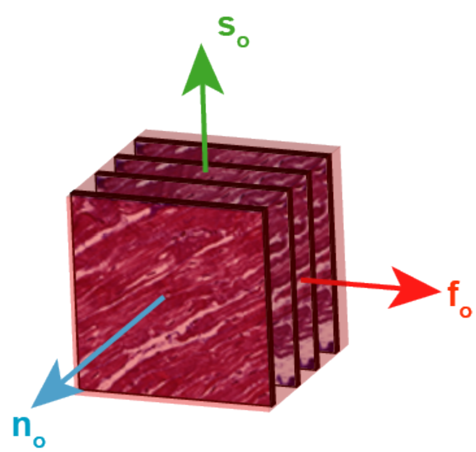
Uncoupled Holzapfel-Ogden Material



Theory and User's Manual

FEBio 3 Material Plugin

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1 Strain Energy Function

This FEBio material plugin implements a nearly incompressible, uncoupled formulation of the constitutive model of passive myocardium, as incorporated in [1] and proposed by Holzapfel and Ogden in [2]. In this implementation, the hyperelastic strain function is decomposed into isochoric and volumetric parts:

$$\Psi(\mathbf{C}) = \tilde{\Psi}(\tilde{\mathbf{C}}) + U(J)\,, \tag{1}$$

where $\tilde{\mathbf{C}}$ is the isochoric right Cauchy- Green deformation tensor and $U(J) = K\,[\ln(J)]^2/2$ with K being the bulk modulus and $J = \det(\mathbf{F})$ [3].

Given a right-handed, orthornormal reference frame describing the fiber ($\mathbf{f_0}$), sheet ($\mathbf{s_0}$) and the sheet-normal ($\mathbf{n_0}$) direction, the following isochoric invariants are defined:

$$\begin{aligned} \tilde{I}_1 = \tilde{\mathbf{C}} : \mathbf{I} \quad &; \quad \tilde{I}_{4f} = \mathbf{f_0}\cdot\tilde{\mathbf{C}}\mathbf{f_0} \quad ; \quad \tilde{I}_{4s} = \mathbf{s_0}\cdot\tilde{\mathbf{C}}\mathbf{s_0} \\ \tilde{I}_{8fs} = \mathbf{f_0}\cdot\tilde{\mathbf{C}}\mathbf{s_0} \quad &; \quad \tilde{I}_{8sn} = \mathbf{s_0}\cdot\tilde{\mathbf{C}}\mathbf{n_0} \quad ; \quad \tilde{I}_{8nf} = \mathbf{n_0}\cdot\tilde{\mathbf{C}}\mathbf{f_0}, \end{aligned} \tag{2}$$

Next, the isochoric part of the strain energy function $\tilde{\Psi}$ is split into the contribution of an isotropic term (matrix) $\tilde{\Psi}_g$, the fiber and sheet constituents $\tilde{\Psi}_i$ and all shear coupling interactions $\tilde{\Psi}_{ij}$ as in [4], which reads

$$\tilde{\Psi} = \tilde{\Psi}_g + \tilde{\Psi}_f + \tilde{\Psi}_s + \tilde{\Psi}_{fs} + \tilde{\Psi}_{sn} + \tilde{\Psi}_{nf} \tag{3}$$

where

$$\begin{aligned} \tilde{\Psi}_g &= \frac{a}{2b}\left[\exp\left(b[\tilde{I}_1 - 3]\right) - 1\right] \\ \tilde{\Psi}_i &= \frac{a_i}{2b_i}\left[\exp\left(b_i[\tilde{I}_{4i} - 1]^2\right) - 1\right] \qquad i \;= f, s \\ \tilde{\Psi}_{ij} &= \frac{a_{ij}}{2b_{ij}}\left[\exp\left(b_{ij}\tilde{I}_{8ij}^2\right) - 1\right] \qquad i,j \;= f, s, n\,. \end{aligned} \tag{4}$$

and all parameters a's or b's ≥ 0. If not combined with any other strain energy terms in a solid mixture, the isotropic modulus a needs to be strictly positive as the material is not stable just on the contribution of fibers. Note that if any of the terms $b_i = 0$ or $b_{ij} = 0$ then the strain energy reads:

$$\begin{aligned} \lim_{b\to 0}\tilde{\Psi}_g &= \frac{a}{2}(\tilde{I}_1 - 3) \\ \lim_{b_i\to 0}\tilde{\Psi}_i &= \frac{a_i}{2}(\tilde{I}_{4i} - 1)^2 \\ \lim_{b_{ij}\to 0}\tilde{\Psi}_{ij} &= \frac{a_{ij}}{2}\tilde{I}_{8ij}^2 \end{aligned} \tag{5}$$

Also note that the $\tilde{\Psi}_i$ terms contribute only when the corresponding fiber or sheet constituents are under tension, i.e. $\tilde{I}_{4i} > 0$ for $i = f, s$.

2 Cauchy Stress Tensor

Using the strain energy function (3), define

$$
\begin{aligned}
\frac{\partial \tilde{\Psi}_g}{\partial \tilde{I}_1} &= \frac{a}{2}\exp(b[\tilde{I}_1 - 3]) \\
\frac{\partial \tilde{\Psi}_i}{\partial \tilde{I}_{4i}} &= a_i(\tilde{I}_{4i} - 1)\exp(b_i[\tilde{I}_{4i} - 1]^2) \qquad \text{(No sum.)} \\
\frac{\partial \tilde{\Psi}_{ij}}{\partial \tilde{I}_{8ij}} &= a_{ij}\tilde{I}_{8ij}\exp(b_{ij}\tilde{I}_{8ij}^2) \qquad \text{(No sum.)}
\end{aligned}
\tag{6}
$$

Let $\tilde{\mathbf{F}} = J^{-1/3}\mathbf{F}$ be the isochoric part of the deformation gradient and $\tilde{\mathbf{b}}$ the isochoric left Cauchy-Green deformation tensor. Define the following vectors in spatial configuration: $\tilde{\mathbf{f}} = \tilde{\mathbf{F}}\mathbf{f}_0$, $\tilde{\mathbf{s}} = \tilde{\mathbf{F}}\mathbf{s}_0$ and $\tilde{\mathbf{n}} = \tilde{\mathbf{F}}\mathbf{n}_0$ [2]. Define also the projection tensor by $\mathbb{P} = \mathbb{I} - \frac{1}{3}\mathbf{I} \otimes \mathbf{I}$ where $\mathbb{I}$ is the fourth order identity tensor [5]. Then the isochoric Cauchy stress tensor reads:

$$\sigma_{\text{iso}} = \text{dev}(\tilde{\sigma}) = \mathbb{P} : \tilde{\sigma} \tag{7}$$

where

$$
\begin{aligned}
\tilde{\sigma} = \quad & 2J^{-1}\frac{\partial \tilde{\Psi}_g}{\partial \tilde{I}_1}\tilde{\mathbf{b}} \\
& + 2J^{-1}\frac{\partial \tilde{\Psi}_f}{\partial \tilde{I}_{4f}}(\tilde{\mathbf{f}} \otimes \tilde{\mathbf{f}}) \\
& + 2J^{-1}\frac{\partial \tilde{\Psi}_s}{\partial \tilde{I}_{4s}}(\tilde{\mathbf{s}} \otimes \tilde{\mathbf{s}}) \\
& + J^{-1}\frac{\partial \tilde{\Psi}_{fs}}{\partial \tilde{I}_{8fs}}(\tilde{\mathbf{f}} \otimes \tilde{\mathbf{s}} + \tilde{\mathbf{s}} \otimes \tilde{\mathbf{f}}) \\
& + J^{-1}\frac{\partial \tilde{\Psi}_{sn}}{\partial \tilde{I}_{8sn}}(\tilde{\mathbf{s}} \otimes \tilde{\mathbf{n}} + \tilde{\mathbf{n}} \otimes \tilde{\mathbf{s}}) \\
& + J^{-1}\frac{\partial \tilde{\Psi}_{nf}}{\partial \tilde{I}_{8nf}}(\tilde{\mathbf{n}} \otimes \tilde{\mathbf{f}} + \tilde{\mathbf{f}} \otimes \tilde{\mathbf{n}}).
\end{aligned}
\tag{8}
$$

3 Elasticity Tensor

The second derivatives of the strain energy function (3) are defined by:

$$
\begin{aligned}
\frac{\partial^2 \tilde{\Psi}_g}{\partial \tilde{I}_1^2} &= \frac{ab}{2} \exp(b[\tilde{I}_1 - 3]) \\
\frac{\partial^2 \tilde{\Psi}_i}{\partial \tilde{I}_{4i}^2} &= a_i \left[2b_i[\tilde{I}_{4i} - 1]^2 + 1 \right] \exp(b_i[\tilde{I}_{4i} - 1]^2) \qquad \text{(No sum.)} \\
\frac{\partial^2 \tilde{\Psi}_{ij}}{\partial \tilde{I}_{8ij}^2} &= a_{ij} \left[2b_{ij} \tilde{I}_{8ij}^2 + 1 \right] \exp(b_{ij} \tilde{I}_{8ij}^2) \qquad \text{(No sum.)}
\end{aligned} \tag{9}
$$

Next, the isochoric part of the spatial elasticity tensor is defined by [5]:

$$
\begin{aligned}
\mathbb{c}_{\mathrm{iso}} &= 4J^{-1} \mathbf{b} \frac{\partial^2 \tilde{\Psi}(\tilde{\mathbf{b}})}{\partial \mathbf{b} \partial \mathbf{b}} \mathbf{b} \\
&= J^{-1}(\mathbb{p} : \tilde{\mathbb{c}} : \mathbb{p}) + \frac{2}{3} \mathrm{tr}(\tilde{\sigma}) \mathbb{p} - \frac{2}{3} \left(\mathbf{I} \otimes \sigma_{\mathrm{iso}} + \sigma_{\mathrm{iso}} \otimes \mathbf{I} \right)
\end{aligned} \tag{10}
$$

where $\tilde{\mathbb{c}}$ as given in [6] reads:

$$
\begin{aligned}
\tilde{\mathbb{c}} = 4 \tilde{\mathbf{b}} \frac{\partial^2 \tilde{\Psi}(\tilde{\mathbf{b}})}{\partial \tilde{\mathbf{b}} \partial \tilde{\mathbf{b}}} \tilde{\mathbf{b}} & \\
= \quad & 4 \frac{\partial^2 \tilde{\Psi}_g}{\partial \tilde{I}_1^2} (\tilde{\mathbf{b}} \otimes \tilde{\mathbf{b}}) \\
& + 4 \frac{\partial^2 \tilde{\Psi}_f}{\partial \tilde{I}_{4f}^2} (\tilde{\mathbf{f}} \otimes \tilde{\mathbf{f}} \otimes \tilde{\mathbf{f}} \otimes \tilde{\mathbf{f}}) \\
& + 4 \frac{\partial^2 \tilde{\Psi}_s}{\partial \tilde{I}_{4s}^2} (\tilde{\mathbf{s}} \otimes \tilde{\mathbf{s}} \otimes \tilde{\mathbf{s}} \otimes \tilde{\mathbf{s}}) \\
& + \frac{\partial^2 \tilde{\Psi}_{fs}}{\partial \tilde{I}_{8fs}^2} (\tilde{\mathbf{f}} \otimes \tilde{\mathbf{s}} + \tilde{\mathbf{s}} \otimes \tilde{\mathbf{f}}) \otimes (\tilde{\mathbf{f}} \otimes \tilde{\mathbf{s}} + \tilde{\mathbf{s}} \otimes \tilde{\mathbf{f}}) \\
& + \frac{\partial^2 \tilde{\Psi}_{sn}}{\partial \tilde{I}_{8sn}^2} (\tilde{\mathbf{s}} \otimes \tilde{\mathbf{n}} + \tilde{\mathbf{n}} \otimes \tilde{\mathbf{s}}) \otimes (\tilde{\mathbf{s}} \otimes \tilde{\mathbf{n}} + \tilde{\mathbf{n}} \otimes \tilde{\mathbf{s}}) \\
& + \frac{\partial^2 \tilde{\Psi}_{nf}}{\partial \tilde{I}_{8nf}^2} (\tilde{\mathbf{n}} \otimes \tilde{\mathbf{f}} + \tilde{\mathbf{f}} \otimes \tilde{\mathbf{n}}) \otimes (\tilde{\mathbf{n}} \otimes \tilde{\mathbf{f}} + \tilde{\mathbf{f}} \otimes \tilde{\mathbf{n}}).
\end{aligned} \tag{11}
$$

4 Compilation Instructions

4.1 Compiled versions

A compiled version of the Holzapfel-Ogden plugin has been included for Windows and Linux operating systems (Holzapfel_Ogden.dll and *.so, respectively) . Both files have been compiled with the libraries included in the FEBio 3.2 SDK version. The libraries required are 'FECore' and 'FEBioMech'. For the Windows version, Visual Studio 2017 community version was used. The Linux version was compiled on Ubuntu 18.04 with g++ 7.5.0.

If an update is needed, the source code is included so that the plugin may be complied again according to the following instructions.

4.2 Windows

As of FEBio 3.2.0, Visual Studio 2017 is required. The steps are the following:

- Install Visual Studio 2017 Community version with all C++ development tools included.
- Start a new 'dll' project. Select 'Project→Add Existing Item' and add header and source files to the respective folders within the project.
- Select the 'Release' configuration and 'x64' platform.
- Right-click on the project → 'Properties'.
- Go to 'VC++ Directories' → 'Include Directories'. Add "`C:\ProgramFiles\FEBioXYZ\sdk\include`".
- Go to 'VC++ Directories' → 'Library Directories'. Add "`C:\ProgramFiles\FEBioXYZ\sdk\vs2017\Release\lib`". Note: adjust for respective FEBio and Visual Studio versions.
- Go to 'C/C++' → 'Preprocessor' → 'Preprocessor Definitions'. Add "WIN32".
- Go to 'Linker' → 'Input' → 'Additional Dependencies'. Add "FECore.lib;FEBioMech.lib".
- Depending on the default settings, the following changes may be required. Go to 'C/C++' → 'Precompiled Headers' and select 'Use(/Yu).Also, ensure the field 'Precompiled Header File' to have 'stdafx.h'.

  Right-click on 'stdafx.cpp' and in the 'Precompiled Header' field enter 'Create(/Yc)'.
- Build the project.

4.3 Linux

Using the g++ compiler, run:

```
g++ -o Holzapfel_Ogden.so -fpic -shared *.h *.cpp -I/.../FEBio-3.2/sdk/include -L/.../
    FEBio-3.2/sdk/lib -lfecore_gcc64 -lfebiomech_gcc64 -g
```

The plugin has been exported as "Holzapfel_Ogden.so".

4.4 OSX

To create a plugin with XCode, compile with the -fpic flag and link with -dynamiclib. For more information, see FEBio Developer's Manual.

4.5 Load plugin

To load the plugin in FEBio, the configuration file "febio.xml" located by default in "/FEBioStudio/bin/" or otherwise in "/FEBioStudio/febio/bin/" has to be modified. For the Windows version add the following line

```
<import>path-to-plugin/Holzapfel_Ogden.dll</import>
```

and for a Linux installation,

```
<import>path-to-plugin/Holzapfel_Ogden.so</import>
```

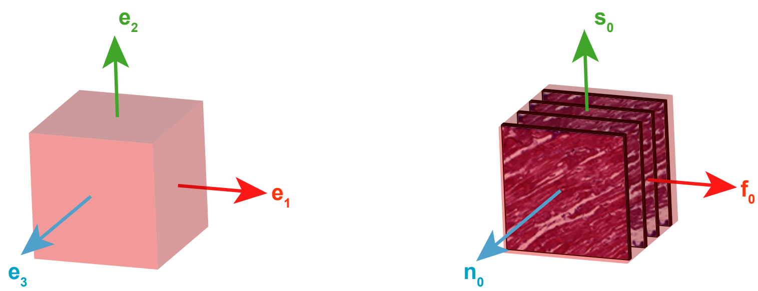
After modifying and saving the configuration file, verify that the plugin is loaded properly by calling FEBio. If loaded successfully, FEBio should print out

```
Success loading plugin Holzapfel_Ogden.dll
```

Finally, run the single-element test model "UniaxialTest.feb", as provided with the plugin. Make sure that it terminates normally, printing the same results as the reference data provided in "DevSed.txt" (deviatoric strain energy) and "Stress.txt" (normal stress).

5 Model implementation

As presented, this material model requires a right-handed, orthonormal reference frame describing the fiber ($\mathbf{f_0}$), sheet ($\mathbf{s_0}$) and the sheet-normal ($\mathbf{n_0}$) direction. The local material axes $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ at each domain also represent the $(\mathbf{f}_0, \mathbf{s}_0, \mathbf{n}_0)$ frame:



Local Material Axes **Fiber, Sheet, Sheet-normal Axes**

This material type is "Holzapfel_Ogden". The following parameters need to be defined:

<a>	a, representing a measure of isotropic matrix modulus
<b>	b, coefficient of exponential argument of isotropic term
<af>	a_f, representing a measure of fiber modulus
<bf>	b_f, coefficient of exponential argument of fiber term
<as>	a_s, representing a measure of sheet modulus
<bs>	b_s, coefficient of exponential argument of sheet term
<afs>	a_{fs}, representing a measure of $\boldsymbol{f} - \boldsymbol{s}$ coupling modulus
<bfs>	b_{fs}, coefficient of exponential argument of $\boldsymbol{f} - \boldsymbol{s}$ coupling term
<asn>	a_{sn}, representing a measure of $\boldsymbol{s} - \boldsymbol{n}$ coupling modulus
<bsn>	b_{sn}, coefficient of exponential argument of $\boldsymbol{s} - \boldsymbol{n}$ coupling term
<anf>	a_{nf}, representing a measure of $\boldsymbol{n} - \boldsymbol{f}$ coupling modulus
<bnf>	b_{nf}, coefficient of exponential argument of $\boldsymbol{n} - \boldsymbol{f}$ coupling term
<k>	bulk modulus

Example 1

```
<material id="1" name="Myocardium" type="Holzapfel_Ogden">
    <a>1</a>
    <b>5</b>
    <af>2</af>
    <bf>5</bf>
    <as>1</as>
    <bs>5</bs>
    <afs>1</afs>
    <bfs>5</bfs>
    <asn>1</asn>
    <bsn>2</bsn>
    <anf>2</anf>
    <bnf>5</bnf>
    <k>1e+3</k>
</material>
```

Note that as of FEBio 3.0, the material parameters can receive an expression instead of a constant value. For example, consider the parameter a_f that if evaluated by an expression, should be defined by `<af type="math"> [...]  </af>`, where `[...]` contains the math expression according to FEBio User's Manual (Appendix C, FEBio v3.2.0).

Also, note that as any other uncoupled material, the Holzapfel-Ogden model can be included within an uncoupled solid mixture. In that case, the bulk modulus should be outside any solid domain. In the following example, the fiber term Ψ_f is substituted by a 2D continuous fiber distribution. Please note that the term $\tilde{\Psi}_{sn}$ is also deactivated by assigning a zero value to the `<asn>` parameter.

Example 2

```
<material id="1" name="Myocardium" type="uncoupled solid mixture">
    <k>1e+3</k>
    <density>1</density>
    <solid type="Holzapfel_Ogden">
        <a>1</a>
        <b>5</b>
        <af>0.0</af>
        <bf>0.0</bf>
        <as>1</as>
        <bs>5</bs>
        <afs>1</afs>
        <bfs>5</bfs>
        <asn>0.0</asn>
        <bsn>0.0</bsn>
        <anf>2</anf>
        <bnf>5</bnf>
    </solid>
    <solid type="continuous fiber distribution uncoupled">
        <fibers type="fiber-exp-pow-uncoupled">
            <alpha>5</alpha>
            <beta>2</beta>
            <ksi>1</ksi>
        </fibers>
        <distribution type="von-Mises-2d">
            <b>3</b>
        </distribution>
        <scheme type="fibers-2d-trapezoidal">
            <nth>31</nth>
        </scheme>
    </solid>
</material>
```

6 Citation Guide

Use of the `"Holzapfel_Ogden"` plugin	[1]
Derivations	[1, 2, 4, 5, 6]
FEBio Software Suite	[3]
FEBio Plugin Feature	[7]

Acknowledgments

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References

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