First SEMESTER–B.TECH- Course Work
MID-SEMESTER EXAMINATION, January 2023

Course Code-FCMT001
Course Title-Mathematics I
Time: 1:30 hours Max Marks-25

Note: Attempt all questions. Missing data/information (if any), may be suitably assumed & mentioned in the answer.

Q. No.	Question	Marks	CO
1 (a)	Test the convergence of following series: $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots, x>0.$	2.5	CO1
(b)	Test the convergence of the series: $\sum\left(\frac{n+1}{n+2}\right)^n.x^n, \quad x>0.$	2.5	CO1
2 (a)	Test the convergence of following series: $\sum[\sqrt{n^4+1}-\sqrt{n^4-1}].$	2.5	CO1
(b)	Test the following series for convergence and absolute convergence: $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ldots.$	2.5	CO1
3 (a)	If $\tan(\theta+i\phi)=\tan\alpha+i\sec\alpha$, then prove that $2\theta=n\pi+\frac{\pi}{2}+\alpha.$	2.5	CO2
(b)	Prove that $\sinh^{-1}(\cot x)=\log(\cot x+\csc x).$	2.5	CO2
4 (a)	Prove that the value of n-th derivative of $\frac{x^3}{x^2-1}$, for $x=0$ is zero, when n is even and $(-n!)$, when n is odd and greater than one.	2.5	CO2
(b)	If $y=a\cos(\log x)+b\sin(\log x)$, show that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0.$	2.5	CO2
5 (a)	Expand $2x^3+7x^2+x-1$ in powers of $x-2$, using Taylor's series.	2.5	CO2
(b)	Expand $\log(1+e^x)$ upto third degree terms using Maclaurin's series.	2.5	CO2

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* csc x = Cosec x.

FIRST SEMESTER- B.TECH-Course Work
END SEMESTER EXAMINATION, MARCH, 2023

Course Code-FCMT001
Course Title- Mathematics-I

Time- 3:00 hours

Max. Marks- 50

Note:- Attempt all the five questions. Missing data/information (if any), may be suitably assumed & mentioned in the answer.

Q. No.	Questions	Marks	CO
Q1	**Attempt any two parts of the following**		
1a	Test the convergence of the following series: $\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+\ldots\ldots$	5	CO1
1b	Test the convergence of the following series: $\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+\frac{3.6.9.12}{7.10.13.16}x^4\ldots\ldots$	5	CO1
1c	Show that the series $\sum(-1)^n[\sqrt{n^2+1}-n]$ is conditionally convergent.	5	CO1
Q2	**Attempt any two parts of the following**		
2a	If $y=e^{\sin^{-1}x}$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+1)y_n=0.$	5	CO2
2b	If $y=cos\,(m\sin^{-1}x)$, using Leibnitz theorem, find $(y_n)_0$.	5	CO2
2c	Using Maclaurin's theorem, expand $\log(x+\sqrt{1+x^2})$ in ascending powers of x as far as the term x^5.	5	CO2
Q3	**Attempt any two parts of the following**		
3a	Show that the pedal equation of the parabola $\frac{2a}{r}=1-cos\theta$ is $p^2=ar$. Hence find the radius of curvature of the given parabola.	5	CO3
3b	Find the angle of intersection of the curves $r=a/(1+cos\theta)$ and $r=b/(1-cos\theta)$.	5	CO3
3c	Find the radius of curvature at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ of the curve $x^3+y^3=3axy.$	5	CO3
Q4	**Attempt any two parts of the following**		
4a	Find all the asymptotes of the curve $y^3-xy^2-x^2y+x^3+x^2-y^2-1=0.$	5	CO4
4b	Find the area of the cardioid $r=a(1+cos\theta)$.	5	CO4
4c	The curves $y^2(a+x)=x^2(3a-x)$ revolves about the axis of x. Find the volume generated by the loop.	5	CO4

Q5	**Attempt any two parts of the following**		
5a	For what values of μ, the system of equations $x + y + z = 1,$ $x + 2y + 4z = \mu,$ $x + 4y + 10z = \mu^2$ have a solution and solve them completely in each case.	5	CO5
5b	Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.	5	CO5
5c	Show that the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies Cayley-Hamilton theorem. Hence find A^{-1}.	5	CO5