Neural Networks and Deep Learning - Assignment 2

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Project Overview

In this project, we developed a feedforward neural network to approximate the XOR function using PyTorch. The network architecture includes a hidden layer with customizable neurons, learning rate, and the option to use a bypass mechanism.

The hidden layer utilizes the sigmoid activation function, and the network's weights and biases are optimized through backpropagation. Experiments involve training the model with different configurations, including varying the number of hidden neurons (k = 1, 2, 4), learning rates (0.1 and 0.01), and bypass options (True/False).

Training is assessed by measuring the network's performance on the XOR dataset, observing metrics such as the average number of epochs to convergence, training and validation loss, and the success rate of runs. The output truth table from the hidden layer reveals the learned behavior of the 9th trial (where k = 1), providing insights into the network's ability to approximate the target function.

The experiments highlight the effects of architectural choices, such as hidden units and learning rates, on the convergence speed, stability, and accuracy of the model.

Result Analysis

The hidden neuron outputs in the truth table behave similarly to the logical **OR** function because of the way the outputs respond to the inputs.

The truth table for OR looks like this:

Input A	Input B	OR Output
0	0	0
0	1	1
1	0	1
1	1	1

The neuron outputs for each pair of inputs align closely with the **OR function** in behavior.

```
Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05864823]
[0. 1.] -> [0.9817817]
[1. 0.] -> [0.9818719]
[1. 1.] -> [0.99997866]
```

Input [0, 0]:

- In most tests, the output is close to 0 (e.g., 0.058, 0.059, etc.), similar to OR's output of 0.

Input [0, 1] or [1, 0]:

- In most tests, the output is close to 1 (e.g., 0.981, 0.999), matching OR's output of 1.

Input [1, 1]:

- In most tests, the output is close to 1 (e.g., 0.999), which aligns with OR's output of 1.

Conclusions

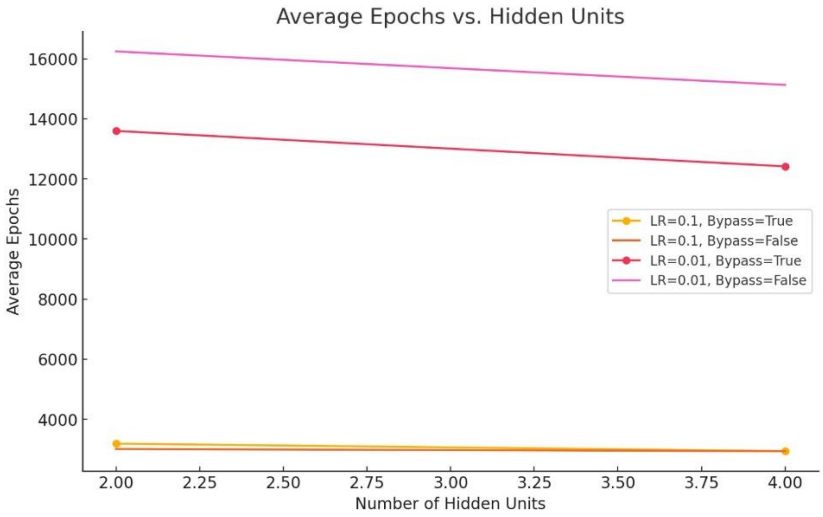
1. How is the average of the epochs until stopping affected by the number of hidden nuerons?

As the number of hidden neurons increases, the average number of epochs decreases slightly with clearer reductions for LR=0.01 compared to LR=0.1. For example:

- With 2 hidden neurons (and LR=0.01), the average epochs are approximately **13599.80** (with a bypass) and **16245.60** (without a bypass).
- With 4 hidden neurons (and LR=0.01), the average epochs drop to **12417.90** (with a bypass) and **15130.70** (without a bypass).

This indicates that adding hidden units can help the model learn faster, likely due to increased model capacity enabling better representation of the problem.

The effect of bypass varies, with LR=0.01 showing improved performance when bypass is enabled.

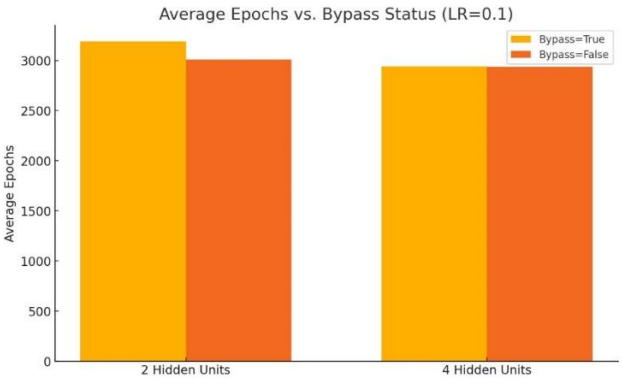


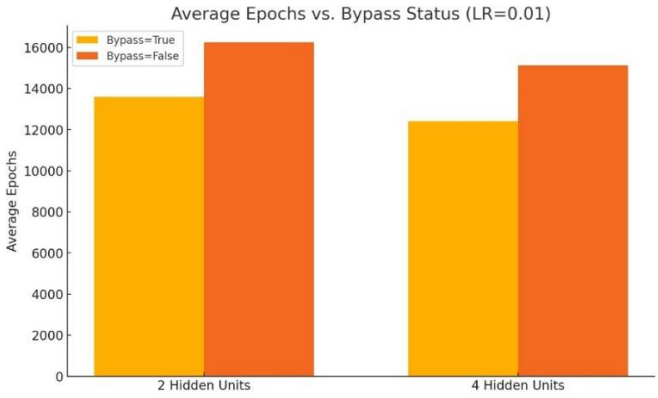
2. How is the average of the epochs affected by the existence of a bridge (bypass)?

The presence of a bypass generally decreases the average number of epochs. For example:

- With 2 hidden neurons (and LR=0.01), the average epochs are approximately **13599.80** (with a bypass) and **16245.60** (without a bypass).
- With 4 hidden neurons (and LR=0.01), the average epochs drop to **12417.90** (with a bypass) and **15130.70** (without a bypass).

While the bypass can help reduce epochs in some cases, its effect is less pronounced with higher learning rates.





3. How is the standard deviation (STD) of the number of epochs affected by the learning rate?

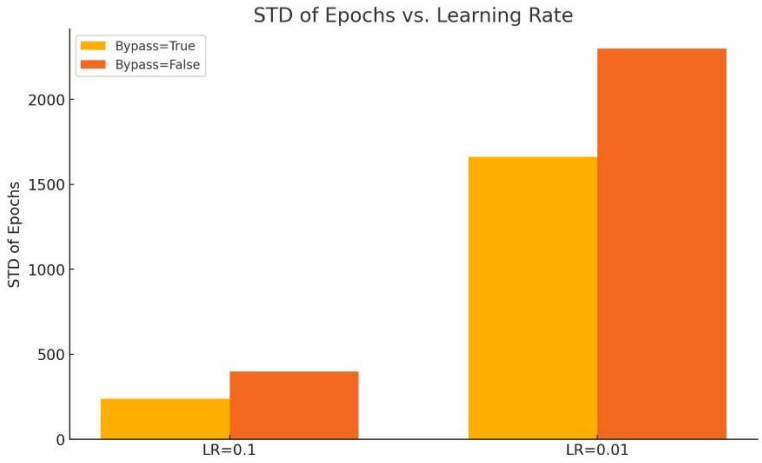
The standard deviation of epochs is higher with a lower learning rate. For example:

- At LR=0.1, the STD is relatively low (e.g., **196.47** for 2 hidden units with a bypass).
- At LR=0.01, the STD increases significantly (e.g., **1724.42** for 2 hidden units with a bypass).

This suggests that lower learning rates cause more variability in the training process, likely due to slower convergence and greater sensitivity to initialization.

The standard deviation is significantly higher for LR=0.01 across bypass settings, highlighting greater variability at lower learning rates.

The bypass slightly stabilizes training (lower STD) for both learning rates.



4. Hidden Neuron Output Analysis

One of the runs demonstrates behavior similar to an **OR gate**, based on the hidden neuron output:

```
Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05864823]
[0. 1.] -> [0.9817817]
[1. 0.] -> [0.9818719]
[1. 1.] -> [0.99997866]
```

Output

```
cuda:0

Running experiment: LR=0.1, Hidden=2, Bypass=True

Running experiment: LR=0.1, Hidden=2, Bypass=False

Running experiment: LR=0.1, Hidden=4, Bypass=True

Running experiment: LR=0.1, Hidden=4, Bypass=False

Running experiment: LR=0.01, Hidden=2, Bypass=True

Running experiment: LR=0.01, Hidden=2, Bypass=False

Running experiment: LR=0.01, Hidden=4, Bypass=True


Running experiment: LR=0.01, Hidden=4, Bypass=False

Running experiment: LR=0.01, Hidden=1, Bypass=True

Run 1 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05893481]
[0. 1.] -> [0.9816969]
[1. 0.] -> [0.981831]
[1. 1.] -> [0.9999784]

Run 2 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.9683643]
[0. 1.] -> [0.03845575]
[1. 0.] -> [0.9999801]
[1. 1.] -> [0.98495704]

Run 3 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05851845]
[0. 1.] -> [0.9818366]
[1. 0.] -> [0.9818781]
[1. 1.] -> [0.9999788]

Run 4 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05833217]
[0. 1.] -> [0.98185986]
[1. 0.] -> [0.981935]
[1. 1.] -> [0.9999789]

Run 5 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05943014]
[0. 1.] -> [0.9817156]
[1. 0.] -> [0.98160374]
[1. 1.] -> [0.99997795]
```

```
Run 6 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05850186]
[0. 1.] -> [0.9818828]
[1. 0.] -> [0.98183]
[1. 1.] -> [0.9999788]

Run 7 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05912386]
[0. 1.] -> [0.98167485]
[1. 0.] -> [0.9817837]
[1. 1.] -> [0.9999782]

Run 8 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05895169]
[0. 1.] -> [0.9817347]
[1. 0.] -> [0.9817878]
[1. 1.] -> [0.9999784]

Run 9 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05931088]
[0. 1.] -> [0.98167855]
[1. 0.] -> [0.9816897]
[1. 1.] -> [0.99997807]

Run 10 (Hidden Layer Output):

Truth Table (Hidden Layer Output):
Input A, Input B -> Hidden Neuron Output
[0. 0.] -> [0.05943364]
[0. 1.] -> [0.98158973]
[1. 0.] -> [0.9817343]
[1. 1.] -> [0.99997795]

Experiment Results:
LR=0.1, Hidden=2, Bypass=True
  Avg Epochs: 3189.40 ± 196.47
  Avg Train Loss: 0.0172 ± 0.0022
  Avg Val Loss: 0.0304 ± 0.0064
  Failed Runs: 0

LR=0.1, Hidden=2, Bypass=False
  Avg Epochs: 3010.90 ± 366.44
  Avg Train Loss: 0.0140 ± 0.0001
  Avg Val Loss: 0.0202 ± 0.0000
  Failed Runs: 2

LR=0.1, Hidden=4, Bypass=True
  Avg Epochs: 2938.70 ± 279.42
  Avg Train Loss: 0.0145 ± 0.0011
  Avg Val Loss: 0.0273 ± 0.0020
  Failed Runs: 0

LR=0.1, Hidden=4, Bypass=False
  Avg Epochs: 2937.50 ± 426.46
  Avg Train Loss: 0.0129 ± 0.0015
  Avg Val Loss: 0.0197 ± 0.0016
```

```
  Failed Runs: 0

LR=0.01, Hidden=2, Bypass=True
  Avg Epochs: 13599.80 ± 1724.42
  Avg Train Loss: 0.0753 ± 0.0108
  Avg Val Loss: 0.1072 ± 0.0130
  Failed Runs: 0

LR=0.01, Hidden=2, Bypass=False
  Avg Epochs: 16245.60 ± 3373.31
  Avg Train Loss: 0.0478 ± 0.0008
  Avg Val Loss: 0.0651 ± 0.0003
  Failed Runs: 8

LR=0.01, Hidden=4, Bypass=True
  Avg Epochs: 12417.90 ± 1597.99
  Avg Train Loss: 0.0603 ± 0.0074
  Avg Val Loss: 0.0908 ± 0.0115
  Failed Runs: 0

LR=0.01, Hidden=4, Bypass=False
  Avg Epochs: 15130.70 ± 1221.48
  Avg Train Loss: 0.0491 ± 0.0039
  Avg Val Loss: 0.0685 ± 0.0049
  Failed Runs: 0

LR=0.01, Hidden=1, Bypass=True
  Avg Epochs: 15526.50 ± 3563.14
  Avg Train Loss: 0.0824 ± 0.0046
  Avg Val Loss: 0.1169 ± 0.0009
  Failed Runs: 0

Process finished with exit code 0
```