Question 2: Evaluate the limit or show that it does not exist:

a)

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+2y^2}$$

Let $y = mx, m \in R, x = x$:

$$\lim_{x\to 0} \frac{m^2x^4}{x^2+2m^2x^2} = \lim_{x\to 0} \frac{m^2x^2}{1+2m^2} = 0$$

Since $m \in R$, it is obvious that the above conclusion passes the two path test, showing that the limit exists. Assume the limit $L = 0$. It follows that:

$$\frac{x^2y^2}{x^2+2y^2} - L \to \frac{x^2y^2}{x^2+2y^2} - 0 \to \frac{x^2y^2}{x^2+2y^2}$$

It then follows that:

$$0 \leq \frac{x^2y^2}{x^2+2y^2} \leq x^2y^2$$

It is obvious that:

$$\lim_{(x,y)\to(0,0)} x^2y^2 = 0$$

Therefore, by Squeeze Theorem,

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+2y^2} = 0$$

b)

$$\lim_{(x,y)\to(0,0)} \frac{2xy\cos y}{x^2+2y^2}$$

It is obvious that:

$$\frac{2xy\cos(y)}{x^2+2y^2} \leq 2xy\cos y$$

It follows that:

$$-2xy \leq 2xy\cos y \leq 2xy$$

$$\lim_{(x,y)\to(0,0)} -2xy = \lim_{(x,y)\to(0,0)} 2xy = 0$$

Therefore, by Squeeze Theorem,

$$\lim_{(x,y)\to(0,0)} \frac{2xy\cos y}{x^2+2y^2} = 0$$