D + \frac{R}{A}\right) q_0(t) = \frac{R}{A} q_i(t)$$

or

$$(D + a)q_0(t) = aq_i(t) \qquad\qquad a = \frac{R}{A} \tag{3}$$

and

$$q_0(t) = \frac{a}{D+a} q_i(t)$$

substituting this in (1) yields

$$\dot{h}(t) = \frac{1}{A}\left(1 - \frac{a}{D+a}\right) q_i(t) = \frac{D}{A(D+a)} q_i(t)$$

or

$$(D + a)h(t) = \frac{1}{A} q_i(t)$$

1.8-6. (a) The order of the system is zero; there are no energy storage components such as capacitors or inductors.

(b) Using KVL on the left loop yields $x(t) = R_1 y_1(t) + R_2(y_1(t) - y_2(t)) = 3y_1(t) - 2y_2(t)$. KVL on the middle loop yields $0 = R_2(y_2(t) - y_1(t)) + R_3 y_2(t) + R_4(y_2(t) - y_3(t)) = -2y_1(t) + 9y_2(t) - 4y_3(t)$. Finally, KVL on the right loop yields $R_4(y_3(t) - y_2(t)) + (R_5 + R_6)y_3(t) = -4y_2(t) + 15y_3(t)$. Combining together yields

$$\begin{bmatrix} 3 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 15 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ 0 \\ 0 \end{bmatrix}.$$

(c) Cramer's rule suggests

$$y_3(t) = \frac{\begin{vmatrix} 3 & -2 & x(t) \\ -2 & 9 & 0 \\ 0 & -4 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 15 \end{vmatrix}}.$$

MATLAB computes the denominator determinant.

```
>> det([3 -2 0;-2 9 -4;0 -4 15])
ans = 297
```

The numerator determinant is easy computed by hand as $0 + 0 + 8x(t) = 8x(t)$. Thus,

$$y_3(t) = \frac{8}{297}x(t) = \frac{8}{297}\left(2 - |\cos(t)|\right)u(t-1).$$

1.10-1. From Figure P1.8-2, we obtain

$$x(t) = q_1/2 + \dot{q}_1 + q_2$$

Moreover, the capacitor voltage $q_1(t)$ equals the voltage across the inductor, which is $\frac{1}{2}\dot{q}_2$. Hence, the state equations are

$$\dot{q}_1 = -q_1/2 - q_2 - x \quad \text{and} \quad \dot{q}_2 = 2q_1$$

1.10-2. The capacitor current $C\dot{q}_3 = \frac{1}{2}\dot{q}_3$ is $q_1 - q_2$. Therefore

$$\dot{q}_3 = 2q_1 - 2q_2 \tag{1}$$

The two loop equations are

$$2q_1 + \dot{q}_1 + q_3 = x \Longrightarrow \dot{q}_1 = -2q_1 - q_3 + x \tag{2}$$

$$-q_3 + \frac{1}{3}\dot{q}_2 + q_2 = 0 \Longrightarrow \dot{q}_2 = -3q_2 + 3q_3 \tag{3}$$

Equations (1), (2) and (3) are the state equations.

For the 2Ω resistor: current is q_1, voltage is $2q_1$.

For the 1H inductor: current is q_1, voltage is $\dot{q}_1 = x(t) - 2q_1 - q_3$.

For the capacitor: current is $q_1 - q_2$, voltage is q_3.

For the $\frac{1}{3}$H inductor: current is q_2, voltage is $\frac{1}{3}\dot{q}_2 = -q_2 + q_3$.

For the 1Ω resistor: current is q_2 and voltage is q_2.

At the instant t, $q_1 = 5$, $q_2 = 1$, $q_3 = 2$ and $x = 10$. Substituting these values in the above results yields

2Ω resistor: current 5A, voltage 10A.

1H capacitor: current 5A, voltage $10 - 10 - 2 = -2$V.

The capacitor: current $5 - 1 = 4$A, voltage 2V.

The $\frac{1}{3}$H inductor: current 1A, voltage $-1 + 2 = 1$V.

The 1Ω resistor: current 1A, voltage 1V.