

Vote theory

Higher Institute of Management

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Overview

1. Introduction

2. Voting Theory

3. Conclusion

4. References

Introduction

Introduction

Voting theory: Voting theory is the **branch of social choice theory** that studies methods, algorithms, and principles for aggregating individual preferences into collective decisions. It seeks to **understand** and **analyze** the mechanisms by which societies make choices through elections or other decision-making processes.

There are several methods for determining a winner.

Voting theory encompasses the study of different procedures :

1. **Methods:** It analyzes the different approaches or methods by which, **individual preferences** are translated into **overall outcomes**. This includes studying systems like plurality voting, IRV , proportional representation, and approval voting.
2. **Algorithms:** Within each voting method, there are specific algorithms for **processing ballots, tallying votes**, determining winners, and **resolving ties**. Voting theory examines the computational aspects of these algorithms.
3. **Principles:** Voting theory also investigates the underlying principles and criteria that guide the design and evaluation of voting systems , like **fairness, accuracy, simplicity, transparency**, and **efficiency** which helps identify trade-offs and strengths.

Introduction

key terms to be taken into consideration for this presentation are :

1. **Electorate:** The group of individuals eligible to vote in an election.
2. **Ballot:** The document used by voters to cast their votes, listing their options.
3. **Plurality:** The candidate or option with the highest number of votes, regardless of achieving a majority.

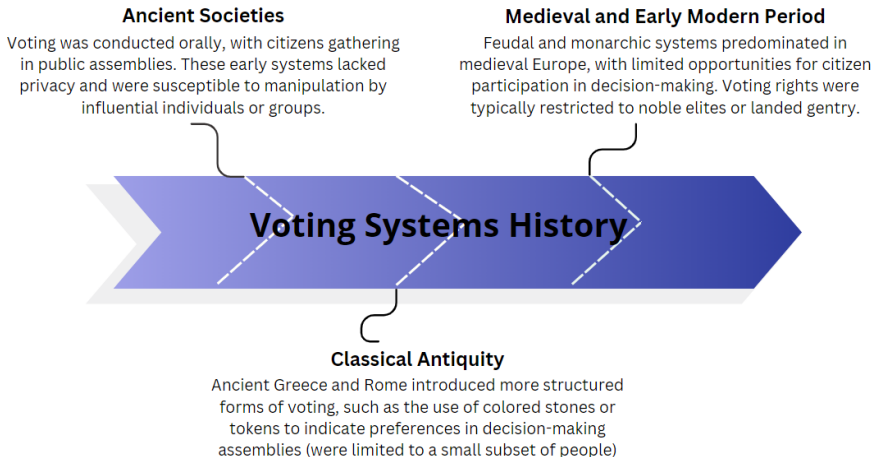
Introduction

Different types of voting systems:

- **Plurality Voting:** voters select a **single candidate**, and the candidate with the **highest number of votes** wins, regardless of whether they have a **majority**. Plurality voting is simple and widely used, but it can result in winners who do not have **broad support** or majority backing.
- **Proportional Representation:** it aims to ensure that the **distribution of seats in a legislative body** reflects the proportion of votes received by each political party or group. PR systems can include methods such as party-list proportional representation and single transferable vote, which prioritize **proportional representation** over **winner-takes-all** outcomes.

- **Instant Runoff Voting:** voters can rank candidates in **order of preference**. If no candidate receives a majority of first-preference votes, the **candidate with the fewest votes is eliminated**, and their votes are redistributed based on the next preference indicated on those ballots. This process continues until one candidate has a majority and is declared the winner. RCV encourages voters to express their genuine preferences and can lead to more **consensus-based outcomes**.
- **Approval Voting:** voters can select **as many candidates as they approve of**. The candidate with the most approvals wins, regardless of whether they have a majority. Approval voting simplifies the voting process and can lead to more inclusive outcomes, but it may not fully capture voter preferences compared to ranked-choice voting.

Introduction



Introduction



Emergence of Representative Democracy

The Enlightenment era brought the idea that elected representatives should act on behalf of the people.

This led to the development of early forms of electoral systems, such as single-member districts

19th and 20th Centuries

This era witnessed significant reforms and innovations in voting systems, including the introduction of secret ballots, universal suffrage, and proportional representation

Voting Systems History



Contemporary Period

In the beginning after the end of World War II, reforms were made such as the adoption of alternative systems like ranked-choice voting, mixed-member proportional representation, and electronic voting.

Introduction

The fundamental principles that voting systems aim to achieve:

- **Fairness**
- **Accuracy**
- **Simplicity**
- **Transparency**
- **Efficiency**

Introduction

Different voting systems prioritize the principles of fairness, accuracy, simplicity, transparency, and efficiency in varying ways, leading to trade-offs in their design:

- **Fairness:** Voting systems strive to ensure fairness by providing **equal representation** and **opportunities** for all voters. IRV systems prioritize fairness by allocating seats in proportion to votes received, thus ensuring that minority viewpoints are represented.
- **Accuracy:** Accuracy refers to the ability of a voting system to reflect **voter preferences** in election outcomes. IRV systems tend to be **more accurate** as they allow voters to express their preferences more fully. In contrast, Plurality Voting may produce **less accurate results**

Introduction

- **Simplicity:** Simple voting procedures are **easier for voters to understand** and administer, enhancing voter confidence and participation. Plurality Voting is often considered the simplest system, as voters **choose only one candidate**, while PR systems and RCV are more complex.
- **Transparency:** Transparent voting systems provide **openness** and **accountability** in the electoral process, contributing to public trust. PR systems and RCV often have transparent processes for counting and tallying votes
- **Efficiency:** Efficient voting systems produce **timely** and **cost-effective** election results. Plurality Voting is often efficient due to its simplicity and ease of implementation. However, PR systems and RCV may require more time and resources for counting and tabulating votes,

Voting Theory

Voting Methods

- **Plurality Voting Method:** The choice with the most first preference votes is declared the winner. It's not necessary for a choice to gain more than 50
- **Approval Voting:** An approval voting strategy involves voters selecting one or more candidates they approve from a list of candidates.
- **Borda Count:** A voting method where points are assigned to candidates based on their ranking: 4 points for First, 3 points for second to last choice, and so on. The candidate with the largest point total is the winner.
- **Instant Runoff Voting (IRV) / Plurality with Elimination:** In this method, voting is done with preference ballots, and a preference schedule is generated. The choice with the least first-place votes is then eliminated from the election, and it continues until a choice has a majority of 50

Plurality Voting



- **A** survey asks to rank which west coast state people would prefer to live and the results are below

Number of votes	75	94	51	12	43	25
1st choice	C	C	O	O	W	W
2nd choice	O	W	W	C	C	O
3rd choice	W	O	C	W	O	C

- **C** = $75 + 94 = 169$ / **O** = $51 + 12 = 63$ / **W** = $43 + 25 = 68$ Votes = 300 $\Rightarrow 169/300 = 56.39$
- **Complexity** : Calculating votes: $O(n)$, where n is the number of voters.
Determining the winner: $O(m)$, where m is the number of candidates.

Approval Voting

- Each column shows the number of people with certain approval vote , approvals are marked with X .

Number of voters	24	21	24	23	22	25
A	X	X				X
B			X	X	X	
C	X	X			X	
D			X		X	X

- $A : 24 + 21 + 25 = 70$ / $B : 24 + 23 + 22 = 69$ / $C : 24 + 21 + 22 = 67$ / $D : 24 + 22 + 25 = 71$
- Complexity:** $O(n)$ where n is the number of voters and $O(m)$, where m is the number of candidates.

Borda Count



- **A**ward points to candidates based on preference schedule, then declare the winner to be the candidate with the most points
- **P**oints : 1st gets 4 points /2nd gets 3 points /3rd gets 2 points /4th gets 1 point

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

For example, Candidate D received

- ▶ 8 first-place votes for $8 \times 4 = 32$ points
- ▶ $4+1 = 5$ second-place votes for $5 \times 3 = 15$ points
- ▶ 10 third-place votes for $10 \times 2 = 20$ points
- ▶ 14 fourth-place votes for $14 \times 1 = 14$ points
- ▶ **Total for Candidate D:** $32 + 15 + 20 + 14 = 81$ points

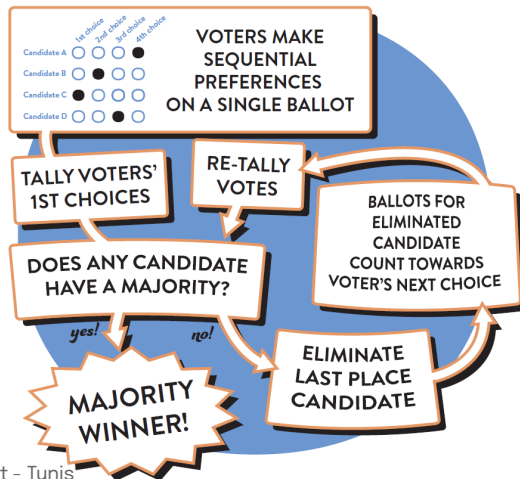
Borda Count



Number of Voters	14	10	8	4	1
1st choice (4 points)	A: 56	C: 40	D: 32	B: 16	C: 4
2nd choice (3 points)	B: 42	B: 30	C: 24	D: 12	D: 3
3rd choice (2 points)	C: 28	D: 20	B: 16	C: 8	B: 2
4th choice (1 point)	D: 14	A: 10	A: 8	A: 4	A: 1

- ▶ Candidate A: $56+10+8+4+1 = 79$ points
 - ▶ Candidate B: $42+30+16+16+2 = 106$ points **Winner!**
 - ▶ Candidate C: $28+40+24+8+4 = 104$ points
 - ▶ Candidate D: $14+20+32+12+3 = 81$ points
- **Complexity:** $O(n.m)$ when summing up the rankings for each candidate across all voters

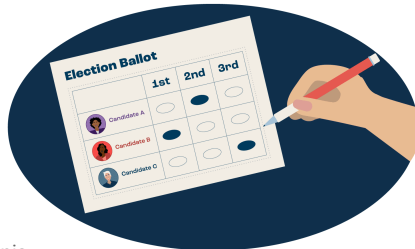
Instant Runoff Voting



Instant Runoff Voting

Instant runoff voting is also called plurality with elimination.

In this method, voting is done with preference ballots and a preference schedule is generated. The choice with the least first-place votes is then **eliminated** from the election, and it continues until a choice has a majority of 50 .



Steps of Instant Runoff Voting



Process of Elimination

- If neither candidate receives a majority of first rankings, then the candidate with the fewest first rankings is eliminated.
- In case of a tie, a random draw decides which of the candidates with the fewest first rankings is eliminated.

Steps of Instant Runoff Voting



Updating Ballots and Declaring a Winner

- Every ballot is then updated, with the candidate ranked next on the ballot taking the place of the eliminated candidate.
- If one of the remaining candidates receives a majority of first rankings on the updated ballots, then this candidate is declared the winner and the process stops.

Steps of Instant Runoff Voting



Iteration Process

- Otherwise, the process continues with the elimination of the remaining candidate who receives the fewest first rankings on the updated ballots.
- This process is iterated until one candidate receives a majority of the first rankings.

EXAMPLE



Consider the preference schedule and determine the winner using IRV Method :

Data					
Voters	1st choice	2nd choice	3rd choice	4th choice	5th choice
9	D	C	A	E	B
5	B	E	A	C	D
2	E	D	B	A	C
5	B	C	D	A	E
8	C	A	D	B	E
6	B	D	C	E	E

Step 1

There are a total of $9+5+2+5+8+6 = 35$ votes, we start by eliminating A since A = 0 first choice and we have : $B = 5 + 5 + 6 = 16$

$C = 8$; $D = 9$; $E = 2$

Voters	1st choice	2nd choice	3rd choice	4th choice	5th choice
9	D	C	A	E	B
5	B	E	A	C	D
2	E	D	B	A	C
5	B	C	D	A	E
8	C	A	D	B	E
6	B	D	C	E	E

Step 2

Eliminate E with 2nd least first ranking $E = 2$ and we have : $B = 5 + 5 + 6 = 16$

$C = 8$; $D = 9 + 2$; $E = 0$;

Voters	1st choice	2nd choice	3rd choice	4th choice	5th choice
9	D	C		E	B
5	B	E		C	D
2	E	D	B		C
5	B	C	D		E
8	C		D	B	E
6	B	D	C	E	E

Step 3

Eliminate C with 3rd least first ranking C = 8 and we have : $B = 5 + 5 + 6 = 16$

$C = 0$ $D = 9 + 2 = 11$ $E = 0$

Number of Voters	1st Choice	2nd Choice	3rd Choice	4th Choice	5th Choice
9	D	C			B
5	B			C	D
2		D	B		C
5	B	C	D		
8	C		D	B	
6	B	D	C		
Total					

Block

D WINS WITH $19/35 = 54.2\%$ (MAJORITY)

Complexity

- Tabulating votes: $O(n * m)$, where n is the number of voters and m is the number of candidates.
- Elimination of candidates $O(m^2)$ in the worst case.
- Redistribution of votes: $O(n * m)$.
- Complexity: It is usually dominated by tabulation but can vary depending on the number of rounds needed to determine a winner.

Conclusion

Conclusion



- To sum up, the study of voting theory includes a **variety of techniques** and Methods that are meant to produce **equitable** and **representative results** in elections and decision-making procedures
- We looked at a number of popular voting Methods, each with a **different approach** and computational complexity
- The election's context, the intended result, and factors like fairness, simplicity, and computational viability all **influence** the voting method selection, voting theory research and development must continue if robust methods supporting democracy, justice, and representation in decision-making are to be developed.

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Thank you for your attention

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