Roll No. [| | | | | | | | | | |] **Total No. of Pages : 03**

Total No. of Questions : 09

B.Tech. (CE/ME/ECE/EE/A&R/CSE/EEE/IT) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-101-18

M.Code : 75353

Date of Examination : 04-12-2023

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Answer Briefly :

a) Test the convergence of the following series $\sum \frac{n!}{n^n}$.

b) State the Raabe's test.

c) State Rolle's theorem.

d) State Langrange's mean value theorem.

e) Prove that $\int_0^{\frac{\pi}{2}} \sin 2x \log \tan x \, dx = 0$

f) Evaluate $\int_0^3 \int_0^1 (x^2 + 3y^2)\, dy\, dx.$

g) Change the order of integration of $\int_0^1 \int_x^{\sqrt{x}} f(x, y) dydx.$

h) Find the first order derivative of $z = x^2 + y^2 + 2x + 3axy.$

i) Find the rank of the following matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

j) Find the determinant of the following matrix $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 7 & 1 \\ 8 & 2 & 3 \end{bmatrix}$

SECTION - B

2. If $z(x + y) = x^2 + y^2$. Show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial x}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial x}\right)$

3. Evaluate $\iint e^{ax+by}\, dxdy$ over the triangle bounded by $x = 0$, $y = 0$, $ax + by = 1$.

4. Test the convergence of the series $\sum \frac{(n!)^2}{(2n)!} x^{2n}$.

5. Verify if the matrix A = $\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is orthogonal and hence find its inverse.

SECTION - C

6. Find the maximum and minimum value of $xy + \frac{a^3}{x} + \frac{a^3}{y}$.

7. a) Solve the simultaneous equations $3x + 2y + 4z = 7$, $2x + y + z = 4$, $x + 3y + 5z = 2$.

   b) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 1 \\ 3 & -2 & 1 \end{bmatrix}$.

8. a) Find the area of the surface of revolution of the solid generated by revolving the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$ about the x-axis.

b) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyzdxdydz$

9. a) Test the convergence of the series $\frac{1^2}{4^2}+\frac{5^2}{8^2}+\frac{9^2}{12^2}+\frac{13^2}{16^2}+$..................

b) Using Maclaurin's series, expand tan z upto the term containing x^5.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.