Investigating Factors Affecting the Resonant Frequency of Cantilever Beams

Research Question: To what extent does changing the length of a circular cantilever beam affect its resonant frequency, and how well can the theoretical model predict this relationship?

International Baccalaureate Physics Extended Essay

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Abstract

Write something here

1 Introduction

Cantilever beams, seemingly simple elements used in construction where one end of a beam is solidly attached and the other loose, has played a major role in engineering, historically they were used for purely mechanical structures such as buildings, cranes and balconies. (Hool and Johnson 1929) While the use of such simple elements is still common in traditional construction, their utility expands far beyond macroscopic architecture, Nowadays cutting edge technology such as MEMS(micro-electromechanical) systems which integrate mechanical systems with electronics in a microscopic level, use such structures as well, microscopic cantilever beams are being used as inertial sensor in gyroscopes to sense tiny acceleration forces. (Bao 2005)

I was fascinated to learn that such simple structures lie at the core of technologies we take for granted today. This pushed me to further investigate their functionality and their dynamic properties. My research question is "To what extent does changing the length of a circular cantilever beam affect its resonant frequency, and how well can the theoretical model predict this relationship?" and I aim to determine how the length of the beam and the initial energy loaded into the beam will affect the frequency that the beam naturally vibrates at when excited using pure mathematics, Finite Element Analysis and Experimentally. As shown in (1), I am expecting to see the frequency is inversely proportional to the square of its length, and to see no relationship between the frequency and the initial energy load.

$$\begin{gathered} f_n \propto \frac{1}{l^2} \\ f_n \not\propto E_{initial} \end{gathered} \tag{1}$$

2 Background Information

2.1 Fourier Transform

2.2 Finite Element Method

The finite element method(FEM) is a computational approach to solving partial differential equations that commonly arise in engineering. The method consists of subdividing a body into smaller, easier to calculate subdomains known as finite elements. The collection of the finite elements is known as the mesh and each finite element is independently calculated, this allows the computation of the differential equations to be much simpler.(Zienkiewicz and Taylor 2000)
The use of this method of engineering analysis is commonly known as Finite Element Analysis(FEA) and will be used as a virtual experiment in 3.2 to simulate the cantilever beams natural frequency. As this method allows to simulate deformation of bodies, this tool is generally built into Computer Aided Design(CAD) suites and Onshape will be used in this paper.

3 Methodology

3.1 Theory

3.2 Finite Element Analysis

3.3 Materials

4 Results And Analysis

4.1 Uncertainty Analysis

5 Discussion

6 Conclusion

References

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