(Please write your Exam Roll No.) Exam Roll No. 25210405617

END TERM EXAMINATION

FIRST SEMESTER [B.TECH] DECEMBER 2017

Paper Code: ETMA-101 **Subject: Applied Mathematics-I**

(Batch 2013 Onwards)

Time: 3 Hours **Maximum Marks: 75**

Note: Attempt any five questions including Q no.1 which is compulsory. Select one question from each unit.

Q1 (a) Assuming the possibility of expansion prove that (2.5)

$$\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{\sqrt{2}}\left[1+\theta-\frac{\theta^2}{2!}-\frac{\theta^3}{3!}+......\right]$$

(b) Test for the convergence of the series (2.5)

$$\sum_{1}^{\alpha}\frac{n^2+2^n}{2^n.n^2}$$

(c) Find the asymptotes of the curve (2.5)

$$y^3-2xy^2-x^2y+2x^3+2x^2-3xy+x-2y+1=0$$

(d) Show that $\int_0^1 (x\log x)^3\,dx=\frac{-3}{128}$. (2.5)

(e) Evaluate $\int_0^1 \left(1-x^{1/n}\right)^{m-1}dx$ (2.5)

(f) Show that the matrix $A=\frac{1}{2}\begin{bmatrix}1+i & -1+i\\ 1+i & 1-i\end{bmatrix}$ is a unitary matrix (2.5)

(g) Test whether the vectors (1, 1, 1, 3), (1, 2, 3, 4) and (2, 3, 4,7) are linearly dependent or not. If dependent, find the relations between them. (2.5)

(h) Solve $(x^4e^x-2mxy^2)dx+2mx^2y\,dy=0$ (2.5)

(i) Show that $\frac{d}{dx}\{J_n^2(x)\}=\frac{x}{2n}\{J_{n-1}^2(x)-J_{n+1}^2(x)\}$. (2.5)

(j) Find the rank of the matrix $\begin{bmatrix}3 & -1 & 2\\ -6 & 2 & 4\\ -3 & 1 & 2\end{bmatrix}$ by reducing it in its normal form. (2.5)

UNIT-I

Q2 (a) If $y=x^x\log x$ prove that (6)

(i) $y_{n+1}=\frac{n!}{x}$ (ii) $y_n=ny_{n-1}+(n-1)!$

(b) Test the convergence of the series $\sum_{1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)$ (6.5)

Q3 (a) Use Maclaurin's theorem to show that (6)

$$\sqrt{1+x+2x^2}=1+\frac{x}{2}+\frac{7}{8}x^2-\frac{7}{16}x^3+.....$$

(b) Test the following series for convergence and absolute convergence. (6.5)

$$1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\frac{9}{16}........$$

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UNIT-II

Q4 (a) If $I_n = \int_0^{\frac{\pi}{4}} \sin^{2n} x\,dx$, show that (6)

$$I_n = \left(1-\frac{1}{2n}\right) I_{n-1} - \frac{1}{n.2^{n+1}}$$

(b) Find the radius of curvature at any point on the curve $x = a\cos^3 t,\ y = a\sin^3 t$. (6.5)

Q5 (a) Trace the curve $a^2x^2 = y^3(2a-y)$. (6)

(b) Prove that the length of an arc of the curve $y^2 = x\left(1-\frac{x}{3}\right)^2$ from the origin to the point (x, y) is given by $l^2 = y^2 + \frac{4}{3}x^2$. (6.5)

UNIT-III

Q6 (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing it to echelon form. (6)

(b) Find the modal matrix of $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ and diagonalize it. (6.5)

Q7 (a) Use Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & -1 \end{bmatrix}$ to find A^{-1}. (6)

(b) Investigate whether the set of equations (6.5)

$$2x - y - z = 2$$
$$x + 2y + z = 2$$
$$4x - 7y - 5z = 2$$

is consistent or not; if consistent, solve it.

UNIT-IV

Q8 (a) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x\sin x$. (6)

(b) Solve $\frac{d^2y}{dx^2} + y = -\cot x$ by the method of variation of parameters. (6.5)

Q9 (a) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = \frac{x^3}{1+x^2}$ (6)

(b) Prove that $\int_{-1}^{1} (x^2-1)P_{n+1} P'_n\,dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$ (6.5)

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