Final Project
Dec. 10, 5 PM
500 points + 50 extra points
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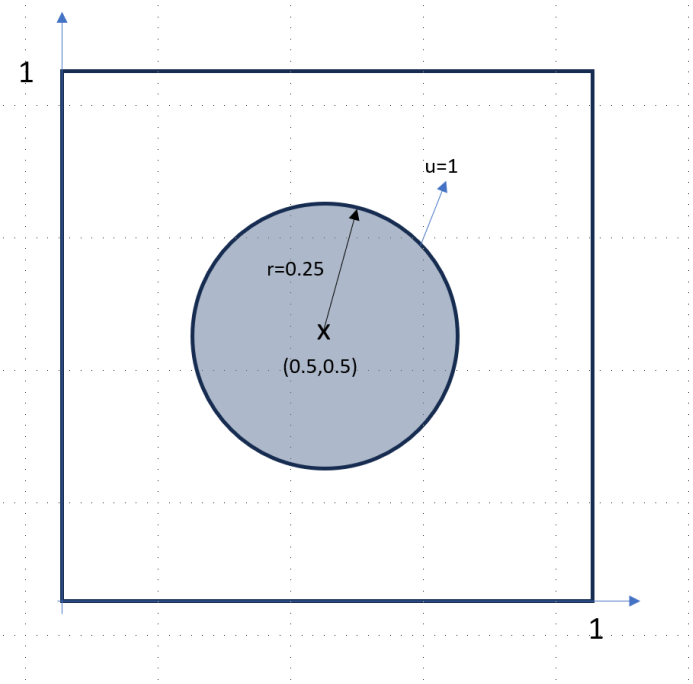
Consider the 2-D Laplace equation

$$\left(u_{xx} + u_{yy}\right) = 0$$

on the domain below with u=0 on the outer boundary and the following internal boundary with u=1



(a) Use a second-order central-difference scheme, develop an iterative solver to solve the above problem. Use the stair-step method to implement the boundary conditions on the internal boundary. Use point GS with SOR. Follow the algorithm shown in class.

(b) Draw a flowchart of your solver. [10 points]

(c) Solve the above on grids with 32x32, 96x96, 160x160 grid cells, and a 224x224 grid also. Use a random number generator to generate the starting guess for the field in the range of [-1,+1]. Plot the residual versus iteration index and the contour plots of the converged solution for each grid for an overrelaxation parameter of 1.0. [140 Points]

(d) How can you check the correctness of your solution? Are there some symmetry lines in the solution where you can obtain an exact solution and compare it to the numerical solution? Is there a different boundary condition you could use on the outer boundary for which you could obtain an exact solution and then compare your numerical solution? [25 points]

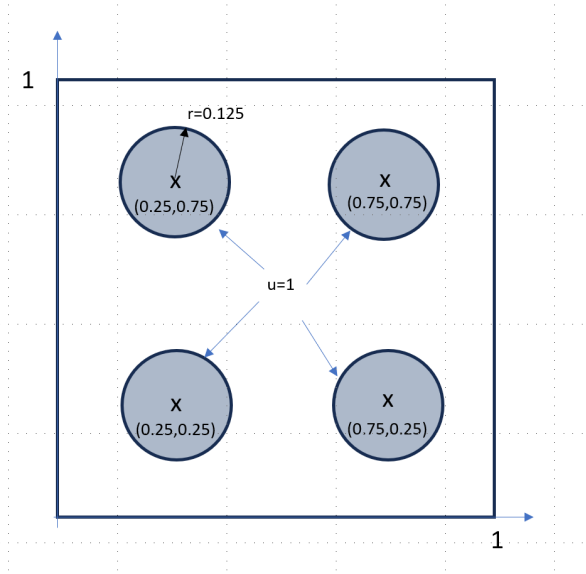
(e) Discuss the observed performance of the iterative solver for each grid simulated here. Consider questions like:

I. How does the convergence change with overrelaxation parameter? What is the optimal value of this parameter? Does this optimal value change with grid size? [25 points]

II. How does the CPU time increase with the number of grid points? Is the expected trend observed? [25 points]

III. Does your solution exhibit the expected second-order accuracy? [50 points]

(f) Repeat (e) for the configuration below. Also comment on how the convergence and CPU time is different from that for the previous configuration. [100 Points]



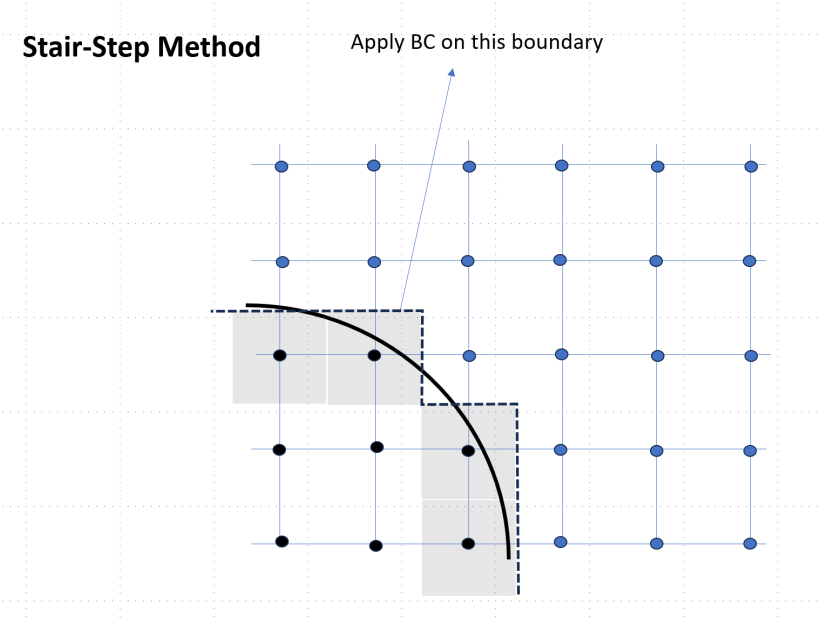
(g) Solve the second configuration with Line-SOR and compare the convergence rate and CPU-time performance with point SOR. [75 points]

(h) **Challenge problem for extra points** – Modify the solver to compute the potential flow past the 4-cylinder configuration in a 4X larger computational domain where you can impose free-stream conditions at the outer boundary. Plot streamlines and pressure contours of the flow. Comment on the correctness of your solution [50 points]

(i) Include a printout of your program as an Appendix to your report.

Note

- *The report needs to be typed up.*
- *Equations may be written neatly by hand and incorporated if needed.*
- *Graphs and plots need to have clear captions.*
- *All symbols should be clearly defined.*
- *Points are not for the length of the report but the quality of your analysis and discussion.*



Hints

- If you are having trouble getting the circular cylinder simulation to work, can you make it work for a square cylinder with boundaries aligned with the grid?