PES UNIVERSITY
(Established under Karnataka Act No. 16 of 2013)
100 Feet Ring Road, BSK III Stage, Bengaluru-560 085
Department of Electronics and Communication Engineering

Course Title: Control Systems
Course Code: UE21EC241B

PROJECT REPORT:

TOPIC: OPTIMIZATION OF DRONE MOVEMENT USING PID CONTROLLERS

TEAM MEMBER:

ARCHANAA A CHANDARAGI (PES1UG21EC056)
ACHYUTH S S (PES1UG21EC010)
ADITHYA R G (PES1UG21EC017)

INTRODUCTION

A quadcopter or quadrotor is a flying machine that uses four rotors to stabilize itself during flight. Quadcopters have been increasingly used in commercial and industrial applications in recent times. Due to its flexible, and inexpensive nature, it serves as an excellent platform to perform a variety of tasks. One of the major challenges in the construction of drones and autonomous vehicles is the design of the controller. However, due to its simple construction, developing a mathematical model and designing a controller for the quadcopter is quite easy.

In this project we are using SIMULINK to simulate a Parrot MINIDRONE and generate an open loop transfer function to answer the questions expected to be answered for the project.

HARDWARE OVERVIEW:

The parrot minidrone is equipped with four different sensors,

Ultrasound sensors: Measure the distance of the drone above the surface.
Camera: Uses optical flow algorithm to help deducing the rotational and translational motion of the drone.
Pressure Sensor: Helps to know the altitude of the drone.
Inertial Measurement Unit: Has a 3-axis accelerometer that measures linear acceleration, and a 3-axis gyroscope that measures the angular rate.

The drone has four actuators(propellers) arranged in an X configuration.
In the X configuration opposing motors rotate in the same direction. Which allows the drone to maintain 6 degrees of freedom (motion along X,Y,Z axes and pitch ,roll and yaw which are rotations about Y,X and Z axes) As the drone has 6 degrees of freedom and only 4 actuators, it makes the drone an underactuated system. Hence a certain motor mixing algorithm is used to bring about motion in the 6 degrees for the drone.

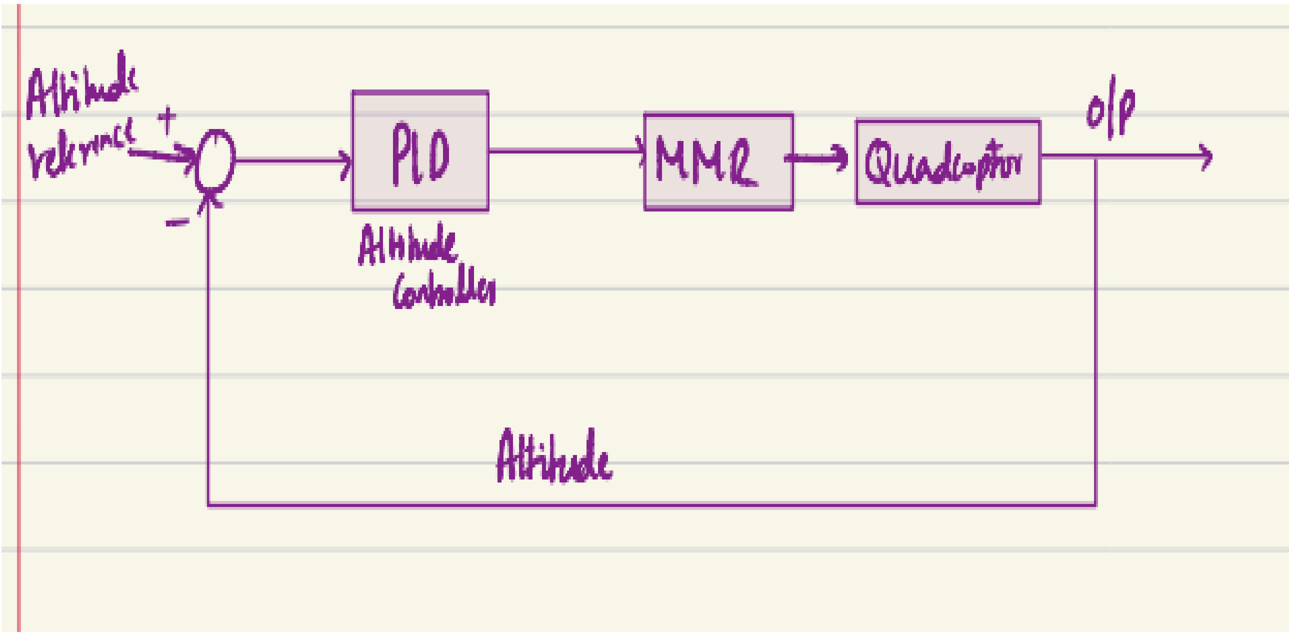
THE CONTROL PROBLEM:

The optimal control of drone movement can be challenging, particularly in complex environments with varying external factors such as wind and turbulence.

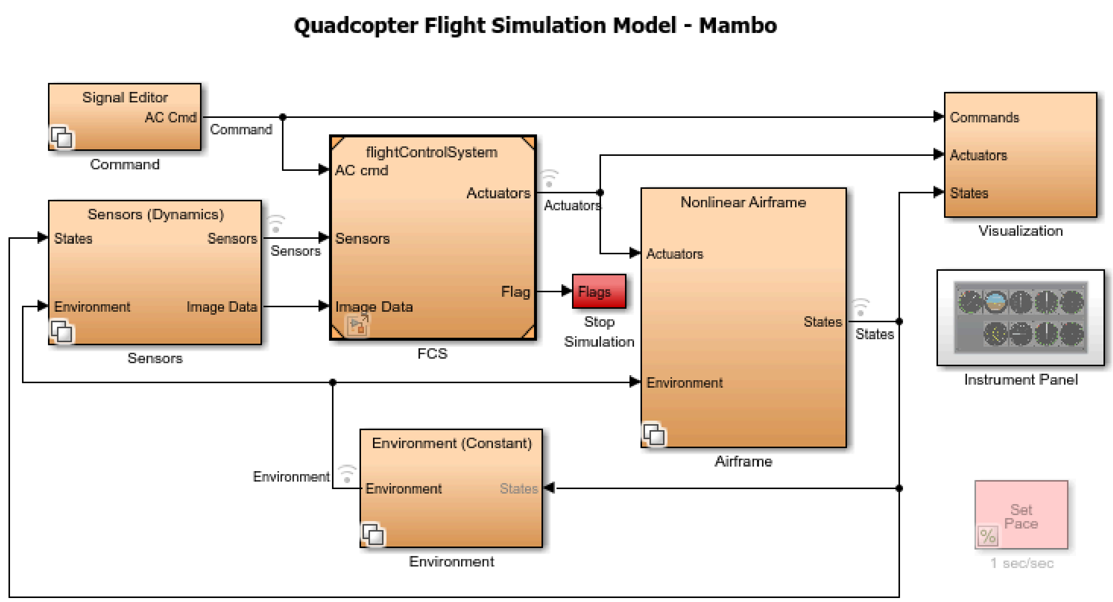
To address this challenge, one of the most widely used control techniques for drone movement is the Proportional-Integral-Derivative (PID) control method. PID controllers are designed to continuously monitor and adjust the drone's movement by using feedback from sensors, which helps maintain stability and accuracy in its motion.

In this project, we aim to optimize drone movement using PID controllers by analyzing the effects of various PID parameters such as proportional gain, integral gain, and derivative gain. Through simulations and experiments, we will evaluate the performance of the PID controllers and determine the optimal parameters that result in stable and efficient drone movement.
The outcomes of this project are expected to provide valuable insights into the design and optimization of drone control systems using PID controllers, which can have significant implications for various applications such as aerial photography, surveying, and inspection.

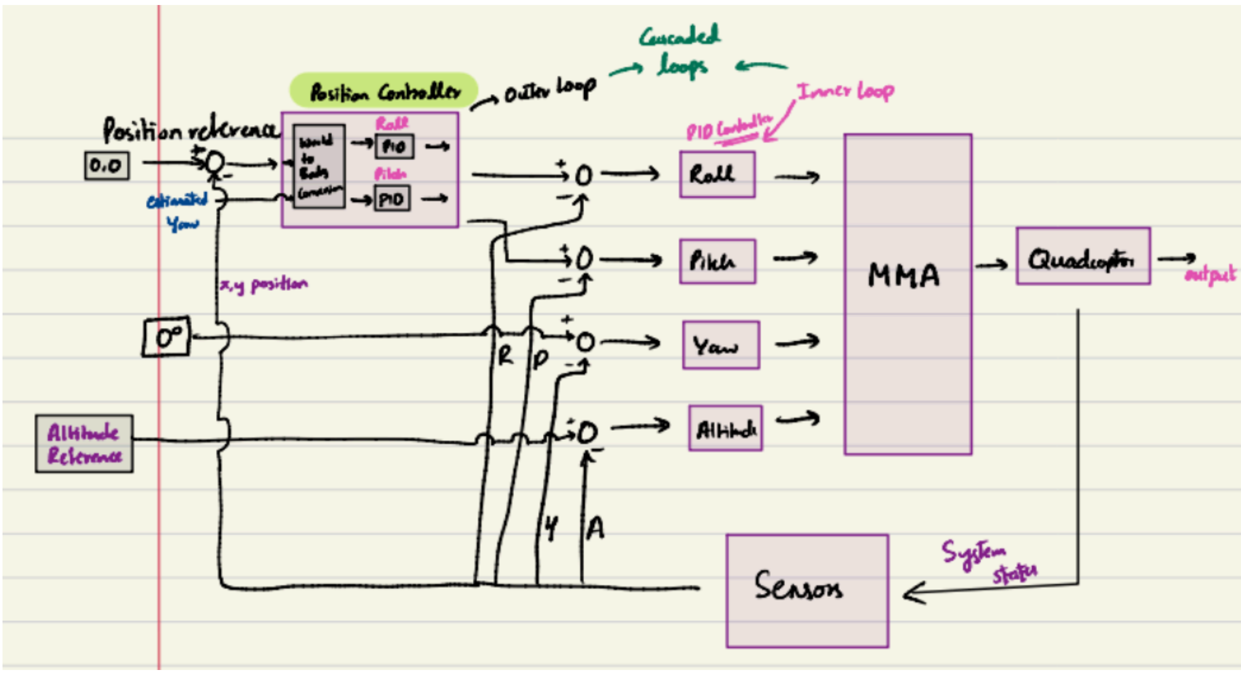
The system states are the altitude, angular position and horizontal velocity of the drone. As we are incorporating the motor mixing algorithm and only configuring the altitude of the drone and control problem reduces to the following flow.



SIMULATION ON SIMULINK:



As we are manipulating only the altitude of the drone for PID Tuning we will be setting the roll,pitch and yaw measurements to zero and only use an Altitude Reference.



Transfer function derivation:

We used a tool on MATLAB called System Identification toolbox to derive the transfer function for the simulated Drone.

The tool generated the following transfer function:

$$\frac{3.3928s^2 - 340.09s + 39451}{s^3 + 74.38s^2 + 5589s + 42107}$$

Matlab questions:

Based on the generated transfer function the following questions were solved:

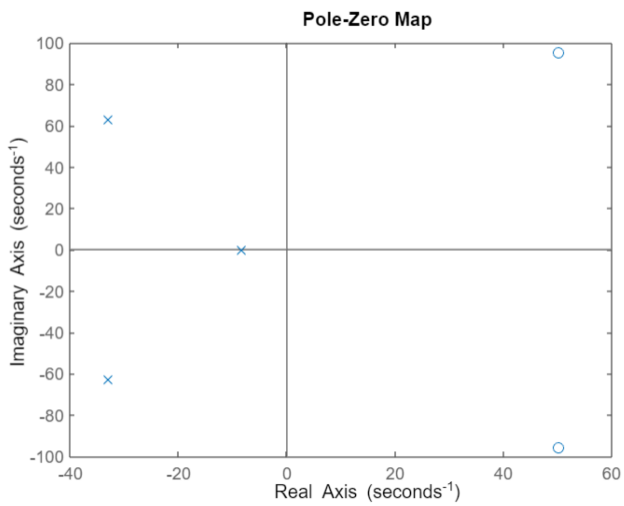
1. The objective of this experiment is to analysis and design of control systems specific to a physical system. Each student will be given a specific physical system, and experiments are to be conducted on that particular physical system. (The specific physical system will be given to a student by the respective Teacher or Student can select the physical system by themselves.)
   a. The objective of this exercise is to obtain the open loop characteristics of the given transfer function of the physical system or plant. (i) Where are the poles and zeros located? Obtain the pole-zero map. (ii) Apply different test signals, and observe the time-domain response. Discuss the results obtained from the viewpoint of pole-zero map.

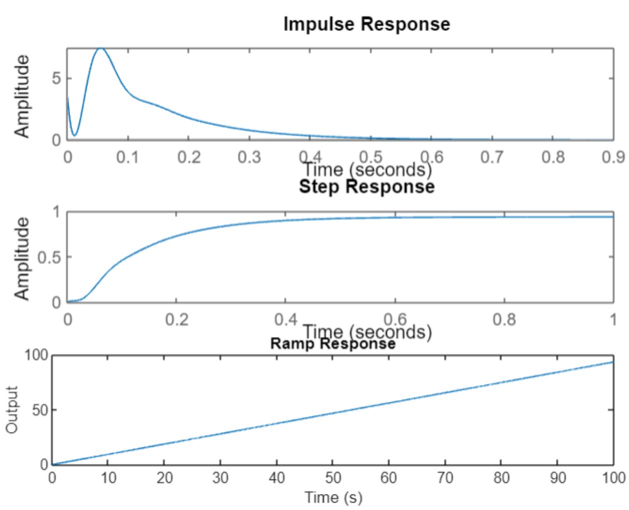
CODE:

```
clc;
clear;
close all;
% to clear all windows and close all images


n1 = [0 3.3928 -340.09 39451];
d1 = [1 74.38 5589 42107];
```

```
sys1 = tf(n1,d1)
figure;
grid on
pzmap(sys1)
%Impulse response
figure;
subplot(311)
impulse(sys1);
title('Impulse Response');


%Step response
subplot(312)
step(sys1);
title('Step Response');


%Ramp Response
t = 0:0.01:100;
ramp = t;
[y, t] = lsim(sys1, ramp, t);
subplot(313)
plot(t, y);
title('Ramp Response');
xlabel ('Time (s)'); ylabel('Output');
```

OUTPUT:





Since the system has all the poles on the RHP so the system is stable.And the responses are also stable.

2. The objective of this exercise is to determine the range of a gain that assures closed loop stability. Assume that the given system is part of a unity negative feedback system, and there is a gain in cascade with the given system in the forward path. Conduct experiments similar to **(project2-1)** and determine the range of k for which the closed loop system is stable.

From theoretical calculations the system is stable for range 0<k<6.9

$$T.f \rightarrow \left(\frac{3.3928s^2 - 340.09s + 39451}{s^3 + 74.38s^2 + 5589s + 42107}\right)k$$

$$1 + G(s)H(s) = s^3 + (74.38 + 3.39k)s^2 + (5589 - 340.09k)s + (42107 + 39451k)$$

$$\begin{array}{c|cc} s^3 & 1 & (5589 - 340.09k) \\ s^2 & (74.38 + 3.39k) & (42107 + 39451k) \\ s^1 & \dfrac{(74.38+3.39k)(5589-340.09k)-(42107+39451k)}{(74.38+3.39k)} & 0 \\ s^0 & & \end{array}$$

$$(5589 - 340.09k) - \frac{(42107 + 39451k)}{(74.38 + 3.39k)} > 0$$

$$(74.38 + 3.39k)(5589 - 340.09k) - (42107 + 39451k) > 0$$

$$415709.82 - 25295.8942k + 18946.71k - 1152.9051k^2 - 42107 - 39451k > 0$$

$$1152.9051k^2 + 45800.1842k - 373602.82 > 0$$

$$k = 6.94, -46.66$$

$$0 < k < 6.9435$$

for $k = 6.94$

$A(s) = (74.38 + 3.39k)s^2 + (45107 + 39451k) = 0 \qquad 97.9066s^2 + 318896.94 = 0$

$s^2 = -3257.1546 \qquad s = \pm 57.07j$

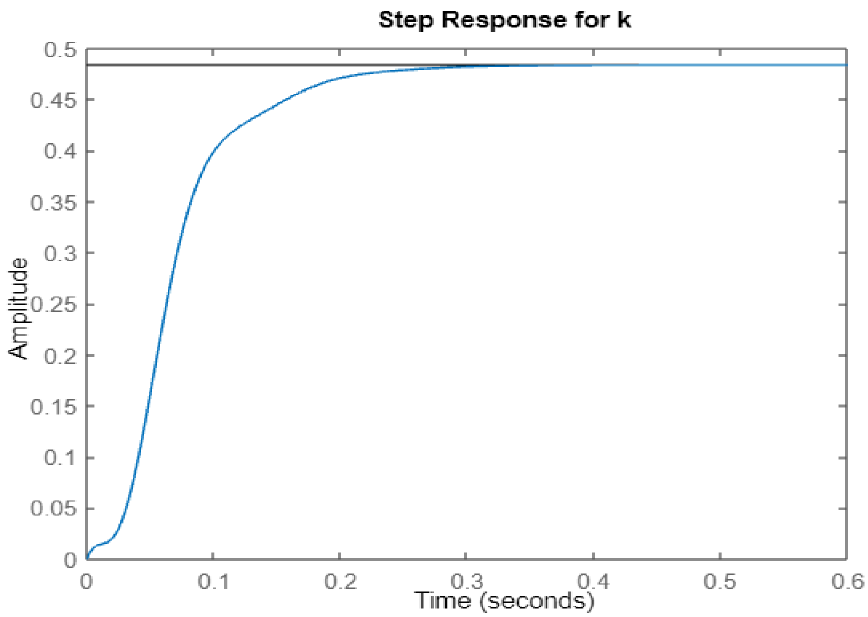
$s^2 = -\omega^2 \qquad \omega = \underline{57.07 \text{ rad/sec}}$

CODE:

```
den= [1 74.38 5589 42107];
num= [0 3.392 -340.09 39451];


G=tf(num,den);
P= 1:1:30;
for k=input("Enter k")
    system=feedback(k*G,1);
    step(system);
    title("Step Response for k");


    poles=pole(system);
    if all(real(poles)<0)
        disp('system is stable');
    else
                disp('system is unstable');
    end
end
```

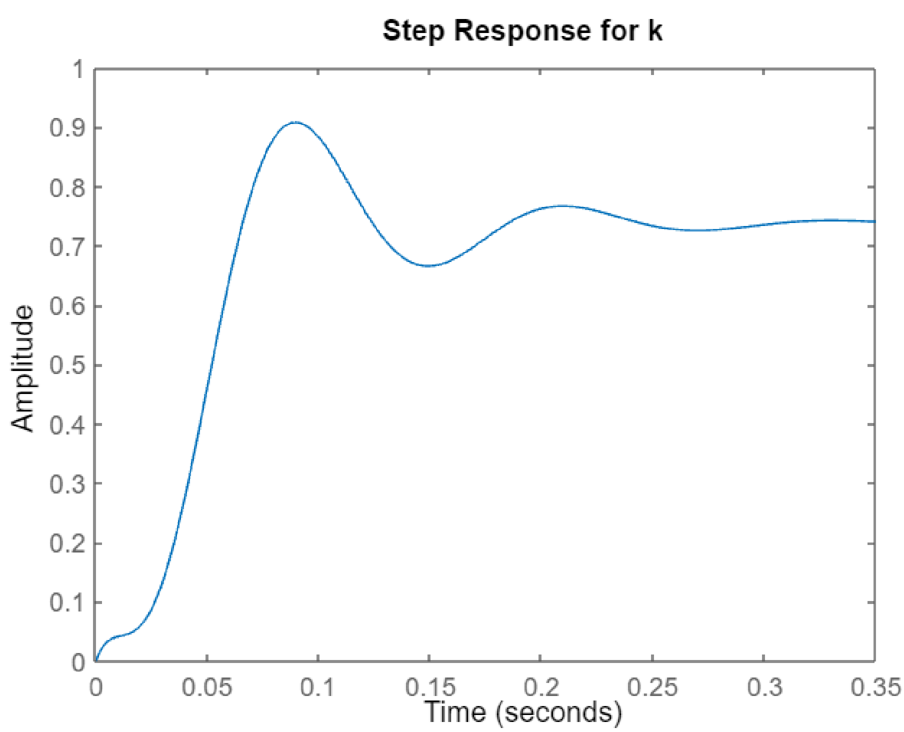
OUTPUT:

For k=1



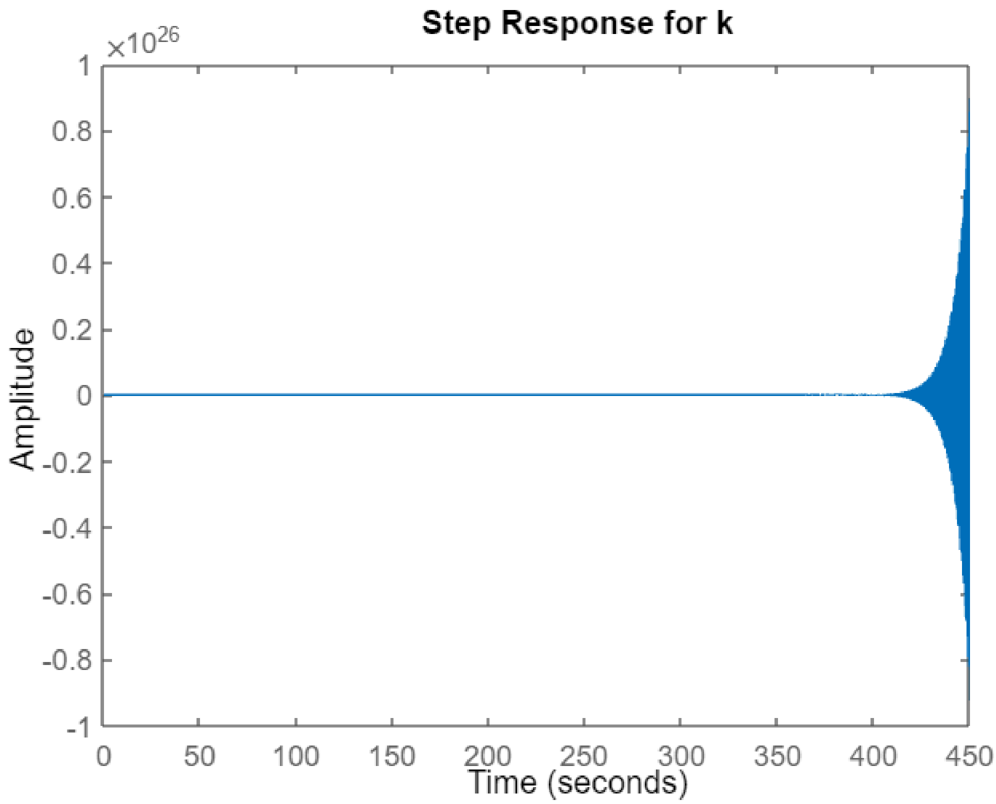

```
system is stable
```

For k=3




```
system is stable
```

For k=7




```
system is unstable
```

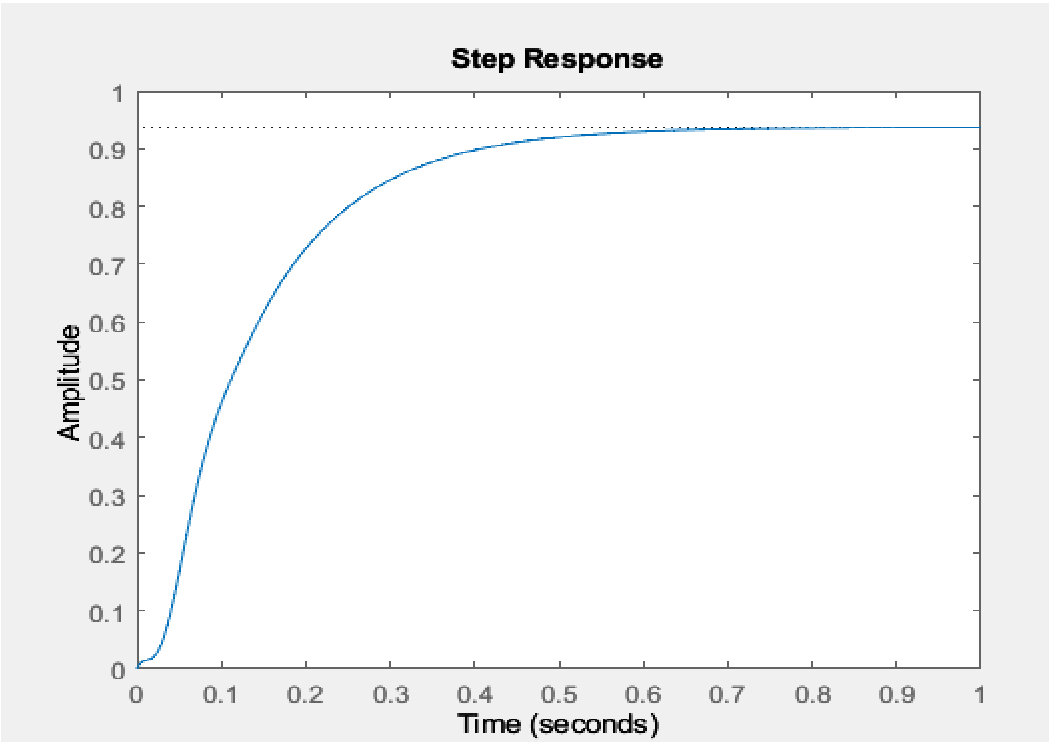
3. The objective of the exercise to analyse the closed loop system behaviour with proportional controller of the system whose transfer function you were given earlier.
    a. Place a gain *k* in the forward path, and close the loop with negative unity feedback. Take different values for *k*. For each value of in this set, obtain the step response. What is the rise time, the settling time? Are there any oscillations? If so, what is the frequency of oscillation? Compare the response of the closed loop system to the open loop system. Compare the closed loop responses. Discuss the results. Can we increase *k* indefinitely?
    b. Obtain the root locus. Mark the earlier choices of *k* on the root locus. Discuss the results obtained from the root locus with reference to those obtained with *k* in the forward path in part 3(a).

CODE:

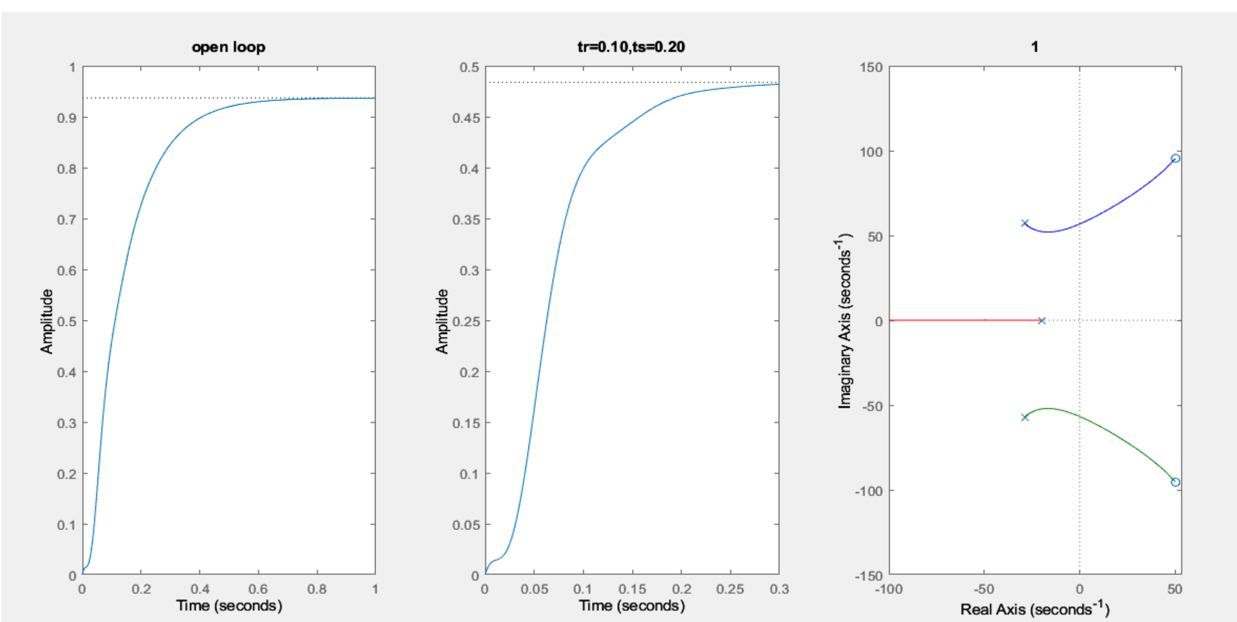
```
den= [0 1 74.38 5589 42107];
num= [0 0 3.392 -340.09 39451];
G=tf(num,den);
figure;
step(G)
for k=1:1:7
   system=feedback(k*G,1);
   figure;
   subplot(1,3,1);
   step(k*G)
   title(sprintf('open loop'))
   [y,t]=step(system);
   stepinfo_str= stepinfo(y,t);
   subplot(1,3,2);
   step(system);

title(sprintf("tr=%.2f,ts=%.2f",stepinfo_str.RiseTime,stepinfo_str.Set
tlingTime))
   subplot(1,3,3);
   rlocus(system);
   title(sprintf("%d",k))
end
```

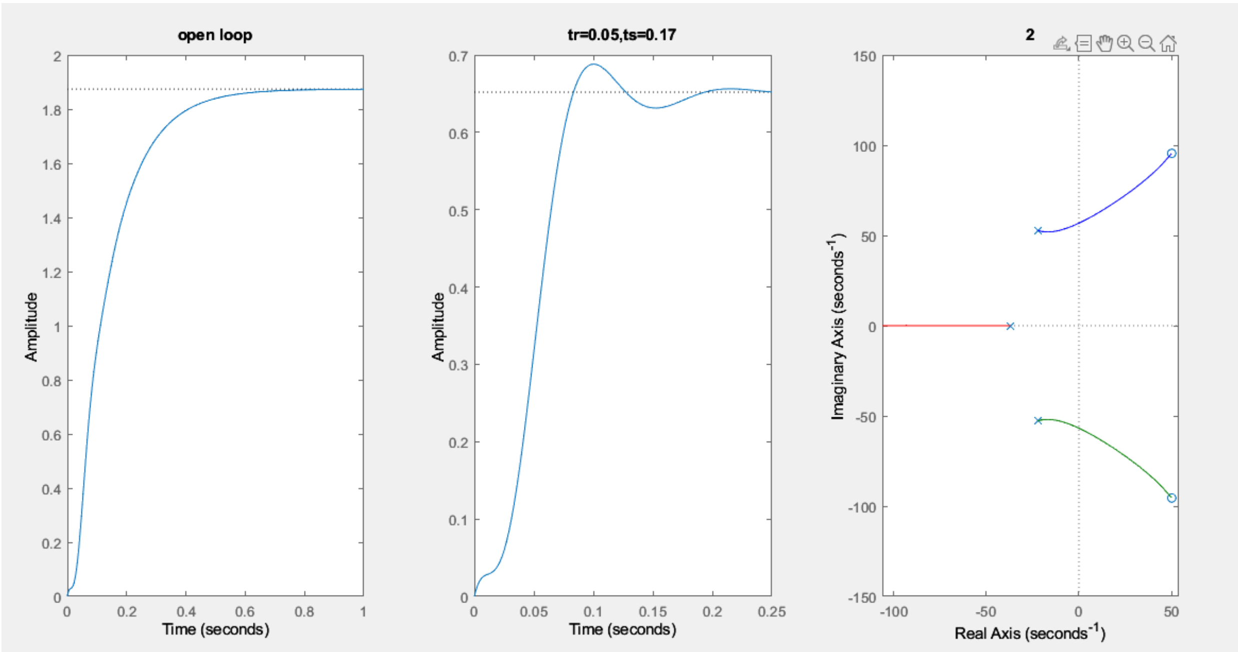
OUTPUT:

Step response of open loop function:

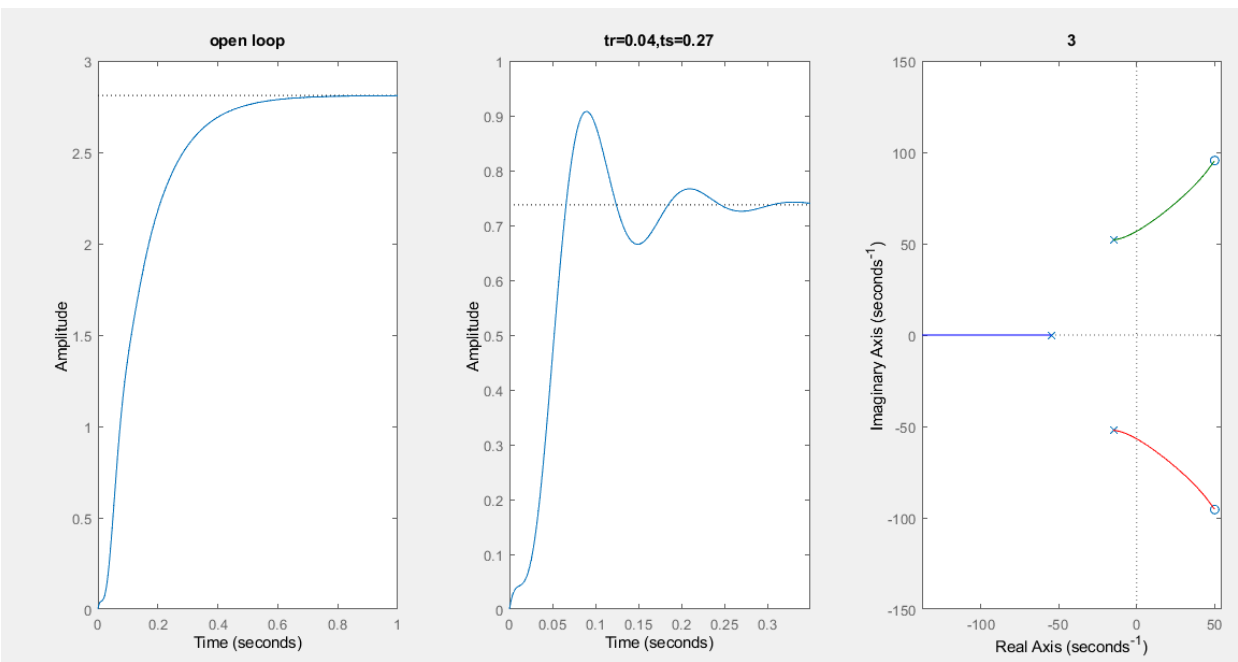


For k = 1:1:7

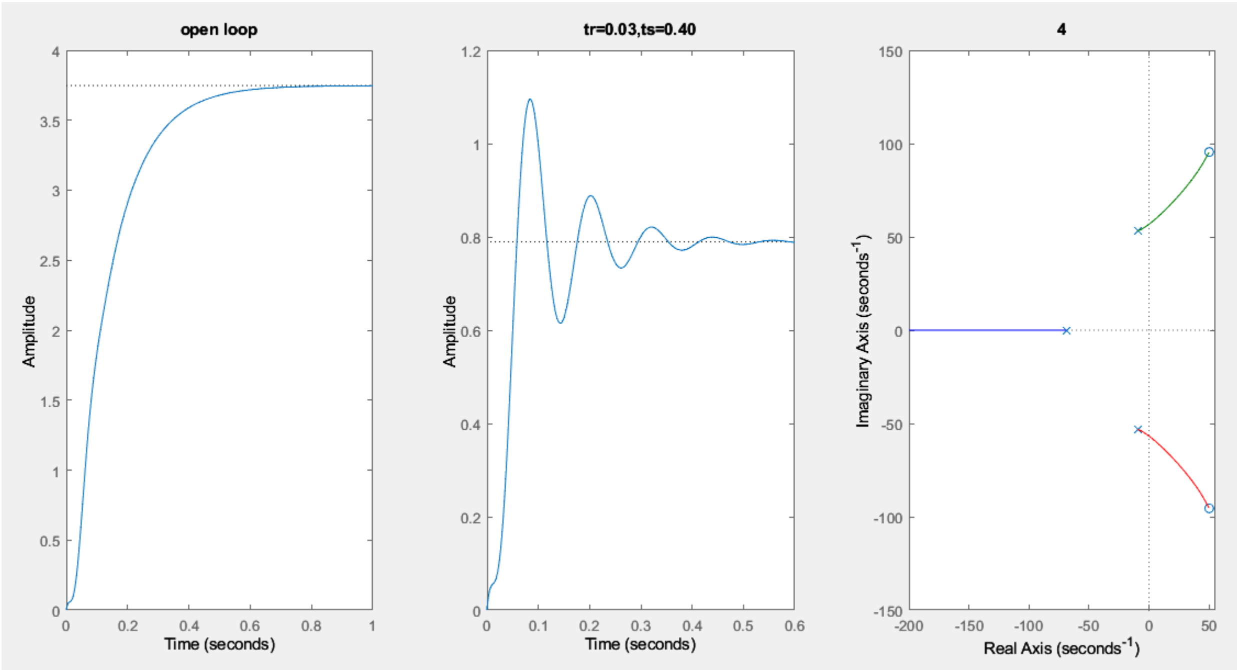


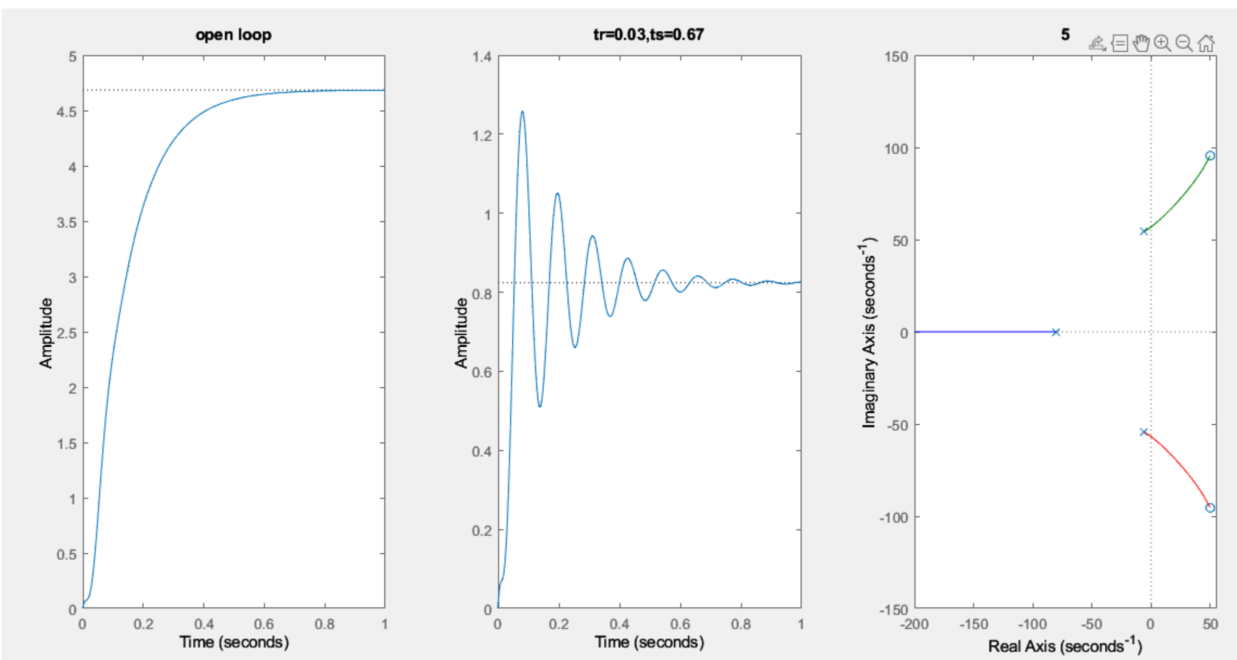


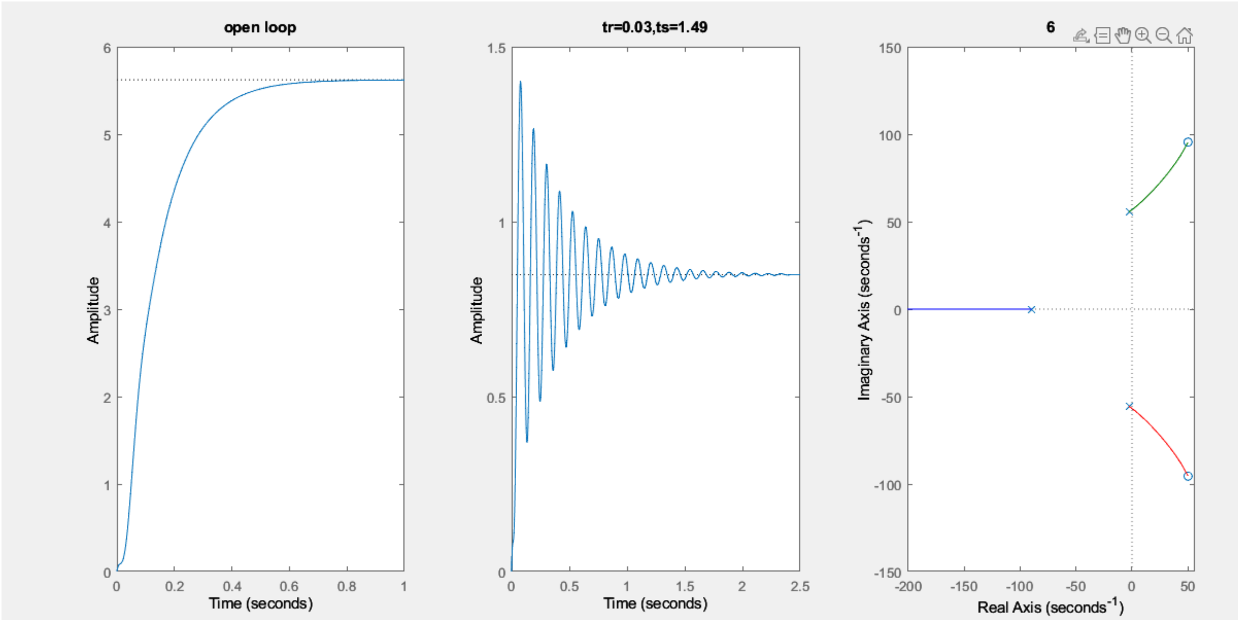
open loop
Amplitude
Time (seconds)
tr=0.05,ts=0.17
Amplitude
Time (seconds)
2
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})

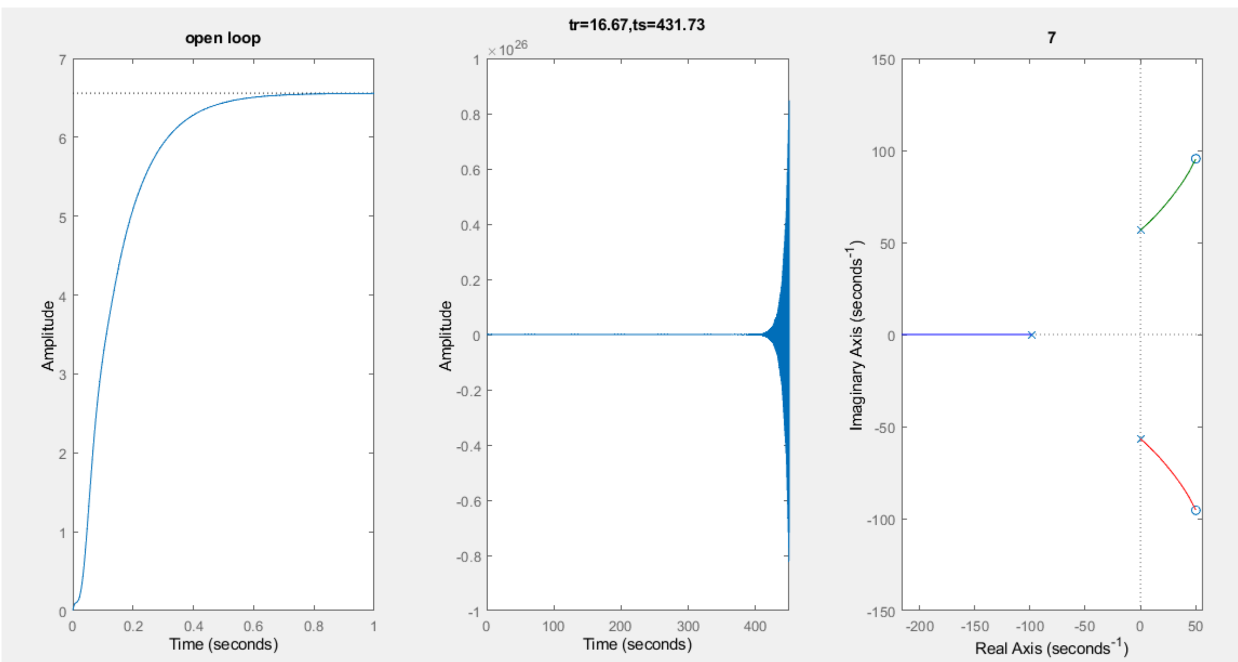


open loop
Amplitude
Time (seconds)
tr=0.04,ts=0.27
Amplitude
Time (seconds)
3
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})


open loop
Amplitude
Time (seconds)
tr=0.03,ts=0.40
Amplitude
Time (seconds)
4
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})


open loop
Amplitude
Time (seconds)
tr=0.03,ts=0.67
Amplitude
Time (seconds)
5
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})





From the outputs it is clear that the theoretical calculations in question two are validated and the system becomes marginally stable at around k=6.9

The frequency of oscillations from the theoretical calculations is obtained as 57.07 rad/sec.

Also by looking at the closed loop and open loop system responses it can be deduced that for closed loop systems it is important to select an ideal range of k for which the system can behave stable.

K cannot be increased indefinitely, for this particular system it is important to choose a k value in the range (0,6.9).

And it is also evident that as the system's stability decreases the root locus is pulled to the right accordingly.

$$T.f \rightarrow \left(\frac{3.3928s^2 - 340.09s + 39451}{s^3 + 74.38s^2 + 5589s + 42107}\right)k$$

$$1 + G(s)H(s) = s^3 + (74.38 + 3.39k)s^2 + (5589 - 340.09k)s + (42107 + 39451k)$$

$$\begin{array}{c|cc}
s^3 & 1 & (5589 - 340.09k) \\
s^2 & (74.38 + 3.39k) & (42107 + 39451k) \\
s^1 & \dfrac{(74.38+3.39k)(5589-340.09k)-(42107+39451k)}{(74.38+3.39k)} & 0 \\
s^0 & &
\end{array}$$

$$(5589 - 340.09k) - \frac{(42107 + 39451k)}{(74.38 + 3.39k)} > 0$$

$$(74.38 + 3.39k)(5589 - 340.09k) - (42107 + 39451k) > 0$$

$$415709.82 - 25295.8942k + 18946.71k - 1152.9051k^2 - 42107 - 39451k > 0$$

$$1152.9051k^2 + 45800.1842k - 373602.82 > 0$$

$$k = 6.94, -46.66$$

$$0 < k < 6.9435$$

for $k = 6.94$

$$A(s) = (74.38 + 3.39k)s^2 + (45107 + 39451k) = 0 \qquad 97.9066s^2 + 318896.94 = 0$$

$$s^2 = -3257.1546 \qquad s = \pm 57.07j$$

$$s^2 = -\omega^2 \qquad \omega = \underline{\underline{57.07 \text{ rad/sec}}}$$

4. The objective of this exercise is to obtain the closed loop behaviour with proportional plus derivative controller of the system you were given earlier. Place a function K (s + z) in the forward path, and close the loop with negative unity feedback. Take different values for K and z. For each sets of (K,z) obtain the step response, and the rlocus. Compare the step response for each case, and compare with the case of putting only a gain K in the forward path. What is therefore the effect of adding a zero in the forward path? Are there any additional insight to be gained from the rlocus: Obtain the root locus for each case. Compare the three loci, and discuss the results.

Theoretical results:

(4)

$$G(s) = K(s+z)\left(\frac{3.3928s^2 - 340.09s + 39451}{s^3 + 74.38s^2 + 5589s + 42107}\right)$$

$1 + G(s)H(s) =$

$$s^3(1 + 3.3928K) + (74.38 - 340.09K + 3.3928zK)s^2 + (5589 + 39451K - 340.09zK)s + (42107 + 39451Kz)$$

s^3	$(1+3.3928K)$	$(5589+39451K-340.09Kz)$
s^2	$(74.38-340.09K+3.39Kz)$	$(42107+39451Kz)$
s^1		0
s^0		

$z = -1 : 1 : 1 \qquad z = -1, 0, 1$

for $z = -1$

$$(74.38 - 340.09k - 3.39k)(5589 + 39451k + 340.09k)$$
$$-$$
$$(1 + 3.3928k)(42107 - 39451k) = 0$$

$$(74.38 - 343.4828k)(5589 + 39791.09k)$$
$$- (1 + 3.3928k)(42107 - 39451k) = 0$$

$$415709.82 + 2959661.274k - 1919725.369k - 13667555.01k^2$$
$$- 42107 + 39451k - 142860.6296k + 133849.3528k^2 = 0$$

$$-13533705.66k^2 + 936526.2754k + 373602.82 = 0$$

$$k = 0.204, -0.135$$

$$k = 0.204 > 0$$

for $z = 0$

$$(74.38 - 340.09k)(5589 + 39451k)$$
$$-$$
$$(1 + 3.3928k)(42107) = 0$$

$$415709.82 + 2934365.38k - 1900763.01k - 13416890.59k^2$$
$$- 42107 + 142860.6296k = 0$$

$$-13416890.59k^2 + 1176463k + 373602.82 = 0$$

$$k = 0.216, -0.128$$

for $z=1$

$$(74.38 - 340.09k + 3.39k)(5589 + 39451k - 340.09k) - (1 + 3.3928k)(42107 + 39451k) = 0$$

$$(74.38 - 336.6972k)(5589 + 39110.91k) - (1 + 3.3928k)(42107 + 39451k) = 0$$

$$415709.82 + 2909069.486k - 1881800.651k - 13168533.89k^2 - 42107 - 39451k - 142860.6296k - 133849.3528k^2 = 0$$

$$-13302383.24k^2 + 844957.2054k + 373602.82 = 0$$

$$k = 0.202\ ,\ -0.13$$

$$0 < k < 0.2$$

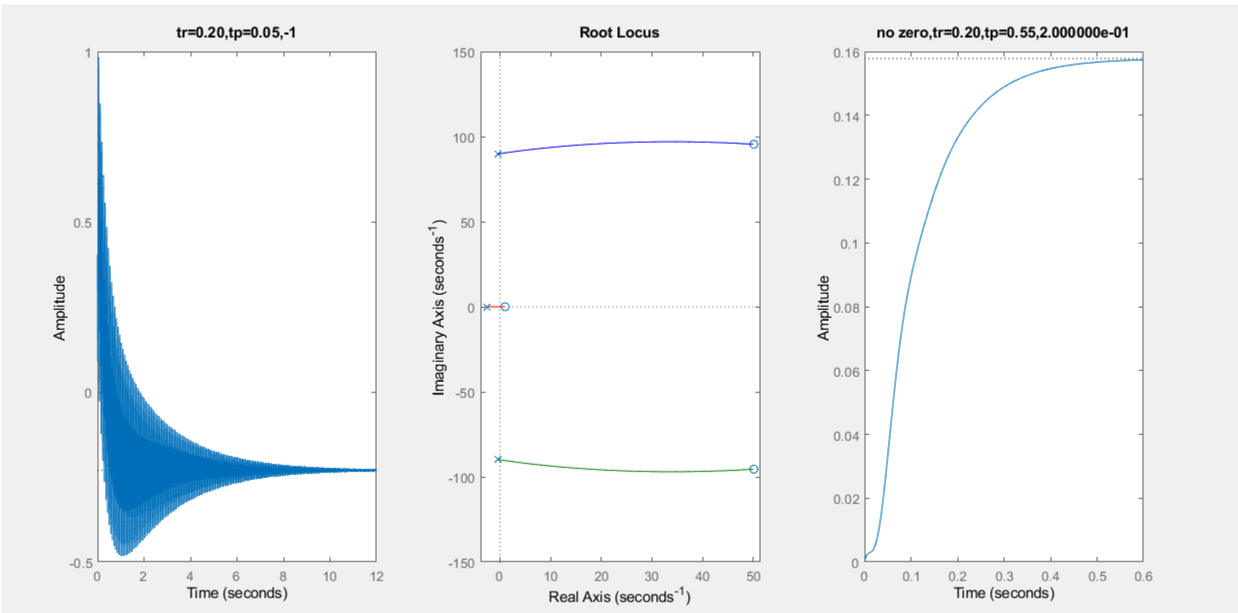
CODE:

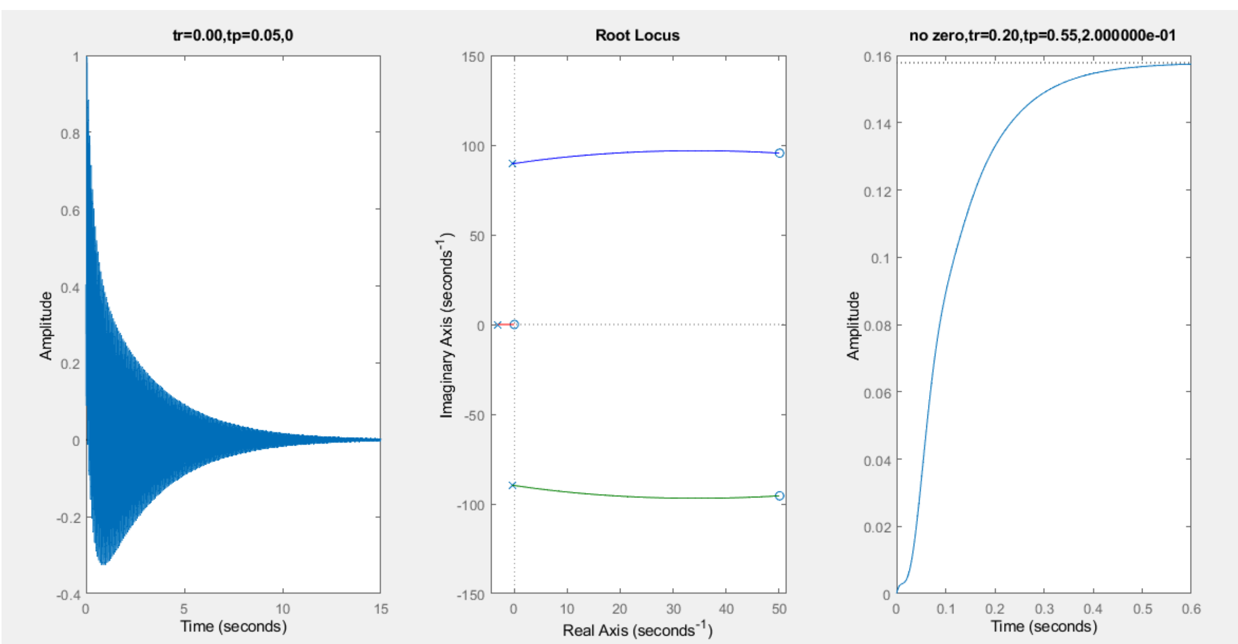
```
den= [0 1 74.38 5589 42107];

num= [0 0 3.392 -340.09 39451];

G=tf(num,den);

s=tf('s');

for k = 0.2

   for z = -1:1:1

       sys=k.*(s+z);

       system=feedback(sys*G,1);

       figure;

       [y,t]=step(system);
```

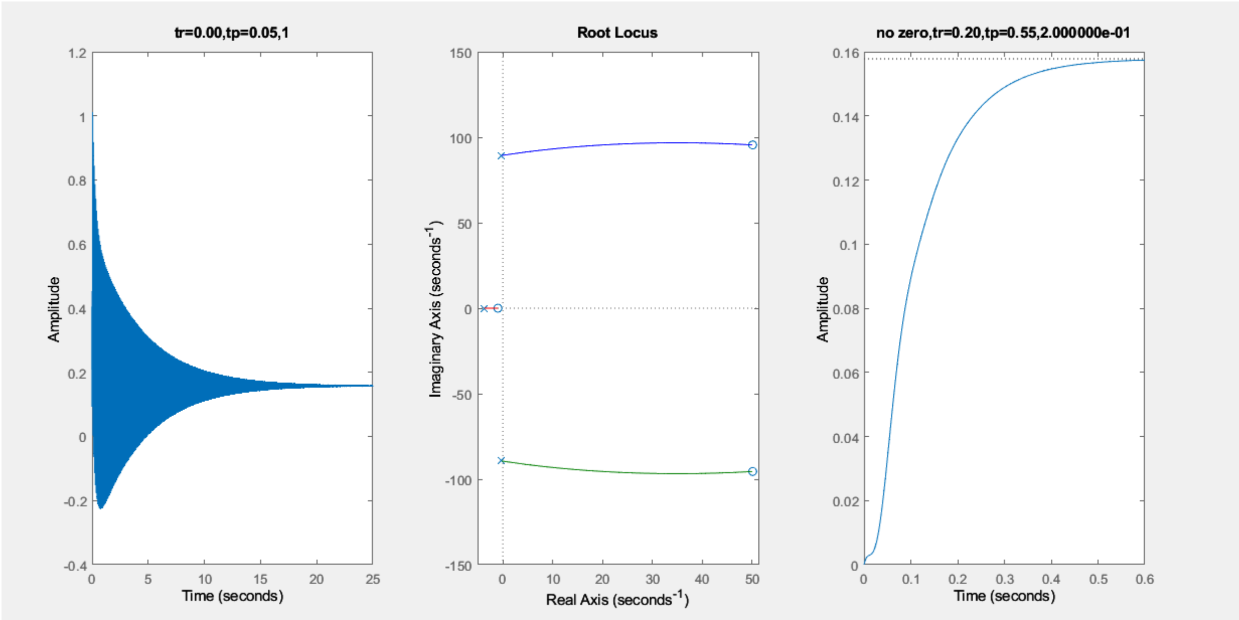
```
        stepinfo_str= stepinfo(y,t);

        subplot(1,3,1);

        step(system);

title(sprintf("tr=%.2f,tp=%.2f,%d",stepinfo_str.RiseTime,stepinfo_str.PeakTime,z));

        subplot(1,3,2);

        rlocus(system);

        subplot(1,3,3);

        F = feedback(k*G,1);

        [r,x]=step(F);

        stepinfo_kstr= stepinfo(r,x);

        step(F);

        title(sprintf("no
zero,tr=%.2f,tp=%.2f,%d",stepinfo_kstr.RiseTime,stepinfo_kstr.PeakTime,k))

            poles=pole(system);

            if all(real(poles)<0)

                disp('system is stable');

            else

                        disp('system is unstable');

            end

   end

end
```

OUTPUT:

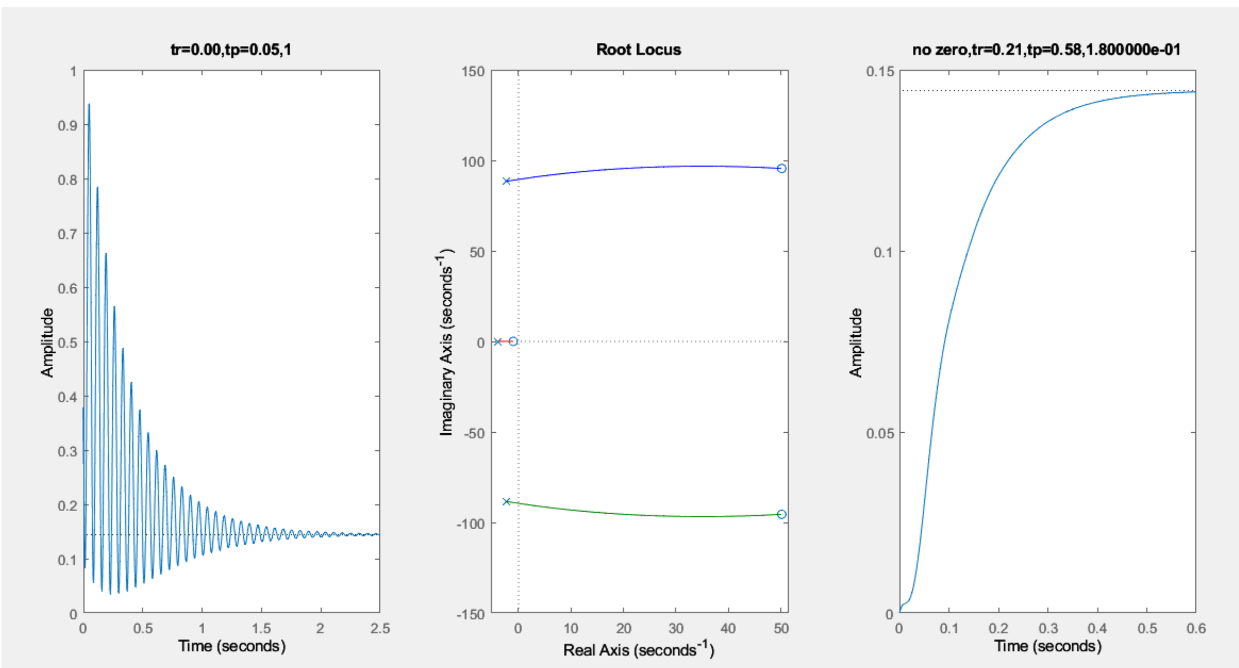
For k=0.2

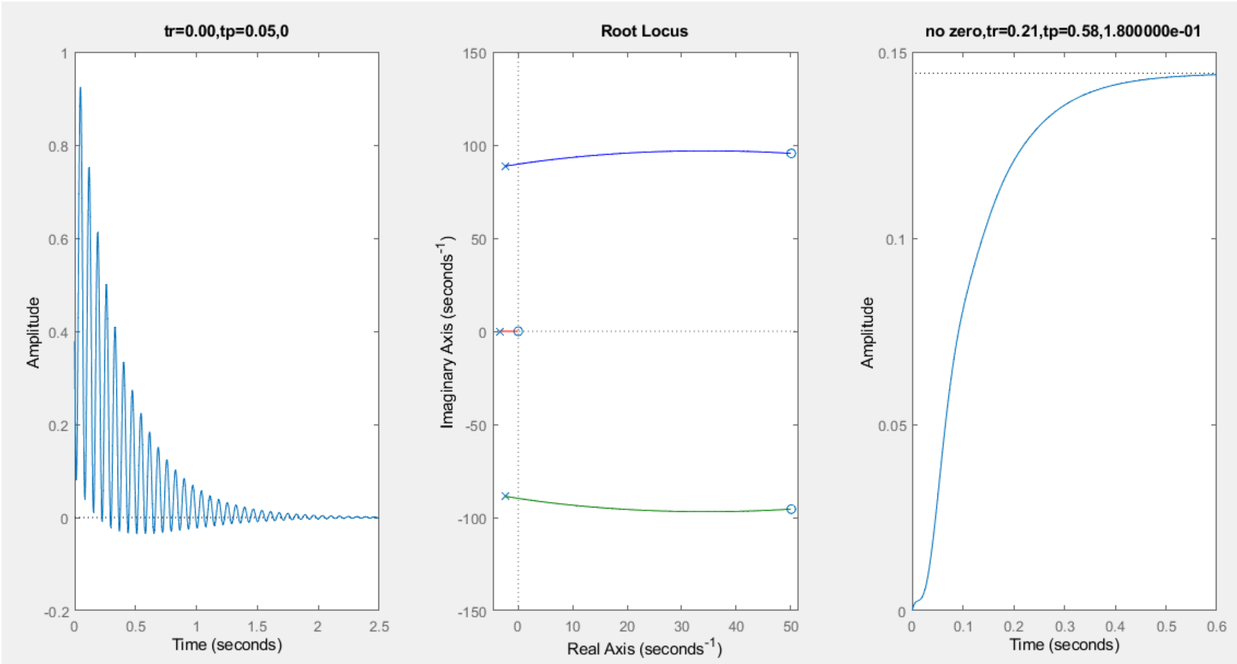


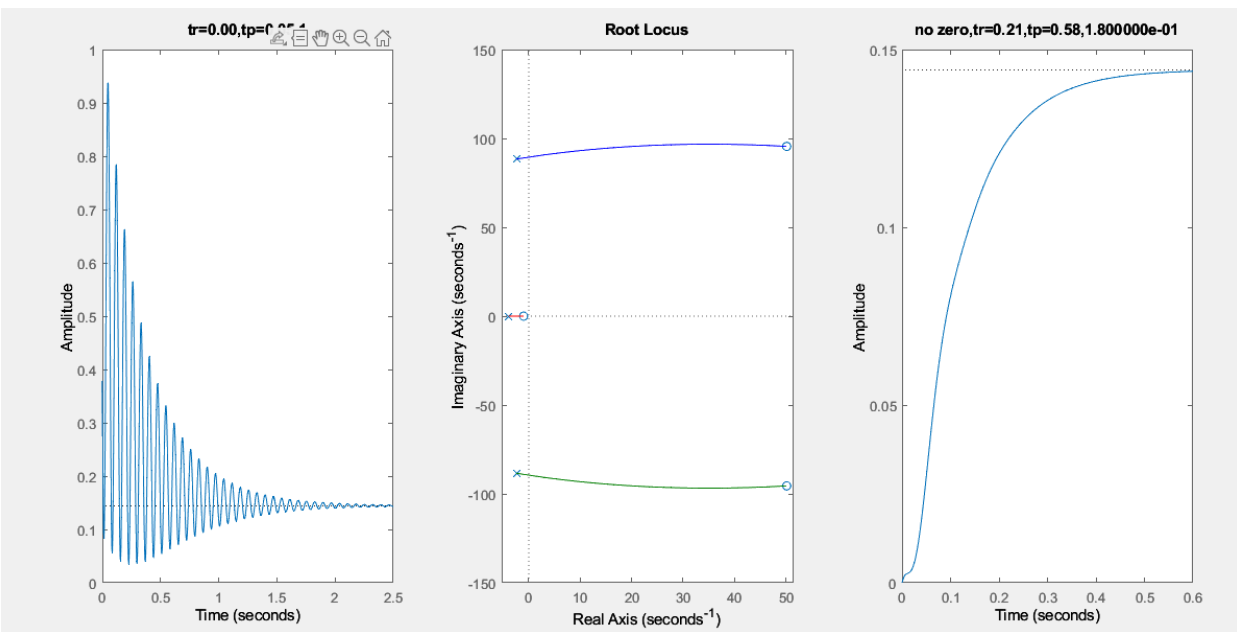



tr=0.00,tp=0.05,1
Amplitude
Time (seconds)
Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
no zero,tr=0.20,tp=0.55,2.000000e-01
Amplitude
Time (seconds)

For k=0.18


tr=0.00,tp=0.05,1
Amplitude
Time (seconds)
Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
no zero,tr=0.21,tp=0.58,1.800000e-01
Amplitude
Time (seconds)


tr=0.00,tp=0.05,0
Amplitude
Time (seconds)
Root Locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
no zero,tr=0.21,tp=0.58,1.800000e-01
Amplitude
Time (seconds)


tr=0.00,tp=
Amplitude
Time (seconds)
Root Locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
no zero,tr=0.21,tp=0.58,1.800000e-01
Amplitude
Time (seconds)

The effect of adding a zero in the forward path is that it can help to improve the transient response of the system. By placing a zero at a suitable location, we can decrease the rise time and the peak time also reduces.

And adding a zero on the right half of s-plane increases the undershoot of the system.

From the root locus, we can observe the variation of the closed-loop poles as we vary K and z. For example, as we change K, the root locus moves towards the left half of the s-plane, indicating an improvement in the stability of the system.

And as we decrease K, the poles move away from the imaginary axis, indicating a decrease in the oscillations of the system.

By comparing the step response and root locus for each case, we can determine the optimal values of K and z that can help us analyze the frequency response of the system.

5. (a)The objective of this exercise is to obtain the closed loop behaviour with proportional plus integral controller of the system you were given earlier. (i) Place a function $K((s+z)/s)$ in the forward path, and close the loop with negative unity feedback. Take different values for K and z, For each sets of (K,z)obtain the step response, and the root locus. Compare the step response for each case, and compare with the case of putting only a gain Kin the forward path and $K(s+z)$. What is therefore the effect of adding a pole in the forward path? Are there any additional insights to be gained from the root locus. Compare the three loci, and discuss the results. Hence (ii) infer the

results if the following function $K_1 + \frac{K_2}{s} + K_3 s$ is placed in the forward path. Substantiate your answer for suitable choices of K_1, K_2 and K_3.

Theoretical calculations:

5

$$G(s) = \frac{K(s+z)}{s}\left(\frac{3.3928s^2 - 340.09s + 39451}{s^3 + 74.38s^2 + 5589s + 42107}\right)$$

(i)

$1 + G(s)H(s) =$

$$s^4 + s^3(74.38 + 3.3928k) + s^2(5589 - 340.09k + 3.3928kz) + s(42107 + 39451k - 340.09kz) + 39451kz$$

s^4	1	$(5589 - 340.09k + 3.3928kz)$	$39451kz$
s^3	$(74.38 + 3.3928k)$	$(42107 + 39451k - 340.09kz)$	0
s^2	$\frac{(74.38+3.3928k)(5589-340.09k+3.3928kz) - (42107+39451k-340.09kz)}{(74.38+3.3928k)}$	$39451kz$	
s^1		0	
s^0			

$$b - \frac{(39451k)(74.38 + 3.3928k)^2}{(74.38 + 3.3928k)(5589 - 340.09k + 3.3928kz) - (42107 + 39451k - 340.09kz)}$$

for $z = -1$

$$b - \frac{(39451k)(74.38 + 3.3928k)^2}{(74.38 + 3.3928k)(5589 - 340.09k - 3.3928k) - (42107 + 39451k + 340.09k)} >$$

$$(74.38+3.3928k)(5589-343.4828k)(42107+39791.09k)$$
$$-(39451k)(74.38+3.3928k)=0$$

$$(415709.82-25548.25066k+18962.3592k-1165.36844k^2)$$
$$(42107+39791.09k)-(2190565.38k+133849.3528k^2)=0$$

$$(415709.82-6585.89146k-1165.36844k^2)(42107+39791.09k)$$
$$-2190565.38k-133849.3528k^2=0$$

$$1.750429\times10^{10}-277312131.7k-49070168.9k^2$$
$$+1.65415\times10^{10}k-262059799.8k^2-46371280.48k^3$$
$$-2190565.38k-133849.3528k^2=0$$

$$-46371280.48k^3-311263818.1k^2+1.62619\times10^{10}k+1.750429\times10^{10}=0$$

$$k\simeq-1.05$$

CODE:

```
den= [0 1 74.38 5589 42107];

num= [0 0 3.392 -340.09 39451];

G=tf(num,den);

s=tf('s');

for k=-1

   for z=-1:1:1

       sys=k*(((s+z).*s^-1));

       sys1=feedback(sys*G,1);

       figure();
```

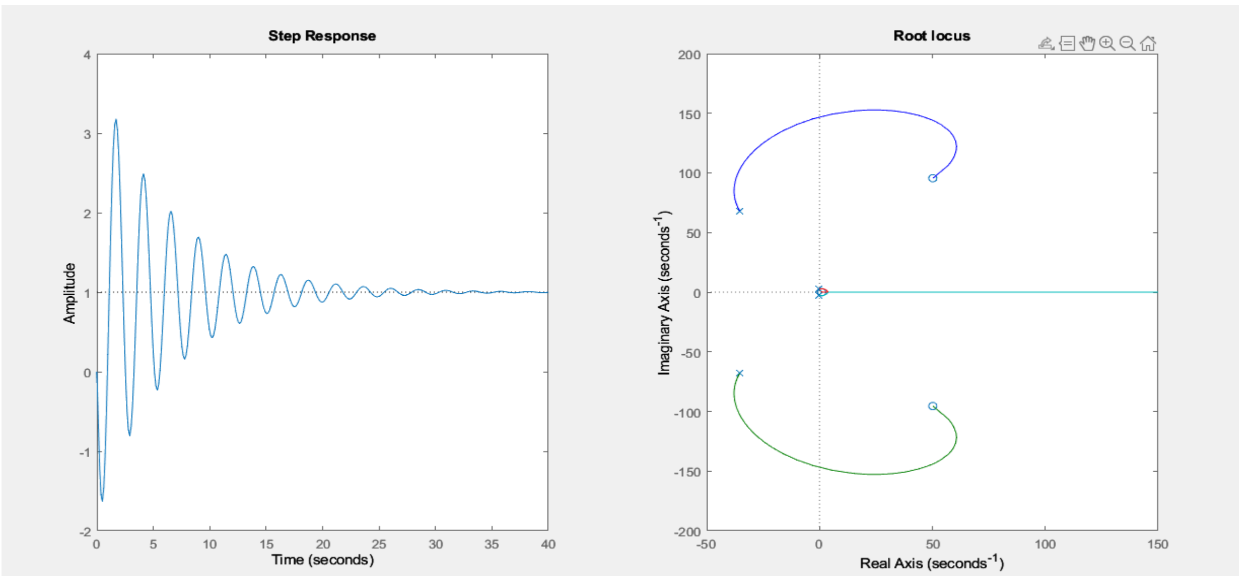
```
        subplot(1,2,1);

        step(sys1);

        title(sprintf('Step Response',k,z));

        subplot(1,2,2);

        rlocus(sys1);

        title(sprintf('Root locus',k,z));

        poles=pole(sys1);

            if all(real(poles)<=0)

                disp('system is stable');

            else

                        disp('system is unstable');

            end

    end

end
```

OUTPUT:

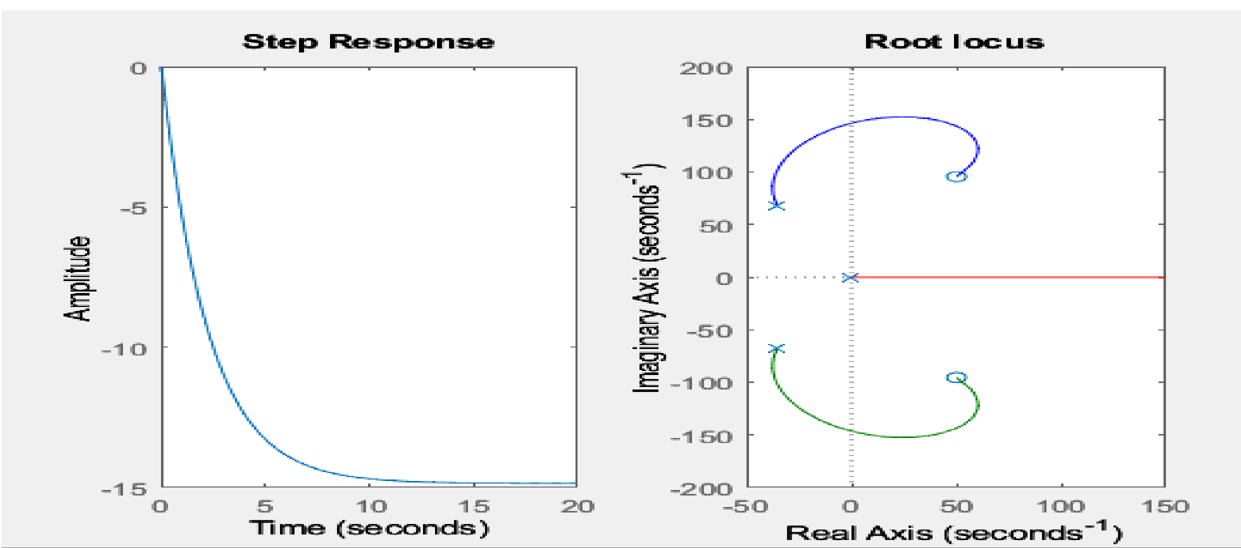
For k = -1 the system is marginally stable for z = -1

But unstable when z=1

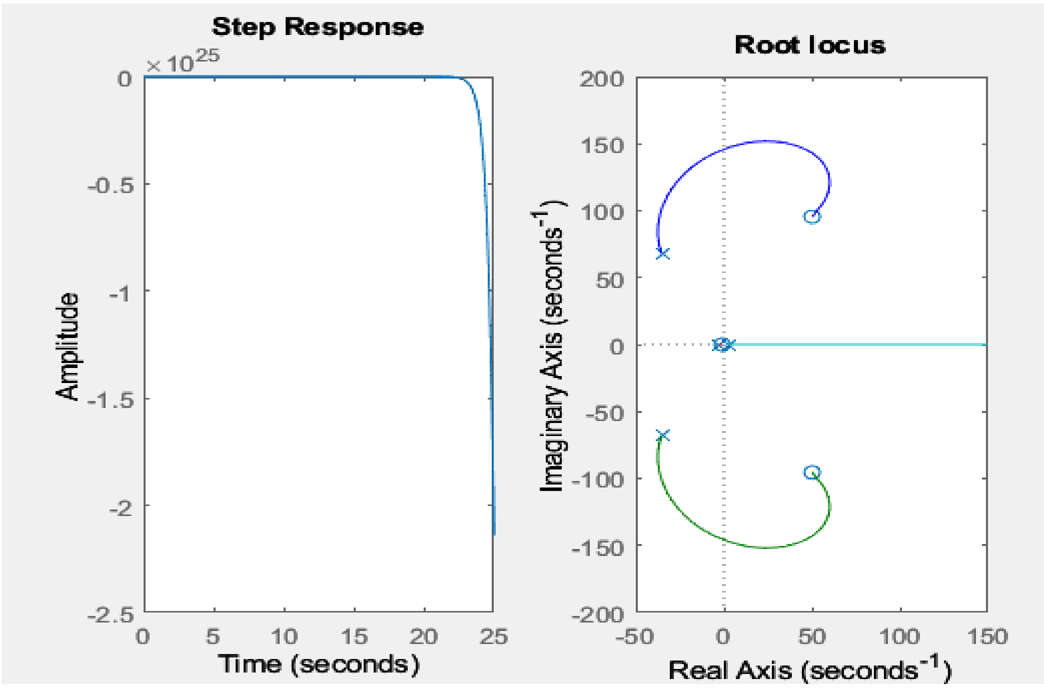
z=-1



z= 0



z=1



System is unstable

The stability depends on the location of zero.Adding a pole to a system increases the type of the system which leads to increase instability of the system.

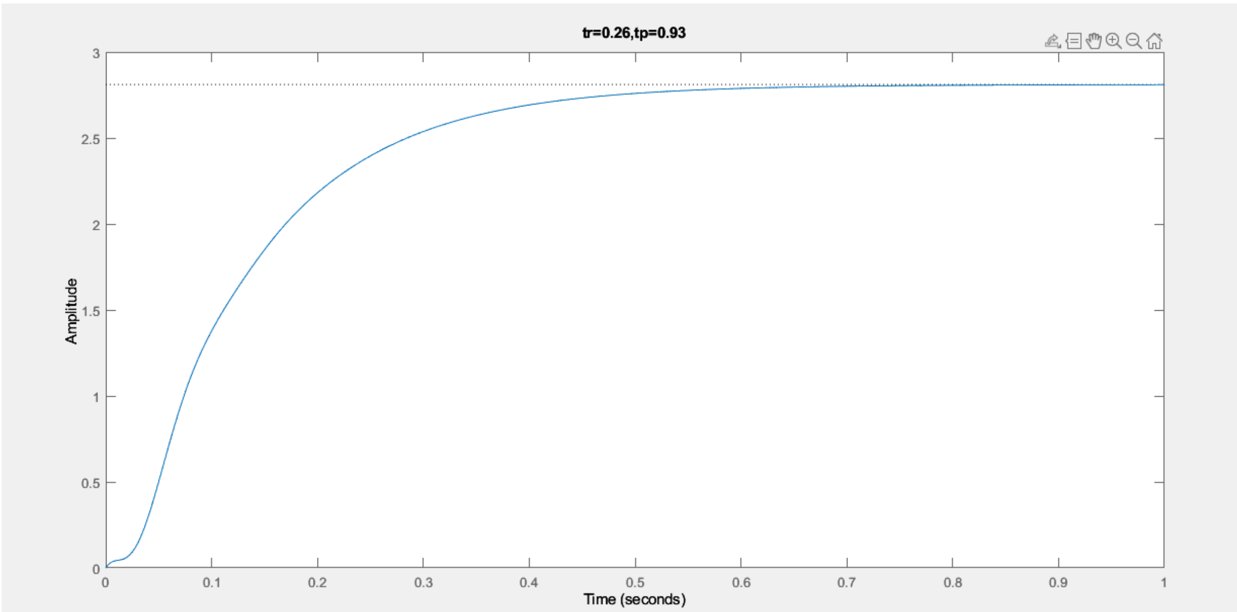
b)

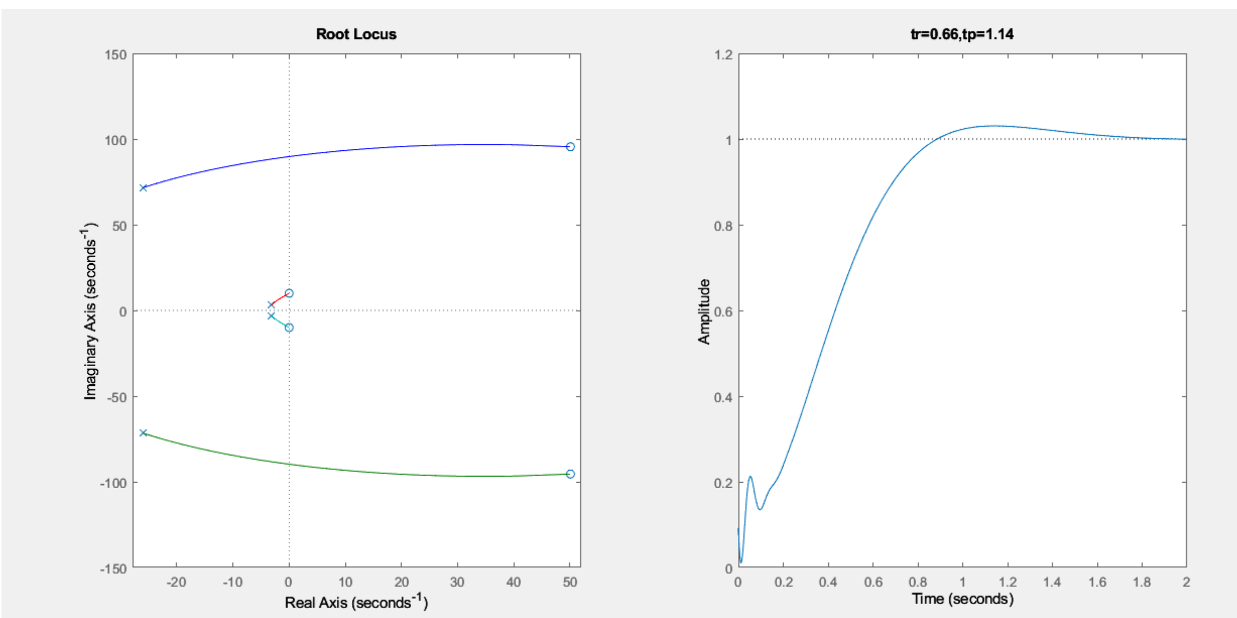
CODE:

```
clc;

clear all;

close all;

den= [0 1 74.38 5589 42107];

num= [0 0 3.392 -340.09 39451];

G=tf(num,den);
```

```
figure;
[y,t]=step(1*G);
stepinf_str= stepinfo(y,t);
step(3*G)
title(sprintf("tr=%.2f,tp=%.2f",stepinf_str.RiseTime,stepinf_str.PeakTime));
for k1 = 3
   for k2=0
       for k3= 0.03
           den= conv([0 1 74.38 5589 42107],[1 0]);
           num= conv([0 0 3.392 -340.09 39451],[0 k3 k2 k1]);
           sys = tf(num,den);
           system = feedback(sys,1);
           figure;
           subplot(1,2,1);
           rlocus(system)
           subplot(1,2,2);
           inf=stepinfo(system);
           step(system)
           title(sprintf("tr=%.2f,tp=%.2f",inf.RiseTime,inf.PeakTime));
           poles=pole(system);
   if all(real(poles)<0)
       disp('system is stable');
   else
               disp('system is unstable');
       end
   end
   end
end
```

OUTPUT:


tr=0.26,tp=0.93
Amplitude
Time (seconds)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
150
100
50
0
-50
-100
-150
-20
-10
10
20
30
40
50
tr=0.66,tp=1.14
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
1.4
1.6
1.8
2

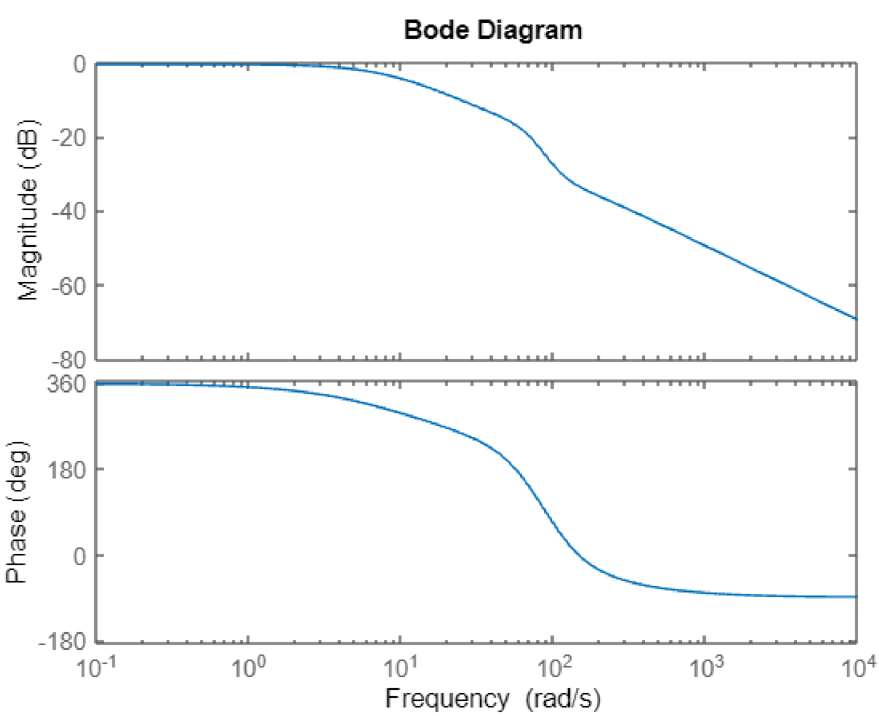
The addition of a pole has increased the rise time, decreased the overshoot, and increased the damping ratio of the system.

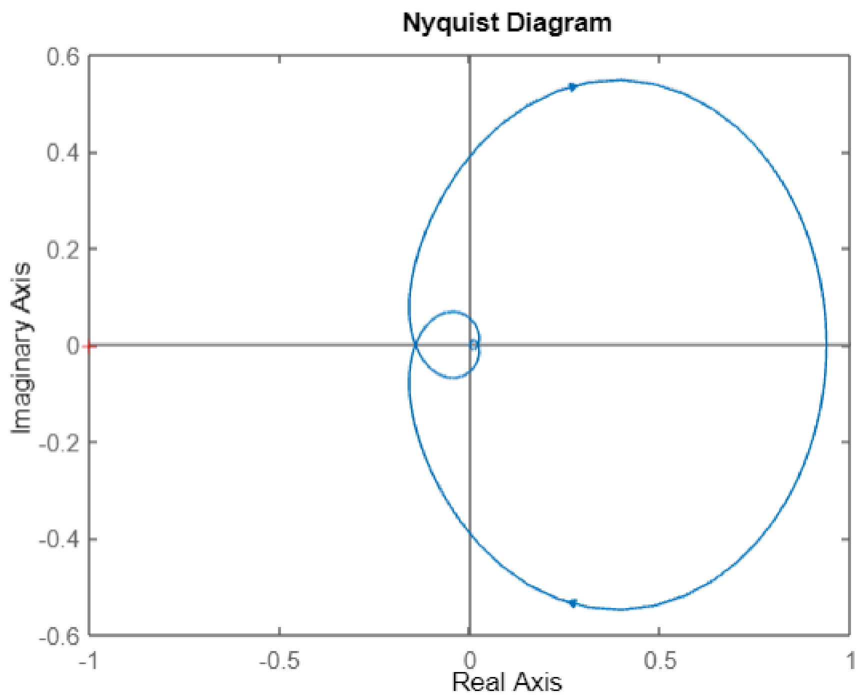
From the root locus, we can observe the variation of the closed-loop poles as we vary K. Here, as we increase K, the poles move towards the right half of the s-plane, indicating a decrease in the stability of the system. Similarly, as we move the pole towards the left half of the s-plane, the poles move closer to the imaginary axis, indicating an increase in the oscillations of the system.

From the root locus, we can observe the variation of the closed-loop poles as we vary K1, K2, and K3. As we increase K1, the poles move towards the left half of the s-plane, indicating an improvement in the stability of the system. Similarly, as we increase K2, the poles move towards the right half of the s-plane, indicating a decrease in the stability of the system. As we increase K3, the poles move away from the imaginary axis, indicating a decrease in the oscillations of the system.

6. (b) The objective of this exercise is to analyse the system in frequency-domain. Obtain the Bode and Nyquist plot for the system you were given earlier. What are the gain and phase margins? What can you conclude about the stability of your system?

```
den= [0 1 74.38 5589 42107];
num= [0 0 3.392 -340.09 39451];
G=tf(num,den);
figure;
bode(G);
margin(G);
figure;
nyquist(G);
```





From the Bode plot, we deduce that the Gain and Phase margins are as below

Gain margin: Inf Phase margin: 64.44 degrees

Since the gain margin is infinite, we can conclude that the system is stable. The phase margin of 64.44 degrees indicates that the system has a good phase margin and is well-damped.

6)

(a). For lead compensator

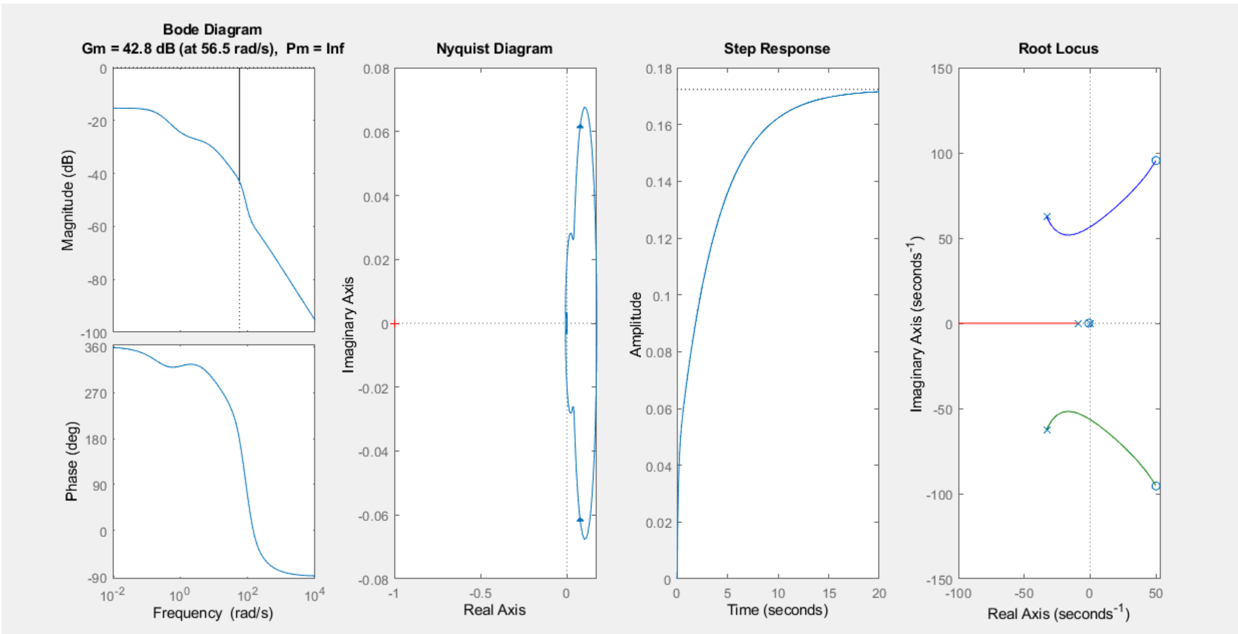
```
den= [0 1 74.38 5589 42107];

num= [0 0 3.392 -340.09 39451];

G=tf(num,den);

for a1=

  for t=0

    k=5;

    nlag=[k*t*a1,k];

    dlag=[0 0 a1*t a1];

    lead=tf(nlag,dlag)

    sys=feedback(lead*G,1);

    figure;

    subplot(4,1,1);

    margin(sys)

    subplot(4,1,2);

    nyquist(sys);

    subplot(4,1,3);

    step(sys);

    subplot(4,1,4);

    rlocus(sys);

  end

end
```

(b.) The lag compensator

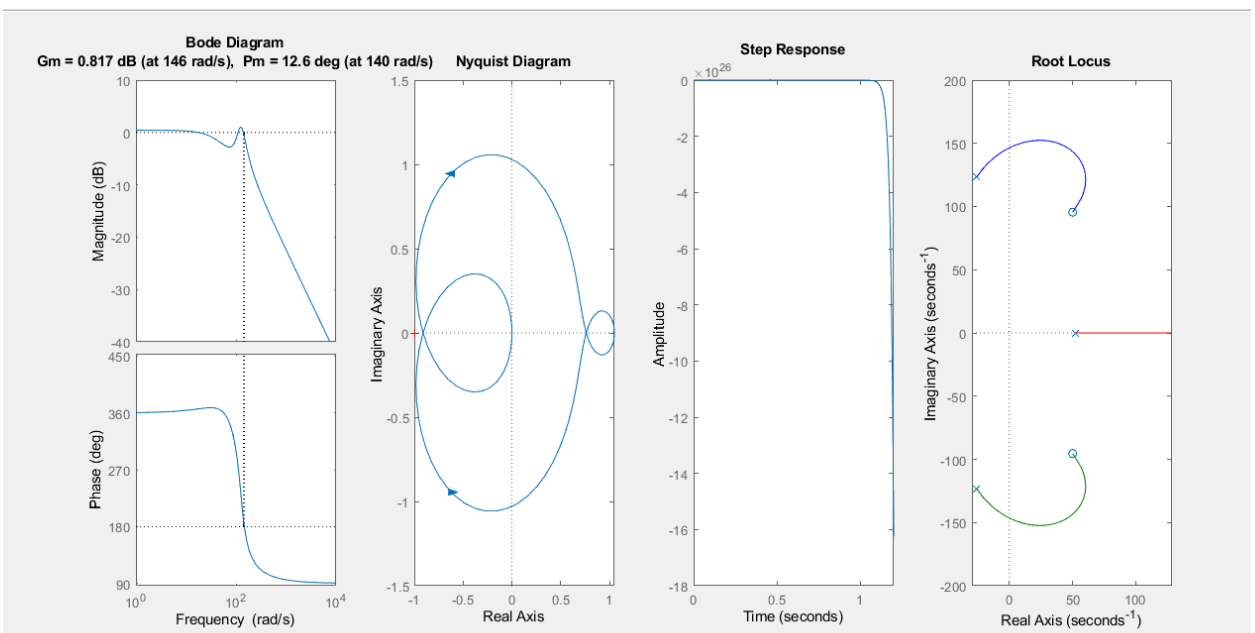
```
den= [0 1 74.38 5589 42107];

num= [0 0 3.392 -340.09 39451];

G=tf(num,den);

for a1=0.5

   for t=1;

      k=1;

      nlag=[k*t,k];

      dlag=[0 0 a1*a1*t a1];

      lag=tf(nlag,dlag)

      sys=feedback(lag*G,1);

      figure;

      subplot(1,4,1);

      margin(sys)

      subplot(1,4,2);

      nyquist(sys);

      subplot(1,4,3);

      step(sys);

      subplot(1,4,4);

      rlocus(sys);

   end

end
```

OUTPUT:

Lag compensator:



Lead compensator:



Manual Calculations:

(6) Assume $k=1$

lag

$$G(s)=\frac{(\tau s+1)}{\alpha(\alpha\tau s+1)}\times\left(\frac{3.3928s^2-340.09s+39451}{s^3+74.38s^2+5589s+42107}\right)$$

$$=\frac{(\tau s+1)(3.3928s^2-340.09s+39451)}{(\alpha^2\tau s+\alpha)(s^3+74.38s^2+5589s+42107)}$$

$$=\frac{3.3928\tau s^3+(3.3928-340.09\tau)s^2+(39451\tau-340.09)s+39451}{\alpha^2\tau s^4+(74.38\alpha^2\tau+\alpha)s^3+(5589\alpha^2\tau+74.38\alpha)s^2+(42107\alpha^2\tau+5589\alpha)s+42107\alpha}$$

$$\alpha^2\tau s^4+(74.38\alpha^2\tau+\alpha+3.3928\tau k)s^3+(5589\alpha^2\tau+74.38\alpha+3.3928k-340.09\tau k)s^2+(42107\alpha^2\tau+5589\alpha+39451\tau k-340.09k)s+(39451+42107\alpha)$$

s^4	$\alpha^2\tau$	$(5589\alpha^2\tau+74.38\alpha+3.3928k-340.09\tau k)$	$(39451+42107\alpha)$
s^3	$(74.38\alpha^2\tau+\alpha+3.3928\tau k)$	$(42107\alpha^2\tau+5589\alpha+39451\tau k-340.09k)$	
s^2	$\dfrac{(74.38\alpha^2\tau+\alpha+3.3928\tau)(5589\alpha^2\tau+74.38\alpha-336.6972)-\alpha^2\tau(42107\alpha^2\tau+5589\alpha+39110.91)}{(74.38\alpha^2\tau+\alpha+3.3928\tau)}$	$(39451+42107\alpha)$	
s^1			
s^0			

$$\alpha^2\tau > 0$$

$$(74.38\alpha^2\tau + \alpha + 3.3928\tau) > 0$$

$$(74.38\alpha^2\tau + \alpha + 3.3928\tau)(5589\alpha^2\tau + 74.38\alpha - 336.6972)$$
$$-\alpha^2\tau(42107\alpha^2\tau + 5589\alpha + 39110.91) > 0$$

$$\alpha > 0$$
(or) $\tau \geqslant 0$

$$(\alpha)(74.38\alpha - 336.6972) > 0$$

$$74.38\alpha^2 > 336.6972\alpha$$

$$\alpha > 4.5 \quad \tau \geqslant 0$$

(6)

<u>lead</u>

$$G(s)=\frac{K(\alpha\tau s+1)}{\alpha(\tau s+1)}\left(\frac{3.3928s^2-340.09s+39451}{s^3+74.38s^2+5589s+42107}\right)$$

Assume $K=1$

$$=\frac{(\alpha\tau s+1)(3.3928s^2-340.09s+39451)}{(\alpha\tau s+\alpha)(s^3+74.38s^2+5589s+42107)}$$

$=$

$$\alpha\tau s^4+(74.38\alpha\tau+\alpha)s^3+(5589\alpha\tau+74.38\alpha)s^2+(42107\tau\alpha)s+42107\alpha$$

$$+(3.3928\tau\alpha)s^3+(3.3928-340.09\alpha\tau)s^2+(39451\alpha\tau-340.09)s-39451$$

$$=\alpha\tau s^4+(74.38\alpha\tau+\alpha+3.3928\tau\alpha)s^3+(5589\alpha\tau+74.38\alpha+3.3928-340.09\alpha\tau)s^2+(42107\tau\alpha+39451\alpha\tau-340.09)s+(42107\alpha-39451)$$

$\alpha\tau>0$

$\alpha=0$ or $\tau=0$

$(74.38\alpha\tau+\alpha+3.3928\tau\alpha)>0$

$\Rightarrow\alpha>0$

$$(74.38\alpha\tau+\alpha+3.3928\alpha\tau)(5589\alpha\tau+74.38\alpha+3.3928-340.09\alpha\tau)-\alpha\tau(42107\tau\alpha+39451\alpha\tau-340.09)>0$$

$\tau = 0$

$$\alpha(74.38\alpha + 3.3928) = 0$$

$$74.38\alpha^2 + 3.3928\alpha = 0$$

$$\alpha = -0.045$$

Inferences:

> Settling time is better for the Lead compensator

> There is a small variation in rise time between the lead and lag compensator

> Internally for each compensator there is not much variation of rise time.

> Hence, we chose to use any of the compensators for our system keeping in mind the output needed at the moment.

References:

1. “Modern Control Systems” by Richard C. Dorf.
2. “Control system Engineering” by I.J.Nagrath
3. www.mathworks.com
4. Control systems by Ganesh rao
5. “Design and Analysis of Quadcopter Classical Controllers” by Ahmed.H. Ahmed and others.
6. “Design and Testing of PID Controllers on a Parrot Minidrone” by Hrishitva Patel