Second Semester-B.Tech Course work
Mid Semester Examination, May, 2023

Course code: FCMT007
Course title: Mathematics II

Time: 1 hour 30 min.

Maximum Marks. 25

Note: Attempt all five questions. Missing data/information(if any), may be suitably assumed and mentioned in the answer.

Q. No.	Question	Marks	CO
1a	Find the general solution of the following differential equation: $$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=9\frac{e^{-3x}}{x^3}$$	2.5	CO1
1 b	Solve the differential equation: $$x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$$	2.5	CO1
2a	Find the general solution of the following differential equation: $$\frac{d^3y}{dx^3}+y=\sin x$$	2.5	CO1
2b	Find the roots of indicial equations and recurrence relation of the coeficient of the series solution of following equation: $$4x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0.$$	2.5	CO1
3a	If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then prove that $$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u.$$	2.5	CO2
3b	If $u=f(y-z,z-x,x-y)$, then show that $$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0.$$	2.5	CO2
4a	Find the Jacobian $\frac{\partial(u,v,w)}{\partial(r,\theta,\phi)}$ of the following functions $$u(r,\theta,\phi)=r\cos\theta\cos\phi, v(r,\theta,\phi)=r\cos\theta\sin\phi, w(r,\theta,\phi)=r\sin\theta.$$	2.5	CO2
4b	Using Taylor's formula, find the quadratic approximations for the function $e^{-x^2-2y^2}$ at origin.	2.5	CO2
5a	Discuss the maximum or minimum values for the function: $u(x,y)=x^3+y^3-3xy$.	2.5	CO2
5b	Using Lagrange's method of undteremined multiplier, detremine the maximum or minimum value of $x^2+y^2+z^2$ when $ax^2+by^2+cz^2=1$.	2.5	CO2

Roll no.......................

Second Semester-B.Tech Course
End Semester Examination, July, 2023

Course code: FCMT007
Course title: Mathematics II
Time: 3 hours.

Maximum Marks. 50

Note: Missing data/information(if any), may be suitably assumed and mentioned in the answer.

Q. No.	Question	Marks	CO
Q1	Attempt any 2 parts of the followings:		
1a	Solve the following initial value problem: $$(D^3 - D^2 + 2D - 2)y = 0, \quad y(0) = 1, \quad y'(0) = 0 = y''(0), \quad \text{where} \quad D \equiv \frac{d}{dx}.$$	5	CO1
1b	Find the complete solution of the differential equation: $$x^2y'' - 3xy' + 5y = x^2 \sin\log x.$$	5	CO1
1c	Using Frobenius method, find the series solution about the point $x = 0$ of the differential equation: $$9x(1-x)y'' - 12y' + 4y = 0.$$	5	CO1
Q2	Attempt any 2 parts of the followings:		
2a	If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$ (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u.$	5	CO2
2b	Find the maximum and minimum of the function $$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy.$$	5	CO2
2c	Transform the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates.	5	CO2
Q3	Attempt any 2 parts of the followings:		
3a	Using change the order of integration, evaluate $\int_0^2 \int_0^{4-x^2} \frac{2xe^{3y}}{4-y} dydx.$	5	CO3
3b	Evaluate $\iiint_D 2(z^2x^2+z^2y^2)\, dxdydz$, where $D = \{(x,y,z) \in \mathbb{R}^3,\ x^2+y^2 \leq 1,\ -1 \leq z \leq 1\}.$	5	CO3
3c	Prove that $$\int_0^2 (8-x^3)^{-1/3} dx = \frac{2\pi}{3\sqrt{3}}.$$	5	CO3

$y = 4 - x^2$

~~$y = 4 - x$~~

$x^2 = 4 + y$

Q4 Attempt any 2 parts of the followings:

4a In each of the following parts, perform only one iteration: — 5 — CO4

(i) Use Newton-Raphson method, to find a root of the equation $xe^x - 1 = 0$, with $x_0 = 0.6$.

(ii) Solve the system of equations

$$27x + 6y - z = 85$$
$$6x - 15y + 2z = 72$$
$$x + y + 54z = 110$$

with initial approximation $(x_0, y_0, z_0) = (0, 0, 0)$, using Gauss-Siedel method .

4b Compute the value of the integral $\int_{0.2}^{1.4} (x + e^x)dx$, taking 6 intervals using (i) Trapezoidal rule (ii) Simpson's 1/3 rule. — 5 — CO4

4c Consider $\frac{dy}{dx} = x^2 + y^2$, where $y(0) = 1$. Find $y(0.1)$ correct to four decimal places, by fourth order Runge-Kutta method. — 5 — CO4

Q5 Attempt any 2 parts of the followings:

5a With the usual notations, find p for a binomial variate X, if $n = 6$ and $9P(X = 4) = P(X = 2)$. Further, find (i) mean (ii) $P(X \leq 3)$ (iii) $P(X > 3)$? — 5 — CO5

5b The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 and (iii) between 65 and 75.
(Use $P(0 < Z < 1) = \phi(1) = 0.3413$). — 5 — CO5

5c Find the correlation coefficient and obtain the equations of two lines of regression for the following data: — 5 — CO5

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71