

MID-SEMESTER EXAMINATION, September 2023

Max Marks: 15

Duration: 1:30 Hours

Course Title: Design and Analysis of Algorithms

Course Code: COCSC06/ CACSC06/CDCSC06

CMCSC06

Note: Attempt all questions in given order only. Missing data/information (if any) may be suitably assumed & mentioned in the answer.

Q. No.	Question	Marks	CO
1a.	Let A be a unsorted array of n integers. Devise the most efficient algorithm which determine whether two elements whose sum is less than 10^6 exists in A or not. Moreover, compute asymptotic tight bound of time complexity for the devised algorithm.	1.5	CO5
1b.	<p>The following diagram shows the flowchart for a recursive function f(n). Assume that all statements, except for the recursive calls, have O(1) time complexity. If the <u>worst case</u> time complexity of this function is O(n^a), then compute the least possible value (accurate up to two decimal positions) of a.</p>	1.5	CO1
2a.	1. Consider the following c function: double foo (int n)	1.5	CO1

```

{
    int i;
    double sum;
    if (n == 0) return 1.0;
    else
    {
        sum = 0.0;
        for (i = 0; i <= n; i++)
            sum += foo(i);
        return sum;
    }
}

```

Compute the time complexity and space complexity of above function.

2b.	Suppose we modify the above function <i>foo()</i> and store the values of <i>foo(i)</i> , $0 \leq i < n$, as and when they are computed. After this modification in above program, what will the time and space complexity <i>foo()</i> ? Is there be any change in time and space complexity of this case concerning Q.2 (a).	1.5	CO1
3a.	Given the sequence of numbers 36, 96, 101, 29, 32, 55, 117, and 205, demonstrate how you would insert them into an empty red-black tree while maintaining the tree's properties.	1	CO3
3b.	Provide a pseudocode representation illustrating the heapify procedure for creating a max-heap from an array. Describe the essential steps involved in the procedure and discuss its time complexity.	2	CO3
4a.	How does the merge sort algorithm use the divide-and-conquer approach? Provide a brief pseudocode explanation of how it divides, conquers, and combines to achieve sorting. Discuss the time complexity of merge sort and highlight its <u>stability and efficiency</u> compared to other sorting algorithms -	2	CO4
4b.	How do you construct a B tree of order 3 from the following data 7, 2, 9, 11, 15, 5, and 37?	1	CO3
5a.	You are given the following list of integers to sort using Counting Sort and Radix Sort. Compare the time complexity of these two algorithms for this specific given data. Determine the number of steps/iteration for both the algorithms. Data: [235, 468, 129, 782, 932, 123, 634, 346, 349, 568]	2	CO5
5b.	Consider the following data set: [42, 19, 27, 14, 35, 10, 5, 30, 22, 15, 9, 12, 2, 45, 7]. Perform <u>Shell Sort</u> on this data set using the shell increments (gaps) of (7, 4, 2, 1). i). Perform the first step of the Shell Sort with a gap of 7 and show the intermediate result after this step. ii). Perform the second step of the Shell Sort with a gap of 4 and show the intermediate result after this step. iii). Calculate the exact number of comparisons made during above steps.	1	CO3

Degree: B. Tech Semester: 3rd - Course work
END-SEMESTER EXAMINATION, Nov-Dec 2023
 Course Title: Design and Analysis of Algorithms
 Course Code: COCSC06/ CACSC06/ CDCSC06
 CMCSC06

Time: 03 Hours

Max. Marks: 40

Note: - Attempt all the five questions. Missing data/ information (if any), maybe suitably assumed & mentioned in the answer.

	Question	Marks	CO																														
Q 1	Attempt any 2 parts of the following																																
1a	Solve the recurrence relation with the help of master method, also show the process/reason: a. $T(n) = 3T(n/3) + \sqrt{n}$ b. $T(n) = 16T(n/4) + n!$ c. $T(n) = 16T(n/4) + n$	4	CO1																														
1b	Solve the recurrence relation using Recursion Tree, also present the number of leaf nodes, depth of tree, cost at each level, and final cost: $T(n) = T(n/5) + T(4n/5) + n$	4	CO1																														
1c	Find the minimum number of comparisons needed to locate both the minimum and maximum values among 100 numbers, and elucidate the process involved.	4	CO5																														
Q 2	Attempt any 2 parts of the following																																
2a	Create an AVL-tree for the following list of elements: 80, 40, 60, 20, 10, 30, 70, 50, 90, 100, 55, 35. Identify the number of rotations required for making it balanced tree.	4	CO3																														
2b	A. Derive the recurrence relations of the Best, Worst, and Average-case time complexities of the Quicksort algorithm and solve them to derive the asymptotic bounds in all cases. B. Give proof of correctness of insertion sort that it sorts its argument.	4	CO2																														
2c	What is the difference between linear sorting and comparison-based sorting? Explain the different steps of the counting sort algorithm with an example. Discuss its time complexity.	4	CO3																														
Q 3	Attempt any 2 parts of the following																																
3a	A weighted graph indicating (non-negative) distances of roads between adjacent towns is given. A town planner wishes to identify the shortest set of roads to connect the towns. Devise an efficient algorithm, along with its complexity, that will enable the town planner to determine this.	4	CO4																														
3b	You are given an array $A[1..n]$ representing a min-heap and an integer k . You have to design an algorithm to output all the keys in the array that are less than k . (For example, if $k = 6$ and the array A is as shown below, then your algorithm should output the keys 3 and 5.)	4	CO5																														
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Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14																			
Key	3	10	5	13	17	6	11	15	16	21	18	9	8	23																			

Give pseudocode for your algorithm and discuss running time. Let m be the number of keys in A that are smaller than k . The running time of your algorithm should be $O(m)$.

3c ✓ Solve the following instance of the 0-1 knapsack problem given the knapsack capacity in $W = 5$ using dynamic programming and explain it.

Item	Weight	Cost
1	4	10
<u>2</u>	3	20
<u>3</u>	2	15
4	5	25

} 25 ✓

Q 4 Attempt any 2 parts of the following

4a ✓ Assess the suitability of randomized algorithms for the following scenarios and provide explanations for each case:

- a) Merge Sort
- b) Minimum Spanning Tree
- c) Aircraft Control System
- d) Knapsack Problem

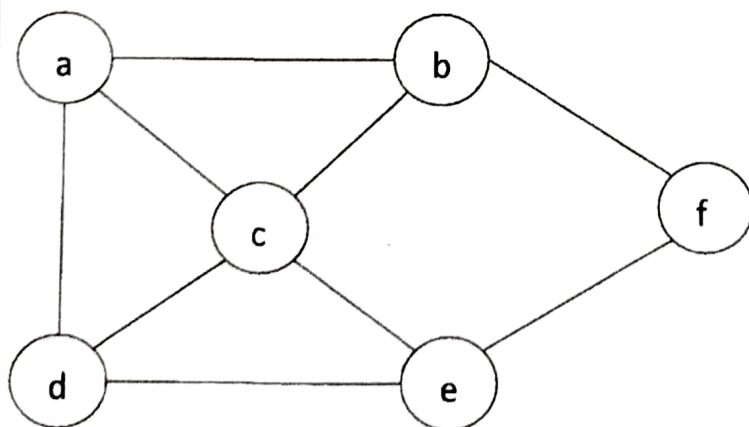
4b Design a randomized algorithm to efficiently find the k th smallest element in an array containing distinct elements. Ensure that the algorithm achieves linear average case time complexity. Explain the algorithm's steps and analyse its time complexity.

4c ✓ Elucidate the concept of randomized algorithms and their utility, using the Quick Sort algorithm as an example. Additionally, perform an analysis of the randomized Quick Sort algorithm, considering both its average and worst-case scenarios.

Q 5 Attempt any 2 parts of the following

5a ✓ Explain P, NP, NP-hard, and NP-complete class problems, give an example of each class of problems.

5b Find the Hamiltonian cycle by using the backtracking approach for the following graph.



5c ✓ Explain 8-queens problem and apply backtracking to solve this problem.