Question 1b Find and sketch the domain of:

$$f(x,y) = \sqrt{sin[\pi(x^2+y^2)]} + ln(2-|x|)$$

Let $g(x,y) = \sqrt{sin[\pi(x^2+y^2)]}$ and let $h(x,y) = ln(2-|x|)$. The domain of $g(x,y)$ is as follows:

$$sin(\pi(x^2+y^2)) \geq 0$$

$$2\pi m \leq \pi(x^2+y^2) \leq 2\pi m + 1, m \in Z$$

$$2m \leq x^2+y^2 \leq 2m+1, m \in Z$$

By choosing a variety of m, a family of circles appear. This is also obvious from the resulting equation. I thought of it as an infinite circle with holes cut out from every interval from $[2m+1, 2m+2]$, hence the way that I drew the accompanying graph for $g(x,y)$. The domain of $h(x,y)$ is as follows:

$$2-|x| > 0 \rightarrow -2 < x < 2$$

The graph for this domain is obvious. Since $f(x,y) = g(x,y) + h(x,y)$, the domain of $f(x,y)$ is the union of the domains of $g(x,y)$ and $h(x,y)$. This results in:

$$D(f(x,y)) = \{(x,y)|2m \leq x^2+y^2 \leq 2m+1, m \in Z\} \cap \{(x,y)| -2 < x < 2\}$$

See the following attached page for the sketches of the graphs of the domains of $g(x,y), h(x,y)$, and finally $f(x,y)$.