(Please write your Exam Roll No.) *Exam Roll No. ....*

END TERM EXAMINATION

FIRST SEMESTER [BCA] DECEMBER 2016

Paper Code: BCA-101	***Subject: Mathematics-I***
Time: 3 Hours	***Maximum Marks: 75***

Note: Attempt any five questions including Q no.1 which is compulsory. Select one question from each unit.

Q1 (a) Prove that every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew symmetric matrix. **(5)**

(b) For what value of x, the matrix **(5)**

$$A = \begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix} \text{ is singular.}$$

(c) Using properties without expanding prove that: **(5)**

$$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

(d) Show that $f(x) = \begin{cases} 2x-1; & x<2 \\ 3; & x=2 \\ x+1; & x>2 \end{cases}$ is continuous at x = 2. **(5)**

(e) Show that function $f(x) = \sin x(1+\cos x)$ is maximum when $x = \frac{\pi}{3}$. **(5)**

UNIT-I

Q2 (a) If the matrix is orthogonal, then find the values of a, b and c where matrix is

$$A = \begin{bmatrix} a & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}.$$ **(6.5)**

(b) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} -2 & 2 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Also, find A^{-1}. **(6)**

Q3 (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **(6)**

(b) Examine the following system of vectors for linearly dependence . If dependent, find the relation between them

$X_1 = (1, -1, 1)$; $X_2 = (2, 1, 1)$; $X_3 = (3,0,2)$. **(6.5)**

P.T.O.

UNIT-II

Q4 (a) Find the value of a so that the function $f(x)=\begin{cases} ax+5 & if \quad x \le 2 \\ x-1 & if \quad x>2 \end{cases}$ is continuous at $x = 2$. **(6.5)**

(b) Evaluate:-

(i) $\lim_{x\to 2}\left(\frac{x^3-2}{x-2}\right)$; (ii) $\lim_{x\to 0}\frac{|x|}{x}; x \ne 0$ **(6)**

Q5 (a) Evaluate:-

(i) $\lim_{x\to \sqrt{2}}\left(\frac{x^4-4}{x^2+3\sqrt{2}x-8}\right)$; (ii) $\lim_{x\to 0}\frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}$ **(6)**

(b) If the function $f(x)=\begin{cases} 3ax+b; & for \quad x>1 \\ 11; & for \quad x=1 \\ 5ax-b; & for \quad x<1 \end{cases}$ is continuous at x = 1, find the values of a and b. **(6.5)**

UNIT-III

Q6 (a) Find $\frac{dy}{dx}$ if:-

(i) $y=\sin\sqrt{x}$ (ii) $x^y.y^x=k$, where k is a constant (iii) $y=\sin^3 2x$ **(6)**

(b) Find the nth derivative of log(2x+3) **(6.5)**

Q7 (a) Find all the asymptotes of the curve $y^2(x-2a)=x^3-a^2$. **(6.5)**

(b) If $y=\sin(m\sin^{-1}x)$, then prove that $(1-x^2)y_{n+2}=(n^2-m^2)y_n+(2n+1)xy_{n+1}$ **(6)**

UNIT-IV

Q8 (a) Solve the following integrals:- https://www.ggsipuonline.com **(6)**

(i) $\int xe^{-x}dx$ (ii) $\int\frac{x^4+1}{x^2+1}dx$ (iii) $\int x^n\log x dx$

(b) Prove that $\beta(m,n)=\frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$ **(6.5)**

Q9 (a) Find out the reduction formulae for $\int_0^{\pi/4}\sin^n x dx$, n being a positive integer. **(6.5)**

(b) If $\int_0^{\pi/4}\tan^n x dx$, then prove that $I_n - I_{n-1}=\frac{1}{n-1}$; n being a positive integer >1. Hence, evaluate I_5. **(6)**

****