```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.cluster.hierarchy as shc
from scipy.spatial.distance import squareform, pdist
```

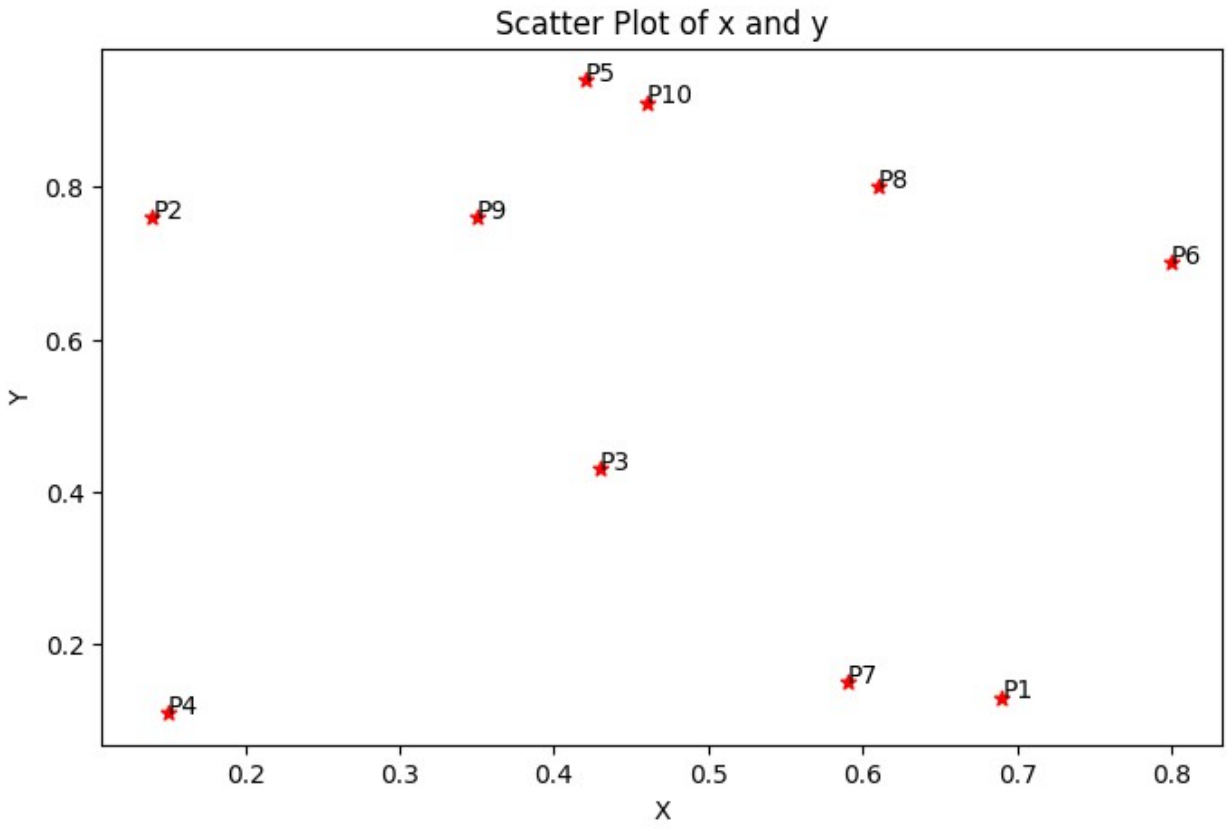
Random Dataset

```
X = np.random.random_sample(size = 10)
Y = np.random.random_sample(size = 10)

point = ['P1','P2','P3','P4','P5','P6','P7','P8','P9','P10']

data = pd.DataFrame({'Point':point, 'X':np.round(X,2),
'Y':np.round(Y,2)})
data = data.set_index('Point')

data
```

Point	X	Y
P1	0.69	0.13
P2	0.14	0.76
P3	0.43	0.43
P4	0.15	0.11
P5	0.42	0.94
P6	0.80	0.70
P7	0.59	0.15
P8	0.61	0.80
P9	0.35	0.76
P10	0.46	0.91

Visualize Dataset

```
plt.figure(figsize=(8,5))
plt.scatter(data['X'], data['Y'], c='r', marker='*')
plt.xlabel('X')
plt.ylabel('Y')
plt.title('Scatter Plot of x and y')
for i in data.itertuples():
    plt.annotate(i.Index, (i.X, i.Y))
```



Distance Matrix

```
dist = pd.DataFrame(squareform(pdist(data[['X', 'Y']]), 'euclidean'),
columns=data.index.values, index=data.index.values)
dist
```

```
           P1        P2        P3        P4        P5        P6
P7  \
P1   0.000000  0.836301  0.396989  0.540370  0.853815  0.580517
0.101980
P2   0.836301  0.000000  0.439318  0.650077  0.332866  0.662722
0.758024
P3   0.396989  0.439318  0.000000  0.425206  0.510098  0.458039
0.322490
P4   0.540370  0.650077  0.425206  0.000000  0.872812  0.877838
0.441814
P5   0.853815  0.332866  0.510098  0.872812  0.000000  0.449444
0.808084
P6   0.580517  0.662722  0.458039  0.877838  0.449444  0.000000
0.588727
P7   0.101980  0.758024  0.322490  0.441814  0.808084  0.588727
0.000000
P8   0.674759  0.471699  0.411461  0.829277  0.236008  0.214709
0.650308
```

```
P9   0.715891  0.210000  0.339559  0.680074  0.193132  0.453982
0.655515
P10  0.813204  0.353412  0.480937  0.857963  0.050000  0.399625
0.771038

           P8        P9       P10
P1   0.674759  0.715891  0.813204
P2   0.471699  0.210000  0.353412
P3   0.411461  0.339559  0.480937
P4   0.829277  0.680074  0.857963
P5   0.236008  0.193132  0.050000
P6   0.214709  0.453982  0.399625
P7   0.650308  0.655515  0.771038
P8   0.000000  0.263059  0.186011
P9   0.263059  0.000000  0.186011
P10  0.186011  0.186011  0.000000
```

Single-Link Clustering

```
def single_linkage(dist_matrix):
    n = len(dist_matrix)
    while n > 1:
        min_val = float('inf')
        min_index = None
        for i in range(n):
            for j in range(i+1, n):
                if dist_matrix.iloc[i, j] < min_val and
dist_matrix.index[i] != dist_matrix.columns[j]:
                    min_val = dist_matrix.iloc[i, j]
                    min_index = (i, j)

        if min_val == float('inf'):
            break

        i, j = min_index
        cluster1, cluster2 = dist_matrix.index[i],
dist_matrix.columns[j]
        print(f'Merging clusters {cluster1} and {cluster2} with
distance {min_val}')

        new_cluster = f'({cluster1},{cluster2})'

        dist_matrix[new_cluster] = dist_matrix[[cluster1,
cluster2]].min(axis=1)
        dist_matrix.loc[new_cluster] = dist_matrix.loc[[cluster1,
cluster2]].min(axis=0)
        dist_matrix = dist_matrix.drop([cluster1, cluster2], axis=0)
        dist_matrix = dist_matrix.drop([cluster1, cluster2], axis=1)
```

```
        n -= 1

        print(dist_matrix)
        print("")

print("Single-Link Clustering:")
single_linkage(dist.copy())
```

```
Single-Link Clustering:
Merging clusters P5 and P10 with distance 0.04999999999999998
                P1        P2        P3        P4        P6
P7  \
P1         0.000000  0.836301  0.396989  0.540370  0.580517  0.101980

P2         0.836301  0.000000  0.439318  0.650077  0.662722  0.758024

P3         0.396989  0.439318  0.000000  0.425206  0.458039  0.322490

P4         0.540370  0.650077  0.425206  0.000000  0.877838  0.441814

P6         0.580517  0.662722  0.458039  0.877838  0.000000  0.588727

P7         0.101980  0.758024  0.322490  0.441814  0.588727  0.000000

P8         0.674759  0.471699  0.411461  0.829277  0.214709  0.650308

P9         0.715891  0.210000  0.339559  0.680074  0.453982  0.655515

(P5,P10)   0.813204  0.332866  0.480937  0.857963  0.399625  0.771038


                P8        P9  (P5,P10)
P1         0.674759  0.715891  0.813204
P2         0.471699  0.210000  0.332866
P3         0.411461  0.339559  0.480937
P4         0.829277  0.680074  0.857963
P6         0.214709  0.453982  0.399625
P7         0.650308  0.655515  0.771038
P8         0.000000  0.263059  0.186011
P9         0.263059  0.000000  0.186011
(P5,P10)   0.186011  0.186011  0.000000

Merging clusters P1 and P7 with distance 0.10198039027185567
                P2        P3        P4        P6        P8
P9  \
P2         0.000000  0.439318  0.650077  0.662722  0.471699  0.210000

P3         0.439318  0.000000  0.425206  0.458039  0.411461  0.339559

P4         0.650077  0.425206  0.000000  0.877838  0.829277  0.680074
```

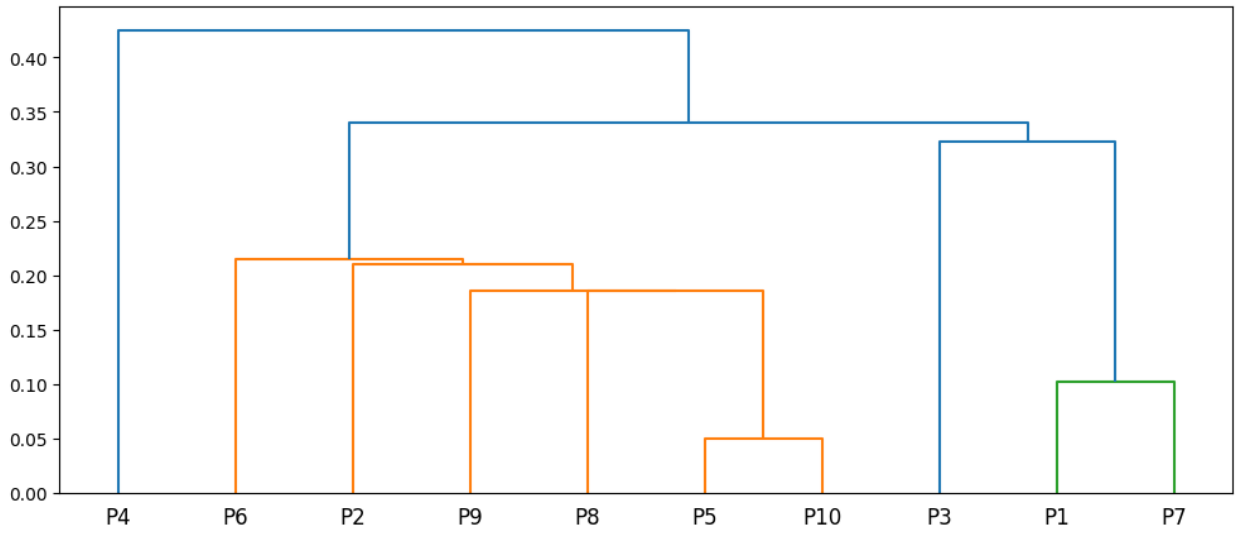
```
P6         0.662722  0.458039  0.877838  0.000000  0.214709  0.453982

P8         0.471699  0.411461  0.829277  0.214709  0.000000  0.263059

P9         0.210000  0.339559  0.680074  0.453982  0.263059  0.000000

(P5,P10)   0.332866  0.480937  0.857963  0.399625  0.186011  0.186011

(P1,P7)    0.758024  0.322490  0.441814  0.580517  0.650308  0.655515


           (P5,P10)   (P1,P7)
P2         0.332866  0.758024
P3         0.480937  0.322490
P4         0.857963  0.441814
P6         0.399625  0.580517
P8         0.186011  0.650308
P9         0.186011  0.655515
(P5,P10)   0.000000  0.771038
(P1,P7)    0.771038  0.000000

Merging clusters P8 and (P5,P10) with distance 0.1860107523773827
                     P2        P3        P4        P6        P9
(P1,P7)  \
P2              0.000000  0.439318  0.650077  0.662722  0.210000
0.758024
P3              0.439318  0.000000  0.425206  0.458039  0.339559
0.322490
P4              0.650077  0.425206  0.000000  0.877838  0.680074
0.441814
P6              0.662722  0.458039  0.877838  0.000000  0.453982
0.580517
P9              0.210000  0.339559  0.680074  0.453982  0.000000
0.655515
(P1,P7)         0.758024  0.322490  0.441814  0.580517  0.655515
0.000000
(P8,(P5,P10))   0.332866  0.411461  0.829277  0.214709  0.186011
0.650308

                (P8,(P5,P10))
P2                   0.332866
P3                   0.411461
P4                   0.829277
P6                   0.214709
P9                   0.186011
(P1,P7)              0.650308
(P8,(P5,P10))        0.000000

Merging clusters P9 and (P8,(P5,P10)) with distance 0.1860107523773828
                          P2        P3        P4        P6
```

```
(P1,P7)  \
P2                   0.000000  0.439318  0.650077  0.662722  0.758024

P3                   0.439318  0.000000  0.425206  0.458039  0.322490

P4                   0.650077  0.425206  0.000000  0.877838  0.441814

P6                   0.662722  0.458039  0.877838  0.000000  0.580517

(P1,P7)              0.758024  0.322490  0.441814  0.580517  0.000000

(P9,(P8,(P5,P10)))  0.210000  0.339559  0.680074  0.214709  0.650308


                     (P9,(P8,(P5,P10)))
P2                             0.210000
P3                             0.339559
P4                             0.680074
P6                             0.214709
(P1,P7)                        0.650308
(P9,(P8,(P5,P10)))             0.000000

Merging clusters P2 and (P9,(P8,(P5,P10))) with distance
0.20999999999999996
                                P3        P4        P6   (P1,P7)  \
P3                        0.000000  0.425206  0.458039  0.322490
P4                        0.425206  0.000000  0.877838  0.441814
P6                        0.458039  0.877838  0.000000  0.580517
(P1,P7)                   0.322490  0.441814  0.580517  0.000000
(P2,(P9,(P8,(P5,P10))))  0.339559  0.650077  0.214709  0.650308

                          (P2,(P9,(P8,(P5,P10))))
P3                                       0.339559
P4                                       0.650077
P6                                       0.214709
(P1,P7)                                  0.650308
(P2,(P9,(P8,(P5,P10))))                  0.000000

Merging clusters P6 and (P2,(P9,(P8,(P5,P10)))) with distance
0.21470910553583897
                                   P3        P4   (P1,P7)  \
P3                           0.000000  0.425206  0.322490
P4                           0.425206  0.000000  0.441814
(P1,P7)                      0.322490  0.441814  0.000000
(P6,(P2,(P9,(P8,(P5,P10)))))  0.339559  0.650077  0.580517

                             (P6,(P2,(P9,(P8,(P5,P10)))))
P3                                             0.339559
P4                                             0.650077
(P1,P7)                                        0.580517
```

```
(P6,(P2,(P9,(P8,(P5,P10))))))                          0.000000

Merging clusters P3 and (P1,P7) with distance 0.322490309931942
                                        P4  (P6,(P2,(P9,(P8,
(P5,P10)))))  \
P4                                 0.000000                          0.650077

(P6,(P2,(P9,(P8,(P5,P10)))))  0.650077                          0.000000

(P3,(P1,P7))                       0.425206                          0.339559


                                   (P3,(P1,P7))
P4                                     0.425206
(P6,(P2,(P9,(P8,(P5,P10)))))           0.339559
(P3,(P1,P7))                           0.000000

Merging clusters (P6,(P2,(P9,(P8,(P5,P10))))) and (P3,(P1,P7)) with
distance 0.3395585369269929
                                                          P4  \
P4                                                  0.000000
((P6,(P2,(P9,(P8,(P5,P10)))))),(P3,(P1,P7)))  0.425206

                                                    ((P6,(P2,(P9,(P8,
(P5,P10)))))),(P3,(P1,P7)))
P4
0.425206
((P6,(P2,(P9,(P8,(P5,P10)))))),(P3,(P1,P7)))
0.000000

Merging clusters P4 and ((P6,(P2,(P9,(P8,(P5,P10)))))),(P3,(P1,P7)))
with distance 0.425205832509386
                                                          (P4,((P6,(P2,(P9,
(P8,(P5,P10)))))),(P3,(P1,P7))))
(P4,((P6,(P2,(P9,(P8,(P5,P10)))))),(P3,(P1,P7))))
0.0
```

Visualize Dendogram for Single-Link Cluster

```
plt.figure(figsize=(12,5))
dend = shc.dendrogram(shc.linkage(data[['X', 'Y']], method='single'),
labels=data.index)
```



Complete-Link Clustering

```
def complete_linkage(dist_matrix):
    n = len(dist_matrix)
    while n > 1:
        min_val = float('inf')
        min_index = None
        for i in range(n):
            for j in range(i+1, n):
                if dist_matrix.iloc[i, j] < min_val and
dist_matrix.index[i] != dist_matrix.columns[j]:
                    min_val = dist_matrix.iloc[i, j]
                    min_index = (i, j)

        if min_val == float('inf'):
            break

        i, j = min_index
        cluster1, cluster2 = dist_matrix.index[i],
dist_matrix.columns[j]

        print(f'Merging clusters {cluster1} and {cluster2} with
distance {min_val}')
        new_cluster = f'({cluster1},{cluster2})'

        dist_matrix[new_cluster] = dist_matrix[[cluster1,
cluster2]].max(axis=1)
        dist_matrix.loc[new_cluster] = dist_matrix.loc[[cluster1,
cluster2]].max(axis=0)
        dist_matrix = dist_matrix.drop([cluster1, cluster2], axis=0)
        dist_matrix = dist_matrix.drop([cluster1, cluster2], axis=1)

        n -= 1
```

```
        print(dist_matrix)
        print("")

print("Complete-Link Clustering:")
complete_linkage(dist.copy())
```

```
Complete-Link Clustering:
Merging clusters P5 and P10 with distance 0.04999999999999998
                P1        P2        P3        P4        P6
P7  \
P1         0.000000  0.836301  0.396989  0.540370  0.580517  0.101980

P2         0.836301  0.000000  0.439318  0.650077  0.662722  0.758024

P3         0.396989  0.439318  0.000000  0.425206  0.458039  0.322490

P4         0.540370  0.650077  0.425206  0.000000  0.877838  0.441814

P6         0.580517  0.662722  0.458039  0.877838  0.000000  0.588727

P7         0.101980  0.758024  0.322490  0.441814  0.588727  0.000000

P8         0.674759  0.471699  0.411461  0.829277  0.214709  0.650308

P9         0.715891  0.210000  0.339559  0.680074  0.453982  0.655515

(P5,P10)   0.853815  0.353412  0.510098  0.872812  0.449444  0.808084


                P8        P9  (P5,P10)
P1         0.674759  0.715891  0.853815
P2         0.471699  0.210000  0.353412
P3         0.411461  0.339559  0.510098
P4         0.829277  0.680074  0.872812
P6         0.214709  0.453982  0.449444
P7         0.650308  0.655515  0.808084
P8         0.000000  0.263059  0.236008
P9         0.263059  0.000000  0.193132
(P5,P10)   0.236008  0.193132  0.050000

Merging clusters P1 and P7 with distance 0.10198039027185567
                P2        P3        P4        P6        P8
P9  \
P2         0.000000  0.439318  0.650077  0.662722  0.471699  0.210000

P3         0.439318  0.000000  0.425206  0.458039  0.411461  0.339559

P4         0.650077  0.425206  0.000000  0.877838  0.829277  0.680074

P6         0.662722  0.458039  0.877838  0.000000  0.214709  0.453982
```

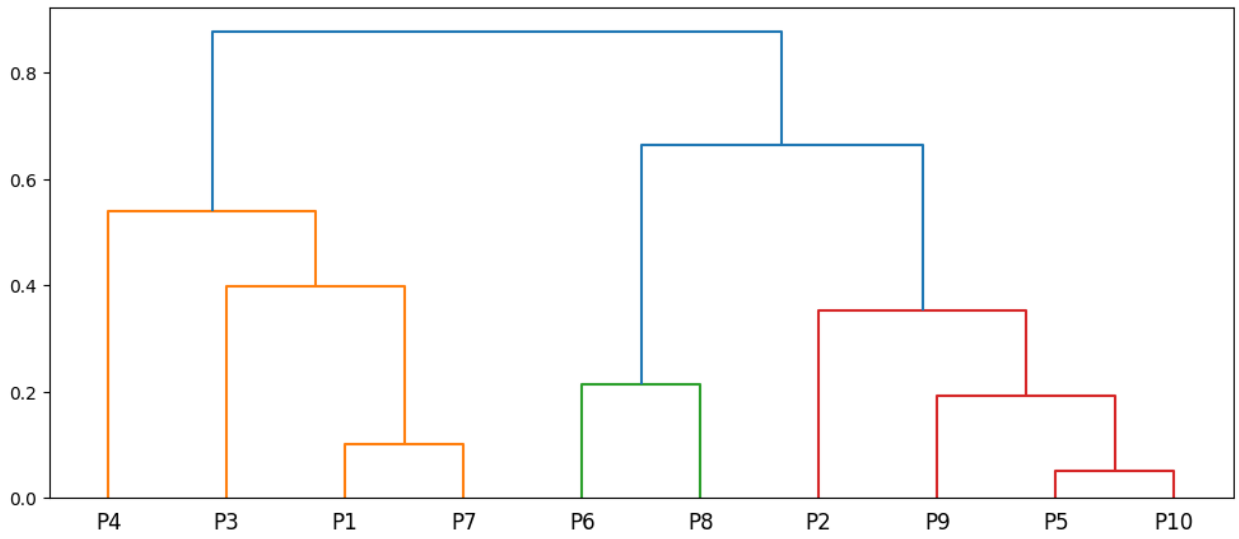
```
P8        0.471699  0.411461  0.829277  0.214709  0.000000  0.263059

P9        0.210000  0.339559  0.680074  0.453982  0.263059  0.000000

(P5,P10)  0.353412  0.510098  0.872812  0.449444  0.236008  0.193132

(P1,P7)   0.836301  0.396989  0.540370  0.588727  0.674759  0.715891


          (P5,P10)   (P1,P7)
P2        0.353412  0.836301
P3        0.510098  0.396989
P4        0.872812  0.540370
P6        0.449444  0.588727
P8        0.236008  0.674759
P9        0.193132  0.715891
(P5,P10)  0.050000  0.853815
(P1,P7)   0.853815  0.101980

Merging clusters P9 and (P5,P10) with distance 0.1931320791582796
                     P2        P3        P4        P6        P8
(P1,P7)  \
P2             0.000000  0.439318  0.650077  0.662722  0.471699
0.836301
P3             0.439318  0.000000  0.425206  0.458039  0.411461
0.396989
P4             0.650077  0.425206  0.000000  0.877838  0.829277
0.540370
P6             0.662722  0.458039  0.877838  0.000000  0.214709
0.588727
P8             0.471699  0.411461  0.829277  0.214709  0.000000
0.674759
(P1,P7)        0.836301  0.396989  0.540370  0.588727  0.674759
0.101980
(P9,(P5,P10))  0.353412  0.510098  0.872812  0.453982  0.263059
0.853815

               (P9,(P5,P10))
P2                  0.353412
P3                  0.510098
P4                  0.872812
P6                  0.453982
P8                  0.263059
(P1,P7)             0.853815
(P9,(P5,P10))       0.193132

Merging clusters P6 and P8 with distance 0.21470910553583897
                     P2        P3        P4   (P1,P7)  (P9,(P5,P10))
(P6,P8)
```

```
P2              0.000000  0.439318  0.650077  0.836301       0.353412
0.662722
P3              0.439318  0.000000  0.425206  0.396989       0.510098
0.458039
P4              0.650077  0.425206  0.000000  0.540370       0.872812
0.877838
(P1,P7)         0.836301  0.396989  0.540370  0.101980       0.853815
0.674759
(P9,(P5,P10))   0.353412  0.510098  0.872812  0.853815       0.193132
0.453982
(P6,P8)         0.662722  0.458039  0.877838  0.674759       0.453982
0.214709

Merging clusters P2 and (P9,(P5,P10)) with distance
0.35341194094144585
                           P3        P4   (P1,P7)   (P6,P8)  (P2,(P9,
(P5,P10)))
P3                   0.000000  0.425206  0.396989  0.458039
0.510098
P4                   0.425206  0.000000  0.540370  0.877838
0.872812
(P1,P7)              0.396989  0.540370  0.101980  0.674759
0.853815
(P6,P8)              0.458039  0.877838  0.674759  0.214709
0.662722
(P2,(P9,(P5,P10)))   0.510098  0.872812  0.853815  0.662722
0.353412

Merging clusters P3 and (P1,P7) with distance 0.3969886648255841
                           P4   (P6,P8)  (P2,(P9,(P5,P10)))  (P3,
(P1,P7))
P4                   0.000000  0.877838            0.872812
0.540370
(P6,P8)              0.877838  0.214709            0.662722
0.674759
(P2,(P9,(P5,P10)))   0.872812  0.662722            0.353412
0.853815
(P3,(P1,P7))         0.540370  0.674759            0.853815
0.396989

Merging clusters P4 and (P3,(P1,P7)) with distance 0.5403702434442518
                     (P6,P8)  (P2,(P9,(P5,P10)))  (P4,(P3,(P1,P7)))
(P6,P8)              0.214709            0.662722           0.877838
(P2,(P9,(P5,P10)))   0.662722            0.353412           0.872812
(P4,(P3,(P1,P7)))    0.877838            0.872812           0.540370

Merging clusters (P6,P8) and (P2,(P9,(P5,P10))) with distance
0.6627216610312356
                               (P4,(P3,(P1,P7)))  ((P6,P8),(P2,(P9,
(P5,P10))))
```

```
(P4,(P3,(P1,P7)))                          0.540370
0.877838
((P6,P8),(P2,(P9,(P5,P10))))               0.877838
0.662722

Merging clusters (P4,(P3,(P1,P7))) and ((P6,P8),(P2,(P9,(P5,P10))))
with distance 0.8778382538941898
                                                        ((P4,(P3,(P1,P7))),
((P6,P8),(P2,(P9,(P5,P10)))))
((P4,(P3,(P1,P7))),((P6,P8),(P2,(P9,(P5,P10)))))
0.877838
```

Visualize Dendogram for Complete-Link Cluster

```
plt.figure(figsize=(12,5))
dend = shc.dendrogram(shc.linkage(data[['X', 'Y']],
method='complete'), labels=data.index)
```



Model Interpretation

Single-Linkage:

- The single-link clustering algorithm merges clusters based on the minimum distance between individual data points in the clusters.
- As the merging progresses, clusters with smaller distances are merged first, indicating high similarity or proximity between the data points within those clusters
- The process continues, gradually merging clusters with larger distances, until all data points are included in a single cluste

Complete-Linkage:

- Complete-link clustering merges clusters based on the maximum distance between any two points in the clusters.

- The merging process starts with clusters having smaller maximum distances, indicating high similarity or proximity between the data points within those clusters
- As the process continues, clusters with larger maximum distances are merged, suggesting either dissimilarity between some points within those clusters or the presence of outlierts.r.