Assignment 3:
Self-organizing systems

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Exercise: Self-organization of vegetation in arid ecosystems.

4.6.1

The term describing water infiltration into the soil is expressed as follows:

$$Infiltration = \alpha * O * \frac{P + k_2 * W_0}{P + k_2} \tag{1}$$

Water infiltration into the bare soil is calculated setting $P = 0$. The result is

$$Infiltration = \alpha * O * W_0 \tag{2}$$

When plant density goes to infinity, the infiltration is given by

$$Infiltration = \alpha * O \tag{3}$$

Keeping the value of the surface water O constant, the relationship between water infiltration and plant density is reported in Figure 1.

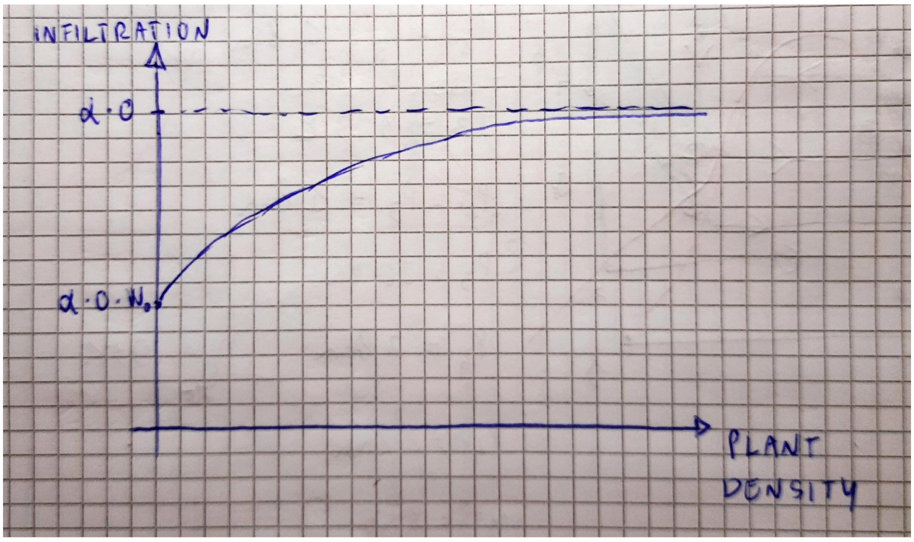

Figure 1: *Vegetation s*

Looking at equations 1a and 1b in Rietkerk et al. 2002, more infiltration translates into more soil water, which in turn increases the plant density. Since higher plant density implies higher infiltration, a positive feedback loop in created. However, when the soil reaches the saturation point this loop is interrupted.

4.6.2

The dimension of flows are respectively

$$[Flow\ P] = \frac{m^2}{d} * \frac{g}{m^2} * \frac{1}{m} = \frac{g}{d * m} \tag{4}$$

$$[Flow\ O] = \frac{m^2}{d} * mm * \frac{1}{m} = \frac{mm * m}{d} \tag{5}$$

$$[Flow\ W] = \frac{m^2}{d} * mm * \frac{1}{m} = \frac{mm * m}{d} \tag{6}$$

while for the net flows we have

$$[NetFlow\ P] = \frac{g}{d * m^2} \tag{7}$$

$$[NetFlow\ O] = \frac{mm}{d} \tag{8}$$

$$[NetFlow\ W] = \frac{mm}{d} \tag{9}$$

The dimensions of the updates are

$$[Update\ P] = \frac{g}{m^2} + \frac{g}{d * m^2} * d + \frac{g}{d * m^2} * d = \frac{g}{m^2} \tag{10}$$

$$[Update\ O] = mm + \frac{mm}{d*} * d + \frac{mm}{d} * d = mm \tag{11}$$

$$[Update\ W] == mm + \frac{mm}{d*} * d + \frac{mm}{d} * d = mm \tag{12}$$

The map of the vegetation density after 1000 days is given in Figure 2

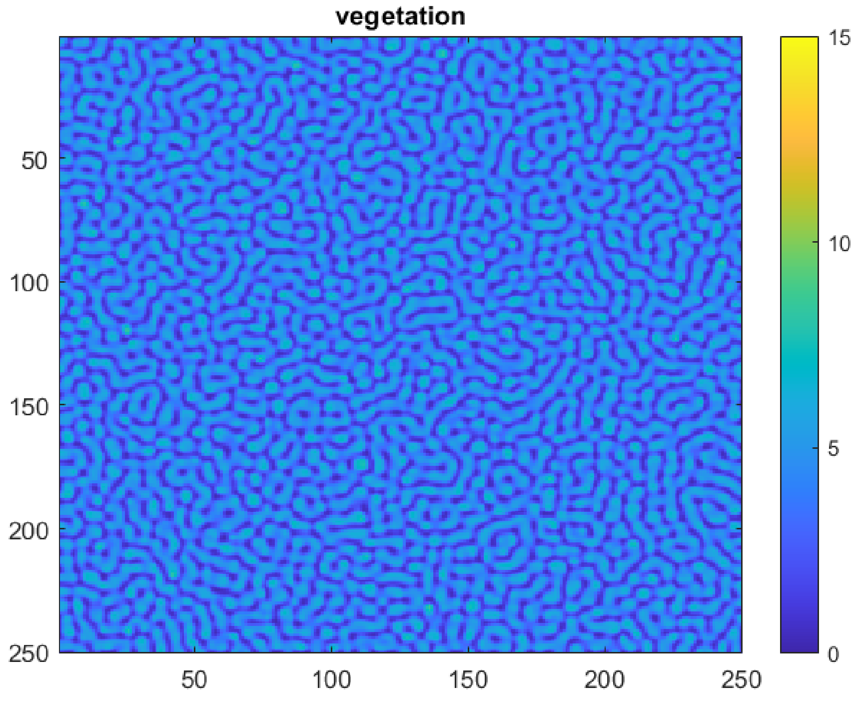

Figure 2: *Vegetation on the ground after 1000 days.*

The dynamics of the simulation shows that at first the system goes towards a higher density of vegetation. At a certain point, vegetation density is too high to be sustained by the amount of rainfall. In this moment, vegetation density crashes and the soil almost reaches the bare state. Afterwards, the state reaches equilibrium with pattern vegetation of a labyrinth with spots (Figure 2).

The main assumptions of the used model are: flatness of the ground; homogeneity of soil composition; the amount of rain is constant; the type vegetation is the same throughout the entire ground; all the needed nutrients for vegetation growth are assumed to be present in sufficient amount and they don't pose any limitation.

4.6.3

The parameter W_0 describes the share of water that infiltrate in bare soil. The more porous is the soil, the higher it will be W_0.

Running the model with $W_0 = 0.5$ (Figure 3) shows that, after 1000 days, the soil is completely bare. This phenomenon is explained in the following way: when the infiltration in the bare soil is high, there is less runoff, which means that the vegetation areas do not have enough water to sustain themselves.

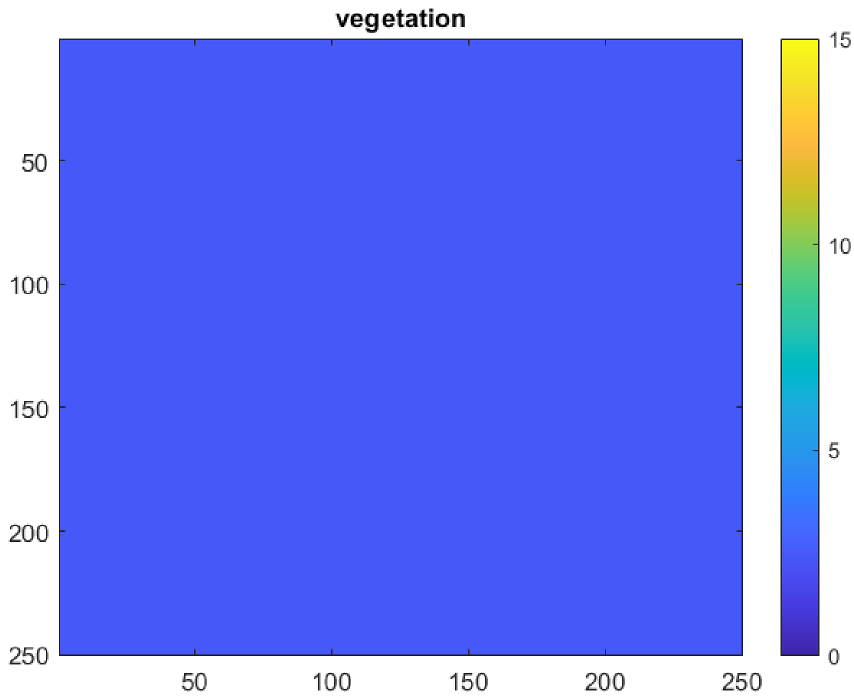

Figure 3: *Vegetation after 1000 days with a bare soil infiltration* $W_0 = 0.5$

The plants grow in spatial patterns because the runoff of water on the ground changes as the plant density changes. If the soil is bare, the runoff is greater and therefore there is less water infiltration. On the other hand, the spots with higher plant density will have higher soil water (leading to greater plant growth around these spots). In the model this behaviour is represented by the term

$$\alpha * O * \frac{P + k_2 * W_0}{P + k_2}$$

in equation 1.c in Rietkerk et al. 2002.

This phenomenon could be experimentally verified creating artificial rain on a portion of soil which is partly bare soil and partly covered by vegetation. Measuring then the runoff water in each of the two parts the behaviour that the model predicts can be verified.

4.6.4

When the rainfall parameter is increased to 1.5, the equilibrium state is reached faster. In this case, the vegetation density increases at first reaching values around 15. After that, the density slightly decreases approaching an equilibrium state where the ground is completely filled with vegetation of constant density of ~ 8 (Figure 4. This means that the rainfall is enough to sustain the vegetation on the entire ground.

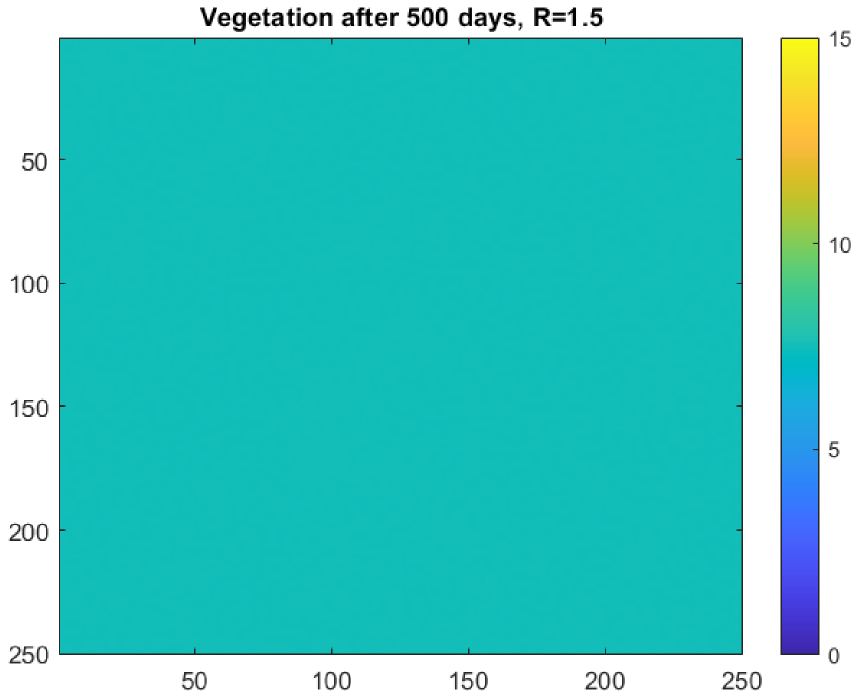

Figure 4: *Equilibrium state of the vegetation after 500 days with the rainfall parameter R=1.5*

On the other hand, when the rainfall is decreased to 1.1, the system takes more time to reach the equilibrium. The vegetation in the system increases at first. However, the rainfall in not enough to sustain the vegetation and its density drops to near 0. After that, some spots develop and start "reproducing" leading to a spotted pattern equilibrium state (Figure 6).

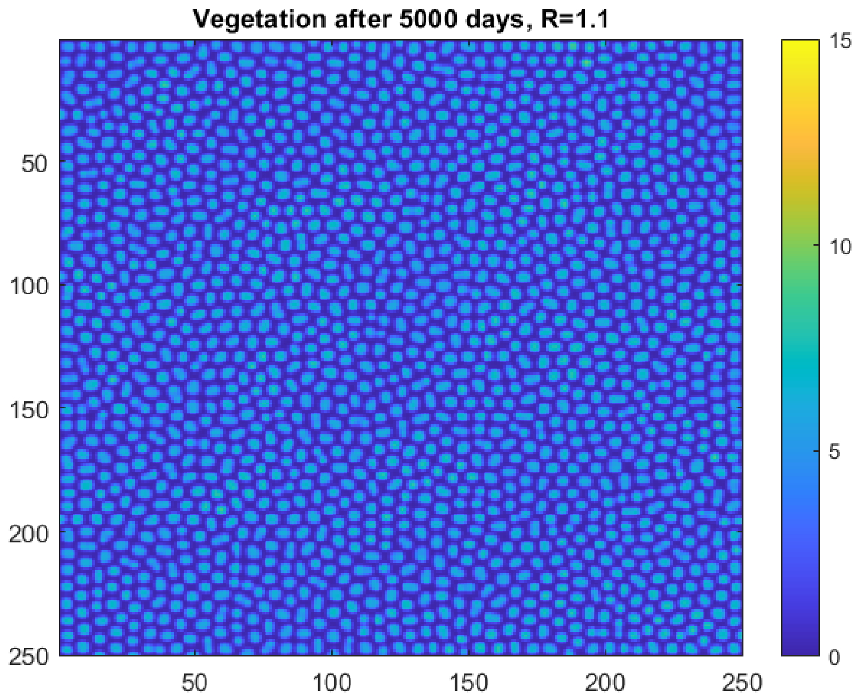

Figure 5: *Equilibrium state of the vegetation after 5000 days with the rainfall parameter R=1.1*

Increasing the grazing from 0.4 to 0.45 quickly leads the system towards the bare soil state. This means that the system is extremely sensitive to the grazing parameter.

Reducing the amount of initial biomass from 90 to 0.1, the system takes more time to approach an equilibrium state. However, looking at Figure 7, it is possible to notice that the equilibrium state of the system does not change. The underlying reason behind this behaviour is that the equilibrium state is determined only by the parameters in equations 1a, 1b and 1c in Rietkerk et al. 2002. Therefore, the initial condition affect only the time scale of the system evolution, but not its fundamental properties.

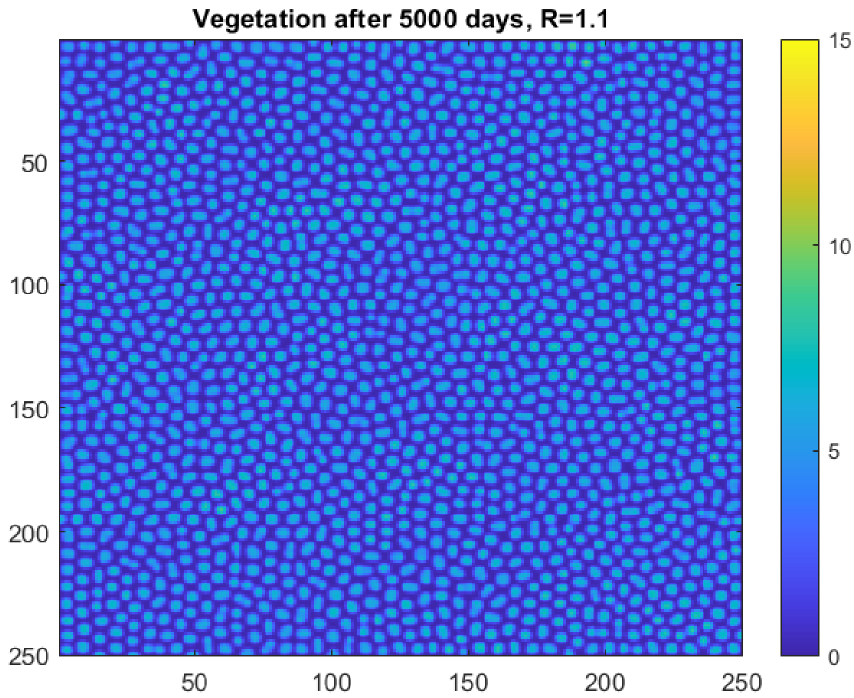

Figure 6: *Equilibrium state of the vegetation after 500 days with the grazing parameter d=0.45*

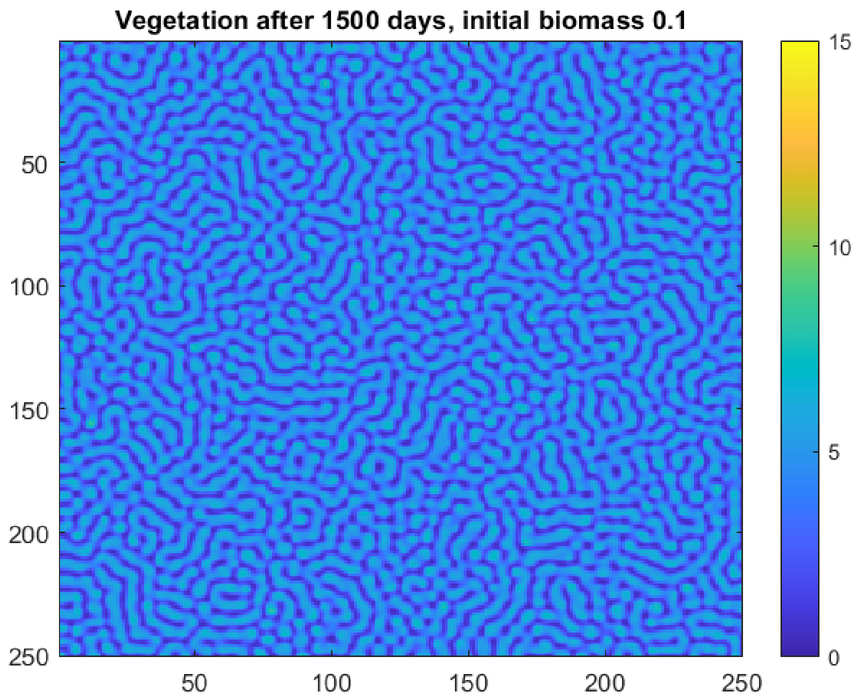

Figure 7: *Equilibrium state of the vegetation after 1500 days with initial biomass 0.1*

4.6.5

In the case of a non-flat ground, boundaries conditions are needed. Since we are dealing with a small, finite system for computational reasons, without any boundary conditions the system would behave like an island (completely disconnected from the rest of the environment). This is an unrealistic scenario. Boundaries conditions connect each pair of extremities of the system, approximating an infinite (that means, much larger) system.

Running the model with a slope (Figure 8), the vegetation creates a regular band pattern. However, the system never reaches an equilibrium state: the vegetation is continuously moving in the opposite direction of the water flow.

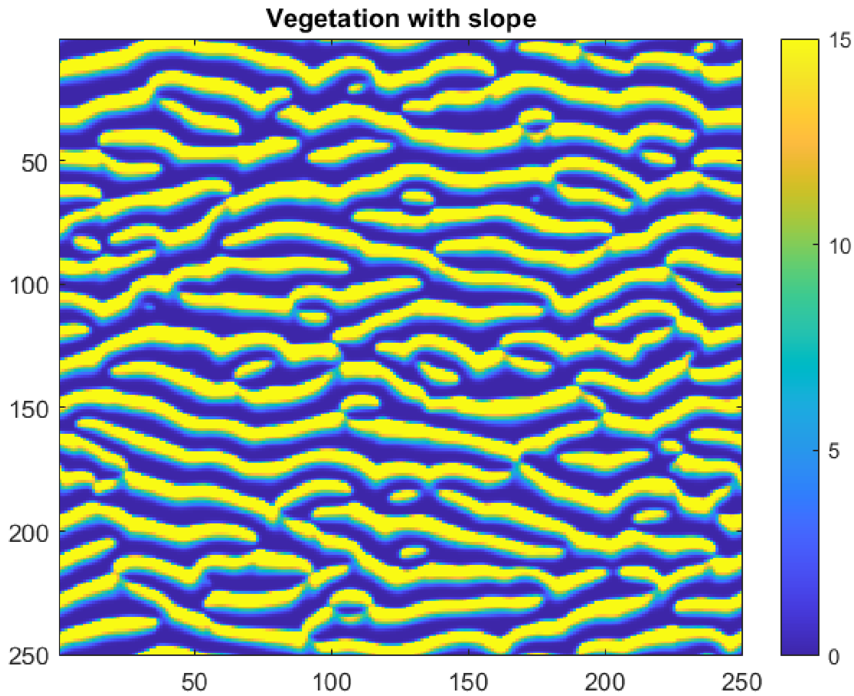

Figure 8: *Vegetation state after 1500 days with a slope*

The direction of the water flow can be changed by changing the sign of the velocity v (from positive to negative).