Assignment 2: PDE - Groundwater flow

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3.2 Exercise Hooghoudt and two-dimensional discretization

3.2.1

Table 1 shows if the variables are valid within the cell of between two cells.

3.2.2

Given that the level of the ditches is given by the following code `H(1:NPixY,1) = 0; H(1:NPixY,NPixX) = 0;`

the code responsible for maintaining this level constant in in the dynamic loop: `% calculate new H values by forward integration H(1:NPixY,2:NPixX-1) = ...`

In fact, in the calculations for the new values of H, the X values start from 2 and finish at NpixX-1. This means that the boundaries are excluded from the recalculation of the height level.

In a hydrological context this means that the flow rate of water in the ditches is much higher than the flow rate of water coming from the ground.

3.2.3

The distance between ditches is about 8 m because in the X-direction there are 100 pixels and each pixel has 0.079 length. Thus

$$Distance\ Ditches = NPixX * PixX \tag{1}$$

3.2.4

In the statement `KDX = K * (0.5*(H(:,1:NPixX-1) + H(:,2:(NPixX))) -DemClay) * PixY;` we multiply by 0.5 because we are calculating the average height. The dimension of ':' in `H(:,1:NPixX-1)` is PixY, the entire column. We use `NPixX-1` instead of `1:NPixX` because in order to calculate the average height we need to confront two values with the same length: from the first to the penultimate column and from the second to the last column.

3.2.5

In the equation `KDX = K * (0.5*(H(:,1:NPixX-1) + H(:,2:(NPixX))) -DemClay) * PixY;` we multiply by `PixY` because it is a component of the area through which the

	Within a cell	Between cells
PixX	X	
DemClay	X	
Net		X
PorVol	X	
PixY	X	
Q		X
I	X	
Prec	X	
KDX		X

Table 1: *Variables validation.*

water flows. In this case, since water flows in the x-direction, the area is on the y-z plane: this means that we should multiply the average height times PixY.

Differently, in the equation `FlowX(:,2:NPixX) = -1 * KDX .* (H(:,2:(NPixX))-H(:,1:NPixX-1)) / PixX;` we divide by `PixX` because of the hydraulic gradient in the x-direction: the difference of height in two consequent cells has to be divided by the length of the cell in the x-direction, that is PixX.

3.2.6

Running the model for both 50 and 100 days we obtained Figure 1 and Figure 2.

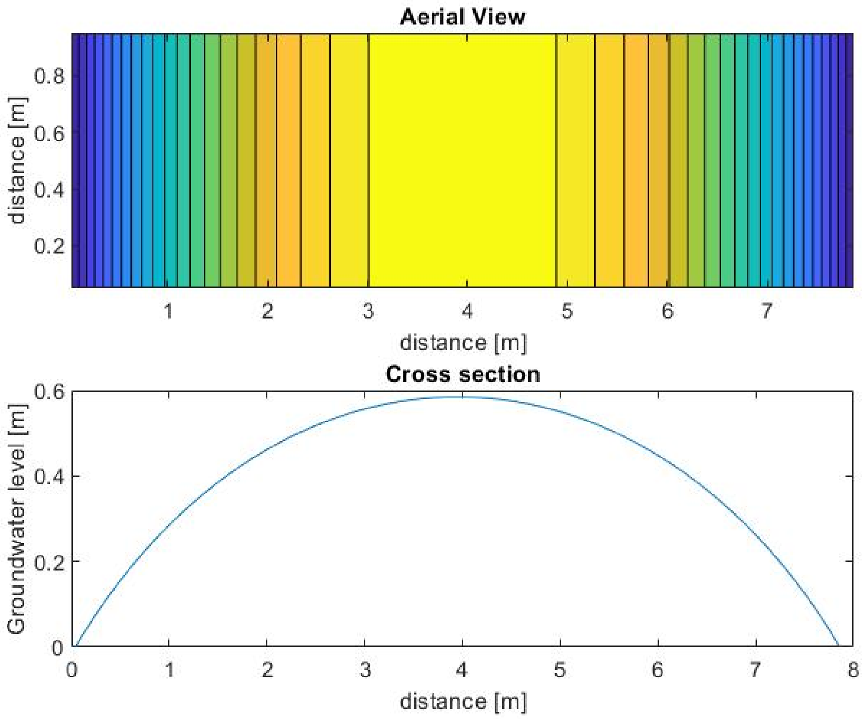

Figure 1: *Groundwater level for 50 days.*

In the second figure a series of peaks is present. The solution to this is o reduce the `dt`. This way the curve does not have peaks anymore.

Increasing the size of each pixel would also lead to a smoother curve, but at the expense of the accuracy of the model.

3.2.7

The equilibrium state is when the maximum groundwater level is around 0.6 m. If the time span is 10 days, the equilibrium is not reached, while after 100 days it is really close to it.

An equilibrium is reached because as the water head increases, the average height gradient increases. According to the Darcy's law, this implies a greater flow rate out to the ditches. As a results, when the volumetric flow rate at the edges is equal to the total amount of precipitation per unit time the equilibrium is reached.

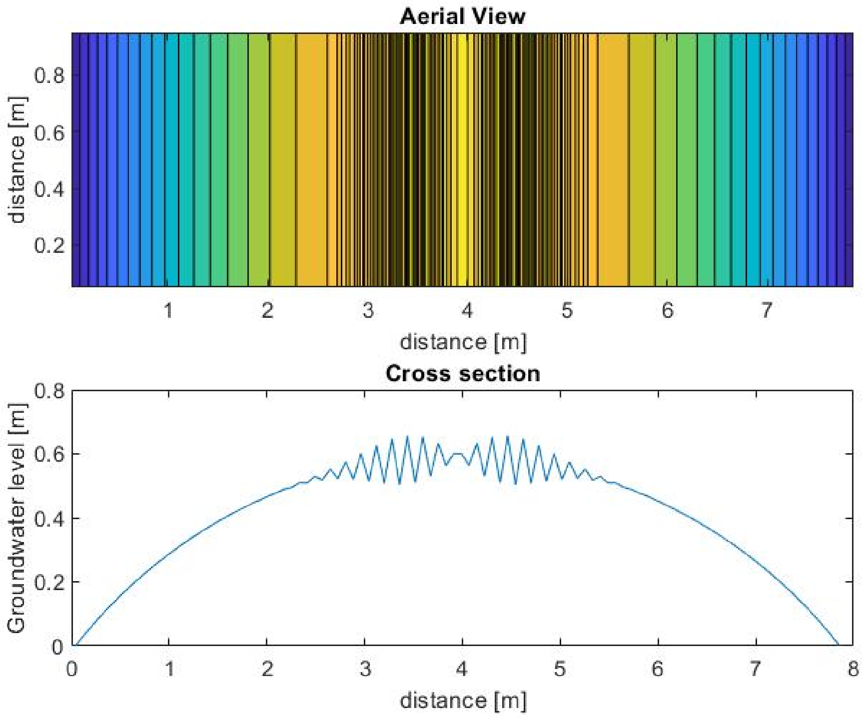

Figure 2: *Groundwater level for 100 days.*

In case the precipitation rate increases, the equilibrium state would also increase. In fact, higher precipitations means that the flow rate at the edges should be higher. This is achieved when the average height gradient is sufficiently large (i.e., since the distance between the ditches is fixed, when the maximum groundwater level is high enough).

If the hydraulic conductivity of the system increases (from 0.1 m/d to 0.5 m/d) the groundwater level decreases, as can be seen in Figure 3, due to the fact that soil lets more water flow out.

3.2.8

In order to check whether the water balance is 0, the following code lines had to be implemented in the model:

```
while Time <= EndTime
...
TotFlow=TotFlow-sum(FlowX(:,2))*dt+sum(FlowX(:,NPixX))*dt;
...
end
```

```
TotPrec = 0.01*PixX*PixY*(NPixX-2)*(NPixY)*(EndTime-StartTime);

Storage=(sum(sum(H-InitH)))*PixX*PixY*PorVol;

Balance = TotPrec - Storage - TotFlow;
```

The balance is $-1.9 * 10^{-4}$.

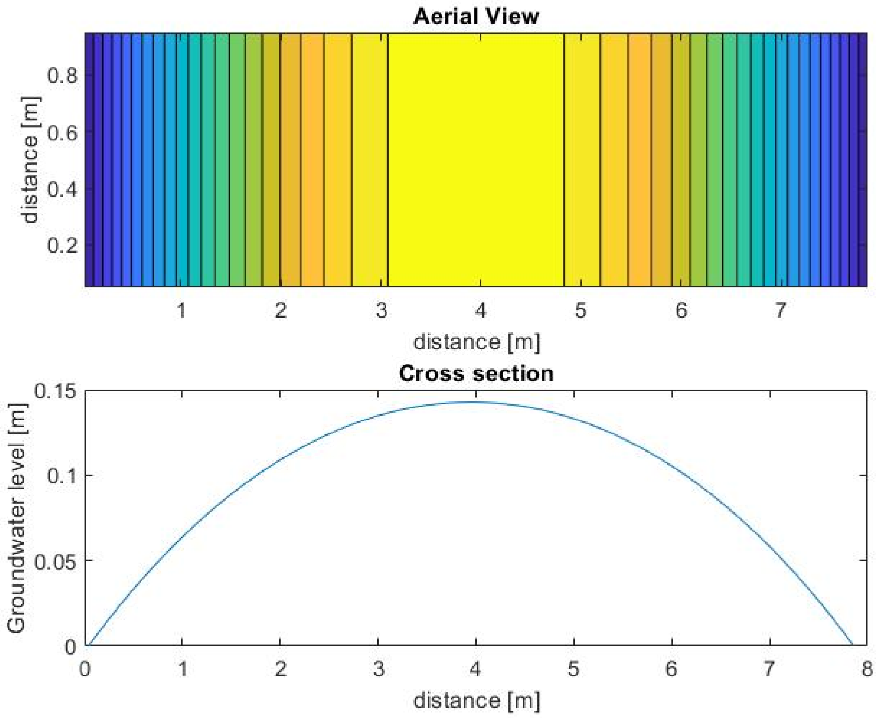

Figure 3: *Groundwater level for 100 days with higher hydraulic conductivity.*

3.2.9

In order to include a slope in the impermeable layer the following code is used:

```
% system constants
DemClay(1:NPixY,1:NPixX) = -1;
for i=1:NPixY
DemClay(i,:)   = -0.5/(NPixY-1)-1+i*0.5/(NPixY-1);
end
```

Whitin the dynamic loop, KDX and KDY are modified as follows:

```
KDX=K*(0.5*(H(:,1:NPixX-1)+H(:,2:(NPixX)))-DemClay(:,1:NPixX-1))*PixY;
KDY=K*(0.5*(H(1:NPixY-1,:)+H(2:(NPixY),:))-DemClay(1:NPixY-1,:))*PixX;
```

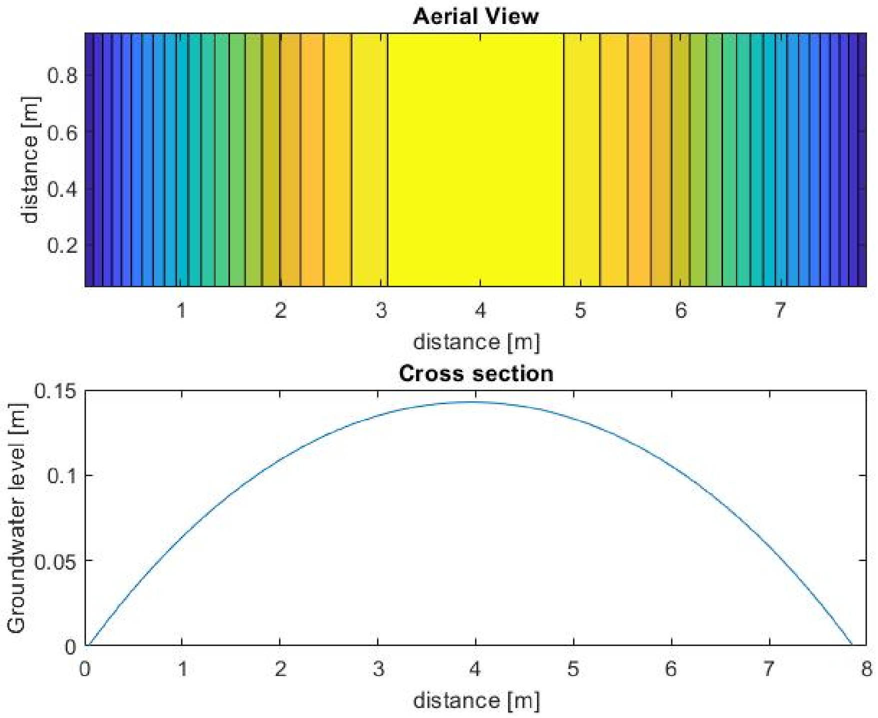

Figure 4: *Cross-section after 10 days at y=3.*

3.2.10

Running the model for 10 days the groundwater level at y=3 is shown in Figure 5, while at y=29 is shown in Figure 6. From these figures it is possible to see that the runoff happens mainly on the right side of the ground. In the other parts, no runoff is present after 10 days.

If we run the model for 100 days we obtain the groundwater level shown in Figure 7.

After 10 days the runoff is spread all around the ground area in a non-ordered way. This means that there is a continuous exchange of water in the ground. Looking at the borders, the water level is so low that no runoff takes place.

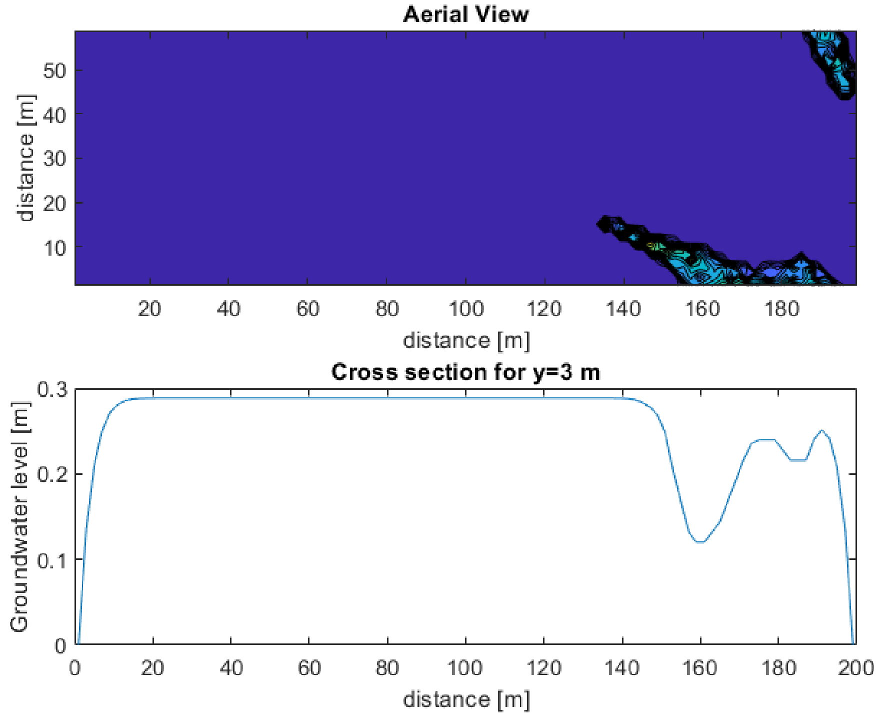

Figure 5: *Runoff and cross-section after 10 days at y=3.*

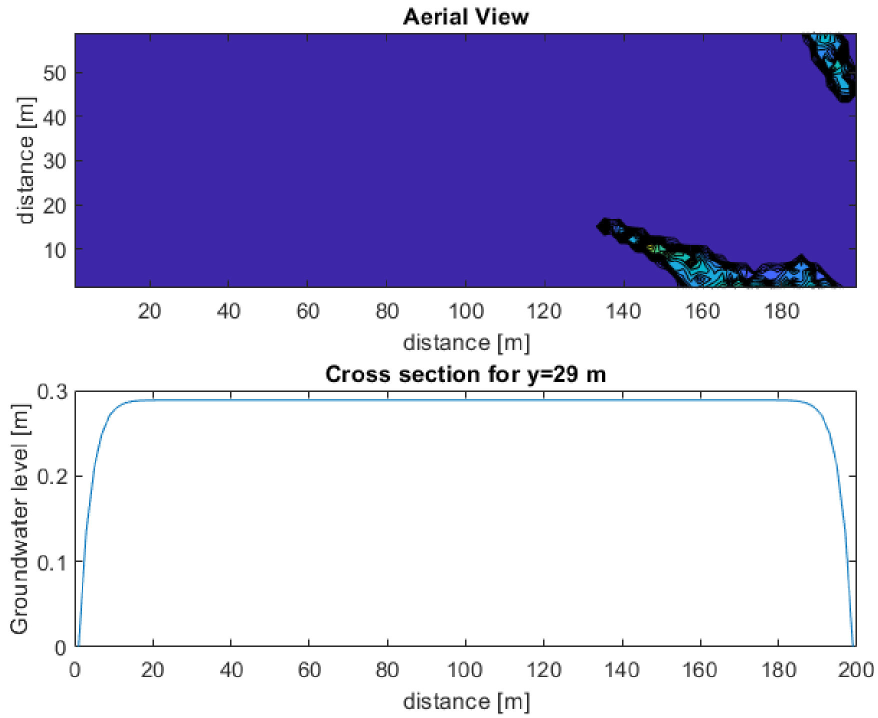

Figure 6: *Runoff and cross-section after 10 days at y=29.*

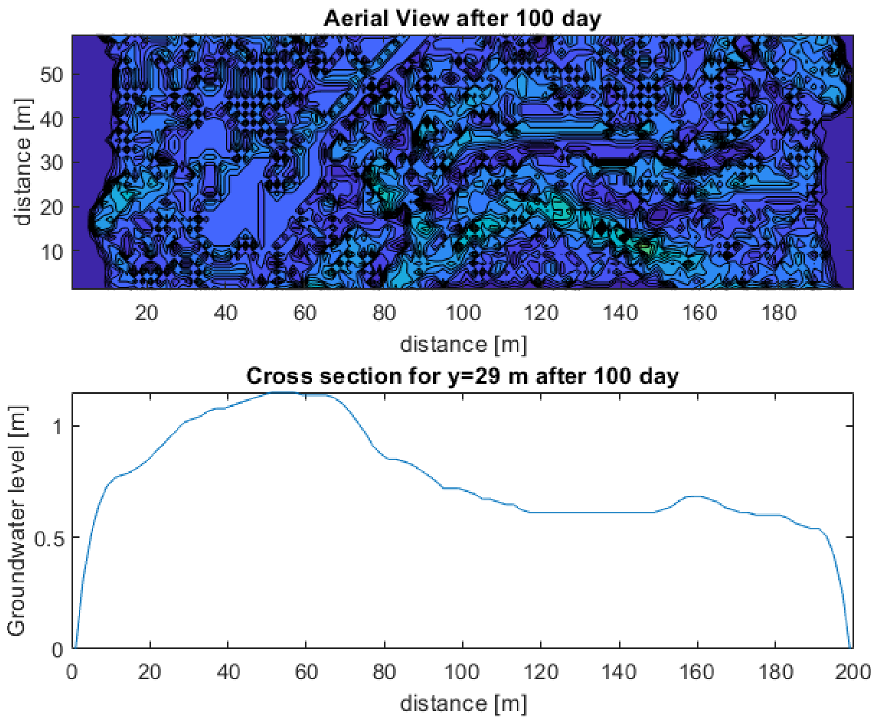

Figure 7: *Runoff and cross-section after 100 days.*