Quantum Benchmarking Of Molecular Ground-State Energy Estimation





Southern California





- ¹ Department of Physics, Computer Science and Mathematics, University of Modena and Reggio Emilia, Italy
- ² CNR-NANO-Istituto for Nanosciences, Modena, Italy
- ³ University of Southern California, Los Angeles, California, 90089, USA

Overview

- Ground-state energy estimation (GSEE) and excited-state energy estimation (ESEE) are at the core of predicting and controlling chemical reactions, with huge impact in various technologies.
- Classical algorithms for GSEE scale exponentially with system size and are elusive to even the most powerful supercomputers for useful system sizes.
- Hence, quantum computation (QC) emerges as a powerful alternative with a potentially linear scaling. Results of QC algorithms must be benchmarked on reliable data.
- The Variational Quantum Eigen solver (VQE) is a hybrid quantum-classical algorithm for approximating molecular ground-state energies on near-term quantum devices.
- We demonstrate VQE for methylene (CH₂) and Benzene (C₆H₆) using quantum software and running on CPU and GPU devices. While we cannot draw conclusions on energy accuracy, we prove the GPU acceleration of VQE.

Electronic Configuration

$$\hat{H} = \sum_{i=1}^{N} \left(-\frac{1}{2} \nabla_i^2 - \sum_{A} \frac{Z_A}{r_{iA}} \right) + \sum_{i < j} \frac{1}{r_{ij}}$$

$$\hat{H} = \sum_{pq} h_{pq} a_p^{\dagger} a_q + \frac{1}{2} \sum_{pqrs} h_{pqrs} a_p^{\dagger} a_q^{\dagger} a_r a_s$$
3. Second Quantization Hamiltonian

1. Electronic Hamiltonian

$$H^{AS} = \sum_{pq}^{\text{active}} \tilde{h}_{pq} \hat{a}_p^{\dagger} \hat{a}_q + \frac{1}{2} \sum_{pqrs}^{\text{active}} g_{pqrs} \hat{a}_p^{\dagger} \hat{a}_q^{\dagger} \hat{a}_r \hat{a}_s + E_{\text{core}}$$

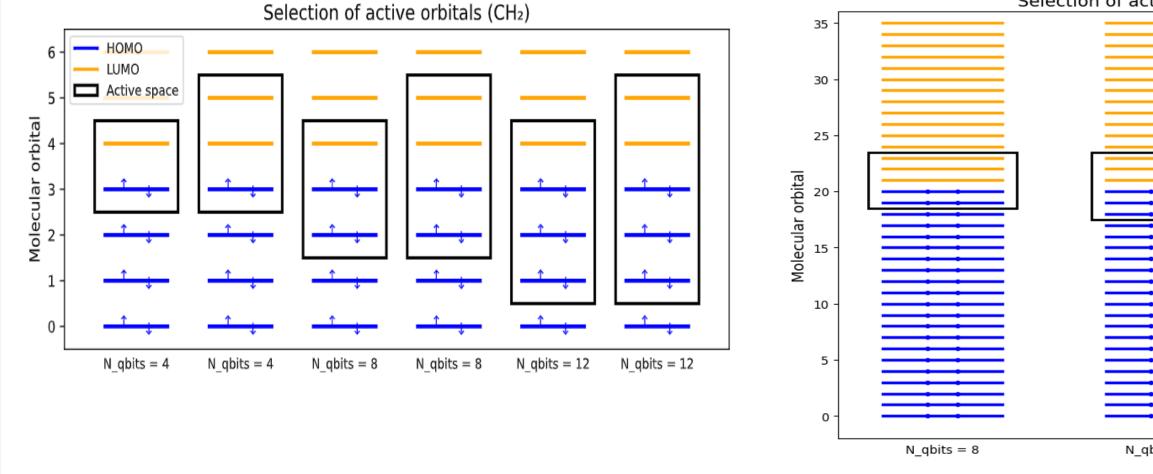
2. Hartree-Fock Approximation

 $\Psi_{HF} = \frac{1}{\sqrt{N!}} \det[\psi_1(\mathbf{x}_1) \, \psi_2(\mathbf{x}_2) \, \cdots \, \psi_N(\mathbf{x}_N)]$

5. Active Space Hamiltonian

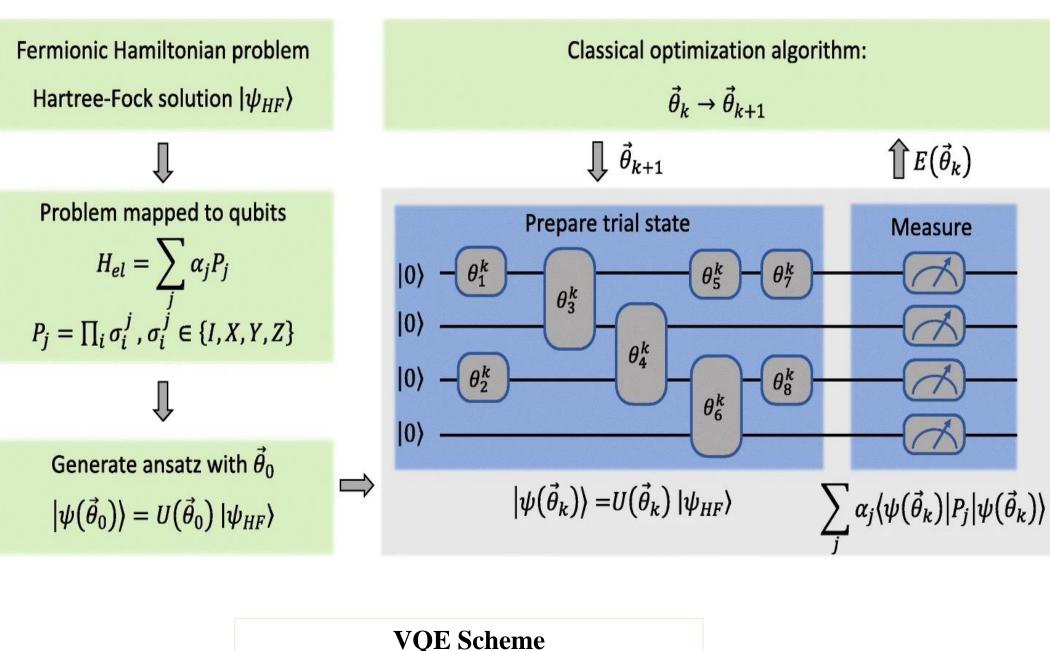
Methodology

- We use the CUDA Quantum (CUDA-Q) framework, enabling simulations on both CPU and GPU backends.
- Within the chosen basis set (sto-3g) for the molecular Hamiltonian, we downsize the active space by freezing "core" orbitals and restricting the simulation to chemically relevant "reactive" orbitals, thus balancing computational cost with chemical accuracy.



The molecular Hamiltonian is constructed via PySCF and OpenFermion libraries, and the UCCSD ansatz is implemented as the variational circuit within CUDA-Q.

Overview of the Variational **Eigensolver (VQE)** Quantum workflow. electronic problem first is structure fermionic mapped operators to qubit-based Pauli operators. wavefunction is prepared using a parameterized quantum circuit (ansatz) based on the Hartree-Fock reference. Expectation values of the Hamiltonian are measured on a quantum device, and the energy is evaluated classically. A classical optimizer updates parameters iteratively hybrid energy, quantum-classical optimization loop.



We systematically evaluate energy convergence, qubit and parameter counts, and computational runtimes across various active space configurations.

Conclusion

- Our results show a significant acceleration in runtime for GPU based simulations compared to CPU implementations, while maintaining comparable energy accuracy within mHa differences.
- The gain increases monotonically with the size of the active space.
- The approach is generalizable to other molecular species, including open-shell molecules.

Methylene (CH₂):

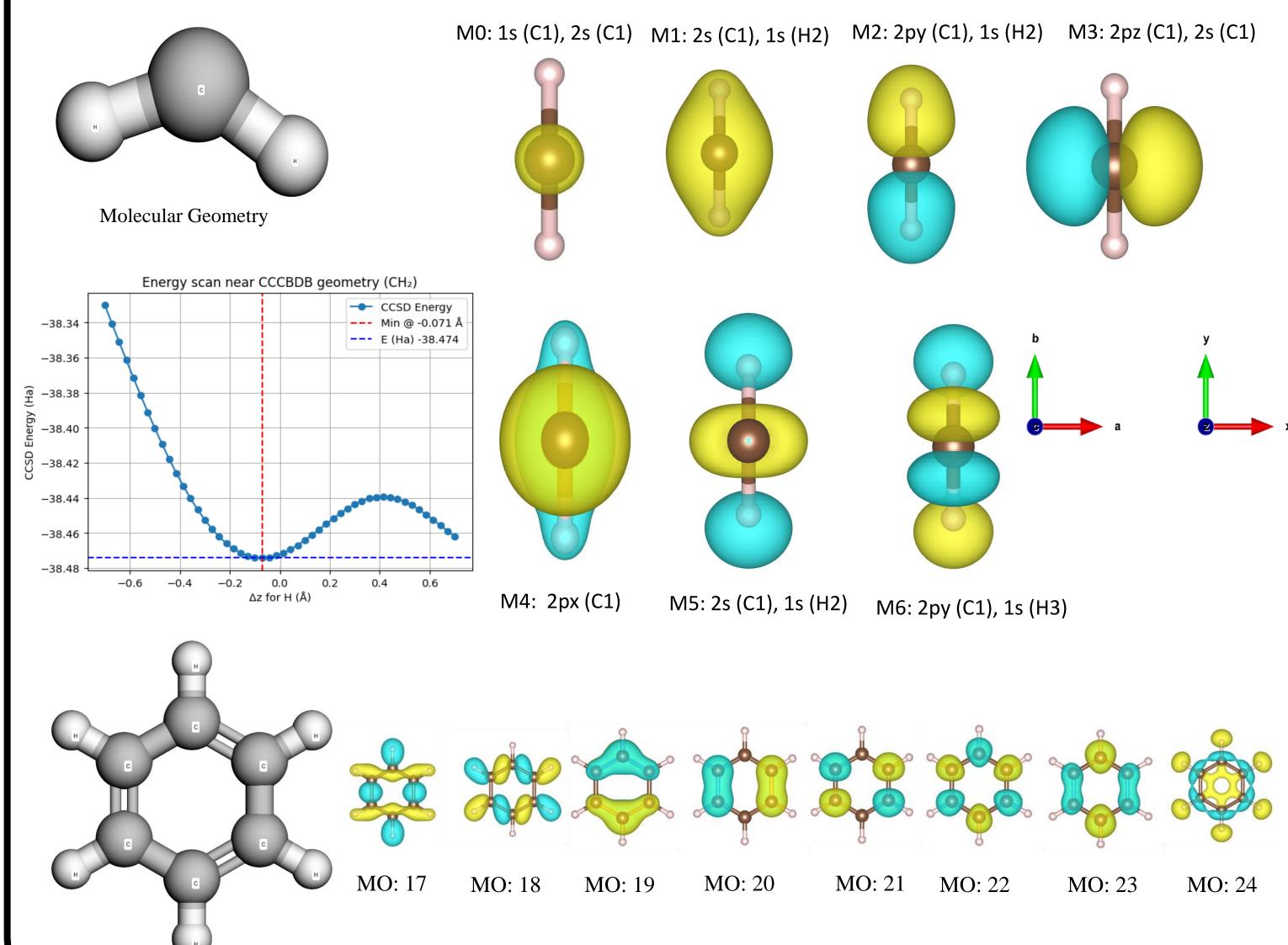
- ✓ For large active space (e.g. nele_cas=6, norb_cas=5), GPU can be ~30× faster than CPU.
- ✓ For small active space, the speedup is marginal or negligible.
- ✓ In one case, CPU was even slightly faster (likely due to overhead/delay in GPU scheduling or under-utilization).

Benzene (C₆H₆):

- ✓ The large active space (e.g. ncore=14, nele_cas=14, norb_cas=8) shows over 260× speedup, highlighting GPU superiority for large-scale quantum simulations.
- ✓ For the smallest case, GPU is still about 2.3× faster than CPU.

Geometry

- We take initial geometry coordinates from the Computational Chemistry Comparison and Benchmark Database (CCCBDB).
- Then we scanned more less energy near by it.
- The Isosurface plot of Molecular Orbital using basis set (sto-3g).



Molecular orbitals used in the active space Molecular Geometry Result Methylene Results (CH₂) Benzene Results (C₆H₆) C₆H₆ - VQE Computation Runtime Data C₆H₆ Energy Convergence during VQE Optimization CH2 Energy Convergence during VQE Optimization — CPU [ncore=18, nele cas=6, norb cas=6 — GPU [ncore=1, nele cas=6, norb cas=5] — GPU [ncore=2, nele cas=4, norb cas=3] — GPU [ncore=2, nele_cas=4, norb_cas=4] GPU [ncore=14 nele cas=14 norb cas=8] — GPU [ncore=3, nele_cas=2, norb_cas=2] GPU [ncore=3, nele cas=2, norb cas=3] - CPU [ncore=1, nele_cas=6, norb_cas=4] — CPU [ncore=1, nele_cas=6, norb_cas=5 -227.2 -CPU [ncore=2, nele_cas=4, norb_cas=3 — CPU [ncore=2, nele cas=4, norb cas=4] — CPU [ncore=3, nele cas=2, norb cas=2] — CPU [ncore=3, nele_cas=2, norb_cas=3] --- Min VQE Energy: -38.398 Energy Convergence on GPU [ncore=14, nele_cas=14, norb_cas=8] — CH₂ VQE Energy — CH2 VQE Energy Min VQE Energy (Ha): -227.8161 Min VQE Energy (Ha): -227.9175 --- Min VQE Engergy: -38.3752 - Total Runtime (s): 45409.74 --- Total Runtime (s): 17.06 Total Runtime (s): 182.97 - Total Runtime (s): 20.03 Quantum Runtime (s): 0 --- Quantum Runtime (s): 45409.26 --- Quantum Runtime (s): 17.02 Quantum Runtime (s): 19.9 - Optimizer Runtime (s): 0.48 --- Optimizer Runtime (s): 0.04 Optimizer Runtime (s): 0.06 Energy Convergence on CPU [ncore=1, nele_cas=6, norb_cas=4] — CH₂ VQE Energy — CH₂ VQE Energy --- Min VQE Energy (Ha): -227.893 --- Min VOE Engergy: -38.367 --- Total Runtime (s): 859.18 --- Total Runtime (s): 60.22 Total Runtime (s): 7.01 Quantum Runtime (s): 84.4 --- Quantum Runtime (s): 859.03 --- Quantum Runtime (s): 60.12 - Quantum Runtime (s): 6.98 Optimizer Runtime (s): 0.15 -227.5 Energy Convergence on CPU [ncore=1, nele_cas=6, norb_cas=5] — C6H6 VOE Energy — C6H6 VOE Energy — CH₂ VQE Energy --- Min VQE Energy (Ha): -227.927 Min VQE Energy: -38.3941 Min VQE Engergy: -38.3736 --- Min VQE Energy (Ha): -227.848 --- Total Runtime (s): 39.16 Total Runtime (s): 2093.21 Total Runtime (s): 171.77 --- Quantum Runtime (s): 39.09 --- Quantum Runtime (s): 171.43 Quantum Runtime (s): 0 - Quantum Runtime (s): 65.06 Optimizer Runtime (s): 0.07 Optimizer Runtime (s): 0.3-Optimizer Runtime (s): 0.3 Comparative Analysis of VQE w.r.t Epoch Comparative Analysis of VQE w.r.t Epoch C6H6 Quantum Simulation Runtime(Log Scale) vs Epoch CH2 Quantum Simulation Runtime(Log Scale) vs Epoch nvidia | ncore=1, nele_cas=6, norb_cas=4 nvidia | ncore=1, nele_cas=6, norb_cas=5 qpp-cpu | ncore=19, nele_cas=4, norb_cas=4 nvidia | ncore=2, nele_cas=4, norb_cas=3 gpp-cpu | ncore=18, nele cas=6, norb cas=6 nvidia | ncore=2, nele_cas=4, norb_cas=4 qpp-cpu | ncore=16, nele_cas=10, norb_cas=6 vidia | ncore=3, nele_cas=2, norb_cas=2 qpp-cpu | ncore=14, nele_cas=14, norb_cas=8 nvidia | ncore=3, nele cas=2, norb cas=3 idia | ncore=19, nele_cas=4, norb_cas=4 qpp-cpu | ncore=1, nele cas=6, norb cas=4 nvidia | ncore=18, nele_cas=6, norb_cas=6 qpp-cpu | ncore=1, nele cas=6, norb cas=5 nvidia | ncore=16, nele_cas=10, norb_cas=6 qpp-cpu | ncore=2, nele_cas=4, norb_cas=3 - nvidia | ncore=14, nele_cas=14, norb_cas=8 qpp-cpu | ncore=2, nele_cas=4, norb_cas=4 qpp-cpu | ncore=3, nele_cas=2, norb_cas=2 qpp-cpu | ncore=3, nele_cas=2, norb_cas=3

References