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Question #1

$l_1 = l_2 = m_1 = m_2 = 1$

given $\mathbb{x}(0) = [q(0)\ \dot{q}(0)]^T$, and τ...

find $\mathbb{x}(t)$

Method #1: Newton-Euler for RR Manipulator

$$\mathbb{u} = K^T \circ \mathbb{I}$$

$$\mathbb{u}_2 = (\ddot{\theta}_1 + \ddot{\theta}_2) + \ddot{\theta}_1 \cos\theta_2 + \dot{\theta}_1^2 \sin\theta_2 + g\cos(\theta_1 + \theta_2)$$

$$\mathbb{u}_1 = (1 + \cos\theta_2)\ddot{\theta}_1 + \ddot{\theta}_2 + \dot{\theta}_1^2 \sin\theta_2 + g\cos(\theta_1 + \theta_2) + \ddot{\theta}_1 + (\ddot{\theta}_1 + \ddot{\theta}_2)\cos\theta_2$$
$$- (\dot{\theta}_1 + \dot{\theta}_2)^2 \sin\theta_2 + g\cos\theta_1 + \ddot{\theta}_1 + g\cos\theta_1$$

Put into control equation form...

$$\mathbb{u} = D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q)$$

$$\mathbb{u} = \underbrace{\begin{bmatrix} 2 + 2\cos\theta_2 & 1 + \cos\theta_2 \\ 1 + \cos\theta_2 & 1 \end{bmatrix}}_{D} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} -2\sin(\theta_2)\dot{\theta}_2 & -\sin(\theta_2)\dot{\theta}_2 \\ \sin(\theta_2)\dot{\theta}_1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 2g\cos\theta_1 + g\cos(\theta_1 + \theta_2) \\ g\cos(\theta_1 + \theta_2) \end{bmatrix}}_{G}$$

given $\mathbb{x}(0)$ and $\mathbb{u}$... solve for $\ddot{q} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$

$\downarrow$

$$q_{i+1} = q_i + \dot{q}_i * \Delta t + \tfrac{1}{2}\ddot{q}_i * \Delta t^2 \quad \text{(joint variable at next timestep)}$$

timestep

$$\dot{q}_{i+1} = \frac{q_{i+1} - q_i}{\Delta t}$$