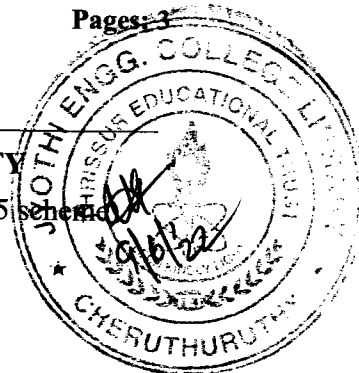A

Reg No.:_____   Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree (S,FE) Examination June 2022 (2015 scheme)

Course Code: MA202
Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

Normal distribution table is allowed in the examination hall.
PART A (MODULES I AND II)
Answer two full questions.

1 a) A random variable X has the following probability function

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) Find k, (ii) Find the minimum value of k such that $P(X \le k) > \frac{1}{2}$ (iii)   8mark

Find the distribution function of X

b) The mean and variance of a binomial variate are 8 and 6. Find $P(X \ge 2)$ 7mark

2 a) A continuous random variable has a probability density function   8mark

$f(x) = kx^2e^{-2x}$, $x \ge 0$. Find mean and variance.

b) The manufacturer of cotter pins knows that 5% of his product is defective. If he

sells cotter pins in boxes of 100 and guarantees that not more than 10 pins will be   7mark

defective. What is the probability that a box will fail to meet the guaranteed

quality?

3 a) In a normal distribution 7% of items are under 35 and 89% of items are under 63.

Find mean and the standard deviation of the distribution   8mark

b) If the mileage which a car owner get with a certain kind of radial tyre is a random

variable having an exponential distribution with mean 40,000 km. Find the   7mark

probabilities that one of these tyres will last (i) at least 20,000 km (ii) at most

30,000 km.

PART B (MODULES III AND IV)
Answer two full questions.

4 a) Using Fourier integral representation show that

$$\int_0^\infty \frac{2\,Cos\,x\omega + \omega Sin\,x\omega}{4 + \omega^2}\,d\omega = \begin{cases} 0 & if\ x < 0 \\ \dfrac{\pi}{2} & if\ x = 0 \\ \pi e^{-2x} & if\ x > 0 \end{cases}$$

8mark

b) Find the Fourier Sine transform of $f(x) = \begin{cases} x^2 & if\ 0 < x < 2 \\ 0 & if\ x > 2 \end{cases}$

7mark

5 a) Find the Fourier transform of $f(x) = \begin{cases} e^x & if\ -a < x < a \\ 0 & otherwise \end{cases}$

8mark

b) Find Laplace transform of $\dfrac{1-e^{2t}}{t}$

7mark

6 a) Using the Laplace transform , solve

$y'' + 6y' + 8y = e^{-3t} - e^{-5t}, \qquad y(0) = 0, \quad y'(0) = 0$

8mark

b) Find the inverse Laplace Transform, using Convolution theorem

$\dfrac{240}{(s^2+1)(s^2+25)}$

7mark

PART C (MODULES V AND VI)

Answer two full questions.

7 a) Find the positive solution of $\ln x - \cos x = 0$ using Newton's method starting from $x_0 = 1$. (Correct to three decimal places).

6mark

b) Compute $Cosh\ 0.56$ using Newton's forward difference formula from the following values correct to five places of decimal. $Cosh\ 0.5 = 1.127626,$
$Cosh\ 0.6 = 1.185465,\ Cosh\ 0.7 = 1.255169,\ Cosh\ 0.8 = 1.337435$

7mark

c) Using Lagrange's Formula, find the interpolating polynomial from the given data.

x	0	2	3	6
$f(x)$	659	705	729	804

7mark

8 a) The following table gives the population of town during the last six censuses. Estimate using Newton's interpolation formula the population in the year 1946.

year	1911	1921	1931	1941	1951	1961
Population(in thousands)	12	15	20	27	39	52

7mark

b) Evaluate $\int_{-5}^{5} x^4\ dx$ using Trapezoidal rule with $n = 10$

7mark

c) Solve the linear system by Gauss elimination.

$-3x_1 + 6x_2 - 9x_3 = -46.725$

$$x_1 - 4x_2 + 3x_3 = 19.571 \qquad \text{6mark}$$

$$2x_1 + 5x_2 - 7x_3 = -20.073$$

9 a) Evaluate $\int_0^1 \frac{1}{1+x^2}\, dx$ using Simpson's rule by taking $h = 0.2$ 5mark

b) Find $y(0.5)$ if $\frac{dy}{dx} = (y + x)^2$, $y(0) = 0$, using Euler method(take $h = $ 7mark 0.1).Correct to three decimal places.

c) Given that $y' = 1 + y^2$, $y(0) = 0$ use the fourth order Runge Kutta method to   8mark find $y(0.2)$ by taking $h = 0.1$

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