

MSPA PREDICT 400

Discussion Topic: Week 3 Solving Minimization and Maximization Problems

Introduction

This document presents the results of the third weeks discussion topic for the Masters of Science in Predictive Analytics course: PREDICT 400. This assessment required the student to take a system of equations and extend it into a minimization or maximization problem by adding an extra variable (or variables) and constraints(s). The student was then required to solve the problem using graphical and/or simplex method(s).

Minimization/Maximization Problem

For this assessment, I formed an optimization function with constraints based on the system of linear equations which I used as part of the second weeks discussion topic. The original system of equations is shown below.

$$x - y + \frac{29}{100}z = \frac{-169}{100} \quad (1)$$

$$\frac{-17}{20}x + y + \frac{-1}{5}z = \frac{243}{100} \quad (2)$$

$$\frac{-47}{20}x + \frac{119}{100}y + z = \frac{-161}{50} \quad (3)$$

Optimize according to 'z'.

$$z = \frac{47}{20}x + \frac{-119}{100}y + \frac{-161}{50}$$

$$z = \frac{47}{20}\left(y + \frac{-29}{100}z + \frac{-169}{100}\right) + \frac{-119}{100}\left(\frac{1}{5}z + \frac{17}{20}x + \frac{243}{100}\right) + \frac{-161}{50}$$

$$z = \frac{23500y - 10115x - 100832}{19195}$$

Utilize the existing system of equations to form constraints for 'x' and 'y', whilst enforcing an additional maximum constraint on 'y'.

$$y \geq \frac{17}{20}x + \frac{243}{100}$$

$$y \leq \frac{493697479}{250000000}x + \frac{-2705882353}{1000000000}$$

$$y \leq 10$$

Graphical Method

The optimization problem and inequality constraints are formalized below:

Find the maximum value of :

$$z = \frac{23500y - 10115x - 100832}{19195}$$

Subject to the constraints :

$$y \geq \frac{17}{20}x + \frac{243}{100}$$

$$y \leq \frac{493697479}{250000000}x + \frac{-2705882353}{1000000000}$$

$$y \leq 10$$

Plot the inequality constraints.

```

In [1]: #Source: http://stackoverflow.com/questions/17576508/python-matplotlib-drawing-linear-inequality-functions

import numpy as np
import matplotlib.pyplot as plt
from sympy.solvers import solve
from sympy import Symbol
%matplotlib inline

def f1(x):
    return (17/20)*x+(243/100)
def f2(x):
    return (493697479/250000000)*x+(-2705882353/1000000000)
def f3(x):
    return -0*x+10

x = Symbol('x')
x1, = solve(f1(x)-f2(x))
x2, = solve(f1(x)-f3(x))
x3, = solve(f2(x)-f3(x))

y1 = f1(x1)
y2 = f1(x2)
y3 = f2(x3)

plt.plot(x1,f1(x1),'go',markersize=10)
plt.plot(x2,f1(x2),'go',markersize=10)
plt.plot(x3,f2(x3),'go',markersize=10)

plt.fill([x1,x2,x3,x1],[y1,y2,y3,y1],'red',alpha=0.5)

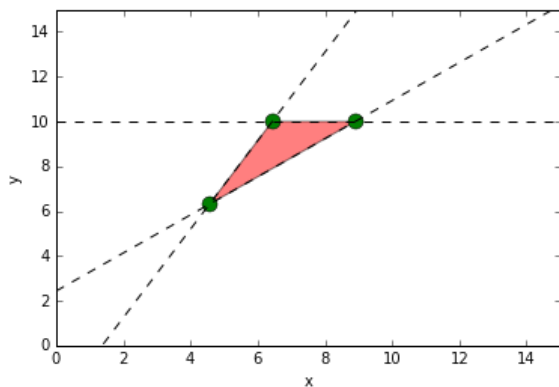
xr = np.linspace(0,15,100)
y1r = f1(xr)
y2r = f2(xr)
y3r = f3(xr)

plt.plot(xr,y1r,'k--')
plt.plot(xr,y2r,'k--')
plt.plot(xr,y3r,'k--')

plt.xlim(0,15)
plt.ylim(0,15)

plt.ylabel("y")
plt.xlabel("x")
plt.show()

```



```
In [2]: import pandas as pd
import numpy as np

y_values = [y1, y2, y3]
x_values = [x1, x2, x3]
op_values = np.array([y_values, x_values, range(0,3)])

df_optimization = pd.DataFrame(op_values.T, columns = ["y", "x", "z"])

for index, row in df_optimization.iterrows():
    x = row["x"]
    y = row["y"]
    z = (-10115*x+23500*y-100832)/19195
    row["z"] = z

df_optimization
```

Out[2]:

	y	x	z
0	6.31116921920377	4.56608143435738	0.0674427164763631
1	10.00000000000000	8.90588235294118	2.29669184683511
2	10.00000000000000	6.43404255311176	3.59925290832376

```
In [3]: print("Maximum value of z: ", df_optimization['z'][2])
print("when y: ", df_optimization['y'][2])
print("and x: ", df_optimization['x'][2])

Maximum value of z:  3.59925290832376
when y:  10.00000000000000
and x:  6.43404255311176
```