Numerical Methods
EN 530.766
HW 3

Consider the unsteady conduction problem

$$u_t = u_{xx} \text{ for } 0 \le x \le 2\pi \text{ given } u(0,t) = u(2\pi,t) = 0 \text{ and } u(x,0) = \sin(mx).$$

(a) Derive the exact solution to this problem.

(b) Write computer programs to obtain the numerical solutions for the above equation using the:

(i) Forward Euler method,

(ii) Backward Euler method*, and

(iii) Crank-Nicolson method*

**Write your own TDMA solver and use it for the BE and CN methods.*

(c) For m=2,

(i) Obtain numerical solutions using a mesh with $\Delta x = 2\pi / 20$ and r=1/3. Plot and compare the numerical and exact solutions for the three methods at time t=0.1, 0.5 and 1.0. Discuss your observations regarding this comparison.

(ii) Based on your numerical solutions, show that the error and stability characteristics for each method behave as expected. Specifically,

a) does the spatial error scale as $O(\Delta x^2)$?.

b) Is the stability predicted by VN stability analysis verified in your simulations?

c) What happens to the accuracy of the solution when you operate at r > 1/2 for the implicit schemes?

You will have to conduct a grid refinement study and other "numerical experiments" to answer this question.

(d) Obtain a numerical solution of the above problem using a mesh with $\Delta x = 2\pi / 20$ and r=0.5, and for m=3, 5, and 7.

(i) Make plots to compare the solution for the three methods with the exact solution and

(ii) interpret the results within the context of the amplification factor for these methods, i.e. show that the numerical results are consistent with what you expect from the amplification factor for these methods.

NOTE:

1) Prepare your report using a word processor (MS Word, TeX, etc.). You can attach a hand-written derivation for the problem (a).

2) Plots should only be as big as they need to be for clear viewing. Do not make full page plots because none of the plots you will make need to be that big. Plots should be numbered and have clear captions that describe the plot.

3) Please include the source code of your programs in the report.