

First SEMESTER-B.TECH- Course Work
MID-SEMESTER EXAMINATION, January 2023

Course Code-FCMT001

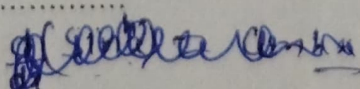
Course Title-Mathematics I

Time: 1:30 hours

Max Marks-25

Note: Attempt all questions. Missing data/information (if any), may be suitably assumed & mentioned in the answer.

Q. No.	Question	Marks	CO
1 (a)	Test the convergence of following series: $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots, x > 0.$	2.5	CO1
(b)	Test the convergence of the series: $\sum \left(\frac{n+1}{n+2}\right)^n \cdot x^n, \quad x > 0.$	2.5	CO1
2 (a)	Test the convergence of following series: $\sum [\sqrt{n^4 + 1} - \sqrt{n^4 - 1}].$	2.5	CO1
(b)	Test the following series for convergence and absolute convergence: $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots$	2.5	CO1
3 (a)	If $\tan(\theta + i\phi) = \tan \alpha + i \sec \alpha$, then prove that $2\theta = n\pi + \frac{\pi}{2} + \alpha.$	2.5	CO2
(b)	Prove that $\sinh^{-1}(\cot x) = \log(\cot x + \csc x).$	2.5	CO2
4 (a)	Prove that the value of n -th derivative of $\frac{x^3}{x^2-1}$, for $x = 0$ is zero, when n is even and $(-n!)$, when n is odd and greater than one.	2.5	CO2
(b)	If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$	2.5	CO2
5 (a)	Expand $2x^3 + 7x^2 + x - 1$ in powers of $x - 2$, using Taylor's series.	2.5	CO2
(b)	Expand $\log(1 + e^x)$ upto third degree terms using Maclaurin's series.	2.5	CO2

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* $\csc x = \operatorname{Cosec} x.$

FIRST SEMESTER- B.TECH-Course Work
END SEMESTER EXAMINATION, MARCH, 2023

Course Code-FCMT001
 Course Title- Mathematics-I

Time- 3:00 hours

Max. Marks- 50

Note:- Attempt all the five questions. Missing data/information (if any), may be suitably assumed & mentioned in the answer.

Q. No.	Questions	Marks	CO
Q1	Attempt any two parts of the following		
1a	Test the convergence of the following series: $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \dots$	5	CO1
1b	Test the convergence of the following series: $\frac{3}{7}x + \frac{3 \cdot 6}{7 \cdot 10}x^2 + \frac{3 \cdot 6 \cdot 9}{7 \cdot 10 \cdot 13}x^3 + \frac{3 \cdot 6 \cdot 9 \cdot 12}{7 \cdot 10 \cdot 13 \cdot 16}x^4 + \dots$	5	CO1
1c	Show that the series $\sum (-1)^n [\sqrt{n^2 + 1} - n]$ is conditionally convergent.	5	CO1
Q2	Attempt any two parts of the following		
2a	If $y = e^{\sin^{-1} x}$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + 1)y_n = 0.$	5	CO2
2b	If $y = \cos(m \sin^{-1} x)$, using Leibnitz theorem, find $(y_n)_0$.	5	CO2
2c	Using Maclaurin's theorem, expand $\log(x + \sqrt{1 + x^2})$ in ascending powers of x as far as the term x^5 .	5	CO2
Q3	Attempt any two parts of the following		
3a	Show that the pedal equation of the parabola $\frac{2a}{r} = 1 - \cos\theta$ is $p^2 = ar$. Hence find the radius of curvature of the given parabola.	5	CO3
3b	Find the angle of intersection of the curves $r = a/(1 + \cos\theta)$ and $r = b/(1 - \cos\theta)$.	5	CO3
3c	Find the radius of curvature at the point $(\frac{3a}{2}, \frac{3a}{2})$ of the curve $x^3 + y^3 = 3axy$.	5	CO3
Q4	Attempt any two parts of the following		
4a	Find all the asymptotes of the curve $y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0.$	5	CO4
4b	Find the area of the cardioid $r = a(1 + \cos\theta)$.	5	CO4
4c	The curves $y^2(a + x) = x^2(3a - x)$ revolves about the axis of x . Find the volume generated by the loop.	5	CO4

Q5	Attempt any two parts of the following		
5a	<p>For what values of μ, the system of equations</p> $\begin{aligned}x + y + z &= 1, \\x + 2y + 4z &= \mu, \\x + 4y + 10z &= \mu^2\end{aligned}$ <p>have a solution and solve them completely in each case.</p>	5	CO5
5b	<p>Find the eigenvalues and eigenvectors of the matrix</p> $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$	5	CO5
5c	<p>Show that the matrix</p> $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ <p>satisfies Cayley-Hamilton theorem. Hence find A^{-1}.</p>	5	CO5