(Please write your Exam Roll No.) **Exam Roll No. ....**

END TERM EXAMINATION

FIRST SEMESTER [BCA] NOVEMBER-DECEMBER 2017

Paper Code: BCA-101	***Subject: Mathematics-I***
Time: 3 Hours	***Maximum Marks: 75***

Note: Attempt any five questions including Q.no.1 which is compulsory. Select one question from each unit.

Q1 (a) Solve the following system of equations by Cramer's rule:
$2y - 3z = 0, \quad x + 3y = -4, \quad 3x + 4y = 3.$ **(5)**

(b) Solve: $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$ **(5)**

(c) Find the maximum and minimum values of $f(x) = x + \sin 2x$ in $[0, 2\pi]$. **(5)**

(d) Evaluate $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx.$ **(5)**

(e) Show that $\lim_{x \to 0} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$ does not exist. **(5)**

Unit-I

Q2 (a) Find eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}$. **(6.5)**

(b) Find whether or not the following set of vectors are linearly dependent or independent.
$X_1 = (1,1,0), \quad X_2 = (1,0,1), \quad X_3 = (0,1,1).$ **(6)**

Q3 (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}. **(6.5)**

(b) Find the rank of matrix $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 3 & 4 & 7 & 10 \\ 5 & 7 & 11 & 17 \\ 6 & 8 & 13 & 16 \end{bmatrix}$. **(6)**

Unit-II

Q4 (a) Discuss the continuity of the function $f(x) = \begin{cases} -x, & x \le 0 \\ x, & 0 < x \le 1 \\ 2 - x & 1 < x \le 2 \\ 1 & x > 2 \end{cases}$

at each point $x = 0, 1, 2$. **(6.5)**

(b) $\lim_{x \to 0} \left(\frac{\cos mx - \cos nx}{x^2} \right)$. **(6)**

Q5 (a) Let $f(x) = \begin{cases} 1, & x \le 3 \\ ax + b, & 3 < x < 5 \\ 7, & 5 \le x \end{cases}$. Find the values of a and b so that $f(x)$ is continuous. **(6.5)**

(b) Evaluate: (i) $\lim_{x\to\infty} \frac{x}{\sqrt{4x^2+1}}$ (ii) $\lim_{x\to 0} \frac{x^3 \cot x}{1-\cos x}$. **(6)**

Unit-III

Q6 (a) Verify the hypothesis and conclusion of Lagrange's mean value theorem for the function $f(x) = \frac{1}{4x-1}, 1, \le x \le 4$. **(6.5)**

(b) Expand $\log \sin x$ in powers of (x-2) by Taylor's series. **(6)**

Q7 (a) If $y = \left[x + \sqrt{1+x^2}\right]^m$, show that $(1+x^2)\, y_{n+2} + (2n+1)\, x y_{n+1} + (n^2 - m^2)\, y_n = 0$. Also find $y_n(0)$. **(6.5)**

(b) Find asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + x^2 - y^2 - 2x - 3y = 0$. **(6)**

Unit-IV

Q8 (a) Evaluate: (i) $\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}}\, dx$ (ii) $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$. **(6)**

(b) Show that $\beta(p,q) = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}}\, dx$. **(6.5)**

Q9 (a) If $I_{m,n} = \int \cos^m x \sin nx \, dx$, prove that $(m+n)\, I_{m,n} = -\cos^m x \cos nx + m I_{m-1,n-1}$. Hence evaluate $\int_0^{\pi/2} \cos^5 x \sin 3x \, dx$. **(7.5)**

(b) Evaluate $\int_0^1 x^{3/2} (1-x)^{3/2}\, dx$. **(5)**

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