Agent-Based-Model: if you want to choose, meet an expert. Don't talk.

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Abstract

Though decision-making is a pervasive and intriguing research subject, little is known on the factors that influence the dynamic of authority, community and individuals. Citizens may consult with the "front-desk" services that are offered by the authorities or share and receive information within their social networks. For our purposes, as a private case, we focus on *how individuals choose the optimal school for their children?* Choosing a school is a complex problem that is characterized by uncertainty and partial information. In this paper, we present an agent-based-model (ABM) of heterogeneous interactions: citizen-citizen and citizen-human agent (authority), which account for aggregate behavior of decision-making. The model is designed from the authority's perspective, in which the availability of the human agents represents the authority's budget and resources. The aim is to explore the complex system and help policy-makers to hold a better understanding of the dynamic patterns of social interactions, and by that a better management. The following summarizes our insights from the results: (1) the interactions with proximate-citizens do not affect significantly the results of making an optimal decision. (2) The percentage of social hubs has a strong negative impact on making the optimal decision, while the prevalence of human agents (and by that the authority's involvement) has a strong positive impact. (3) The increasing of human agents' availability might have an indirect effect that helps the network to become more informative and more helpful for other citizens.

1. Introduction

Every once in a while an individual has to make a decision regarding his life or his relatives' life which involves the authorities or any other governmental organization. Some of these choice-tasks, such as choosing health care insurance, schools and training programs, require a complex decision-making that is hard to do with the enormous amount of information. Moreover, studies have shown that the individual capabilities of integrating and processing a large amount of data is limited (Häubl, Gerald & Murray 2003; Payne, Bettmann & Johnson 1993), and that he tends to feel dissatisfied and overwhelmed by the information overload (Huffman & Kahn, 1998; Malhotra, 1982). Hence, the city authorities often offer "front-desk" services, namely, the services of human representatives to provide help through face-to-face or phone meetings (we will name them *human agents*, hereafter). Such services are costly and are by nature limited in capacity. The human agents are not necessarily on behalf of the government; in some cities, such as New York City and London citizens are hiring the services of expensive private consultants to assist with decisions such as school choices (Harris, Elizabeth & Fessenden 2017). The citizens' decisions are not based solely on information that is originated or delivered by the authorities. In fact, there is a large body of literature on the importance of social interactions and Word-of-Mouth (WOM) communications in individual behavior, and consequently, on decisions (e.g. Van den Bulte, Iyengar and Valente 2011, Goldenberg, Libai & Muller 2001, Eliashberg, Jonker, Sawhney & Wierenga 2000). For example, more than 40% of the United States' citizens are asking for advice from family and friends when choosing services such as lawyers and doctors (Walker, 1995).

Moreover, for the last few decades considerable researches have focused on individual decision-making in various aspects, e.g. analytical processing, intuitive responses, heuristics and etc. (Rangel, Camerer & Montague, 2008; Greene & Haidt, 2002), and on the development of automated assisting programs, such as recommendation system, in order to improve individuals to their optimal decision (Häubl & Trifts, 2000).

However, it is not clear how and at what extent do the social interactions integrate into a decision process with the assistance given by the authorities. The aim of this paper is to address this question, to explore the complex system which consists of individuals, community-structure and the human agents, and might help policy-makers exploit a better understanding of the dynamic patterns of social interactions, and by that a better management. For our purposes we preferred to focus on *how* *individuals choose the optimal school for their children?* As a private case of the intriguing interplay between the community and the authorities, with the assumption that this can easily be adjusted in other cases similar to the one depicted above. Choosing school is a multidimensional problem with intertwined variables, which is characterized by uncertainty and partial information (Cahill & Hall, 2014; Rosen, 2017). Despite of the fact that many families take their responsibilities for choosing schools seriously, they might fail in achieving their objectives (David, West & Ribbens, 1994). Since every individual has uncountable optional paths to make his decision in, and the deep uncertainty that relies in this process, we get, at the macro-level a very complex system. This system belongs to the stream of *aggregate complexity*, an approach that focuses on how complex systems arise from interactions among individual entities (Manson, Sun & Bonsal, 2012). It has shown that such systems cannot be treated with the traditional methods of decision analysis (Lampert, 2002).

Agent based model (ABM) is a computer simulation made up of agents or entities that are acting autonomously. These agents may or may not interact with other agents or entities and behave according to internal rules for decision-making, movement and action. The aggregate behavior of the system is the result of the interactions, decisions and actions of the relatively simple behaviors of the individual agents (Sanchez & Lucas, 2002). ABM offers a virtual laboratory that allows us to navigate between the empirical and theoretical research, where the key of ABM and complexity research is to capture the core characteristics among the system entities and their interrelationships (Manson, Sun & Bonsal, 2012).

In this paper, we present an ABM of heterogeneous interactions: citizen-citizen and citizen-human agent (authority), which account for aggregate behavior of decision-making and unique social effects. The model is designed from the authority's perspective, in which the availability of the human agents represents the authority's budget and resources. The attempt is to optimize to maximum the amount of citizens who made their most suitable choice for them, under partial information and few constraints, without considering citizen's budget. We examine the influences of

(1) the prevalence of interactions with human agents (2) a community's structure and the relative percentage of social hubs in the network (3) the time-condition of making a decision (4) the profile of fellow-citizen with which the citizen consults.

Preliminary insights gained from our results:

1. The interactions with proximate-citizens do not affect significantly the results of making an optimal decision.
2. Among the investigated parameters the percentage of social hubs has a strong negative impact on making the optimal decision, while the prevalence of human agents (and by that the authority's involvement) has a strong positive impact.
3. The increasing of human agents' availability might have an *indirect effect* that helps the network to become more informative and more helpful for other citizens.

The rest of this paper is organized as follows: In section 2 we describe the metrics we used and the implementation of the agent based model. Section 3 presents the results of the simulations. Suggestions for further potential complex model are discussed at the end of this paper.

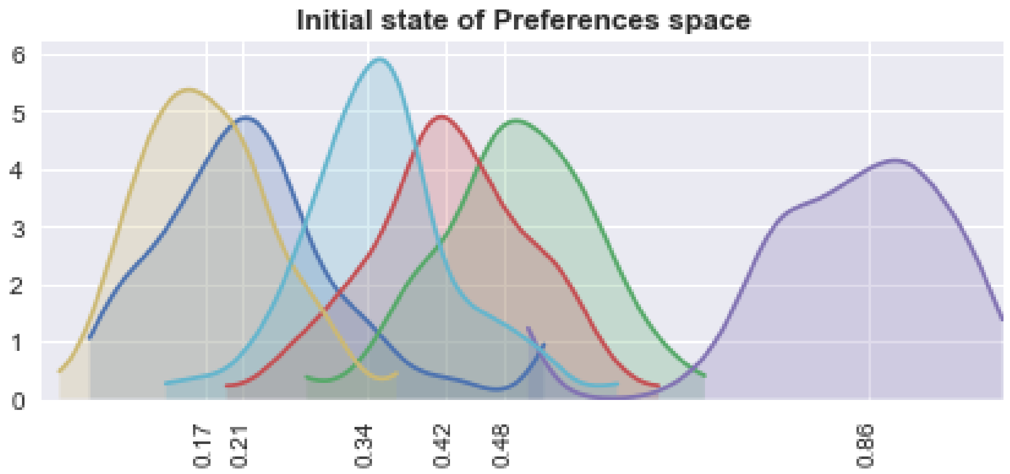
2. Agent-Based Model

In the presented model's implementation we made several simplifying assumptions, due to low model sensitivity and the lack of empirical data. For instance, each citizen has one and only one optimal decision (school) for him which is not changing over time; The information is processed without memory, based solely on the exact location and the citizen's neighborhood in the preferential space; The capacity of the schools is unlimited and the authority (human agents) knows approximately what is the optimal option for the citizen and operates in his favor when interacting with him. In addition, for the sake of simplicity we construct a one dimensional space with hard boundaries $[0,1]$ that serves as a representation of the preferential space. At the initialization of the simulation, several *Options* are drawn independently from a uniform distribution $U\sim(0,1)$.

Table 1: Summary of model variables and parameters

Parameter / Variable	Value	Description	Range
N	1000	Number of Citizens	
m	6	Number of options (schools)	
$O_{j=1..m}$		Location of Options	[0,1]
σ_I	$\frac{1}{2*m}$	Std citizen's initial preferences	$\sigma_I > \sigma_A > 0$
σ_A	$\frac{1}{15*m}$	Std human agent interaction	$\sigma_I > \sigma_A > 0$
R_{HA}	0.8	Influenced by HA interaction	$2 > R_{HA} > R_{WOM} > 0$
R_{WOM}	0.1	Influenced by WOM interaction	$2 > R_{HA} > R_{WOM} > 0$
P_{HA}		Probability to meet HA	$0.1 \geq P_{HA} \geq 0$
P_{WOM_a}		Probability to meet WOM_a	$0.3 \geq P_{\mathrm{WOM}_a} \geq 0$
P_{WOM_d}		Probability to meet WOM_d	$0.3 \geq P_{\mathrm{WOM}_d} \geq 0$
Per_{SH}		Percentage of Social Hubs	$0.032 \geq Per_{SH} \geq 0$
D_p		Decision parameter decreasing linearly with time	$12\sigma_A \geq D_p \geq 3\sigma_A$

We examine a population consisting of $N > 1$ independent individual citizens. Each citizen agent in the ABM has several associated attributes: [1] *Optimal decision* (O_j) - assigned randomly with the initialization and stays static; [2] *Initial location* in the preferential space $\left(C_i(0)\right)$ – drawn from a truncated normal distribution $\mathcal{N}\sim(O_j, \sigma_I^2)$ and updated along the simulation; [3] *Correct Human agent* (R_{HA}) – a weighting parameter for the acceptance of human agent's suggestion – equivalent and constant for all citizens; [4] *Correct WOM agent* (R_{WOM}) – a weighting parameter for the acceptance of WOM citizen's suggestion – equivalent and constant for all citizens; [5] *Decision Parameter* $(D_p(t))$ – defines the condition of when to make a decision – decreasing linearly with time and equivalent for all citizens. In addition, each citizen agent has a "social network" with whom he interacts and, sharing and receiving information.



<u>Fig 1:</u> An example of initial preferences space
The numbers at x axis are the options and the colored Gaussian describes the citizens' distributions

At any iteration t based on: the citizen's value in the preferential space, his neighborhood and the decision parameter $D_p(t)$, a sequential testing is preformed - whether to decide or not:

$$\begin{array}{ll} Decide\ O_j\ , & if\ |C_i(t) - O_j| < D_p\ ,\ \exists_{=1}\ for\ all\ O_{j=1..m} \\ Sample\ next\ C_i(t+1)\ , & else \end{array}$$

In other words, a citizen will make a decision only if in his neighborhood, which is defined by D_p, exists one and only one option. Otherwise the decision cannot be made. Since no one really knows how to measure the impact that interactions have upon citizens, which can be very subjective and vary in many aspects, we'd rather simplify the interaction effect, describing it as a *basin of attraction*. Thus, the citizen may interact with an "advisor" - human agent or citizen through WOM, and will be attracted to their value according to:

$$C_i(t+1) = C_i(t)\left(1 - \frac{R}{2}\right) + A_i * \frac{R}{2}$$

Where:

$$A_i\ , R = \begin{cases} 0\ ,\ 0 & w.p. \quad 1 - P_{HA} - P_{WOM_a} - P_{WOM_d} \\ \mathcal{N} \sim (O_c, \sigma_A^2),\quad R_{HA} & w.p. \quad P_{HA} \\ C_{WOM_a}\ ,\ R_{WOM} & w.p. \quad P_{WOM_a} \\ C_{WOM_d}\ ,\ R_{WOM} & w.p. \quad P_{WOM_d} \end{cases}$$

In probability P_{HA} the citizen will encounter a human agent. The interaction is mainly affected by the preliminary assumption that the authorities are operating in the favor of the citizen and that the human agent knows approximately what is the optimal option for the specific citizen. A human agent will draw in each interaction a different number from a truncated normal distribution around the citizen's optimal decision $\mathcal{N} \sim (O_c, \sigma_A^2)$ and attract the citizen closer to his value, and consequently to the citizen's optimal decision ($\sigma_A < \sigma_I$). In the extreme case of abundance of human agents, and no other kinds of interactions taking place, citizens will fluctuate around their optimal option (σ_A – the size of the fluctuations), until they will make a decision.

Each citizen also has a social network comprising of other citizens of similar initial preferences (proximity to other citizens in the preferential space). This network structure is motivated by social science theories of *"Homophily"* – people tend to associate with others who are resemble to them in attitudes, intrapersonal characteristics and behaviors (McPherson , Smith-Lovin & Cook, 2001). Social network analysis has shown that social networks composed of a large number of highly-connected small-clustered groups with relatively low-degree nodes, i.e. the vast majority of citizens has only a few connections each. In addition, this type of social networks belongs, in many cases, to scale-free graphs family, which contain a low percentage of nodes with a very large number of social connections, also named as *Social Hubs* (Mislove, Marcon, Gummadi, Druschel & Bhattacharjee, 2007; Nekovee, Moreno, Bianconi, & Marsili, 2007).

We applied a method of variation of the "random graph with a planted partition" suggested by graph theorists (Fortunato, 2010; Condon & Karp, 2001). This variation allows us to generate networks with pre-selected values for the average degree, the relative percentage of social hubs and the clustering coefficient (Peres, 2014). According to the method, the N nodes are organized into two separate bins of sizes $N_1,\ N_2$ ($N = N_1 + N_2$). The probability that a citizen from bin i and a citizen from bin j are connected is $p_{i,j}$, where in our model it is translated to the distance in the preferential space.

The average degree of the nodes is given by:

$$Proximate \text{ ("Homophily")} \qquad D_1 = p_{1,1} * (N_1 - 1) + p_{1,2} * N_2 = 6$$

$$Social\ Hubs \qquad D_2 = p_{2,2} * (N_2 - 1) + p_{1,2} * N_1 = 180$$

We held the average degree for both, the proximate connections and the social hubs connections, and examined the number of social hubs - N_2 in the network. For this purpose we referred N_2 as a variable and manipulated the equations above:

$$p_{1,1} = p_{2,2} = p = \frac{D_1(N - N_2) - N_2 D_2}{(N-1)(N-2N_2)}$$

$$P_{1,2} = \frac{D_1 - p(N - N_2 - 1)}{N_2}$$

This in turn, led us to define upper bounds to the percentage of social hubs in the network:

$$\frac{D_1 N}{D_1 + D_2} \geq N_2 = 0.032$$

The contribution of social hubs to complex-information's processes, such as spreading information and decision making in large scale, is a matter of ongoing debate (Peres, 2014; Deng, Liu & Xiong, 2013). Here, by varying systematically the number of social hubs in the network, we are testing the influence of this variable, while controlling other topological metrics. We measure the number of social hubs in the network as the percent of the total number of citizens -Per_{SH}. In figure 2 there is an example of degree histogram of the population in one given simulation.

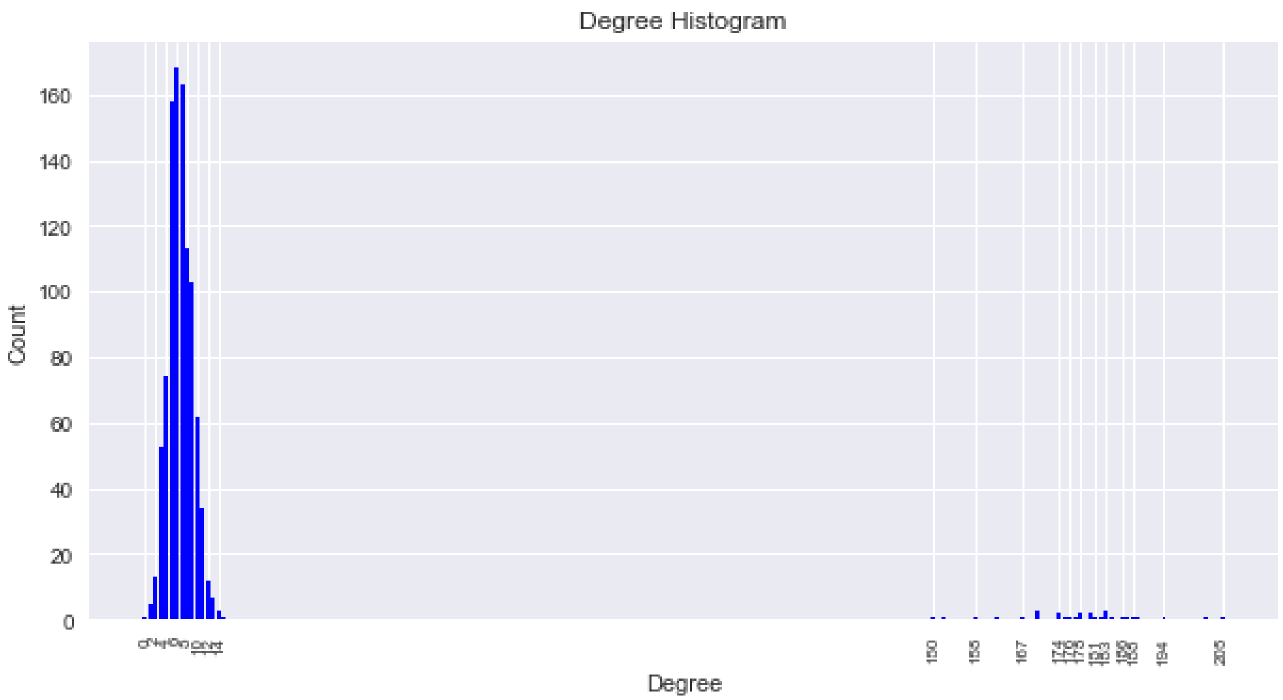


Fig 2: An example of a degree histogram of the population

Following Maki and Thompson, we assume that the information spreads by directed contact of the "*information carrier*" with others in the network (Maki & Thompson, 1973). In this model we propose two types of *information carrier*:

- a citizen who met a human agent - C_{WOM_a}.
- a citizen who already decided - C_{WOM_d}. Notice that this metric solely can raise a problem where interactions will catalyze citizens into making decisions and the network to convergence, but without taking into consideration the citizens' optimal choices. Thus, when the preferential space is dense, we might be witnessing a situation where certain options will not be chosen (Figure 3).

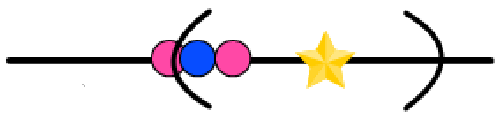


Fig. 3: A scenario in dense space with C_{WOM_d}
The citizen (star) cannot choose the middle option (blue circle) since interactions with C_{WOM_d} will bounce him between the edges and he will have to choose one of the pink circles

In the probabilities P_{WOM_a}, P_{WOM_d} and if there are *information carriers* within the citizen's social network he will choose one randomly and interact with him.

The R parameter is a weighting parameter that determines how much the citizen will "adopt" and will be attracted to the advisors' location value. It is easy to see that R is ranging between 0 to 2. While 0 means that the interaction has not influenced the citizen at all and 2 states that the citizen fully grasped the advisor's suggestion and their values will be equal. Consistent to pervious literature as specified above, we set the probability's range to interact with human agent lower than the probability's range of interacting with someone within the social network.

In addition, due to the fact that each simulation is particular and singular, for each set of parameters we ran a number of 20 different trial-runs, which were later averaged.

3. Results

We tested the model's sensitivity where all combinations of the parameters were considered in a full factorial design experiment. Four of the input variable parameters - P_{HA}, P_{WOM_d}, P_{WOM_a} & Per_{SH} were manipulated on five levels and D_p – the decision parameter with the rate of change was manipulated on three levels. Parameter ranges were set at described above (Table 1). Including 20 runs for each parameter's set, the experiment produces an overall of $5^4 * 3 * 20 = 37{,}500$ information process simulations. This procedure allows us to explore the effect of different combinations of their sizes, by generating all possible outcomes of the above manipulations. At first, only by applying *distance minimization* metric for every initialization set of the system and without running the simulation, we end up with $61 \pm 9\%$ of the population choosing the optimal option for them. While it can be an acceptable result, we should notice that with minor interventions from the authorities, at the end of the simulations the results are poorer and stands on $55 \pm 10\%$ of the population.

OLS regression was conducted with *probability for optimal decision* as the dependent variable, and the results presenting the effect of different parameters (Table 2, figure 4).

The following main results raised some interesting observations:

Parameter	Value
$Const$	0.686 ± 0.003
P_{HA}	1.72 ± 0.03
D_p	0.17 ± 0.02
P_{WOM_d}	-0.51 ± 0.01
P_{WOM_a}	-0.17 ± 0.01
Per_{SH}	-2.90 ± 0.08

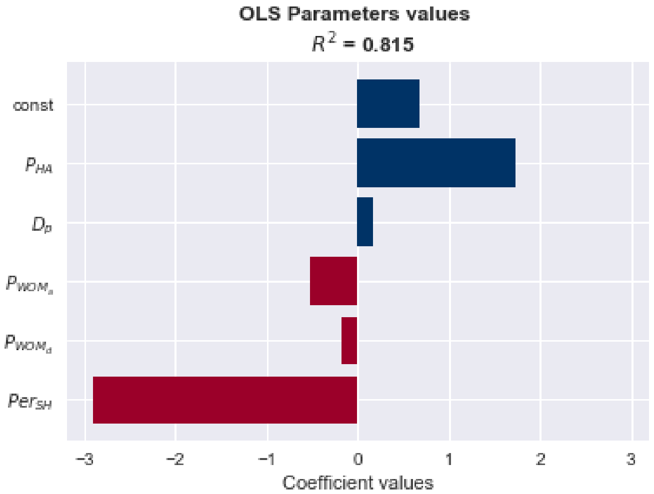


Table 2 & Fig. 4: OLS regression results
$R^2 = 0.815$, And all the variables are significant at $p \leq 0.0001$ level

- **Result 1:** The influence of WOM on the optimal decisions is weak and it tends to be negative

Despite of the fact that the probabilities for interaction with a citizen who decided or met a human agent, P_{WOM_d} and P_{WOM_a} respectively, are higher than the probability for meeting a human agent, their impact on the decision is negligible. In practice, in each simulation every citizen had plenty of interactions within his social network, whether it's with proximate citizens or with social hubs, which relocate him on the preferential space. However, it turns out that, practically, those interactions are not advancing the citizens toward their optimal decision. We are familiar with the benefits and the mechanisms of sharing and consulting in a social environment (Batson, 2014), nonetheless according to our model those interactions can be considered as "empty talks", in which the interactions are occurring frequently but are not changing the situation significantly.

- **Result 2:** The two major factors for optimal decisions are the probability to meet human agent (positively strong) and the percentage of social hubs in the population (negatively strong).

The results point out that two major factors are the most influential for making optimal decisions in the macro-level: the availability of a human agent and the amount of social hubs in the network. In fact, they are affecting the results as two opposing forces: while social hubs have connections all over the preferential space and, as a result, interactions will increase the entropy of the system, the human agents are operating to minimize the citizens' fluctuations around the optimal decisions. For the use of policy makers; while the amount of social hubs in communities cannot be adjusted, they can be explored and mapped to assist better management and efficient resources setting. For instance, in figure 5, we can see how the optimal match is dependent on the prevalence of the human agents in the system and on the amount of social hubs which exist in the same network. Unlike the amount of social hubs, the prevalence of human agents can be controlled.

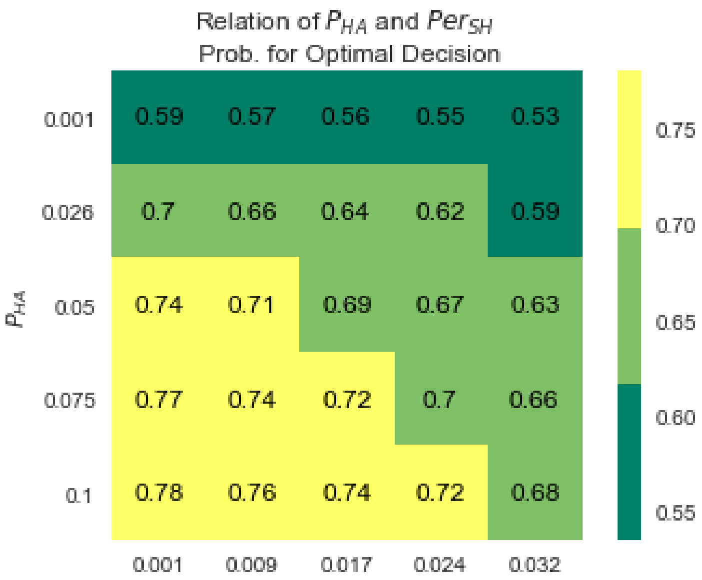


Fig 5. Discretized heat map of the relation between the probability to meet human agent to the percentage of social hubs in the population over the optimal decision at the macro level

- **Result 3:** The higher the probability to meet a human agent, the lower amount of citizens will be influenced by social hubs as *direct effect*

We will define *direct effect* as making a decision right after having an interaction. In such a system, where the citizens have no memory and they are always "correcting" their location on the preferential space according to the last interaction (without calculating the past), decisions can be made in several ways: on the first iteration due to the initial set, by direct effect or by the shrinking of the decision parameter. Based on the OLS regression, we conclude that the pace and the size of the decision parameter have merely a weak effect on the final results. Analyzing the direct effect, which is the main cause for making a decision, we can see in Figure 6 that higher probability for interaction with a human agent helps mainly to diminish the amount of people who are influenced by the social hubs. On the other hand, increasing the probabilities of WOM-interactions, will lead to an opposite effect, enlarging the number of people that decided due to social hubs on account of people that decided after meeting a human agent. Thus we can strengthen our regression's results and deduce that the probability to WOM-interactions has only small impact on the final results. Since most of the population is not making decisions due to interactions with proximate citizens, but rather to interactions with social hubs and human agents, and this situation is not varying dramatically with the change of the probabilities.

Interestingly, despite of the fact that the probability of interacting with social hubs is much smaller than interacting with someone else within an individual's social network (there is a minuscule number of social hubs in each citizen's social network) their influence on decisions is undoubtedly more prominent. This can be explained by the metric of interaction that defined as *basin of attraction*, where the power of interaction is inversely proportional to the distance of the interactors.

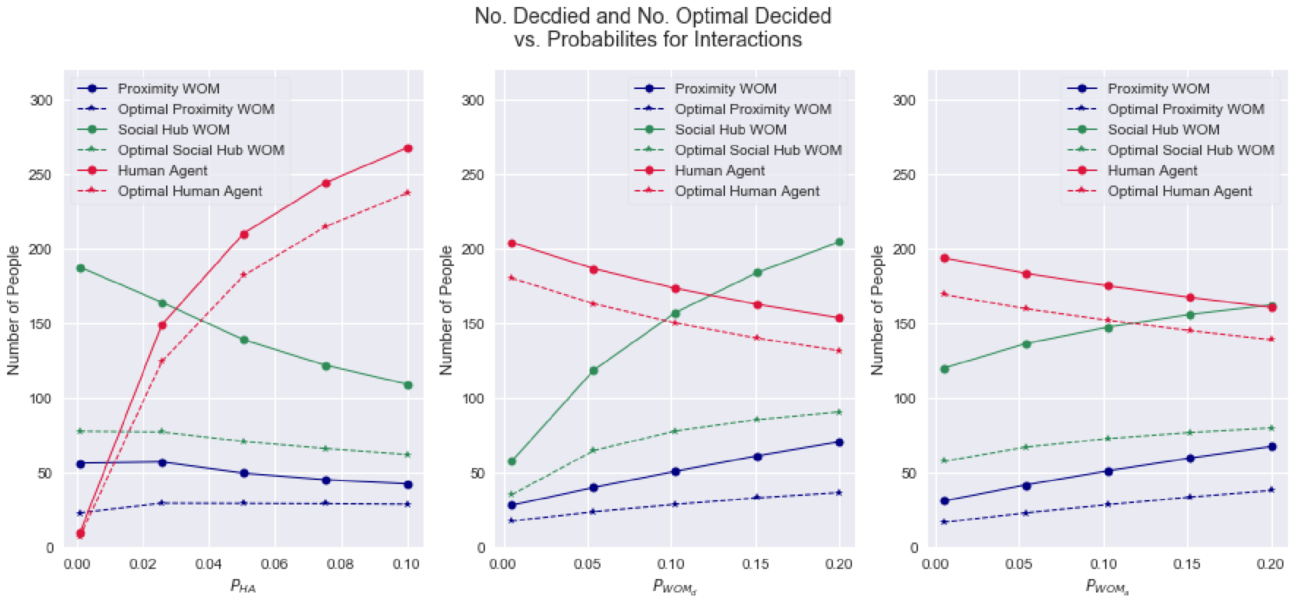


Fig. 6: Number of people that made a decision right after having an interaction and their successes with making an optimal decision, as function of probability to meet human agent, probability to meet citizen who have decided and probability to meet citizen who have met human agent

- **Result 4:** The higher probability to meet human agents can increase the *informative WOM interactions* in the social network

Another interesting result that shed some light on other intrinsic characteristics of the parameters' relation exists in the data of *direct effect*. When calculating, for each category of interaction, the percentage of people that had chosen the optimal option for them, we can see clearly the tendency of the monotonic graphs (figure 7). Higher probability to meet a human agent causes greater success in matching citizens to their optimal options, no matter what was the interaction that encouraged them to make the decision. This non-trivial result cannot be interpreted directly from the decline in the number of people that made a decision through WOM with the increasing of the probability to meet a human agent. It would be reasonable to conclude that the "network became smarter", where interactions with proximate citizens or with social hubs were more informative to the citizen seeking for advice. Since interactions through WOM is only with *information carrier*, we can infer that the prevalence of the human agents enables the *information carriers* to be more precise on their location, and by that more helpful for other citizens. It is surprising because the citizens are not dealing with the same questions and choices, and seemingly their self-preferences should play a role in the decision. Unlike situations of spreading "homogenous" valuable information across the network, such as news in public, here, each group differentiated in their designated optimal option and citizens may receive precise information that unfortunately lead them to incorrect decisions. In this model we assume that every *information carrier* citizen is interacting with others by attracting them to his location, without "actually seeing" what the other is looking for and what is the optimal decision for him. Despite this implementation of the model, it seems that at the macro-level the assistance from the authorities may have an indirect effect, in which citizens who were helped by the authorities will function as better advisors.

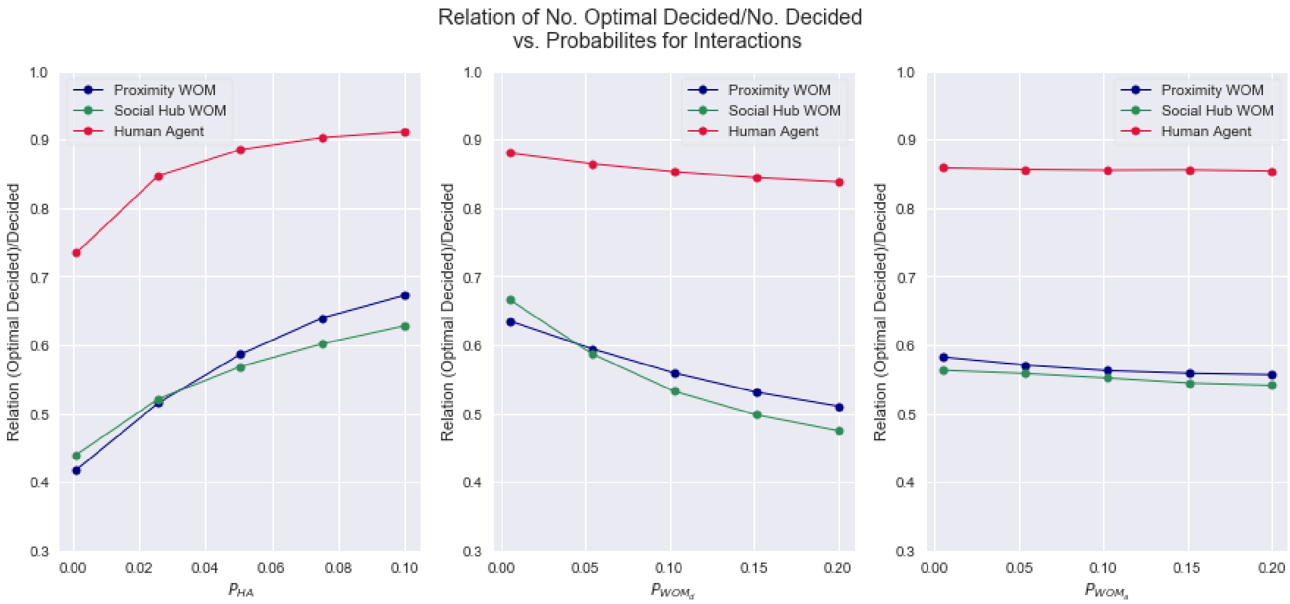


Fig. 7: successful decisions out of total decisions grouped by the last interaction vs. the probability to meet human agent, probability to meet citizen who have decided and probability to meet citizen who have met human agent

4. Discussion

This is a preliminary model of multiplayer decision-making under conditions of uncertainty and partial information. The citizens in the model differ in their preferences, needs and optimal decisions. Although the simplicity of the model, the limiting assumptions and the dependence on several stochastic variables, we manage to point out some important characteristics concerning this issue. For instance, the dynamic between the negative impact of matching optimal decision to a citizen (which is generally created by the social hubs in the network), and the authorities' role to reduce this "damage", by interaction with the human agents they offer. In addition, we are suggesting applicable conclusions for decision-makers which may come into consideration when allocations resources in similar cases.

Furthermore, we propose several directions for further research, mainly enlarging the complexity of the model: (1) converting the *R* weighting parameter to inhomogeneous parameter that also consisting different values for weak ties (social hubs) and strong ties (proximate). (2) Examination of strategies to optimize the allocation of resources; as for now, we focused only on the quantity of human agents disregarding the different ways in which they are distributed. (3) Ranking the optimal options instead of a binary decision based wheatear to take the optimal option or not; this method, which is closer to represent a reality-situation would enable us to implement into the model the limiting of school's capacity.

Moreover, there are accessible empirical data, such as the quantities and types of interactions that took place before the decision was made, which might help us examine this matter more thoroughly.

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