

MIT EECS 6.815/6.865: Assignment 7:  
Harris Corners, Features and Automatic Panoramas

Due Wednesday November 11 at 9pm

## 1 Summary

- Harris corner detection
- Patch descriptor
- Correspondences using nearest neighbors (NN) and the second NN test
- RANSAC
- Fully automatic panorama stitching of two images

This is not an easy assignment. There are many steps that all depend on the previous ones and it's not always trivial to debug intermediate results. We provided you with visualization helpers which you can read about throughout this pdf or in `panorama.cpp`. We will make use of `homography.cpp` from problem set 6.

This problem set is long, but you have two weeks to complete it. Make sure you start early!

## 2 Previous Problem

This Problem Set uses code from Problem Set 6, homographies. Replace the functions in `homography.cpp` and `homography.h` with your own code.

## 3 Harris Corner Detection

The Harris corner detector is founded on solid mathematical principles, but its implementation looks like following a long cookbook recipe. Make sure you get a good sense of where you're going and debug/check intermediate values.

### 3.1 Structure tensor

The Harris Corner detector is based on the structure tensor, which characterizes local image variations. We will focus on greyscale corners and forget color variations.

We start from the gradient of the luminance  $I_x$  and  $I_y$  along the  $x$  and  $y$  directions (where subscripts denote derivatives). The structure tensor is

$$M = \sum w \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad (1)$$

where  $w$  is a weighting function (a Gaussian in our case) and we omit the pixel coordinates for conciseness. For this, we will first compute  $I_x^2$ ,  $I_x I_y$  and  $I_y^2$  at each pixel and convolve them with a Gaussian.

For this, extract the luminance of the image using the `lumiChromi` function from Pset 1.

Using a Gaussian with standard deviation `sigmaG`, blur the luminance to control the scale at which corners are extracted. A little bit of blur helps smooth things out and help extract stable mid-scale corners. More advanced feature extraction uses different scales to add invariance to image scaling.

Next, compute the luminance gradient along the x and y direction. We've added the functions `gradientX` and `gradientY` in `filtering.cpp` for you. Call these functions with the default value of `clamp`.

Then, compute the contribution of each pixel to the structure tensor, using Equation (1) for  $M$ . The matrix is symmetric and we only need to store three values per pixels. You can store them in a `Image` with three channels.

Then, compute the local weighted sum by convolving the above per-pixel contributions using a Gaussian blur with standard deviation `sigmaG*factorSigma`. Use the separable gaussian filtering function with default truncation.

1 Write a function `Image computeTensor(const Image &im, float sigmaG=1, float factorSigma=4)` that returns a 3D array (i.e. an `Image`) of the size of the input where the three channels at each location `x`, `y` store the three values corresponding to the  $I_x I_x$ ,  $I_x I_y$ , and  $I_y I_y$  components of the tensor (in this order).

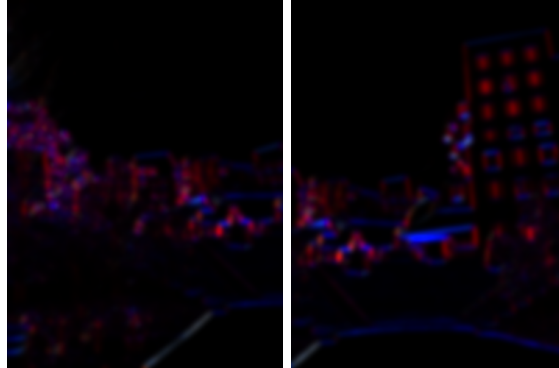


Figure 1: To visualize the results, take a look at the resulting tensor `Image`. These are our results for the Stata pair with RGB channels being `xx`, `xy`, `yy`.

## 3.2 Harris corners

Implementing the Harris corner detectors require a few steps, presented here in order. Read the whole subsection before implementing.

**Corner response** To extract the Harris corners from an image, we need to measure the corner response from the structure tensor. The measure of corner response is  $R = \det(M) - k(\text{trace}(M))^2$ , which compares whether the matrix has two strong eigenvalues, indicative of strong variation in all directions (see class notes). **Only pixels with positive corner responses can be corners.**

```
2 Implement Image cornerResponse(const Image &im,
    float k=0.15, float sigmaG=1, float factorSigma=4).
```



Figure 2: Our stata corner responses, normalized by the maximum values (as done in `testCornerResponse`).

**Non-maximum suppression** We get a strong corner response in a the neighborhood of each corner. We need to only keep the strongest response in this neighborhood. For this, we need to reject all pixels that are not a local maximum in a window of `maxiDiam`. We've written a function `maximum_filter` in `filtering.cpp` that you might find useful.

**Removing boundary corners** Because we will eventually need to extract a local patch around each corner, we can't use corners that are too close to the boundary of the image. Exclude all corners that are less than `boundarySize` pixels away from any of the four image edges.

**Putting it all together** In the end, your function should return a list of `Points` containing the coordinates of each corner. For this assignment, we provide a class `Point`. Each point corresponds to a Harris corner.

You are now ready to implement `HarrisCorners`.

```
3 Implement a function vector<Point> HarrisCorners(const Image  
   &im, float k=0.15, float sigmaG=1, float factor=4,  
   float maxDiam=7, float boundarySize=5) that returns a list of  
   2D Points.
```

Implement the algorithm following the steps previously described.

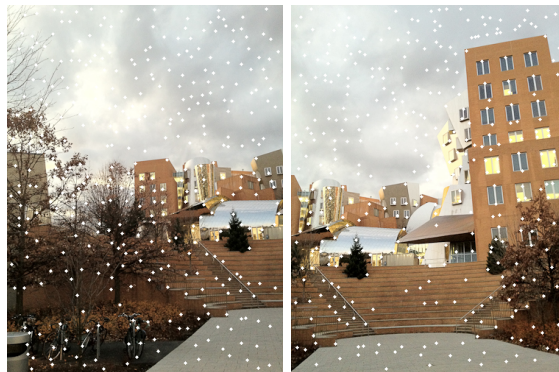


Figure 3: Use the provided function `visualizeCorners` to verify your results `HarrisCorners`. These are our results on Stata.

More bells and whistles such as adaptive maximum suppression or different luminance encoding might help, but this will be good enough for us.

## 4 Descriptor and correspondences

Descriptors characterize the local neighborhood of an interest point such as a Harris corner so that we can match it with the same point in different images.

Points in two images are put in correspondence when their descriptors are similar enough. We will call the combination of an interest point's coordinates and its descriptor a **Feature**, which is implemented as a class in `panorama.cpp`.

Our descriptors will be all the pixels in a `radiusDescriptor*2+1` by `radiusDescriptor*2+1` window around the associated interest point. That is, they will be a small single-channel Image of size  $9 \times 9$  when `radiusDescriptor=4`, whose center pixel has the coordinates of the point.

We also want to address potential brightness and contrast variation between images. For this, we subtract the mean of each patch, and divide the resulting patch by its standard deviation. Note that, as a result of the offset and scale, our descriptors will have negative numbers and might be greater than 1. We have added the `mean` and `var` methods to the `Image` class, in `Image.cpp`.

## 4.1 Descriptors

4 Write a subroutine `Image descriptor(const Image &blurredIm, Point p, float radiusDescriptor=4)` that extracts a single descriptor around interest Point `P`. Here, `im` is a **single-channel** Image (since we will be computing descriptors based on the luminance alone).

## 4.2 Features

We define a **Feature** as a pair  $(p, d)$  where  $p$  is a **Point** and  $d$  is the corresponding **Descriptor** encoded as a single-channel 9x9 Image. See our **Feature** class in `panorama.h` for more information.

5 Write a function `vector<Feature> computeFeatures(const Image &im, vector<Point> cornersL, float sigmaBlurDescriptor=0.5, float radiusDescriptor=4)` that takes as input a list `cornerL` of the above Harris corners associate each of them with a descriptor. The function should return a vector of features.

We compute the descriptor from a blurred single channel image. Specifically, first, extract the luminance of the input image. To avoid aliasing issues, blur the image with a Gaussian blur of standard deviation `sigmaBlurDescriptor`. Then, for each corner, extract the patch descriptor around it.

We provided you with a function `visualizeFeatures` that overlays the descriptors at the location of their interest points, with positive values in green and negative ones in red. The normalization by the standard deviation makes low-contrast patches harder to recognize, but high-contrast ones should be easy to debug, e.g. around the tree or other strong corners.

## 4.3 Best match and 2nd best match test

Now that we have code that can compute a list of features for each image, we want to find correspondence from features in one image to those in a second one. We will use our descriptors and the  $L_2$  norm to compare pairs of features. The procedure is not symmetric (we match from the first to the second image). We will use the **FeatureCorrespondence** class, which you can also see in `panorama.cpp`.

Let us first implemented the Euclidean distance between features:

6 The squared distance between two descriptors is the sum of squared differences between individual values. Implement `float l2Features(Feature &f1, Feature &f2)` that returns this sum of squared distances.

**Second-best test** If you remember the discussion in class, for our matching procedure, not only do we consider the most similar descriptor, but also the

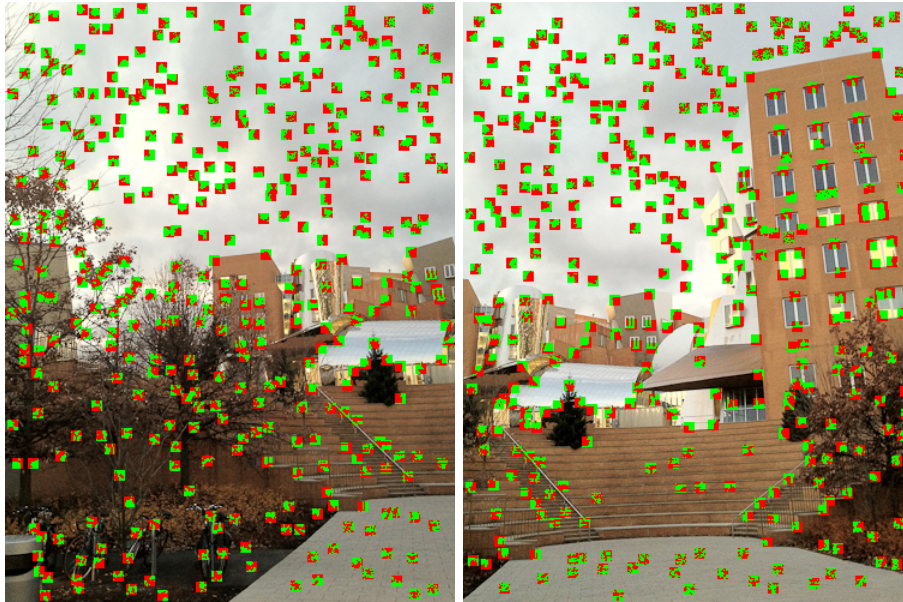


Figure 4: Visualizing features.

second best. If the ratio of distances of the second best to the best is less than **threshold**, we reject the match because it is too ambiguous: the second best match is almost as good as the best one. Be careful between the squared distance and the distance itself. You can compute everything with just the squared distance (it's faster, no need for `sqrt`) but then you need to use the square of the threshold.

Your function `findCorrespondences` should return a vector of `FeatureCorrespondence` (pairs of 2D points) corresponding to the matching interest points that passed the test. The size of this list should be at most that of `listFeatures1`, but is typically much smaller.

```
7 Write a function vector <FeatureCorrespondence> findCorrespondences(
  vector<Feature> listFeatures1, vector<Feature>
  listFeatures2, float threshold=1.7) that computes, for each
  feature in listFeatures1, the best match in listFeatures2, but
  rejects matches when they fail the second-best comparison studied in
  class. As usual, writing helper functions might help. The search for
  the minimum (squared) distance can be brute force.
```



Figure 5: Use the provided `visualizePairs` to debug your matches. Note that not all correspondences are going to be perfect. We will reject outliers in the next section using RANSAC. But a decent fraction should be coherent, as shown here.

## 5 RANSAC

So far, we've dealt with the tedious engineering of feature matching. Now comes the apotheosis of automatic panorama stitching, the elegant yet brute force RANSAC algorithm (RANDOM Sample Consensus). It is a powerful algorithm to fit low-order models in the presence of outliers. Read the whole section and check the slides to make sure you understand the algorithm before starting your implementation. If you have digested its essence, RANSAC is a trivial algorithm to implement. But start on the wrong foot and it might be a path of pain and misery.

In our case, we want to fit a homography that maps the list of feature points from one image to the corresponding ones in a second image, where correspondences are provided by the above `findCorrespondences` function. Unfortunately, a number of these correspondences might be utterly wrong, and we need to be robust to such so-called *outliers*. For this, RANSAC uses a probabilistic strategy and tries many possibilities based on a small number of correspondences, hoping that none of them is an outlier. By trying enough, we can increase the probability of getting an attempt that is free of outliers. Success is estimated by counting how many pairs of corresponding points are explained

by an attempt. Our RANSAC function will estimate the best homography from a `listOfCorrespondences`.

**Random correspondences** For each RANSAC iteration, pick four random pairs in `listOfCorrespondences`. The function `vector<FeatureCorrespondence> sampleCorrespondences(vector<FeatureCorrespondence> listOfCorrespondences)` can help you randomly shuffle the vector, and you can then use the first four entries as the four random pairs.

**Converting correspondences** For each RANSAC iteration, pick four random pairs in `listOfCorrespondences`. Given four pairs of points, you should have a function from problem set 6 that computes a homography. In that problem set you used a `listOfPairs` as the list of pairs of points. We supply you with a function `vector<CorrespondencePair> getListOfPairs(vector<FeatureCorrespondence> listOfCorrespondences)`.

**Singular linear system** In some cases, the four pairs might result in a singular system for the homography. Our first solution was to test the determinant of the system and return the identity matrix when things go wrong. It's not the cleanest solution in general, but RANSAC will have no problem dealing with it and rejecting this homography, so why not? Use the `determinant` method from Eigen's `Matrix` class.

**Scoring the fit** We need to evaluate how good a solution this homography might be. This is done by counting the number of inliers. See the function `inliers` described below.

8.a Implement `vector<bool> inliers(Matrix H, vector<FeatureCorrespondence> listOfCorrespondences, float epsilon=4)`.

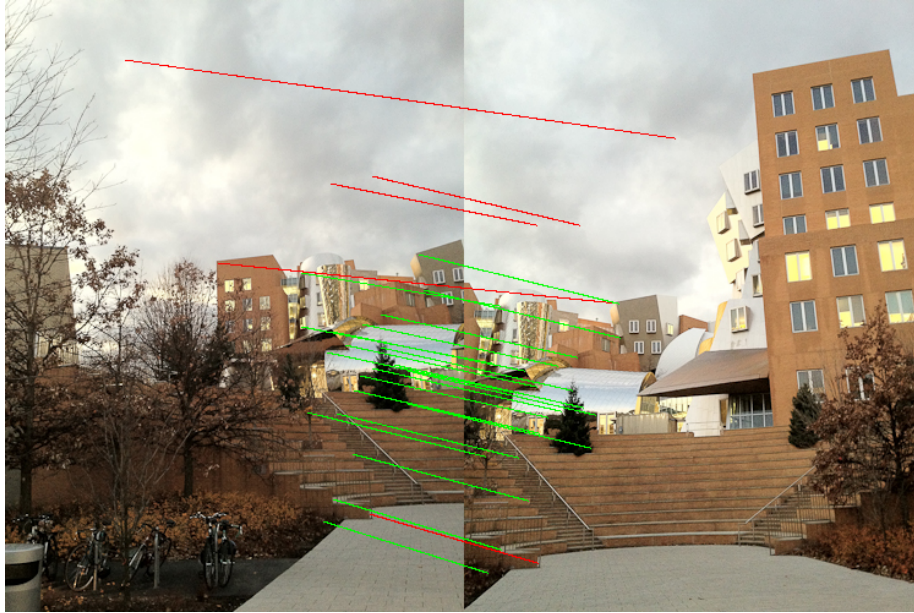
The function should return a list of Booleans of the same length as `listOfCorrespondences` that indicates whether each correspondence pair is an inlier, i.e., is well modeled by the homography. For this, use the test  $\|p' - Hp\| < \text{epsilon}$ . Use the output boolean vector to count the number of inliers. If the number of inliers of the current homography is greater than the best one so far, keep the homography and update the best number of inliers.

8.b Write a function `Matrix RANSAC(vector<FeatureCorrespondence> listOfCorrespondences, int Niter=200, float epsilon=4)` that takes a list of correspondences and returns a homography that best transforms the first member of each pair into the second one. `Niter` is the maximum number of RANSAC iterations (random attempts) and `epsilon` is the precision, in pixel, for the definition of an outlier. vs. inlier. That is, the pair  $p, p'$  is said to be an inlier with respect to



a homography  $H$  if  $\|p' - Hp\|_2 < \epsilon$  ( $\|\cdot\|_2$  indicates L2 norm).

You can use the provided function `visualizePairsWithInliers` to see which correspondences are considered inliers. It outputs an image similar the output of `visualizePairs` except that inliers are in green and outliers are red. But, your mileage will vary because RANSAC is a probabilistic algorithm. Don't freak out if you don't get exactly the same inliers, RANSAC uses randomized trials.



You can also use the provided `visualizeReprojection`, which shows where the homography reprojects feature points. For inlier, detected corners are in green, while those reprojected from the other image are in red. For outliers, the local corners are yellow and the reprojected ones are blue. Our reproductions for Stata are below. The result below further emphasizes that RANSAC is probabilistic: the set of inliers is not exactly the same as above.



## 6 Automatic panorama stitching

9 Write a function `Image autostitch(Image &im1, Image &im2, float blurDescriptor, float radiusDescriptor)` that takes two images as input and automatically outputs a panorama image where the second image is warped into the domain of the first one. You should get a similar-ish result to the last assignment, but automatically.

Try it on the Stata, Boston-skyline and at least another pair of images.

## 7 Second part to come soon !

## 8 Extra credits (10% max)

For any extra credit you attempt (5% each), please write a new test function in your main file, and include the name of the test function in the submission questionnaire. This is a requirement for getting the extra credit.

- Adaptive non-maximum suppression.
- Wavelet descriptor.

- Rotation or Scale invariance
- Full SIFT.
- Evaluation of repeatability.
- Least square refinement of homography at the end of RANSAC
- Reweighted least square.
- Bundle adjustment

## 9 Submission

Turn in your files to the online submission system (link is on Stellar) and make sure all your files are in the `asst` directory under the root of the zip file. If your code compiles on the submission system, it is organized correctly. The submission system will run code in your main function, but we will not use this code for grading. The submission system should also show you the image your code writes to the `./Output` directory

In the submission system, there will be a form in which you should answer the following questions:

- How long did the assignment take? (in minutes)
- Potential issues with your solution and explanation of partial completion (for partial credit)
- Any extra credit you may have implemented and their function signatures if applicable
- Collaboration acknowledgment (you must write your own code)
- What was most unclear/difficult?
- What was most exciting?