ECE-351 Pre-Lab 5

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Febuary 18, 2025

Task 1: Find the transfer function

$$V_{out} = V_{in} \cdot \frac{Z_{LC}}{Z_R + Z_{LC}} : Z_{LC} = (\frac{1}{Z_L} + \frac{1}{Z_C})^{-1} = (\frac{1}{j\omega L} + j\omega C)^{-1} = \frac{1}{\frac{1}{j\omega L} + j\omega C} = \frac{j\omega L}{1 - \omega^2 LC}$$

$$\Rightarrow V_{out} = V_{in} \frac{\frac{j\omega L}{1-\omega^2 LC}}{R + \frac{j\omega L}{1-\omega^2 LC}} = V_{in} \frac{j\omega L}{R(1-\omega^2 LC) + j\omega L} : s = j\omega \quad s^2 = -\omega^2$$

$$\rightarrow \frac{V_{out}}{V_{in}} = \frac{sL}{s^2 RLC + sL + R} = \frac{s\frac{1}{RC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

Task 2: Find the impulse response

$$p = \frac{-\frac{1}{RC} + \sqrt{(\frac{1}{RC})^2 - 4\frac{1}{LC}}}{2} = -\frac{1}{2RC} + \frac{1}{2}\sqrt{\frac{1}{R^2C^2} - \frac{4}{LC}} : \alpha = -\frac{1}{2RC} \quad \beta = \frac{1}{2}\sqrt{\frac{1}{R^2C^2} - \frac{4}{LC}}$$

$$g = s\frac{1}{RC}\bigg|_{s=p} = \frac{-1}{2R^2C^2} + \sqrt{\frac{1}{4R^4C^4} - \frac{1}{R^2C^3L}}$$

$$|g| = \sqrt{\frac{1}{4R^4C^4} + \frac{1}{4R^4C^4} - \frac{1}{R^2C^3L}} = \sqrt{\frac{1}{2R^4C^4} - \frac{1}{R^2C^3L}}$$

$$\angle g = \tan^{-1}\left(\frac{\sqrt{\frac{1}{4R^4C^4} - \frac{1}{R^2C^3L}}}{\frac{-1}{2R^2C^2}}\right) = \tan^{-1}\left(-\sqrt{\frac{4R^4C^4}{4R^4C^4} - \frac{4R^2C^4}{R^2C^3L}}\right) = \tan^{-1}\left(-\sqrt{1 - \frac{4R^2C}{L}}\right)$$

$$h(t) = \frac{\sqrt{\frac{1}{2R^4C^4} - \frac{1}{R^2C^3L}}}{-j\frac{1}{2}\sqrt{\frac{1}{R^2C^2} - \frac{4}{LC}}} exp\left(\frac{-1}{2RC}t\right) \sin\left[-j * \frac{1}{2}\sqrt{\frac{1}{R^2C^2} - \frac{4}{LC}}t + \tan^{-1}\left(-\sqrt{1 - \frac{4R^2C}{L}}\right)\right] u(t)$$

$$h(t) = -j\sqrt{\frac{\frac{1}{2R^4C^4} - \frac{1}{R^2C^3L}}{\frac{1}{4R^2C^2} - \frac{1}{LC}}} exp\left(\frac{-1}{2RC}t\right) \sin\left[-j * \frac{1}{2}\sqrt{\frac{1}{R^2C^2} - \frac{4}{LC}}t + \tan^{-1}\left(-\sqrt{1 - \frac{4R^2C}{L}}\right)\right] u(t)$$

Note: the -j term is to seperate the j component of $p = \alpha + j\omega$