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209508

Dec., 2018
B.Tech. (ME) Vth Semester
MATHS–III
(GA-502C)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Define Dirichlet's Conditions. (1.5)

(b) State and Prove Change of scale property of Fourier Transform. (1.5)

(c) If $\sin(A + iB) = x + iy$, prove that

$$x^2 \operatorname{cosec}^2 A - y^2 \sec^2 A = 1. \qquad (1.5)$$

(d) Show that the function $e^x(\cos y + i \sin y)$ is an analytic function. Also find its derivative. (1.5)

(e) Evaluate $\int_c (12z^2 - 4iz)dz$ along the curve C joining the points (1, 1) and (2, 3). (1.5)

(f) A card is drawn from an ordinary pack and a gambler bets that it is spade or an ace. What are the odds against his winning the bet. (1.5)

(g) If A and B are two events such that $P(A) = 1/4$, $P(B) = 1/3$ and $P(A \cup B) = 1/2$. Show that A and B are independent events. (1.5)

(h) Six coins are tossed 6400 times. Using the Poisson Distribution determine the approximate probability of getting six heads x times. (1.5)

(i) Find a half-range cosine series for the function $f(x) = x^2$ in the range $0 \le x \le \pi$. (1.5)

(j) Find the Fourier Sine transform of $f(x) = e^{-ax}$, $a > 0$. (1.5)

PART-B

2. (a) An Alternating current after passing through rectifier has the form

$$i = \begin{cases} I_0 \sin x & \text{for } 0 \le x \le \pi \\ 0 & \text{for } \pi \le x \le 2\pi \end{cases}$$

where I_0 is the maximum current and the period is 2π. Express i as a fourier series. (8)

(b) Find the fourier series expansion of $f(x) = 1 + |x|$ defined in $-3 < x < 3$. (7)

3. (a) Find the Fourier cosine Transform of $f(x) = \frac{1}{1+x^2}$. Hence derive fourier sine transform of $\phi(x) = \frac{x}{1+x^2}$. (8)

(b) Using finite fourier transform, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the conditions :

(i) $u_x(0, t) = u_x(6, t) = 0$, for $0 < x < 6$, $t > 0$.

(ii) $u(x, 0) = x(6 - x)$, for $0 < x < 6$. (7)

4. (a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z by Milne Thomson Method. (8)

(b) Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2+y^2}$ are harmonic functions of (x, y), but are not harmonic conjugates. (7)

5. (a) Evaluate $\int_c \frac{z^2-2z}{(z+1)^2(z^2+4)}\,dz$ where c is the circle $|z| = 10$. (8)

(b) Expand $f(z)$ as Taylor's or Laurent's series expansion.

$f(z) = \frac{z^2-1}{(z+2)(z+3)}$ when (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$ (7)

6. (a) Two-Thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting a first class is 0.25 and that of a boy getting a first class is 0.28. A student is selected at random and is found to get the first class. What is the probability that the student is a boy ? (8)

(b) A die is thrown three times. Events A and B are defined as below :

A : 4 appears on third throw

B : 6 and 5 appears respectively on first two throws. Find the probability of A given that B has already occurred. (7)

7. (a) Let X be a random variable defined by the density function $f(x) = \begin{Bmatrix} 3x^2, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{Bmatrix}$. Find E(X), E(3X − 2), E(X^2). (8)

(b) Fit a Binomial Distribution to the following frequency distribution :

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

(7)