Problem Chosen	2023 MCM/ICM Summary Sheet	Team Control Number
A		2319576

Plants V.S. Weather

Summary

In recent years, drought and global warming are endangering the living environment of animals and plants. In order to explore the mechanism of community dynamic development, in this paper, we mainly focus on the two influencing factors of temperature and humidity. Combined with the influence of pollutants and habitats, we establish an evolution model of plant communities.

In order to facilitate the establishment of the model, we first establish the population evolution model of a single species—**Logistic Model**, and then introduce inter-species competition to obtain the **Lotka-Volterra competition model**. At this time, we do not consider the influence of environmental factors, so the inherent growth rate of the population remains unchanged.

After obtaining the interspecies competition model, we add the influence of **environmental** factors. In this paper, we consider the influence of **temperature** and **humidity** on the intrinsic growth rate. The influence functions are given by experimental facts. The sources of temperature and humidity data are reliable, so that the model is highly portable.

Then we discuss the impact of the number of species on the survival of the community. According to the facts, we apply three plants with different responses to drought, and find that the community with **high resistance and high resilience** to environmental changes are more likely to survive. In the actual plant community, the increase in the number of species will increase the possibility of the emergence of such environmentally adapted species, and the number of species has an insurance effect on community survival.

In order to explore the impact of **higher frequency drought** conditions on plant communities, we process the actual humidity data to make it have a higher frequency of change, re-establish the impact function of humidity on the inherent growth rate, and analyze the community response through the multi-species competition model. Through the analysis of the results, we find that species with high resistance and high resilience can make certain resistance to this high frequency drought. The species without this ability are likely to become extinct.

Finally, we explore the impact of pollutants and habitats on communities, introduce the Air Pollution Tolerance Index to describe the tolerance of plants to air pollution, and analyze the impact of habitat reduction on plant reproduction rate from two aspects of plant phenology and pollination network.

Keywords: Muti-Species; Interaction; Lotka-Volterra Competition Model; Biodiversity

Contents

1 Introduction

1.1 Problem Background

In recent years, extreme weather around the world has taken turns. Climate change has a great impact on plant growth, especially drought weather poses a long-term threat to plant species diversity.

Different species of plants have different responses to environmental stresses. The plant community contains a wide variety of plant populations, and the plant species diversity plays a role in the adaptation of drought and other weather conditions. In some communities with only one species of plant, the next few generations of plants have relatively weak adaptability to drought and other weather. The schematic diagram is shown in Figure 1.

Figure 1: Plant Diversity

1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement, the main work of this paper is as follows:

- Under various irregular weather cycles, establish a mathematical model to predict the change of plant community with time. In the model, it is necessary to consider the interaction between different species during cycles of drought (including the drought period when the precipitation should be sufficient).
- Analyze the long-term interaction between a community of plants and the larger environment. Find the number of plant species in the community that can benefit the plant population. And predict what will happen after the number of species grows.
- Describe the impact of species types in the community on the results.
- Analyze the impact of more frequent and changing droughts on plant species in future weather cycles, and the impact of infrequent drought on the overall population.

1.3 Literature Review

Xu Chong [1] showed that extreme drought reduced the aboveground net primary productivity of communities, different functional groups, dominant and non-dominant species. However, the response of aboveground net primary productivity is affected by location, drought duration and drought pattern. The response of aboveground net primary productivity to extreme drought in the wet grassland ecosystem is lower than that in the dry grassland ecosystem. The aboveground net primary productivity of desert grassland community is most sensitive to extreme drought. The aboveground net primary productivity of grasses and dominant species in different grassland ecosystems to extreme drought is consistent with the above-ground net primary productivity of communities, while the above-ground net primary productivity of miscellaneous grasses and non-dominant species is significantly different from the above-ground net primary productivity of communities.

Plants can affect the atmosphere through carbon, water and energy exchange, and can also intensify drought through the feedback effect between land and atmosphere. Plant diversity may play a key role in land-atmosphere feedback during drought. Average annual rainfall is an important factor to control the aboveground net primary productivity of grassland ecosystem and its resilience to extreme drought. Temperature is a factor that regulates the response of aboveground net primary productivity to extreme drought. Therefore, other factors may be important factors affecting the recovery rate of aboveground net primary productivity after the end of extreme drought [1,2].

According to Graven D et al. [2], in a larger environment, the limited growth resources change with the gradient of plant species diversity, where there is a reduction in resource availability, biodiversity communities can buffer the decline of productivity through a number of mechanisms, and are more likely to include species that can access scarce resources in times of stress (Including the great likelihood of tolerant species being present [3]. For example, where water is scarce, some plants die due to insufficient water supply, but some can obtain water from deeper layers of soil or nutrients from different depths of the root base. Ditto for other places with scarce resources. Vanda Éva Molnára, Edina Simonb, et al. [7] used Air Pollution Tolerance Index (APTI) to evaluate the tolerance of plant species to air pollution, and they applied Random Forest Regression for statistical model building. The results from the APTI values indicated that plants in polluted environment had higher tolerance than plants grown under better environmental conditions.

1.4 Our Work

The whole modeling process can be shown as follows:

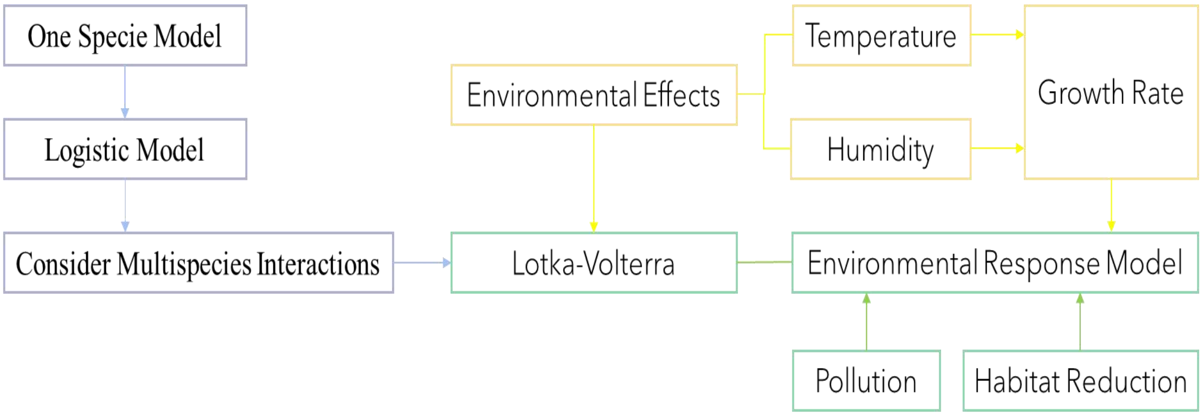

Figure 2: Flow Chart of Our Work

2 Assumptions and Justifications

To simplify the given problem, we make the following basic assumptions in this paper, each of which is properly justified.

- **Over the time scale of this model, we assume that the environmental holding capacity remains constant.**
  For a plant community, when there is no great change in the environment, the total environmental resources fluctuate little, and the environmental capacity remains basically unchanged.
- **For multi-species models, only the competition between plants is considered.**
  For plant communities, different species will compete for light, water and other resources, making competition the most common interspecific relationship.
- **Assuming that environmental factors only affect the intrinsic growth rate of the species, which in turn has an impact on the population development.**

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description
H	Humidity (%)
T	Temperature (℃)
r	Growth rate
s	Competition coefficient
N	Number of populations
y	Leaf water content
$APTI$	Air pollution tolerance index
API	Air pollution index

4 Model and Results

Next, we will describe our model in detail.

4.1 Evolution Model of Plant Communities

4.1.1 Logistic Model for One Specie

We generally use the logistic model to describe the change trend of the population of a certain species. For a single species in a space, the rate of change of the population can be expressed as:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{1}$$

Where N is the number of population, r is the growth rate, K is the environmental capacity of this specie.

This model considers the population change of a single species in an environment with limited resources and intraspecific competition. If the population change of multiple species in a plant community is considered, the impact of interspecific competition needs to be added. For this reason, we introduce Lotka-Volterra Model.

4.1.2 Multispecies Competition Model

For a plant community with multiple species, we use the Lotka-Volterra interspecific competition model to describe the population change. We assume that there is a plant community with n species, and the change in the number of each population can be expressed as:

$$\begin{cases} \dfrac{dN_1}{dt} = r_1 N_1 \left(1 - \dfrac{N_1}{K_1} - \sum s_{i1} \dfrac{N_i}{K_i}\right) \\ \dfrac{dN_2}{dt} = r_2 N_2 \left(1 - \dfrac{N_2}{K_2} - \sum s_{i2} \dfrac{N_i}{K_i}\right) \\ \qquad \cdots \\ \dfrac{dN_j}{dt} = r_j N_j \left(1 - \dfrac{N_j}{K_j} - \sum s_{ij} \dfrac{N_i}{K_i}\right) \end{cases} \tag{2}$$

Where r_j is the inherent growth rate of the jth species, N_j is the number of the jth population, K_i is the environmental capacity of the ith population, and s_{ij} is the competition coefficient of the ith population to the jth population. A high competition coefficient indicates that the plant has a strong ability to compete for resources and is easy to evolve into a dominant species in the improved plant community. Taking a plant community with two plants as an example, if there is competition between two species and one of them has a strong competitiveness (reflected by its higher competition coefficient), the populations of these two species will change, as shown in Figure 3.

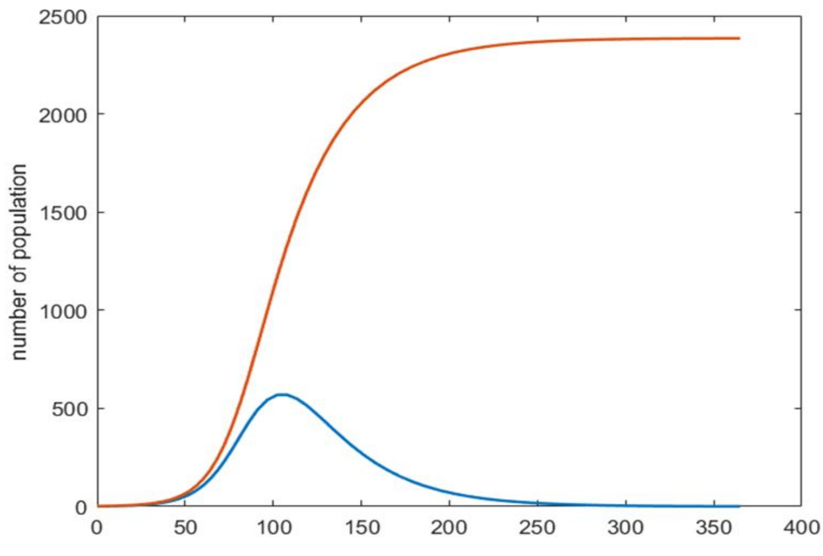

Figure 3: The Change of Two Population Quantity with Time in Constant Environment

In this section, r is the intrinsic growth rate of the species, which is constant in the time scale. We consider that when the plant community is exposed to various irregular weather cycles under the condition that the environmental factors fluctuate little, the growth rate of plants changes significantly with the changes of environmental factors. Therefore, we analyze the effects of different temperature and humidity conditions on the growth rate, and re-establish Models of Community Evolution.

4.2 Environmental Response Model

4.2.1 Data Sources

Using the temperature data of Los Angeles from 2002 to 2022, the temperature data of each year is fitted with a cubic polynomial to obtain the annual temperature change law of Los Angeles. At the same time, the fitting method can eliminate the influence of temperature extremes, making analysis and prediction more accurate. Los Angeles' temperature change pattern is shown in Figure 4.

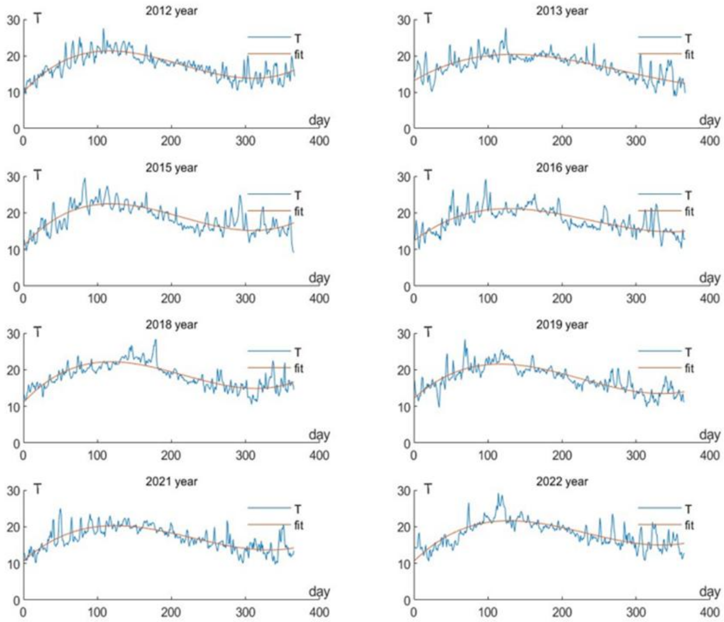

Figure 4: 2012-2022 Temperature and Fitting Results

4.2.2 Analysis of Population Response to Temperature

We know that the effect of temperature on the net photosynthetic rate is mainly achieved by affecting the enzyme activity. For most plants, when the temperature is lower than the critical temperature, the enzyme activity is approximately linear with the temperature. When the temperature reaches the critical temperature, the enzyme deactivates rapidly, and this inactivation is irreversible, so the enzymatic reaction stops rapidly. Figure 5 shows the relationship between photosynthetic rate and temperature.

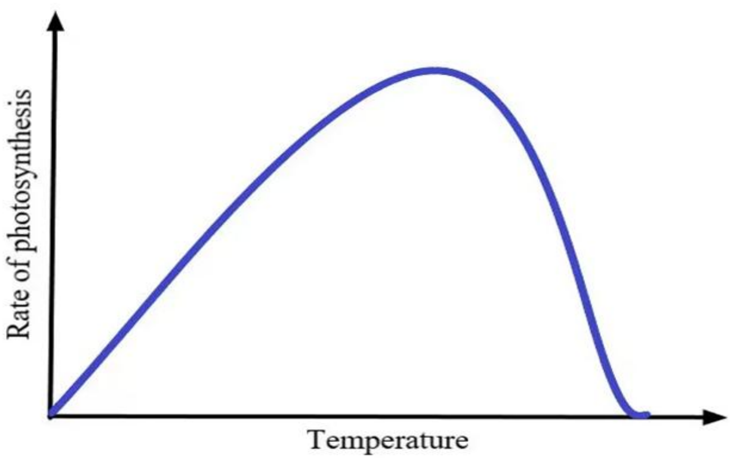

Figure 5: Relationship Between Photosynthetic Rate and Temperature

Therefore, for this model, we treat the relationship between them as linear. Before reaching the critical temperature, the growth rate is positively correlated with temperature, and after reaching the critical temperature, the growth rate is negatively correlated with temperature. Figure 6 shows the correlation between growth rate and temperature.

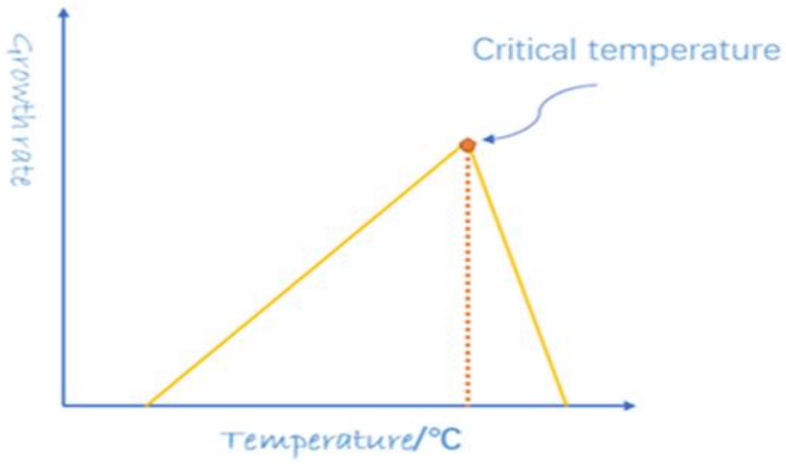

Figure 6: Relationship Between Temperature and Growth Rate

4.2.3 Analysis of Population Response to Soil Moisture

Water is a component of plant cells and an important material transport medium in plants. Plants need water to maintain the stability of cell structure and normal cell metabolism activities. A water-deficient environment will lead to slow plant metabolism and affect plant growth. At the same time, excessive soil moisture will cause oxygen loss in the soil, hinder the respiration of plant roots, and affect the normal development of plants.

For the relationship between soil moisture and leaf water content, we can get the following relationship according to Michaelis equation:

$$y = \frac{aH}{b+H} \tag{3}$$

Where y is leaf water content, x is soil moisture, a, b are constants.

For the relationship between net photosynthetic rate and leaf water content, the following relationship can be obtained by fitting the experimental data:

$$r(y) = -Ay^2 + By + C \tag{4}$$

It can be seen from the formula that the net photosynthetic rate and the water content of leaves are in a quadratic function relationship. From the two equations above, we can get the relationship between plant net photosynthetic rate and soil moisture. Similarly, the humidity data of Los Angeles from 2002 to 2022 is selected as the data source of this model, and polynomial fitting is used to remove the influence of extreme values. This is shown in Figure 7.

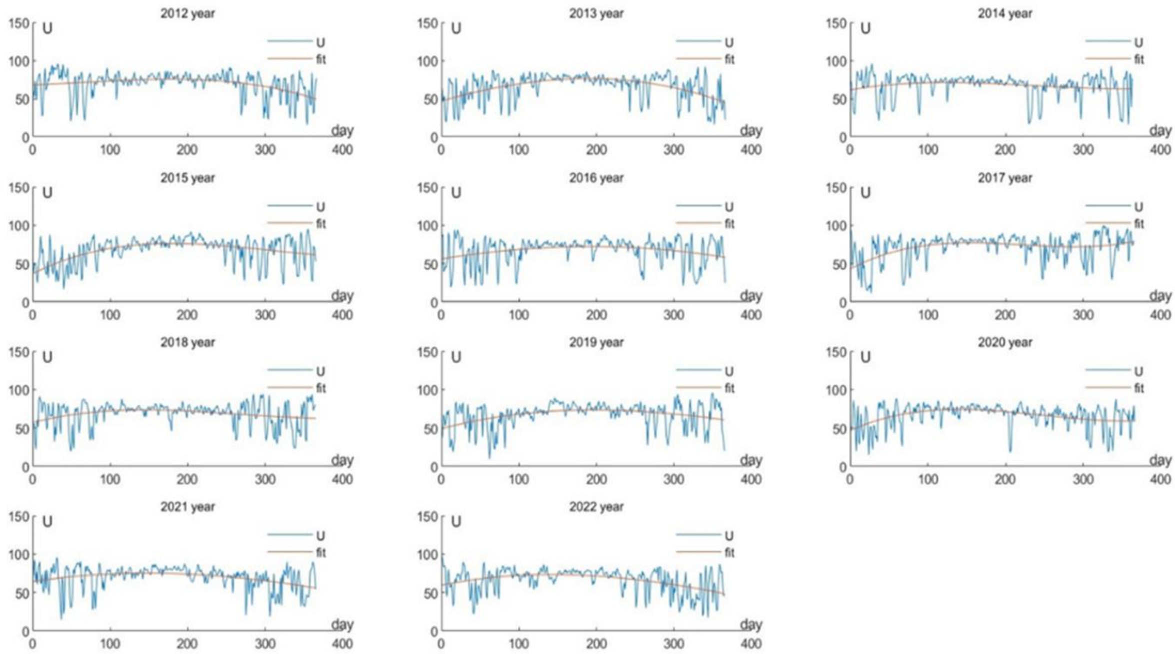

Figure 7: 2012-2022 Humidity and Fitting Results

4.2.4 Global Impact of the Environment

So far, we have obtained the influence of temperature and humidity on the intrinsic growth rate of plants $f(T)$, $g(H)$. Therefore, we can derive the global influence function of environmental variables (temperature, humidity) on the intrinsic growth rate. It can be expressed as:

$$r(T,H)=\alpha f(T)g(H) \tag{5}$$

where α is a constant.

According to the above analysis, we obtain the Lotka-Volterra model of plant interspecific competition under the condition of temperature and humidity changes:

$$\begin{cases} \dfrac{dN_1}{dt}=r_1(T,H)N_1\left(1-\dfrac{N_1}{K_1}-\sum s_{i1}\dfrac{N_i}{K_i}\right) \\ \dfrac{dN_2}{dt}=r_2(T,H)N_2\left(1-\dfrac{N_2}{K_2}-\sum s_{i2}\dfrac{N_i}{K_i}\right) \\ \qquad\qquad \cdots \\ \dfrac{dN_j}{dt}=r_j(T,H)N_j\left(1-\dfrac{N_j}{K_j}-\sum s_{ij}\dfrac{N_i}{K_i}\right) \end{cases} \tag{6}$$

4.2.5 Result Analysis

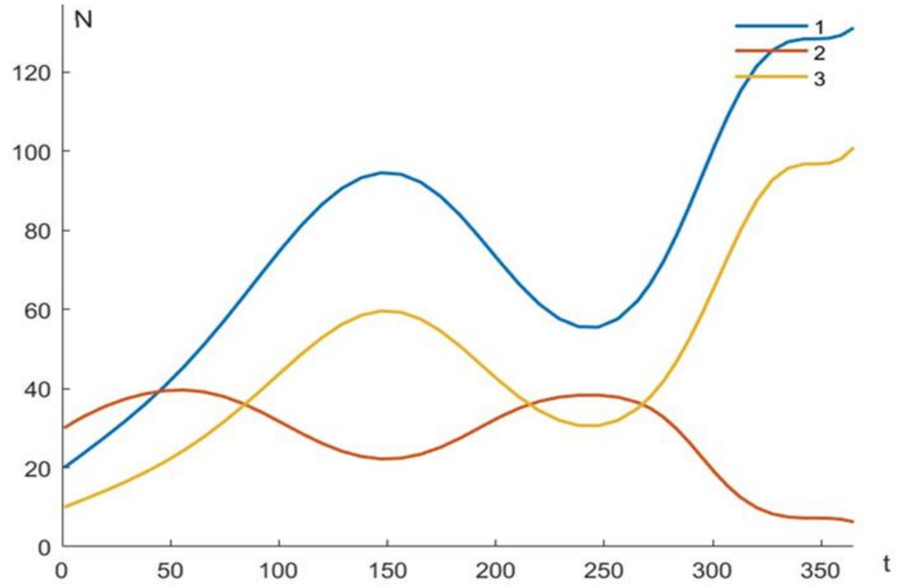

Figure 8: The Change of Population Quantity with Time in Cyclical Environment

- **Parameter setting**

  Set up three different plants, each plant has different inherent growth rate and environmental capacity, and use the fitting data of temperature and humidity in Los Angeles obtained in the previous two sections as the change source of inherent growth rate.

- **Analysis of the figure**

  For populations with different initial population numbers and environmental capacity, environmental factors have a greater impact on the evolution of the community. The dominant species no longer always occupy the dominant position. With the change of environment, the number of dominant species may fluctuate greatly, and even the dominant species may be replaced. The population number change curve is no longer a simple competition relationship curve, and changes with environmental factors. The species at a disadvantage in the competition may have an upward trend (due to the reduction of the number of natural enemies).

4.3 The Insurance Role of Biodiversity on Community Drought Resistance

Different plants respond differently to environmental changes and extreme conditions (such as drought). In plant communities, some plants have strong resistance to drought conditions, showing better high temperature tolerance or higher Survival ability, some plants have strong resilience after extreme drought, and can return to normal quantity levels in a short period of time after drought. To simulate plant species with different drought responses in a plant community, three plant species were defined. This is shown in Figure 9.

Plant / Trait	PLANT A	PLANT B	PLANT C
Resistance	High	Medium	Medium
Resilience	Medium	High	High

Figure 9: Three Plant Species

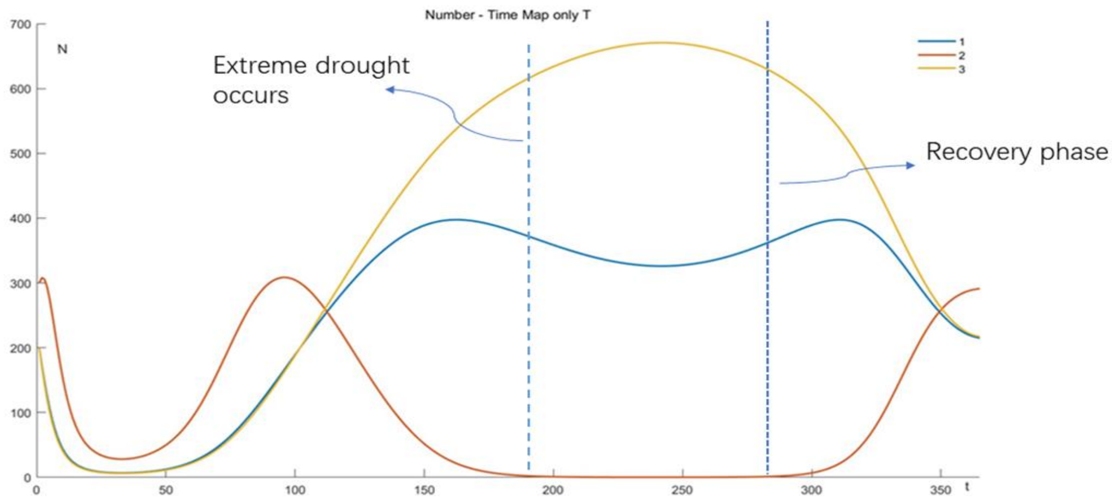

Figure 10: The Change of Population with Time in Cyclical Drought Environment

From the analysis of the results, it can be seen that if a plant community has both high resistance and high recovery species, the community is easy to maintain stability under extreme conditions, and the extinction of the community will not occur.

It can be seen that there is a strong correlation between biodiversity and the drought resistance of plant communities. Figure11 and Figure12 show that the diversity map of local plant populations and the global drought degree map, which can intuitively see this correlation.

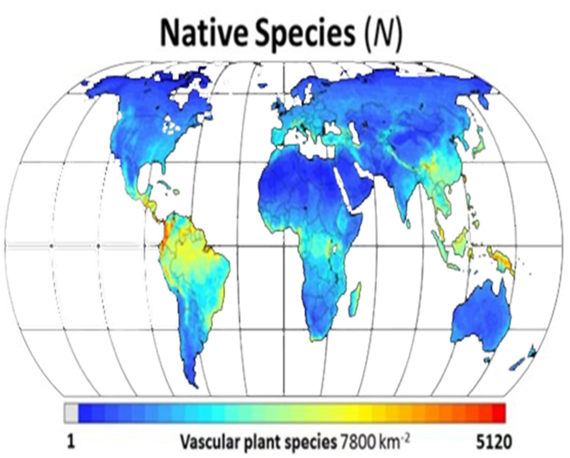

Figure 11: Map of Global Species Distribution [10]

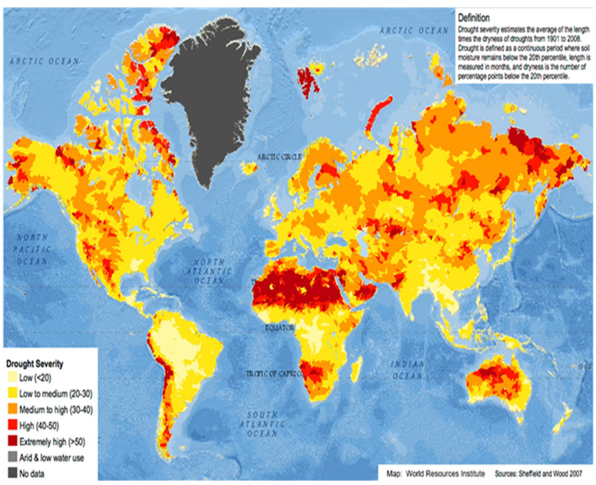

Figure 12: Map of Global Drought Conditions [11]

From these two maps, it can be found that areas with high biodiversity are less arid. Biodiversity and drought resistance are generally positively correlated. In the maps, this correlation is particularly obvious in Africa and Oceania.

4.4 Higher Frequency of Drought

For arid areas, we use the parameter of humidity to represent the degree of drought. During the time of frequent drought and little rainfall, we keep the humidity at a low level to simulate a dry environment, and increase the humidity appropriately when it rains. This reciprocating change in humidity is used to simulate a weather environment that changes at a higher frequency. Figure 13 is an image of simulated humidity. Figure 14 shows the change rule of species groups with time under high-frequency arid environment.

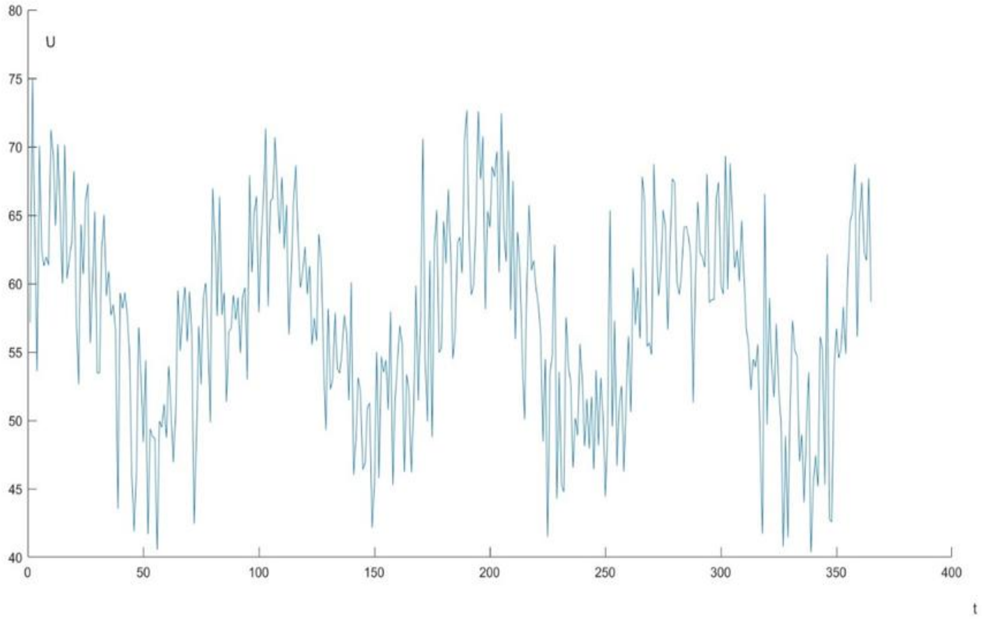

Figure 13: Simulated Humidity Image

Parameter Settings: Species No. 1 has strong resilience to drought, Species No. 2 has strong resistance to drought, and No. 3 is a common plant with neither obvious resilience nor obvious resistance.

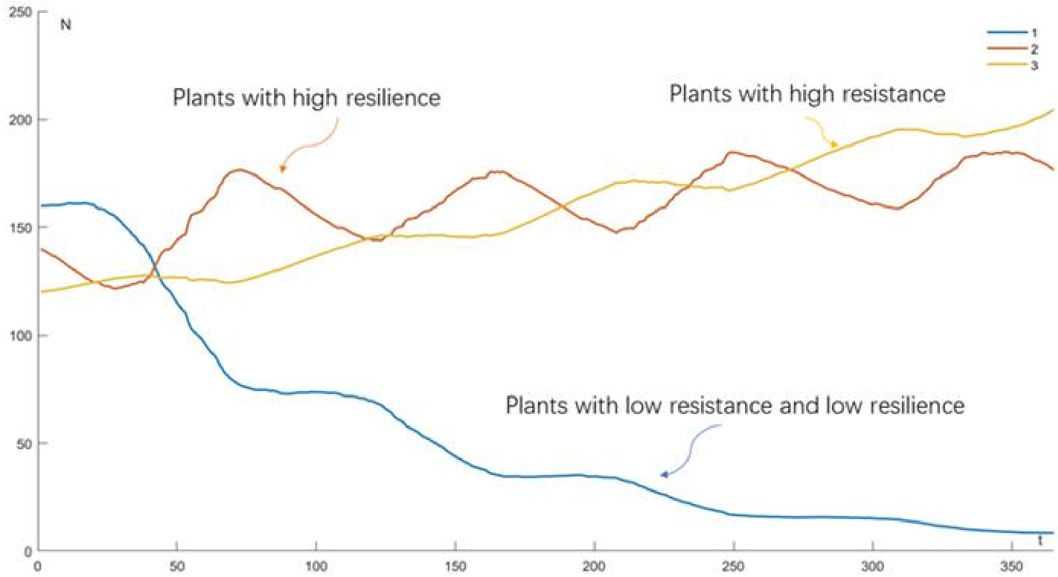

Figure 14: The Change of Population with Time in Higher Frequency Droughts Environment

Result Analysis: It can be seen from the image that the population of species No. 1 continues to decline under high-frequency droughts due to its weak resistance and resilience. Species No. 2 has strong resilience. Although the population decreases during droughts, it will rebound soon, and return to the level before the drought. Species No. 3 has strong resistance, and the drought only slows down the growth rate of this specie.

4.5 Pollution and Habitat Reduction

For the effect of environmental pollution on plant communities, we consider the adverse effects of air pollution and habitat reduction on plants and analyze them separately.

4.5.1 Air Pollution

Air pollution sources are mainly divided into natural pollution sources and man-made pollution sources. Natural phenomena such as volcanic activities and forest fires will cause an increase in airborne dust. Man-made pollution sources such as pollutant emissions from factories and emissions from urban traffic will lead to increased sulfur dioxide and nitrogen dioxide gas content.

Plants can resist air pollution to a certain extent and absorb some harmful gases. However, if the pollutant content that plants accept in an air-polluted environment exceeds the critical amount, it will cause damage to the plants and cause symptoms such as withering, death, and stunting. Figure 15 is a schematic diagram of the impact of urban pollution on plants.

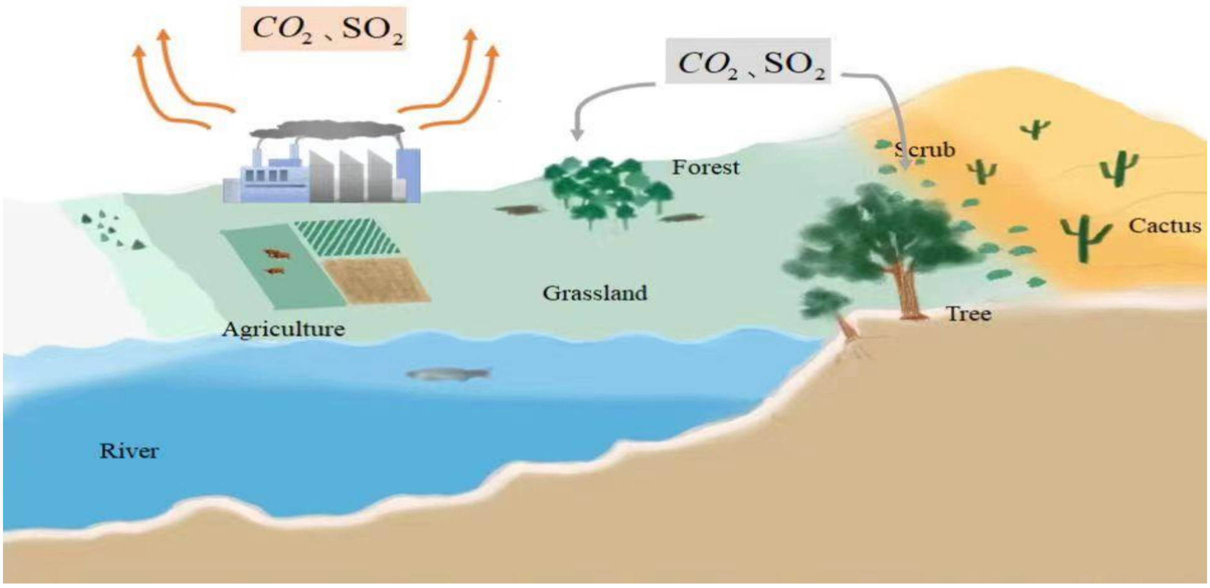

Figure 15: Diagram of urban pollution affecting the environment

We use APTI (Air Pollution Tolerance Index) to indicate the ability of plants to resist the adverse effects of air pollution. APTI can be calculated by the following formula [7].

$$APTI = \frac{A(T+P)+R}{10} \tag{7}$$

Where P is PH of leaf extract, A is ascorbic acid content, R is relative water content, the three parameters can be measured experimentally.

At the same time, we use API to detect the degree of air pollution. In areas with the same API, the APTI values measured by different plants are different. The higher the APTI, the stronger the resistance of the plant to air pollution. From the APTI calculation equation, it can be seen that air pollution affects the content of plant chlorophyll and so on, thus affecting the growth of plants. In addition to temperature and humidity, the intrinsic growth rate of plants should also be a function of API. The corrected intrinsic growth rate can be expressed as:

$$r = r(T, H, \phi(API)) \tag{8}$$

where ϕ is the influence function of API on plants, for different APTI levels, this function is also different.

4.5.2 Habitat Reduction

Habitat reduction mainly has two effects on plant communities. The fragmentation of plant communities will change the plant phenology pattern. At the same time, the reduction of pollinator habitat will affect the pollination network mechanism, resulting in a decrease in plant reproduction speed, which also affects its inherent growth rate. The schematic diagram of the impact of habitat reduction on plant communities is shown in Figure 16:

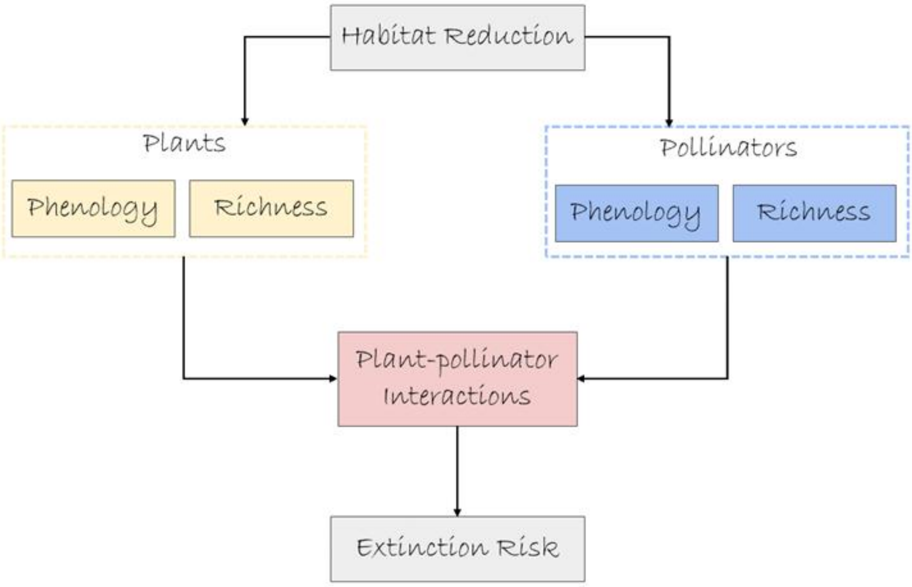

Figure 16: Diagram of the impact of habitat reduction on plant communities

4.6 Long-term Viability

Through modeling of species diversity, we find that a biome can survive complex and variable weather environments only when there are plants with diverse adaptive abilities in the community, such as repeated extreme drought conditions.The long-term survival plants community must contain both high body resistance and high resilience plant populations. If there are no plants with these two abilities, extinction may occur under drought stress.

At the same time, through the modeling and analysis of pollutants and habitats, protecting the surrounding environment of plant communities and reducing the impact of human factors on plant growth and reproduction are also important factors for the long-term survival of plant communities. We have noticed that, plant communities have been dying out due to environmental pollution and habitat destruction caused by human activities in recent years. Therefore, through this model, we call for reducing high-polluting activities and returning a healthy ecological environment to plant communities and ecosystems.

5 Sensitivity Analysis

For the multi-species competition model, the input parameters of the model are temperature and humidity. These two parameters affect the trend of population quantity by affecting the inherent growth rate. Now we keep one parameter unchanged and change the value of the other parameter to explore the sensitivity of the model to these two parameters, and change the temperature and humidity equally. The resulting trend change is shown in Figure17:

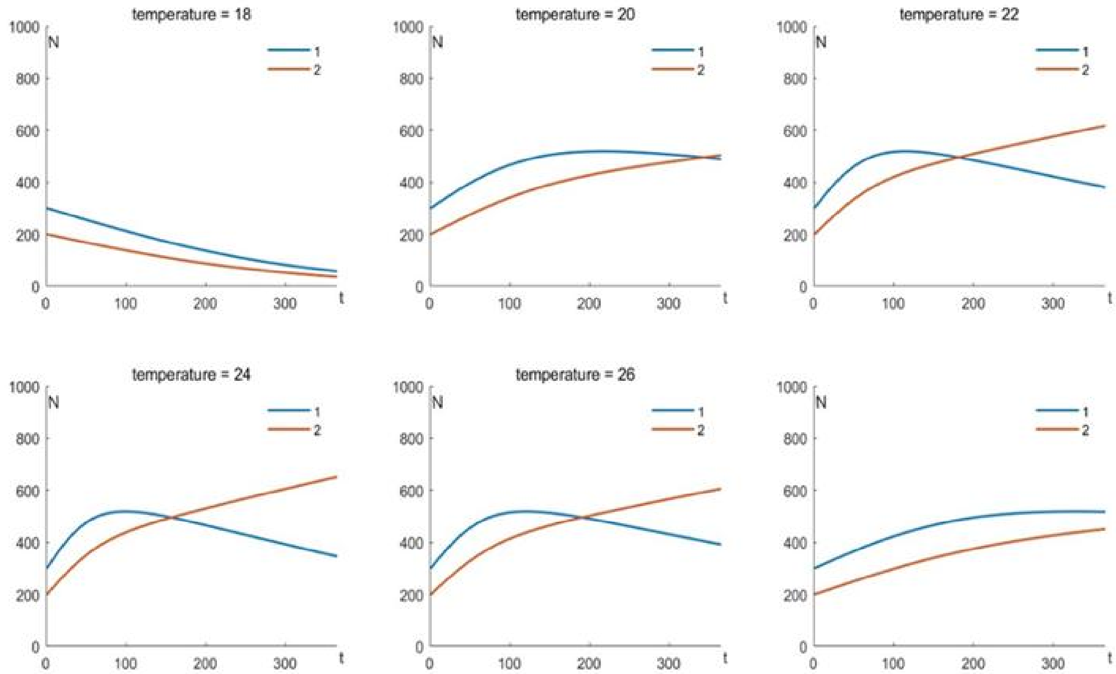

Figure 17: Evolution Trend of Species Number at Constant Humidity

It can be seen from the figure that the model is sensitive to temperature, that is, when the temperature changes and the humidity remains unchanged, the evolution trend of the number of species in the community has a certain degree of change. Similarly, we change the humidity equally, and the trend change is presented in Figure 18:

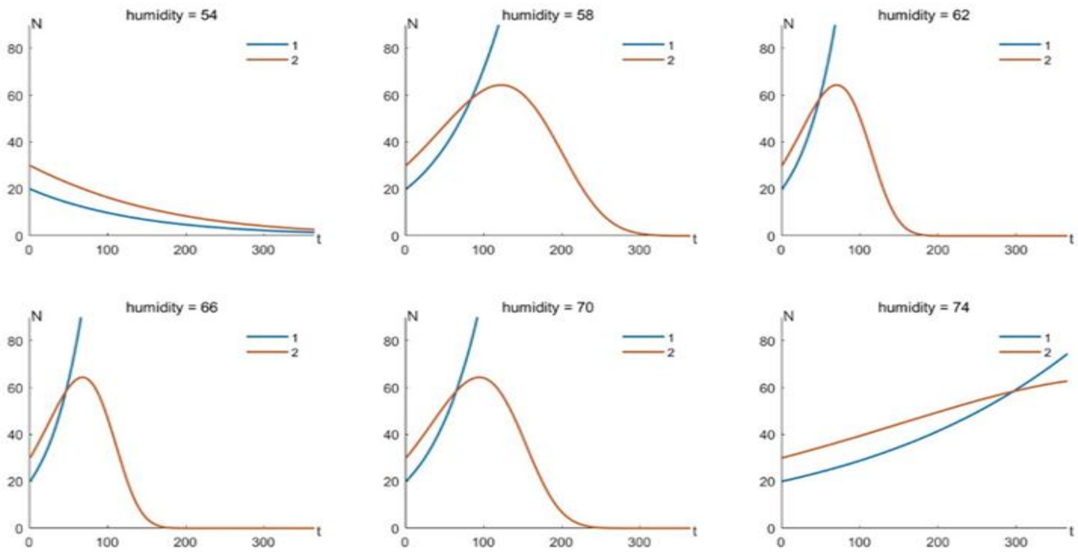

Figure 18: Evolution Trend of Species Number at Constant Temperature

6 Model Evaluation

6.1 Strength

- The model is highly universal. Our model is mainly based on the classic Lotka-Volterra model, which is used to reveal the competitive relationship between species, and in fact, we can also apply this to some correlation predictions. That is to say, once we have the initial data and all the plant characteristics, environmental parameters, and how they will evolve in the future, a reasonable and robust prediction will be given.
- The model has good and expansibility. If we need to consider the influence of other factors, we can modify the differential equation to upgrade the model.
- When considering the influence of environmental factors, the influence function of temperature and humidity is obtained through the actual data of ecological experiments. This method makes the model have strong practical significance and application ability.

6.2 Weaknesses

- For environmental factors, only the influence of temperature and humidity is considered, and some other factors such as predators, insufficient sunlight, etc. are not considered.
- The model uses a linear function to deal with the temperature influence function. Although the model is simplified, the accuracy of the model is weakened to a certain extent.

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[9] "Lotka–Volterra equations" [Online] Available: https://en.wikipedia.org/wiki/Lotka-Volterra_equations

[10] Source: https://anthroecology.org/anthromes/plantbiodiversity/maps (Figure 11)

[11] Source: https://en.wikipedia.org/wiki/File:Global_Water_Security.jpg (Figure 12)

Appendices

Appendix 1
Introduce: Order four Runge-Kutta algorithm to solve Lotka-Volterra competition model

```
function [t,y,n]=ODE4(ufunc,tspan,y0,h)
%% The order of the parameter table is the function name of the differential equations,
% the start and end point of time, the initial value, and the step size (the parameter form refers to the ode45 function).
if nargin<4
    h=0.01;
end
if size(tspan)==[1,2]
    t0=tspan(1);
    tn=tspan(2);
else
    error(message('MATLAB:runge_kuttx0_o4:WrongDimensionOfTspan'));
end
n=floor((tn-t0)/h);% Number of steps
t(1)=t0;%time start
y(:,1)=y0;%The first column assigns initial values, which can be vectors, representing different initial values
for i=1:n
t(i+1)=t(i)+h;
k1=ufunc(t(i),y(:,i));
k2=ufunc(t(i)+h/2,y(:,i)+h*k1/2);
k3=ufunc(t(i)+h/2,y(:,i)+h*k2/2);
k4=ufunc(t(i)+h,y(:,i)+h*k3);
y(:,i+1)=y(:,i)+h*(k1+2*k2+2*k3+k4)/6;
%The numerical solution is carried out according to Rungekutta method
end
```