

# Introduction

ECE 449, Machine Learning



#### Classification

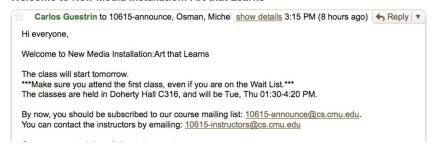
• From data to discrete classes



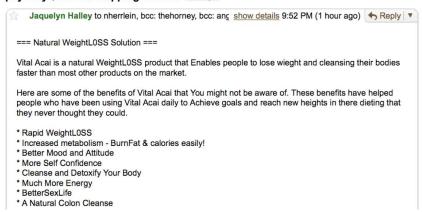
#### Examples: Spam Filtering

#### Data

#### Welcome to New Media Installation: Art that Learns



#### Natural \_LoseWeight SuperFood Endorsed by Oprah Winfrey, Free Trial 1 bottle, pay only \$5.95 for shipping mfw rlk $_{\mbox{\tiny Spam}}$ $|\times$



#### Prediction

Spam/Not Spam



## Examples: Image Classification



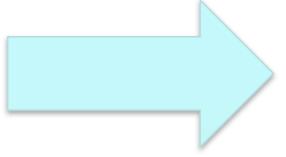






## Examples: Weather Prediction













## Regression

• Predicting a numeric value



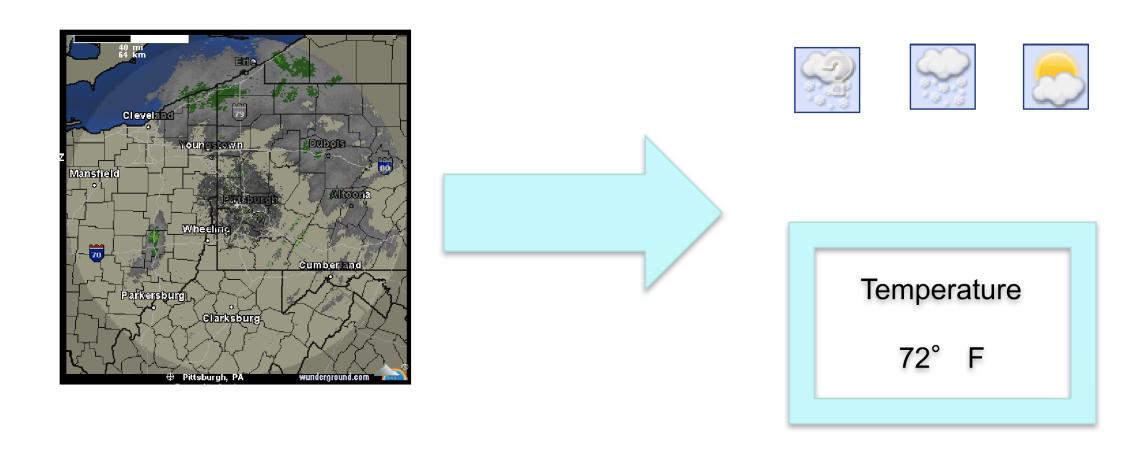
### Examples: Stock Market

Predict stock prices given stock history and today's news





#### Examples: Weather Prediction Revisited



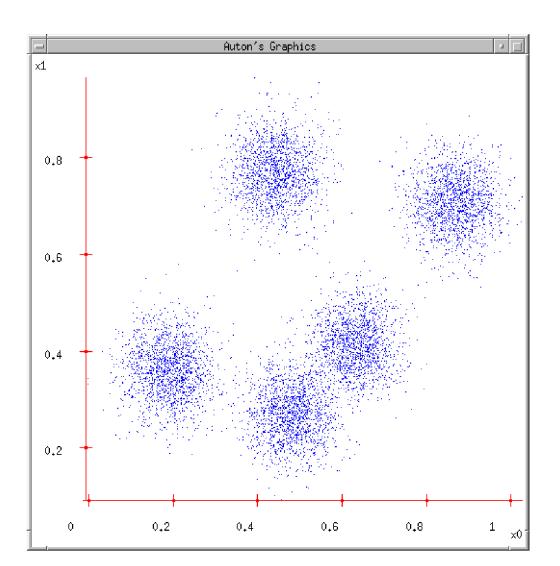


## Clustering

• Discovering structure in data

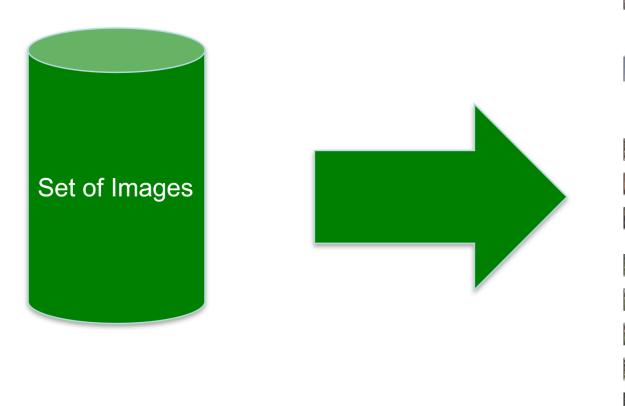


## Examples: Clustering Data











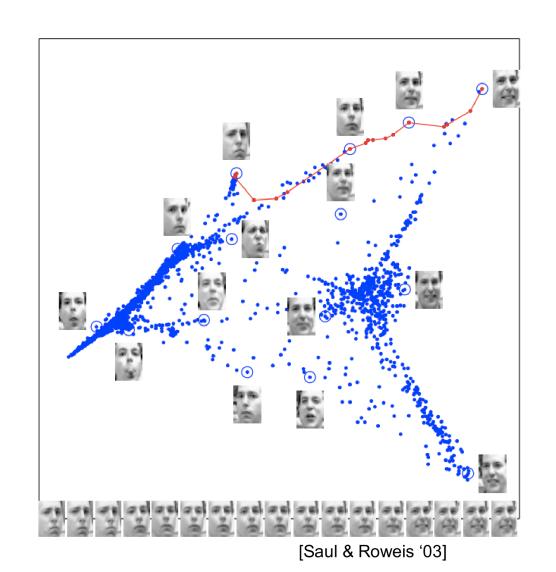


## Embedding

• Visualization and discriminative feature learning



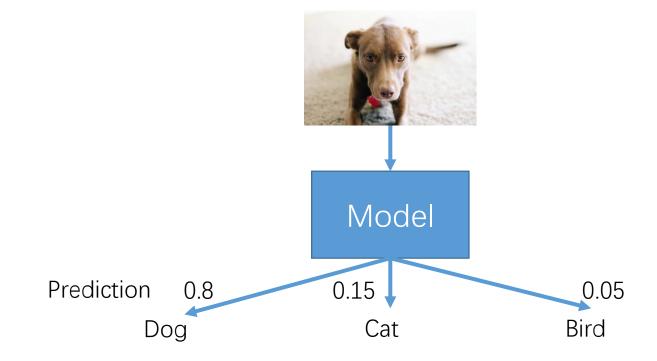
## Examples: Face Embedding





#### Classification

Predict discrete classes

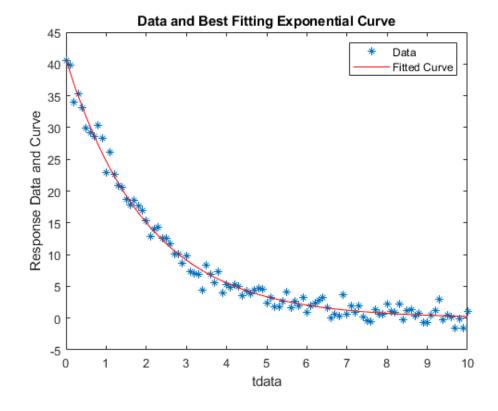




### Regression

• Predicting a numeric value

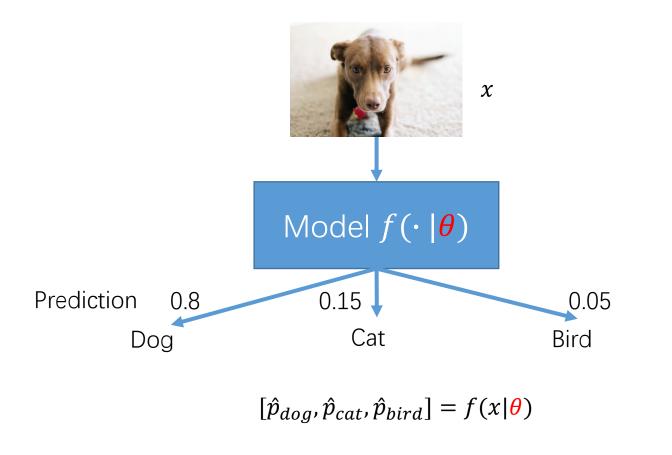


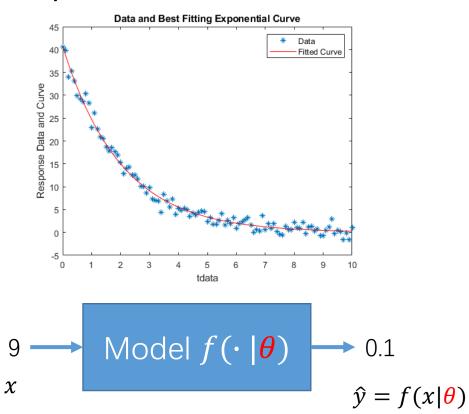




#### Model

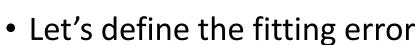
Treat model as a function (linear or nonlinear)





#### Training

- Given data, Learn  $\theta$
- For example, line fitting
  - We have three points (x,y), i.e., (1,2), (2,1), (3,2)
  - $f(x|\theta) = w_0 + w_1 x$ , here  $\theta = \{w_0, w_1\}$

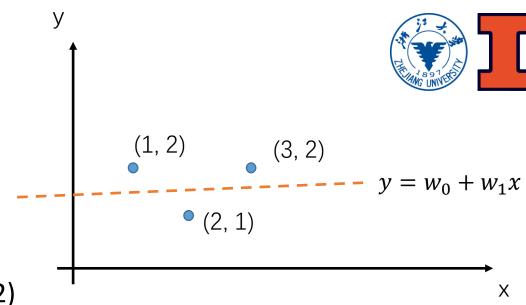


• 
$$loss = (2 - f(1|\theta))^2 + (1 - f(2|\theta))^2 + (2 - f(3|\theta))^2$$
Data (1,2)

Data (2,1)

Data (3,2)

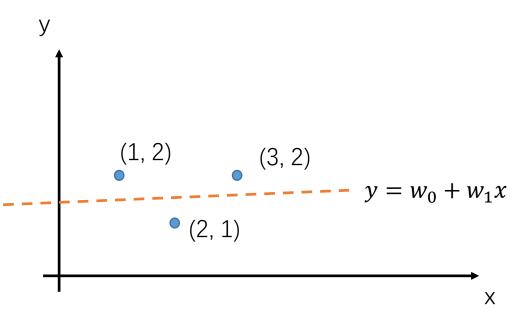
- Define prediction  $\hat{y} = f(x|\theta)$
- $loss = l(y, \hat{y}) = \sum_{i} (y f(x|\theta))^2$





#### Training

- Estimate  $\theta = \{w_0, w_1\}$ 
  - $l = \sum_{i} (y f(x|\theta))^{2} = (y_{1} (w_{0} + w_{1}x_{1}))^{2} + (y_{2} (w_{0} + w_{1}x_{2}))^{2} + (y_{3} (w_{0} + w_{1}x_{3}))^{2}$
- Minimize the loss
  - $\hat{\theta} = arg \min_{\theta} l(y, f(x|\theta))$
- Solve  $\theta$  by setting the gradient to 0
  - $\frac{\partial l}{\partial w_0} = 0, \frac{\partial l}{\partial w_1} = 0$





#### Training

- How about classification?
  - Set classifier  $[\hat{p}_{dog}, \hat{p}_{cat}, \hat{p}_{bird}] = f(x|\theta)$
  - If we have many image-label pairs, we want to estimate  $\theta$  as well.
- Convert class label to one-hot label
  - Set y = [dog, cat, bird], then  $y_1 = [1,0,0]$ ,  $y_2 = [0,1,0]$ ,  $y_3 = [0,0,1]$
- Define loss
  - $loss = -(y_1^T \log(f(x_1|\theta)) + y_2^T \log(f(x_2|\theta)) +$  $y_3^T \log(f(x_3|\theta))$
  - $loss = -\sum_{i} y_{i}^{T} \log(f(x_{i}|\theta))$
- Minimize the loss
  - $\hat{\theta} = arg \min_{\theta} l(y, f(x|\theta))$





$$y_1 = dog$$





$$y_2$$
=cat



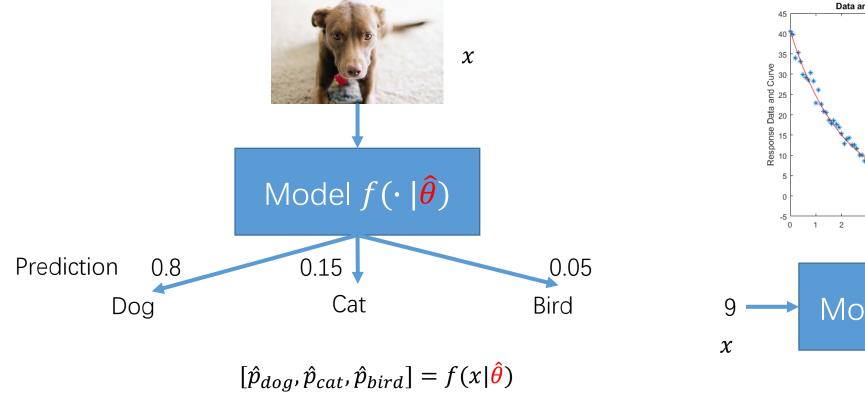


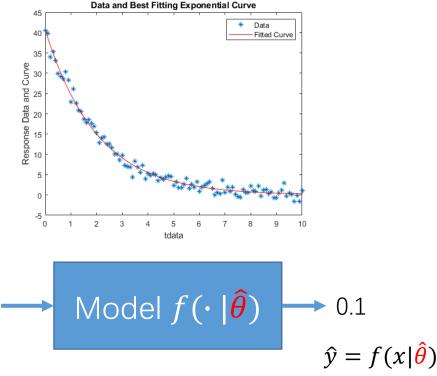
 $y_3$ =bird



#### Testing

• Given the learned model (function)  $f(\cdot | \hat{\theta})$ , we can input any testing data x to get the prediction  $\hat{y} = f(x | \hat{\theta})$ 

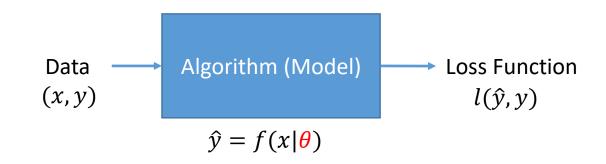






#### Big Picture

- Algorithms that give computers the ability to learn from experience (data) to do specific tasks
  - Different tasks use different types of data, different learning algorithms
  - Performance driven learning: minimize loss function



x: Input data

y: Ground truth label or supervision signal

 $\theta$ : Model parameters

 $f(x|\theta)$ : Mapping from input to the target output

*l*: Loss function



#### Types of Data

- Data can be
  - Binary, numerical or categorical (ordered or not) or a combination
  - A vector/matrix/graph
- Raw input data gets mapped to numerical or indicator form (feature extraction)
- Form of output data impacts loss function



### Types of Learning Algorithms/Models

- We need to define the model (function) configuration  $f(x|\theta)$  before training.
  - Linear model (with respect to  $\theta$ )

```
 f(x|\theta) = w_0 + w_1 x
```

• 
$$f(x|\theta) = w_0 + w_1 x + w_2 x^2$$

• 
$$f(x|\theta) = w_0 + w_1x_1 + w_2x_1x_2$$

• ...

Non-linear function

• 
$$f(x|\theta) = w_0 + w_1 x + w_2 x^2 + w_2 \log(w_1) x^3$$

• ...

Deep learning models



#### Types of Learning Algorithms/Models

- Supervised learning
  - Learning data includes examples with target output, goal is to find a decision function
- Unsupervised learning
  - Learning data has no target output, goal is to learn interesting structure
- Reinforcement learning
  - Sequential decision making in a scenario with changing state and occasional reward/penalty
- Semi-supervised learning, active learning, incremental learning, curriculum learning, federated learning ...



#### Types of Loss Functions

- Mean squared error (usually for regression, i.e., the goal is to predict continuous numerical values)
  - $\frac{1}{N}\sum (y_i \hat{y}_i)^2$
- Cross entropy (usually for classification, i.e., the goal is to predict discrete classes)
  - $-\sum_{i=1}^{C} y_i \log \hat{y}_i$



### Training and Inference/Testing

- Training
  - Given data (x, y), algorithm, minimize the loss function to estimate the model parameters  $\theta$
- Inference/Testing
  - Given data x (without y), algorithm and model parameters  $\theta$ , get the prediction  $\hat{y} = f(x|\theta)$

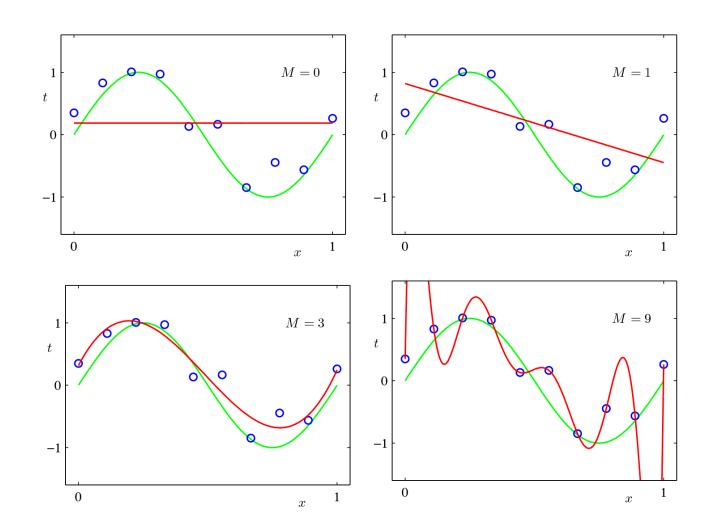


#### Machine Learning Options

- Non-parametric
  - use the data directly
  - Ex: nearest-neighbor
- Parametric
  - Assume a particular distribution
    - Ex: Gaussian → find mean & var from data
  - Assume a functional form
    - Ex: linear  $(a^tx) \rightarrow find coeffs a^t from data$



- Example
  - N-th order regression





- Consider the regression problem
- Assume the perfect decision function exists: y = f(x)

$$MSE = E_{\mathcal{T}} \left[ (\hat{y}_0 - f(x_0))^2 \right]$$

$$= E_{\mathcal{T}} [(\hat{y}_0 - E_{\mathcal{T}} [\hat{y}_0])^2] + (E_{\mathcal{T}} [\hat{y}_0] - f(x_0))^2$$

$$= Var(\hat{y}_0) + Bias^2(\hat{y}_0)$$

• where  $\mathcal{T}$  is the training set (random samples)



$$MSE(x_0) = E_{\mathcal{T}} \left[ \left( \hat{y}_0 - f(x_0) \right)^2 \right]$$

$$= E_{\mathcal{T}} \left[ \left( \hat{y}_0 - E_{\mathcal{T}} [\hat{y}_0] + E_{\mathcal{T}} [\hat{y}_0] - f(x_0) \right)^2 \right]$$

$$= E_{\mathcal{T}} \left[ \left( \hat{y}_0 - E_{\mathcal{T}} [\hat{y}_0] \right)^2 \right] + E_{\mathcal{T}} \left[ \left( E_{\mathcal{T}} [\hat{y}_0] - f(x_0) \right)^2 \right]$$

$$+ 2E_{\mathcal{T}} \left[ \left( \hat{y}_0 - E_{\mathcal{T}} [\hat{y}_0] \right) \left( E_{\mathcal{T}} [\hat{y}_0] - f(x_0) \right) \right]$$

$$= E_{\mathcal{T}} \left[ \left( \hat{y}_0 - E_{\mathcal{T}} [\hat{y}_0] \right)^2 \right] + \left( E_{\mathcal{T}} [\hat{y}_0] - f(x_0) \right)^2$$

$$= Var(\hat{y}_0) + Bias^2(\hat{y}_0)$$

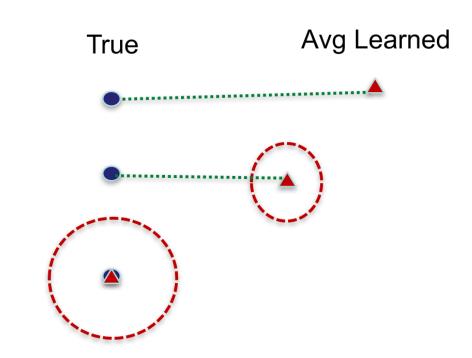


- Bias = distance between average model & theoretical best
- Variance = variability with different training samples

Deterministic classifier  $\alpha(x) = \omega_i \ \forall x$ 

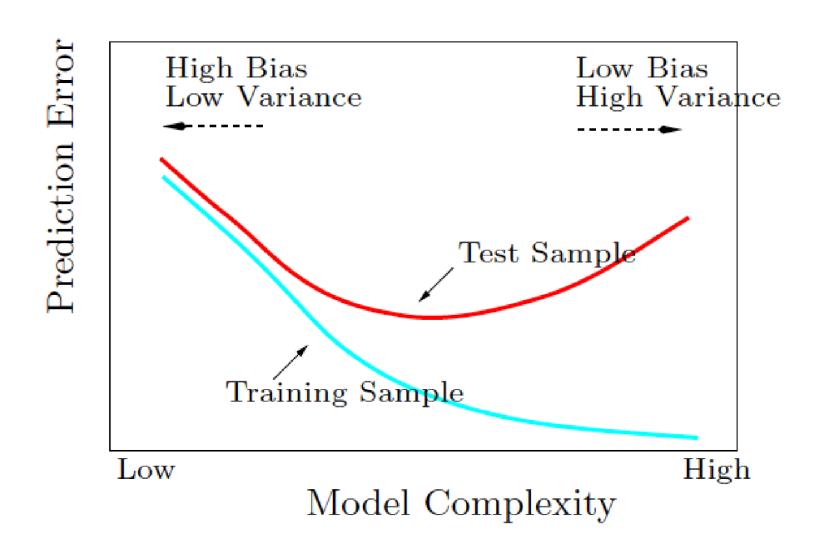
Linear classifier

True n-th order classifier





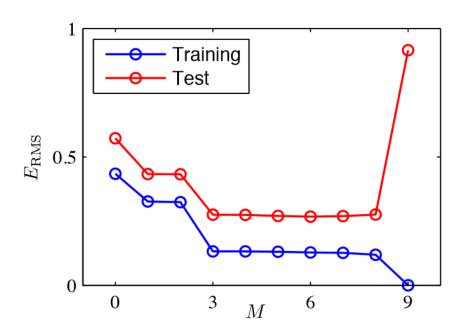
#### Given a Fixed Training Set

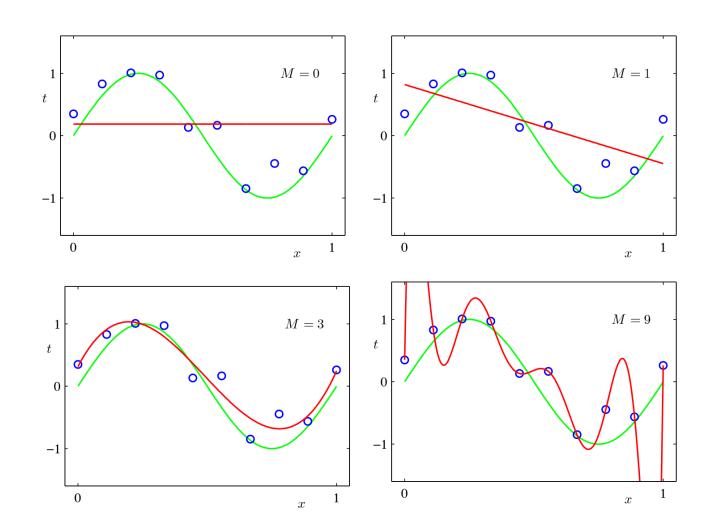




## Examples

• N-th order regression







#### Set the Hyper-Parameters

- The model complexity ≫ #training samples → overfitting
- Whether overfitting → check the testing error
- What happens when # samples N grows?
  - For a specific model: variance ↓as N ↑
- Model complexity also depends on feature dimensionality d (higher d more params)
  - For y=Ax: scalar x,y  $\rightarrow$  scalar A, vector x,y  $\rightarrow$  d<sub>y</sub>d<sub>x</sub> params in A
  - Dimensionality reduction (feature selection or projection, supervised or unsupervised)
  - Feature extraction driven by domain knowledge



#### Practical Implications

- ALWAYS assess performance on data that you haven't looked at in training or model selection (independent test set)
- What does it mean to be "independent"?
  - Two sentences in the same document are not independent
  - Two segments in the same image are not independent
- Use regularization to encourage some parameters to be small

Training Data

**Test Data** 



#### Practical Implications

- Use a held-out validation set or cross-validation for model selection and parameter tuning
- Cross-validation (CV)
  - Partition data into N subsets
  - Train on N-1, validate on Nth
  - Rotate through all N options
  - Choose best configuration
  - Retrain on all the data with best config
- Trade-offs of CV vs. held-out
  - CV makes better use of small data sets
  - CV is more expensive

Training Data

Validation Set

Test Data



### Other Wrong "Model" Problems

- Training data is not representative → impacts all ML approaches
- Could be due to
  - Sampling bias
  - Noisy observations
  - Samples are not independent