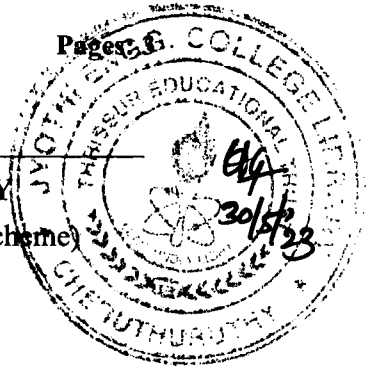


Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S4 (S,FE) / S2 (PT) (S,FE) Examination May 2023 (2015 Scheme)



Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL
METHODS

Max. Marks: 100

Duration: 3 Hours

*Normal distribution table is allowed in the examination hall.***PART A (MODULES I AND II)***Answer two full questions.*

- 1 a) The probability mass function of a random variable X is given below: (7)

X :	-2	-1	0	1	2	3
$f(x)$:	0.1	k	0.2	$2k$	0.3	k

 Find (i) value of k (ii) $P(X \geq -1)$ (iii) $E(X)$ (iv) $P(0 < X \leq 3)$
 (v) Distribution function of X .
- b) The probability that a batsman scores a century in a cricket match is $\frac{1}{3}$. Find the (8)

probability that out of 5 matches, he may score century in

(i) at least 2 matches (ii) at most 2 matches (iii) no match.
- 2 a) A random variable follows Poisson distribution such that $P(X = 0) = \frac{2}{3}P(X = 1)$. (7)

Find (i) $P(X = 3)$ (ii) $P(X > 3)$.
- b) Find the value of k and hence find the mean and variance of the distribution (8)

$$f(x) = \begin{cases} kx^3, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$
- 3 a) Assume that the time between arrivals of customers at a particular bank is (7)

exponentially distributed with a mean of 4 minutes

(i) Find the probability that the time between arrivals is greater than 5 minutes.

(ii) Find the probability that the time between arrivals is between 1 and 4 minutes.
- b) The marks obtained by a batch of students in a certain subject are normally (8)

distributed. 10% of students got less than 45 marks while 5% got more than 75. Find the mean and standard deviation of the distribution.

PART B (MODULES III AND IV)

Answer two full questions.

- 4 a) Find the Fourier cosine transform of (7)

$$f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}. \text{ Hence prove that } f(x) = \frac{4}{\pi} \int_0^\infty \left(\frac{\sin \omega - \omega \cos \omega}{\omega^3} \right) \cos \omega x \, d\omega.$$

- b) Find the Fourier integral representation of the function $f(x) = \begin{cases} 2, & |x| < 2 \\ 0, & |x| > 2 \end{cases}$ and (8)

$$\text{hence evaluate } \int_0^\infty \left(\frac{\sin 2\omega}{\omega} \right) \cos \omega x \, d\omega.$$

- 5 a) Using Fourier sine integral for $f(x) = e^{-kx}$, $x > 0, k > 0$, (7)

$$\text{show that } \int_0^\infty \left(\frac{\omega \sin \omega x}{k^2 + \omega^2} \right) d\omega = \frac{\pi}{2} e^{-kx}.$$

- b) Find the Laplace transform of (8)

$$(i) 3e^{5t} + (t+2)^2 + 2\cos 3t \quad (ii) 4te^{-2t}$$

- 6 a) Using convolution theorem find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$. (7)

- b) Solve the initial value problem, using Laplace transforms (8)

$$y'' - 2y' + 2y = 0 \text{ where } y(0) = y'(0) = 1.$$

PART C (MODULES V AND VI)

Answer two full questions.

- 7 a) Find the value of $\sqrt[3]{24}$ using Newton Raphson Method. (6)

- b) Using Lagrange's formula for interpolation find the value of y when $x = 2$ from the (7)

following table

$$x : -2 \quad -1 \quad 0 \quad 4$$

$$y : -2 \quad 4 \quad 1 \quad 8$$

- c) Using Newton's Interpolation formula find the value of $\tan(0.26)$ from the (7)

following data

x	0.10	0.15	0.20	0.25	0.30
$\tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

- 8 a) Using Euler's method to find $y(0.2)$ and $y(0.4)$, given $y' = x + y$, $y(0) = 1$, $h = 0.2$ (6)

- b) Solve the following by Gauss-Seidel Method (7)

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$2x + 2y + 10z = 14$$

- c) Evaluate $\int_0^2 e^{-x} dx$ by using Simpson's rule with 4 subintervals and compare it with the exact solution. (7)
- 9 a) Find the unique polynomial $p_3(x)$ of degree 3 or less, the graph of which passes through the points $(-1,3)$, $(0,-4)$, $(1,5)$ and $(2,-6)$. (6)
Solve the equations using Gauss elimination method
- b) $x+2y+z = 3$ (7)
 $2x+3y+2z = 5$
 $3x-5y+5z = 2$
 $3x+9y-z = 4$
- c) Obtain the value of y at $x = 0.2$ using Runge- Kutta method of fourth order for the differential equation $\frac{dy}{dx} = 1 + y^2$ with $h = 0.2$, $y(0) = 0$. (7)
