COMP 2613- Fall 2019

Assignment #1

Due no later than Wednesday, October 23rd, 2019, 4:30 pm (during the lecture)

This assignment consists of three (3) pages. Full answer is 7 points. There is in addition 1 bonus point.

1. A regular language (4 points)
Let Σ be an alphabet that contains the symbol a. Define the **thrice** of a string $w = w_1 w_2 \dots w_n$ of length $n = |w|$ over alphabet Σ, denoted $t(w)$, as the string obtained by replacing the 3rd symbol in w by the symbol a,

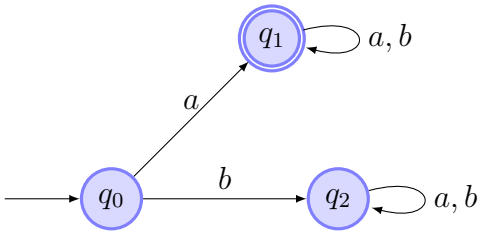
$$t(w) = \begin{cases} w & \text{if } 0 \leq n < 3 \\ w_1 w_2 a & \text{if } n = 3 \\ w_1 w_2 a w_4 \dots w_n & \text{if } n > 3. \end{cases}$$

If the string w has fewer than 3 symbols or if $w_3 = a$, then $t(w) = w$. Given a language L, let $t(L) = \{t(w) \mid w \in L\}$. Show that if L is regular, then so is $t(L)$.
Ensure that your answer includes three parts: (i) your construction for the language $t(L)$, (ii) a demonstration of how your construction works on an example, and (iii) your arguments for why your construction is correct.

2. Flawed theorem (1 point)
The following "theorem" is false, and its "proof" contains one critical flaw. Precisely and succinctly identify the flaw and explain why it is a flaw.

Theorem 1 *The language* $\{awwa \mid w \in \{a, b\}^*\}$ *is regular.*

Proof Let the alphabet be $\Sigma = \{a, b\}$. Let $L_1 = \{aw | w \in \Sigma^*\}$ be the set of strings over the alphabet Σ that start with the symbol a. Similarly, let $L_1^R = \{wa \mid w \in \Sigma^*\}$. Finally, let $L_2 = L_1 L_1^R = \{awwa \mid w \in \Sigma^*\}$ be even-length palindromes that begin and end with an a. The language L_1 is regular since the DFA M_1 drawn here accepts L_1:



That is $L(M_1) = L_1$. Since L_1 is regular, and the class of regular languages is closed under reversal and concatenation, we conclude L_1^R and L_2 are also regular.

3. A regular and a non-regular language (2 points)
For the second question, you may use without proof that the language $L_{cnt} = \{a^i b^i | i \geq 0\}$ is non-regular.

1. Show that the language $L_2 = \{w \in \Sigma^* \mid w$ does not contain the substring $accaabc\}$ is regular. Here $\Sigma = \{a, b, c\}$.

2. Use **two different** techniques and show that the language $L_{even} = \{a^i b^i | i \geq 0$ and i is even$\}$ is non-regular.

4. A non-regular language (1 bonus point)
Please note that no collaboration is permitted on this bonus question. For this problem, you may use without proof that the language $L_{cnt} = \{a^i b^i | i \geq 0\}$ is non-regular. Given a language L over the alphabet $\Sigma = \{a, b\}$, define the language

$countall(L) = \{w \in \Sigma^* \mid$ there exists $v \in L$ such that $|w|_a = |v|_a$ and $|w|_b = |v|_b\}$.
Give a regular language L for which $countall(L)$ is not regular. Show that L is regular, and prove that $countall(L)$ is not regular.

Submission

You must submit the hard copy of your assignment on or before Wednesday, October 23^{rd}, 2019, 4:30 pm (during the lecture). No late submissions will be accepted. The deadline is firm. Use the last page of this assignment as the front page of your assignment. Assignments submitted without the front page will be deducted 1 point.

Collaboration and plagiarism

You are welcome to work and discuss the assignment with other students enrolled in this course (i.e., COMP 2613-Fall 2019). You must clearly state whom your collaborators are, if any, for each problem on the assignment.
Verbal collaboration is allowed. Written collaboration is strictly forbidden. For instance, notes, papers, emails, messages, texting, twitter, chats, blogs, discussion boards, whiteboards, blackboards, and photos used as communication devices are strictly forbidden. All written work that you submit must be your own sole work. Anything else will be considered plagiarism. When you are discussing this assignment with others, do not use any form of writing.
The use of published literature is allowed. If you use any published literature (texts, articles, websites, etc) to complete your assignment, you must quote your sources. I suggest that you develop your own solutions however, without the use of any published materials. You will be asked to answer similar questions on the exams for this course and during the exams no such sources will be available.
Note that no collaborations or sources are allowed for the bonus problem, Problem 4.
If you have any doubt whether a collaboration is allowed or not, ask the lecturer before entering the collaboration.

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Name: _____

My sources and my collaborators, if any, on this assignment were:

Problem 1: _____

Problem 2: _____

Problem 3: _____

Problem 4: _____(no collaboration permitted) _____