MSPA PREDICT 400

Discussion Topic: Week 8 The Fundamental Theorem of Calculus

Introduction

This document presents the results of the eigth w eeks discussion topic for the Masters of Science in Predictive Analytics course: PREDICT 400. This assessment required the student to explain w hat the Fundamental Theorem of Calculus is and w hat it means, using a specific example (or examples) to illustrate their point.

Fundamental Theorem of Calculus

The fundamental theorem of calculus is a theorem that links the concept of the derivative of a function w ith the concept of the function's integral.

First Fundamental Theorem of Calculus

According to w ikipedia, the first fundamental theorem of calculus holds that the definite integration of a function is related to its antiderivative, and can be reversed by differentiation.

That is, if w e have a function, $F(x)$, w hich defines the area betw een some other function, $f(t)$ for all x in [a,b]:

$F(x) = \int_a^x f(t)dt$

then,

$F'(x) = f(x)$

for all x in [a,b].

This page does a good job of explaining the theorm using Python. I have leveraged this code below.

First, find the area under the function $f(x) = x^2$ for the range of $x = 1$ to $x = 3$, using integrals.

In [1]:
```
import scipy.integrate as integrate

a, e = integrate.quad(lambda x: x**2, 1, 3)
print(a)
```

```
8.666666666666668
```

That is,

$F(x) = \int_a^x f(t)dt = 8.\bar{6}$

w here,

$f(x) = x^2$

For this example, w e can define $f(t)$ w ith a variable amount of increments (rectangles) using Riemann sums. We do this by first declaring a function to return area under x^2 for the given range, based on a variable amount of increments.

In [2]:
```
def f(t):
    return t**2
```

Check using four rectangles:

In [3]:
```
delta_x = (3-1)/4

area = 0
for n in range(3, 7):
    area += float(delta_x) * f(delta_x*n)
print(area)
```

```
10.75
```

Check using 50 rectangles:

In [4]:
```python
delta_x = (3-1)/50

area = 0
for n in range(1, 51):
    area += float(delta_x) * f(delta_x*n + 1)
print(area)
```

```
8.827200000000001
```

Using 1000 rectangles:

In [5]:
```python
delta_x = (3-1)/1000

area = 0
for n in range(1, 1001):
    area += float(delta_x) * f(delta_x*n + 1)
print(area)
```

```
8.674667999999999
```

We can see that as w e increase the amount of increments tow ards infinity, $f(t)$ moves closer tow ards the integrated solution.

We then define $F(x)$, w hich calls $\int_a^x f(t)dt$ for any given amount of increments over the desired range.

In [6]:
```python
def F(x):
    delta_x = float((3-1)/x)
    area = 0
    for n in range(1, x+1):
        area += float(delta_x) * f(delta_x*n + 1)
    return area
```

Check using four rectangles:

In [7]:
```python
F(4)
```

Out[7]:
```
10.75
```

Check using 50 rectangles:

In [8]:
```python
F(50)
```

Out[8]:
```
8.827200000000001
```

Check using 1000 rectangles:

In [9]:
```python
F(1000)
```

Out[9]:
```
8.674667999999999
```

We have now proven that w e have w orking definitions w ithin Python for both $F(x)$ and $\int_a^x f(t)dt$.

Finally, it can be show n that $F'(x) = f(x)$:

In [10]:
```python
def F(x, rectangles=1000):
    delta_x = float((x-1)/rectangles)
    area = 0
    for n in range(1, rectangles+1):
        area += float(delta_x) * f(delta_x*n + 1)
    return area

def derivative(f, h=0.1e-5):
    def df(x):
        return (f(x+h/2) - f(x-h/2))/h
    return df

d = derivative(F)
```

In [11]:
```python
print(d(2))
print(f(2))
```

```
4.003500499560886
4
```

```
In [12]: print(d(3))
         print(f(3))
```

```
9.010001999598671
9
```

Second Fundamental Theorem of Calculus

According to w ikipedia, the second fundamental theorem of calculus is that that the definite integral of a function can be computed by using any one of its infinitely-many antiderivatives.

That is, if w e have a function, $F(x)$, w hich defines the area betw een some other function, $f(t)$ for all x in [a,b] and the first theorem holds:

$F'(x) = f(x)$

then,

$\int_a^b f(x)dx = F(b) - F(a)$

for all x in [a,b]

Again, w e first find the area under the function $f(x) = x^2$ for the range of $x = 1$ to $x = 3$, using integrals.

```
In [13]: import scipy.integrate as integrate

         a, e = integrate.quad(lambda x: x**2, 1, 3)
         print(a)
```

```
8.666666666666668
```

By leveraging the definition for $F(x)$ show n above, w e can find $F(b) - F(a)$

```
In [14]: F(3) - F(1)
```

```
Out[14]: 8.674667999999999
```