EN 530.766
Fall 2023
HW 4

Consider the 2-D Laplace equation

$$\left(u_{xx} + u_{yy}\right) = 0 \text{ for } 0 \le x, y \le 2\pi$$

with the following boundary conditions

$$u(0, y) = 0$$

$$u(2\pi, y) = 0$$

$$u(x, 0) = \sin(2x) + \sin(5x) + \sin(7x)$$

$$u(x, 2\pi) = 0$$

(1) Write a computer code and obtain the numerical solution using the point Jacobi and Gauss-Seidel iterative schemes. Use a mesh with $\Delta x = \Delta y = 2\pi / 20$. Track the convergence by calculating the residual, $r^k = \left(\delta_x^{\,2} / \Delta x^2 + \delta_y^{\,2} / \Delta y^2\right) u_{i,j}^k$. Conduct these simulation with two different initial guesses (1) u(i,j) =0; (2) u(i,j)=x_i y_j; and (3) u(i,j)=random number distribution between -1 and 1.

Does the convergence behave as expected? Discuss.

(2) Try the above problem with the SOR point Jacobi and SOR point Gauss-Seidel schemes: Investigate and comment on the convergence properties for various values of the under- and over-relaxation parameter for both schemes. Can you find optimal values of the relaxation parameter for these schemes?

(3) Experiment with a SOR Jacobi method where you alternate between an overrelaxation and an underrelaxation as you iterate. Can you find a pair of relaxation parameters that speed up the solution process compared to the non-relaxed Jacobi method? For more information about this "Scheduled Relaxation Jacobi" Method check out this paper. *Xiang Yang and Rajat Mittal, "Acceleration of the Jacobi iterative method by factors exceeding 100 using scheduled relaxation", Journal of Computational Physics, Vol 274, DOI: 10.1016/j.jcp.2014.06.010.*

(4) Challenge problem – how about experimentally determining a SRJ scheme with 3 different values of the relaxation parameter. How much faster can you get compared to the 2 parameter SRJ scheme.

https://www.dropbox.com/s/sf6qssxurtn48pp/2014-Acceleration%20of%20the%20Jacobi%20iterative%20method%20by%20factors%20exceeding%20100%20using%20scheduled%20relaxation.pdf?dl=0