

ABV- Indian Institute of Information Technology & Management, Gwalior

Discrete Structures (IT201)

Major Semester Examination (Session 2023–24)

Maximum Time: 3 Hours Max Marks: 70

Note: All questions carry equal marks. Assume suitable data wherever required.

Section A is compulsory. In Section B, attempt any 4 questions.

Section A $(10 \times 2 = 20 \text{ Marks})$

Answer all questions in brief. Each question carries 2 marks.

- 1. Define tautology and contradiction with one example each.
- 2. Find the contrapositive of the statement: "If a number is divisible by 6, then it is divisible by 3."
- 3. Write the set of all prime numbers less than 20 using roster form.
- 4. Give an example of a relation that is reflexive and symmetric but not transitive.
- 5. Define injective and surjective functions with one example each.
- 6. What is the degree of each vertex in a complete graph K_5 ?
- 7. State and prove the pigeonhole principle with an example.
- 8. Define a group. Is (Z, +) a group? Justify.
- 9. How many different ways are there to arrange the letters of the word "BANANA"?
- 10. Give an application of graph theory in computer science.

Section B (5 \times 10 = 50 Marks)

Attempt any five questions. Each question carries 10 marks.

11. (a) Construct a truth table for $((p \lor q) \to r) \leftrightarrow ((p \to r) \land (q \to r))$. (b) Using laws of logic, show that $\neg (p \lor q) \equiv (\neg p \land \neg q)$.

- 12. (a) Let $A = \{1, 2, 3\}, B = \{x, y\}$. Find the number of relations from A to B. (b) Define equivalence relation. Prove that the relation "congruence modulo 5" on Z is an equivalence relation.
- 13. (a) Define bijective function with an example. (b) Let $f: R \to R, f(x) = 2x + 1$ and $g: R \to R, g(x) = x^2$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.
- 14. (a) Draw all possible simple graphs with 4 vertices. (b) Prove that in any undirected graph, the sum of the degrees of all vertices is twice the number of edges.
- 15. (a) State and prove Cayley's theorem for groups. (b) Prove that the set of integers under addition forms an abelian group.
- 16. (a) How many 6-digit numbers can be formed using digits 0–9 if repetition is allowed but the number should not start with 0? (b) In how many ways can a committee of 4 members be formed from 8 men and 6 women if at least 2 women must be included?
- 17. (a) Define Hamiltonian path and Eulerian circuit with examples. (b) Show that a simple graph with n vertices and more than $\frac{(n-1)(n-2)}{2}$ edges is connected.