(Please write your Enrollment Number)

Enrollment No. 0 32011 92023

End-Term Examination

(CBCS)(SUBJECTIVE TYPE)(OffLine)

Course Name: B.Tech , Semester: 3rd Sem.

(December, 2024)

Subject Code: BCS 203	Subject: Discrete Structures
Time :3 Hours	Maximum Marks : 60

Note: Q1 is compulsory. Attempt one question each from the Units I, II, III & IV.

Q1		(5*4=20)
	a) Identify the nature of the proposition S, whether it is Tautology/ Contingency/ Contradiction. $S: ((P \wedge Q) \to R) \to ((P \wedge Q) \to (Q \to R))$	
	b) Draw the Hasse Diagram of the following: D_{105}, and D_{72}	
	c) Prove that group G is an Abelian group if and only if $(ab)^{-1} = a^{-1}b^{-1}$, $\forall\, a, b \in G$.	
	d) The chromatic number of the following graph is ________	

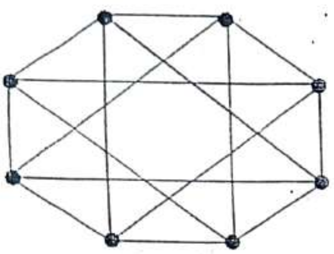


Fig. (a)

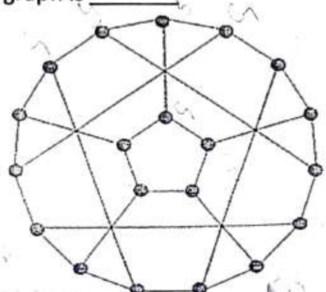


Fig. (b)

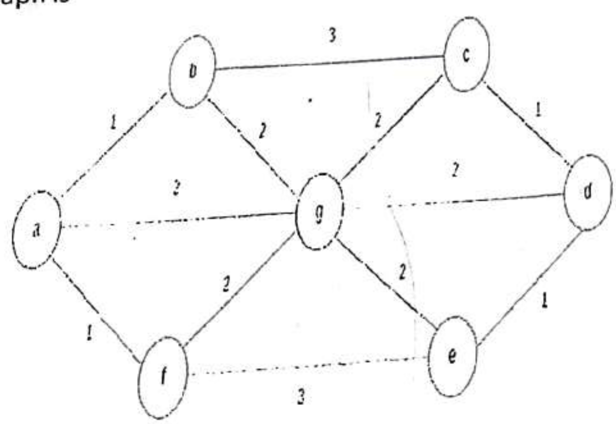
UNIT I

Q2	a) Let p, q, r, s represent the following propositions. p: x∈{8,9,10,11,12} q: x is a composite number r: x is a perfect square s: x is a prime number The integer x≥2 which satisfies: $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is ________. b) The binary operator ≠ is defined by the following truth table.	(5+5)

p	q	p ≠ q
0	0	0
0	1	1
1	0	1
1	1	0

Identify the nature of the binary operator ≠, whether it is associative, commutative, or both?

Q3	a) Translate the following into propositional logic: i) not all rainy days are cold ii) None of my friends are perfect. Note: Where the variables are: rainy(x), cold(x), f(x): friend, p(x): perfect.	(5+5)

	b) Identify the following Boolean expressions which is/are NOT tautology? A. $((a \to b) \wedge (b \to c)) \to (a \to c)$ B. $(a \leftrightarrow c) \to (\sim b \to (a \wedge c))$ C. $(a \wedge b \wedge c) \to (c \vee a)$ D. $a \to (b \to a)$	
	UNIT II	
Q4	**a)** Using the principle of mathematical induction, Show that 2^{2n}- 1 is divisible by 3. **b)** Find the total number of relation on a set R with n elements which is antisymmetric but not reflexive.	(5+5)
Q5	**a)** Prove that the relation congruence modulo m on the set Z of all integers is an equivalence relation. **b)** Explain the following Sets with example: a) Finite b) Infinite, c) Countable d) Uncountable.	()
	UNIT III	(5+5)
Q6	**a)** Define a Field. Prove that the set of integers Z_{11} with addition and multiplication is a Field. **b)** State and prove Lagrange's theorem for finite groups.	
Q7	a) Prove that a group of prime order p is cyclic. b) Let G be the set of all positive rational numbers and * be the binary operation on G defined as $a * b = \frac{ab}{7}, \forall a, b \in G$. Prove that $(G,*)$ be an abelian group.	(5+5)
	UNIT IV	(5+5)
Q8	a) State and prove Euler's formula for connected planar graphs: V–E+R= 2, where V, E, and F represent vertices, edges, and region, respectively. **b)** The number of distinct minimum-weight spanning trees in the following graph is 	
Q9	a) G = (V,E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE? I. If e is the lightest edge of some cycle in G, then every MST of G includes e. II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e. b) Prove that: Any planar graph can be colored using at most five colors such that no two adjacent vertices share the same color.	(5+5)