SCHOOL OF PROFESSIONAL STUDIES

NORTHWESTERN UNIVERSITY

R Lesson 5 - Solutions

MSPA 401 – Introduction to Statistical Analysis

1) Suppose a gambler goes to the race track to bet on four races. There are six horses in each race. He picks one at random out of each race and bets on each of the four selections. Assuming a binomial distribution, answer the following questions.

   i) The gambler wins all four races.

   This is a binomial probability with the number of successes = 4, n = 4 and p = 1/6. The answer is 0.0008 rounded to four decimal places.

   ii) The gambler loses all four races.

   This is a binomial probability with the number of successes = 0, n = 4 and p = 1/6. The answer is 0.4823 rounded to four decimal places.

   iii) The gambler wins exactly one race.

   This is a binomial probability with the number of successes = 1, n = 4 and p = 1/6. The answer is 0.3858 rounded to four decimal places.

   iv) The gambler wins at least one race.

   This can be solved by subtracting from 1.0 the probability of losing all four races. The answer is 1.0 – 0.4823 = 0.5177 rounded to four places.

2) A woman claims she can tell by taste if cream is added before a tea bag is placed in a tea cup containing hot water. An experiment is designed. A series of cups of tea will be prepared with n of them having the cream added prior to the tea bag and n of them with the cream added after the tea bag. This gives a total of 2n cups of tea. The sequence of tea cups is presented in random order. If the woman cannot discriminate it will be expected on average she would guess at random and be correct on half the tea cups. Answer the following questions assuming the number of successes follows a binomial distribution with probability equal to 0.5 and 2n trials.

   v) If the total number of binomial trials is 2n=20, what is the probability the woman is correct more than 15 out of 20 times?

   This is the sum of the probabilities for 16, 17, 18, 19, and 20 successes which can be obtained from the upper tail of the binomial distribution using n = 20 and p = 0.5. The answer is 0.0059 rounded to four decimal places.

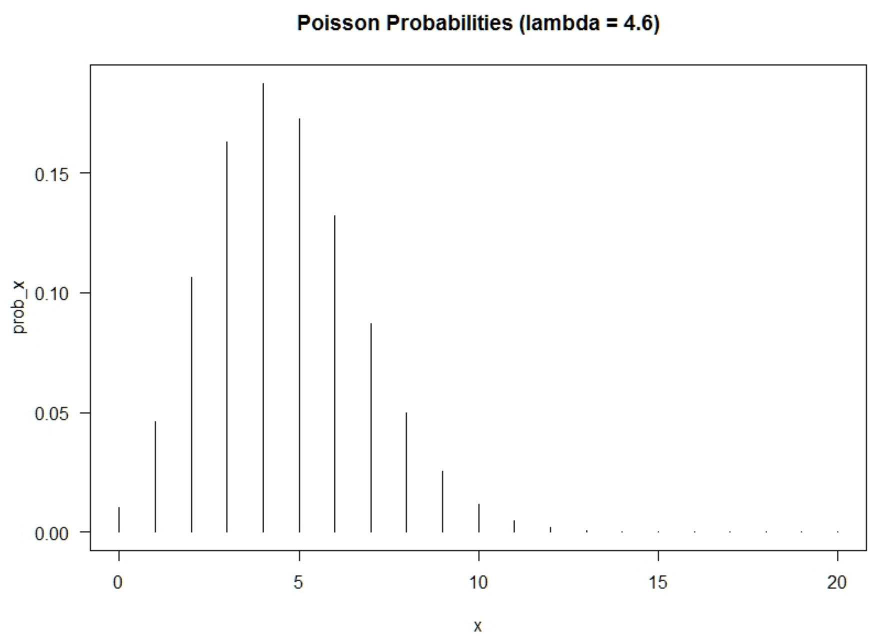
   vi) To reduce the amount of labor, how small can the total number of binomial trials be while keeping the probability of 2n consecutive successes at or below 0.05? (We use 2n = the number of trials since half have the cream first and half after the tea bag.)

This question involves presenting the woman with an even number of cups equally split between the two types of tea. Using a while loop in R, the total number of cups will be increased in increments of two until the probability of 2n consecutive successes is at or below 0.05. This result gives us a statistical test with a defined significance level. The results of the while loop as 2n is increased appear below. Six cups of test is the answer.

Number of Consecutive Cups Correctly Identified: 2 p_value: 0.2500
Number of Consecutive Cups Correctly Identified: 4 p_value: 0.0625
Number of Consecutive Cups Correctly Identified: 6 p_value: 0.0156

3) An emergency room has 4.6 serious accidents to handle on average each night. Using the Poisson distribution, calculate the distribution of accidents per night. (In other words, what is the probability of 0, 1, 2, ... accidents per night?) Plot the result.

This requires calculating probabilities for each possible number of accidents per night.
for (x in 0:20) + cat("\n x:", x, "prob:", sprintf("%.4f", dpois(x, lambda = 4.6)))



4) A production process occasionally produces at random a defective product at a rate of 0.001. If these products are packaged 100 at a time in a box and sold, answer the following questions and compare your answers. Plot the distributions for each type of variable over the range 0, 1, 2, 3, 4. What do you conclude?

i) Using the binomial distribution, calculate the probability a box has zero defectives.

sprintf("%.4f", dbinom(x = 0, size = 100, prob = 0.001)) gives: 0.9048

ii) Using the Poisson distribution, calculate the probability a box has zero defectives.

sprintf("%.4f", dpois(0, lambda = 0.1)) gives 0.9048

iii) This is an example of the Poisson and Binomial distributions converging for small probabilities of binomial success. When n*p is sufficiently small, the Poisson distribution will approximate the Binomial. This can be seen by comparing the two plots which follow. In effect, when this happens, the process being modeled is behaving close to a Bernoulli random variable and the upper tails of both distributions become inconsequential. The plots illustrate that the probability of more than 1 defective in a box is practically zero.