Extra-Análisis factorial

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Se instalan las paqueteria

```
install.packages("datos")
library(datos)
```

1.- Lectura de la matriz de datos

```
C<-data.frame(datos::mtautos)
C<-as.data.frame(mtautos)</pre>
```

1.1.- Exploración de matriz

```
dim(mtautos)
## [1] 32 11
colnames (mtautos)
   [1] "millas"
                       "cilindros"
                                     "cilindrada"
                                                    "caballos"
                                                                   "eje"
                      "velocidad"
   [6] "peso"
                                     "forma"
                                                    "transmision"
                                                                   "cambios"
## [11] "carburadores"
str(mtautos)
## 'data.frame':
                   32 obs. of 11 variables:
## $ millas : num 21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...
## $ cilindros : num
                        6 6 4 6 8 6 8 4 4 6 ...
## $ cilindrada : num 160 160 108 258 360 ...
## $ caballos : num 110 110 93 110 175 105 245 62 95 123 ...
                 : num 3.9 3.9 3.85 3.08 3.15 2.76 3.21 3.69 3.92 3.92 ...
## $ eje
## $ peso
                 : num 2.62 2.88 2.32 3.21 3.44 ...
## $ velocidad : num 16.5 17 18.6 19.4 17 ...
                 : num 0 0 1 1 0 1 0 1 1 1 ...
## $ transmision : num 1 1 1 0 0 0 0 0 0 ...
   $ cambios : num 4 4 4 3 3 3 3 4 4 4 ...
   $ carburadores: num 4 4 1 1 2 1 4 2 2 4 ...
#2.- Quitar los espacios de los nombres
#colnames(C)[1]="Life.Exp"
#colnames(C)[11] = "HS. Grad"
```

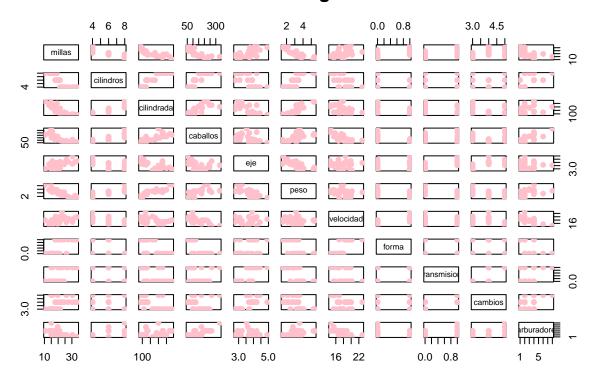
```
esto no va tus nombres estan bien
```

```
#3.- Separa n (autos) y p (variables)
n<-dim(C)[1]
p<-dim(C)[11]
```

4.- Generacion de un scater plot para la visualización de variables originales

```
pairs(C, col="pink", pch=19, main="matriz original")
```

matriz original



Transformación de alguna varibles

```
\#1.\text{-} Aplicamos logaritmo para las columnas 1,3 y 8
```

las cantidades en tus variables son pequeñas no hay necesidad de sacarles logartimo; el logarimo lo utilizamos cuando las cantidades en las variables son muy grandes. # Grafico scater para la visualizacion de la # matriz original con 3 variables que se incluyeron

```
#pairs(C,col="red", pch=19, main="Matriz original")
```

ya no seria necesaari esto por lo antes mencionado. # Nota: Como las variables tiene diferentes unidades # de medida, se va a implementar la matriz de # correlaciones para estimar la matriz de carga

Reduccion de la dimensionalidad

Análsis Factorial de componentes principales (PCFA)

#1.- Calcular la matriz de medias y de correlaciones # Matriz de medias

```
mu<-colMeans(C)
mu
##
         millas
                   cilindros
                                cilindrada
                                               caballos
                                                                  eje
                                                                              peso
##
      20.090625
                    6.187500
                                230.721875
                                             146.687500
                                                             3.596563
                                                                          3.217250
##
      velocidad
                       forma
                              transmision
                                                cambios carburadores
      17.848750
                                  0.406250
                                               3.687500
                    0.437500
                                                             2.812500
#Matriz de correlaciones
R<-cor(C)
R
##
                    millas
                           cilindros cilindrada
                                                    caballos
                                                                                peso
                                                                      eje
## millas
                 1.0000000 - 0.8521620 - 0.8475514 - 0.7761684
                                                              0.68117191 -0.8676594
                -0.8521620
                                                   0.8324475 -0.69993811
## cilindros
                            1.0000000
                                        0.9020329
                                                                           0.7824958
## cilindrada
                -0.8475514
                            0.9020329
                                        1.0000000
                                                   0.7909486 - 0.71021393
                -0.7761684
## caballos
                            0.8324475
                                        0.7909486
                                                   1.0000000 -0.44875912
                                                                           0.6587479
## eje
                 0.6811719 -0.6999381 -0.7102139 -0.4487591
                                                              1.00000000 -0.7124406
                                                   0.6587479 -0.71244065
                                                                          1.0000000
## peso
                -0.8676594 0.7824958
                                       0.8879799
## velocidad
                 0.4186840 -0.5912421 -0.4336979 -0.7082234
                                                              0.09120476 -0.1747159
                 0.6640389 -0.8108118 -0.7104159 -0.7230967
## forma
                                                              0.44027846 -0.5549157
                 0.5998324 -0.5226070 -0.5912270 -0.2432043
                                                              0.71271113 -0.6924953
## transmision
## cambios
                 0.4802848 -0.4926866 -0.5555692 -0.1257043
                                                              0.69961013 -0.5832870
  carburadores -0.5509251
                            0.5269883
                                        0.3949769
                                                  0.7498125 -0.09078980 0.4276059
##
                  velocidad
                                  forma transmision
                                                       cambios carburadores
## millas
                 0.41868403
                             0.6640389
                                         0.59983243
                                                     0.4802848
                                                                 -0.55092507
## cilindros
                -0.59124207 -0.8108118 -0.52260705 -0.4926866
                                                                  0.52698829
## cilindrada
                -0.43369788 -0.7104159 -0.59122704 -0.5555692
                                                                  0.39497686
## caballos
                -0.70822339 -0.7230967 -0.24320426 -0.1257043
                                                                  0.74981247
## eje
                 0.09120476
                             0.4402785
                                         0.71271113 0.6996101
                                                                 -0.09078980
## peso
                -0.17471588 -0.5549157 -0.69249526 -0.5832870
                                                                  0.42760594
                             0.7445354 -0.22986086 -0.2126822
## velocidad
                 1.00000000
                                                                 -0.65624923
## forma
                 0.74453544
                             1.0000000
                                         0.16834512
                                                     0.2060233
                                                                 -0.56960714
## transmision
                -0.22986086
                             0.1683451
                                         1.00000000
                                                     0.7940588
                                                                  0.05753435
## cambios
                -0.21268223
                             0.2060233
                                                     1.0000000
                                                                  0.27407284
                                         0.79405876
## carburadores -0.65624923 -0.5696071 0.05753435
                                                                  1.00000000
                                                     0.2740728
```

2.- Reducción de la dimensionalidad mediante

Análisis factorial de componentes principales (PCFA).

1.- Calcular los valores y vectores propios.

```
eR<-eigen(R)
```

2.- Valores propios

```
eigen.val<-eR$values
eigen.val

## [1] 6.60840025 2.65046789 0.62719727 0.26959744 0.22345110 0.21159612

## [7] 0.13526199 0.12290143 0.07704665 0.05203544 0.02204441
```

3.- Vectores propios

```
eigen.vec<-eR$vectors
eigen.vec
##
             [,1]
                        [,2]
                                    [,3]
                                                [,4]
                                                           [,5]
                                                                     [,6]
   [1,] 0.3625305 -0.01612440 -0.22574419 -0.022540255 -0.10284468 0.10879743
##
   [2,] -0.3739160 -0.04374371 -0.17531118 -0.002591838 -0.05848381 -0.16855369
   [3,] -0.3681852  0.04932413 -0.06148414
                                        0.256607885 -0.39399530
    \begin{smallmatrix} [4,] & -0.3300569 & -0.24878402 & 0.14001476 & -0.067676157 & -0.54004744 & -0.07143563 \end{smallmatrix} 
##
   [5,] 0.2941514 -0.27469408
                             [6,] -0.3461033 0.14303825
                                        0.245899314 0.07502912
                                                               0.46493964
##
                             0.34181851
   [7,]
        0.2004563
                   0.46337482
                             0.40316904 0.068076532
                                                    0.16466591
##
   [8,]
        0.3065113 0.23164699
                             0.42881517 -0.214848616 -0.59953955 -0.19401702
        0.2349429 -0.42941765 -0.20576657 -0.030462908 -0.08978128
                             0.28977993 -0.264690521 -0.04832960
##
        0.2069162 -0.46234863
                                                                0.24356284
  [11,] -0.2140177 -0.41357106 0.52854459 -0.126789179 0.36131875 -0.18352168
##
                                       [,9]
##
               [,7]
                           [,8]
                                                 [,10]
                                                             [,11]
##
        0.367723810 0.754091423 0.235701617
                                            0.13928524
                                                       0.124895628
        0.057277736 0.230824925
                                0.054035270 -0.84641949
##
                                                       0.140695441
##
   [3,] 0.214303077 -0.001142134 0.198427848
                                            0.04937979 -0.660606481
##
   [4,] -0.001495989 0.222358441 -0.575830072
                                            0.24782351
                                                       0.256492062
   [5,] 0.021119857 -0.032193501 -0.046901228 -0.10149369
                                                       0.039530246
##
   [6,] -0.020668302 0.008571929
                               0.359498251 0.09439426
                                                       0.567448697
   [7,] 0.050010522 0.231840021 -0.528377185 -0.27067295 -0.181361780
   [8,] -0.265780836 -0.025935128 0.358582624 -0.15903909 -0.008414634
   0.605097617 -0.336150240 -0.001735039 -0.21382515 0.053507085
```

4.- Calcular la proporcion de variabilidad

```
prop.var<-eigen.val/sum(eigen.val)
prop.var</pre>
```

```
## [1] 0.600763659 0.240951627 0.057017934 0.024508858 0.020313737 0.019236011
## [7] 0.012296544 0.011172858 0.007004241 0.004730495 0.002004037
```

5.- Calcular la proporcion de variabilidad acumulada

```
prop.var.acum<-cumsum(eigen.val)/sum(eigen.val)
prop.var.acum

## [1] 0.6007637 0.8417153 0.8987332 0.9232421 0.9435558 0.9627918 0.9750884
## [8] 0.9862612 0.9932655 0.9979960 1.0000000</pre>
```

Estimacion de la matriz de carga

Nota: se estima la matriz de carga usando los autovalores y autovectores.

se aplica la rotación varimax

Primera estimación de Lamda mayuscula se calcula multiplicando la matriz de los 3 primeros autovectores por la matriz diagonal formada por la raiz cuadrada de los primeros 3 autovalores.

```
L.est.1<-eigen.vec[,1:3] %*% diag(sqrt(eigen.val[1:3]))</pre>
L.est.1
##
               [,1]
                           [,2]
                                      [,3]
   [1,] 0.9319502 -0.02625094 -0.17877989
  [2,] -0.9612188 -0.07121589 -0.13883907
  [3,] -0.9464866 0.08030095 -0.04869285
   [4,] -0.8484710 -0.40502680 0.11088579
   [5,] 0.7561693 -0.44720905 0.12765473
   [6,] -0.8897212  0.23286996  0.27070586
   [7,] 0.5153093 0.75438614 0.31929289
   [8,] 0.7879428 0.37712727 0.33960355
   [9,] 0.6039632 -0.69910300 -0.16295845
## [10,] 0.5319156 -0.75271549 0.22949350
## [11,] -0.5501711 -0.67330434 0.41858505
```

Rotación varimax

```
L.est.1.var<-varimax(L.est.1)
L.est.1.var
```

```
## $loadings
##
##
  Loadings:
##
                       [,3]
         [,1]
                [,2]
##
    [1,] 0.542 -0.664 0.408
   [2,] -0.342 0.617 -0.671
##
   [3,] -0.345 0.715 -0.523
##
    [4,] -0.629 0.296 -0.642
##
    [5,]
                -0.848 0.258
##
    [6,] -0.512 0.782 -0.213
   [7,] 0.284 0.182 0.907
    [8,] 0.232 -0.279 0.864
##
##
   [9,] 0.108 -0.921 -0.145
## [10,] -0.259 -0.914
   [11,] -0.854 -0.107 -0.437
##
##
                   [,1] [,2]
                               [,3]
## SS loadings
                  2.132 4.557 3.197
## Proportion Var 0.194 0.414 0.291
  Cumulative Var 0.194 0.608 0.899
##
## $rotmat
                                     [,3]
##
              [,1]
                          [,2]
## [1.] 0.4405412 -0.69407713 0.5693684
## [2,] 0.4125491 0.71981583 0.5582727
## [3,] -0.7973247 -0.01104967 0.6034494
```

Estimación de la matriz de los errores

```
#1.- Estimación de la matriz de perturbaciones
```

```
Psi.est.1<-diag(diag(R-as.matrix(L.est.1.var$loadings))**% t(as.matrix(L.est.1.var$loadings))))
Psi.est.1
      [,2]
         [,3]
##
   [,1]
            [,4]
              [,5]
                 [.6]
##
 ##
 [3,] 0.00000000 0.00000000 0.09534379 0.0000000 0.0000000 0.00000000
[4,] 0.00000000 0.00000000 0.00000000 0.1037545 0.0000000 0.00000000
##
##
 ##
 ##
##
##
   [,7]
      [,8]
        [,9]
           [,10]
##
 ##
##
##
##
##
```

```
## [8,] 0.00000000 0.1215906 0.000000 0.0000000 0.00000000
## [9,] 0.00000000 0.0000000 0.119928 0.0000000 0.00000000
## [10,] 0.00000000 0.0000000 0.000000 0.0978179 0.00000000
## [11,] 0.00000000 0.0000000 0.0000000 0.06875959
```

2.- Se utiliza el método Análisis de factor principal (PFA)

para estimación de autovalores y autovectores

```
RP<-R-Psi.est.1
R.P
##
                 millas cilindros cilindrada
                                            caballos
                                                                    peso
                                                           eje
## millas
              0.9011826 -0.8521620 -0.8475514 -0.7761684 0.68117191 -0.8676594
## cilindros -0.8521620 0.9482895 0.9020329 0.8324475 -0.69993811 0.7824958
## cilindrada
             -0.8475514 0.9020329 0.9046562 0.7909486 -0.71021393 0.8879799
## caballos
             -0.7761684 0.8324475 0.7909486 0.8962455 -0.44875912 0.6587479
              0.6811719 -0.6999381 -0.7102139 -0.4487591 0.78808373 -0.7124406
## eje
             -0.8676594 0.7824958 0.8879799 0.6587479 -0.71244065 0.9191139
## peso
## velocidad
              0.4186840 -0.5912421 -0.4336979 -0.7082234
                                                     0.09120476 -0.1747159
## forma
              0.6640389 -0.8108118 -0.7104159 -0.7230967
                                                     0.44027846 -0.5549157
## transmision 0.5998324 -0.5226070 -0.5912270 -0.2432043
                                                     0.71271113 -0.6924953
              0.4802848 - 0.4926866 - 0.5555692 - 0.1257043 0.69961013 - 0.5832870
## cambios
## carburadores -0.5509251 0.5269883 0.3949769 0.7498125 -0.09078980 0.4276059
##
               velocidad
                             forma transmision
                                               cambios carburadores
## millas
              0.41868403 0.6640389 0.59983243 0.4802848 -0.55092507
## cilindros
             -0.59124207 -0.8108118 -0.52260705 -0.4926866
                                                       0.52698829
## cilindrada -0.43369788 -0.7104159 -0.59122704 -0.5555692
                                                       0.39497686
## caballos -0.70822339 -0.7230967 -0.24320426 -0.1257043
                                                       0.74981247
             ## eje
## peso
             -0.17471588 -0.5549157 -0.69249526 -0.5832870
                                                       0.42760594
## velocidad
           ## forma
              ## transmision -0.22986086 0.1683451 0.88007201
                                             0.7940588
                                                       0.05753435
## cambios
              -0.21268223 0.2060233
                                  0.79405876
                                             0.9021821
                                                        0.27407284
## carburadores -0.65624923 -0.5696071 0.05753435
                                             0.2740728
                                                        0.93124041
```

Calculo de la matriz de autovalores y autovectores

```
eRP<-eigen(RP)
```

Autovalores

```
eigen.val.RP<-eRP$values
eigen.val.RP

## [1] 6.50827377 2.55158506 0.53989628 0.12815610 0.12047514 0.07098103
## [7] 0.03536869 0.02770690 -0.01082465 -0.01918291 -0.06636999
```

Autovectores

```
eigen.vec.RP<-eRP$vectors
eigen.val.RP

## [1] 6.50827377 2.55158506 0.53989628 0.12815610 0.12047514 0.07098103
## [7] 0.03536869 0.02770690 -0.01082465 -0.01918291 -0.06636999
```

Proporcion de variabilidad

```
prop.var.RP<-eigen.val.RP/ sum(eigen.val.RP)
prop.var.RP

## [1] 0.658328010 0.258099148 0.054611846 0.012963307 0.012186358
## [6] 0.007179907 0.003577631 0.002802622 -0.001094940 -0.001940399
## [11] -0.006713489
```

Proporcion de variabilidad acumulada

```
prop.var.RP.acum<-cumsum(eigen.val.RP)/ sum(eigen.val.RP)
prop.var.RP.acum

## [1] 0.6583280 0.9164272 0.9710390 0.9840023 0.9961887 1.0033686 1.0069462
## [8] 1.0097488 1.0086539 1.0067135 1.0000000</pre>
```

Estimación de la matriz de cargas

con rotación varimax

```
L.est.2<-eigen.vec.RP[,1:3] %*% diag(sqrt(eigen.val.RP[1:3]))</pre>
L.est.2
##
              [,1]
                          [,2]
                                      [,3]
## [1,] 0.9250245 -0.03126684 -0.15888375
## [2,] -0.9611279 -0.06563034 -0.14100963
## [3,] -0.9398412 0.08492069 -0.05055728
## [4,] -0.8425167 -0.39170851 0.09440621
   [5,] 0.7364339 -0.42327193 0.09226618
## [6,] -0.8851139 0.23587577 0.25271063
  [7,] 0.5159435 0.74772017 0.31233902
## [8,] 0.7801603 0.36202748 0.29373198
## [9,] 0.5960270 -0.68373849 -0.13672189
## [10,] 0.5263604 -0.74209587 0.21354024
## [11,] -0.5502616 -0.66520866 0.40348209
```

Rotacion varimax

```
L.est.2.var<-varimax(L.est.2)
```

Estimación de la matriz de covarianzas de los errores.

```
Psi.est.2<-diag(diag(R-as.matrix(L.est.2.var$loadings)%*% t(as.matrix(L.est.2.var$loadings))))
Psi.est.2
##
      [,1]
            [,2]
                [,3]
                     [,4]
                          [,5]
                                [,6]
                                     [,7]
 ##
  ##
 [4,] 0.000000 0.00000000 0.0000000 0.1278176 0.0000000 0.00000000 0.00000000
 ##
 ##
       [,8]
            [,9]
                [,10]
                     [,11]
 [1,] 0.0000000 0.0000000 0.000000 0.0000000
##
 [2,] 0.0000000 0.0000000 0.000000 0.0000000
  [3,] 0.0000000 0.0000000 0.000000 0.0000000
 [4,] 0.0000000 0.0000000 0.000000 0.0000000
##
 [5,] 0.0000000 0.0000000 0.000000 0.0000000
 [6,] 0.0000000 0.0000000 0.000000 0.0000000
##
 [7,] 0.0000000 0.0000000 0.000000 0.0000000
 [8,] 0.1740075 0.0000000 0.000000 0.0000000
 [9,] 0.0000000 0.1585607 0.000000 0.0000000
## [10,] 0.0000000 0.0000000 0.126639 0.0000000
## [11,] 0.0000000 0.0000000 0.000000 0.0919118
```

Obtencion de los scores de ambos métodos

PCFA

```
FS.est.1<-scale(C)%*% as.matrix(L.est.1.var$loadings)
FS.est.1
##
                             [,1]
                                        [,2]
                                                   [,3]
## Mazda RX4
                       -0.0410256 -3.1506899 -0.8884768
                       -0.0854961 -2.8898642 -0.6597321
## Mazda RX4 Wag
                        3.3448166 -4.7100357
## Datsun 710
                                              4.0215781
## Hornet 4 Drive
                        1.9939958 2.1792130
                                              2.4991646
## Hornet Sportabout
                       -0.9971326 4.3474154 -2.7032108
## Valiant
                        1.8024768 3.1134384
                                             2.6501998
## Duster 360
                       -3.3428720 4.8900060 -4.7959068
## Merc 240D
                        2.6729260 -1.9276237
                                              4.7156274
## Merc 230
                        2.7439166 -1.7446688 5.9150571
## Merc 280
                       -0.2339269 -0.7503930
                                             1.6054121
## Merc 280C
                       -0.2646498 -0.5351551
                                             1.8152385
## Merc 450SE
                       -1.8204128 4.7390614 -2.8033838
## Merc 450SL
                       -1.5298686 4.3885003 -2.5668152
## Merc 450SLC
                       -1.6814718 4.7004501 -2.5168213
## Cadillac Fleetwood -4.2018891 7.7980184 -4.5729608
```

```
## Lincoln Continental -4.3705337 7.7837280 -4.7013419
## Chrysler Imperial -4.0676656 6.7908190 -4.5411618
## Fiat 128
                        4.7573865 -6.4262929 5.6219890
## Honda Civic
                        4.4152463 -8.2629727
                                             5.4152887
## Toyota Corolla
                       5.1941016 -7.1097770
                                              6.1303991
## Toyota Corona
                        3.4263878 -0.9002863 4.7132634
## Dodge Challenger
                       -1.0384992 5.0168149 -2.7900049
## AMC Javelin
                       -0.8796376 4.3262084 -2.3262352
## Camaro Z28
                       -3.5689167 4.2896344 -4.8477017
## Pontiac Firebird
                      -1.2766789 4.9610423 -2.9450836
## Fiat X1-9
                       4.3457889 -6.1327204 5.0437596
## Porsche 914-2
                        2.1185070 -7.0091824
                                             1.5982447
## Lotus Europa
                        3.1446320 -7.5320084
                                             3.4307529
## Ford Pantera L
                       -3.7592309 -1.6246832 -5.1731174
                       -2.3768080 -3.7181529 -2.6891172
## Ferrari Dino
## Maserati Bora
                       -6.7112287 -0.3758850 -7.1267213
## Volvo 142E
                        2.2877629 -4.5239583 3.4718175
```

PFA

```
FS.est.2<-scale(C)%*% as.matrix (L.est.2.var$loadings)
FS.est.2
```

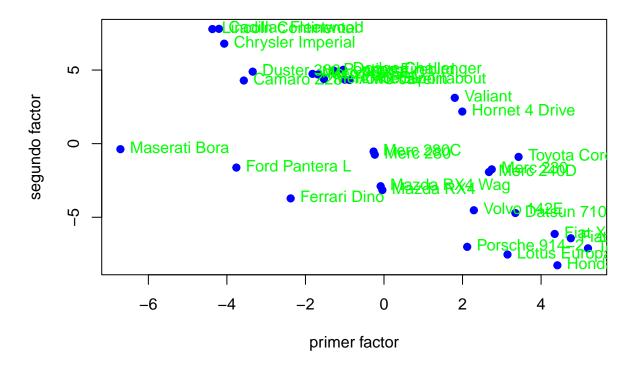
```
##
                             [,1]
                                        [,2]
                                                   [,3]
## Mazda RX4
                       -0.1077303 -3.0993659 -0.8315649
## Mazda RX4 Wag
                       -0.1509015 -2.8387206 -0.6096052
## Datsun 710
                        3.0480740 -4.7362985 4.1391518
## Hornet 4 Drive
                       1.9041708 2.0776248 2.5340651
## Hornet Sportabout
                       -0.8333251 4.3124349 -2.7060136
## Valiant
                       1.7203014 2.9941992 2.6759485
## Duster 360
                       -3.0203632 4.9082231 -4.8995476
## Merc 240D
                       2.4439804 -1.9854245 4.7510058
## Merc 230
                        2.5026052 -1.7917167
                                              5.9315159
## Merc 280
                       -0.2332308 -0.7345388 1.5135711
## Merc 280C
                       -0.2624221 -0.5199431 1.7176870
## Merc 450SE
                       -1.6214524 4.7169260 -2.8517014
## Merc 450SL
                       -1.3565535
                                  4.3620207 -2.6003060
## Merc 450SLC
                       -1.4951107 4.6752591 -2.5611735
## Cadillac Fleetwood -3.8072152 7.8032273 -4.7306551
## Lincoln Continental -3.9612694
                                  7.7967285 -4.8688271
## Chrysler Imperial
                       -3.6844879 6.8158259 -4.6958614
## Fiat 128
                        4.3300998 -6.4606071 5.7966780
## Honda Civic
                        4.0105895 -8.2370654
                                             5.5567925
## Toyota Corolla
                        4.7287845 -7.1424163 6.3213950
## Toyota Corona
                       3.1931881 -0.9877093 4.7885040
## Dodge Challenger
                       -0.8782235 4.9578913 -2.7882747
## AMC Javelin
                       -0.7296103 4.2877292 -2.3312551
## Camaro Z28
                       -3.2183860 4.3426408 -4.9770702
## Pontiac Firebird
                      -1.0873987 4.9252398 -2.9593636
## Fiat X1-9
                        3.9602386 -6.1610267 5.2012300
## Porsche 914-2
                        1.8677790 -6.9526900 1.7349393
## Lotus Europa
                        2.8053883 -7.5327158 3.5596593
## Ford Pantera L
                       -3.4921201 -1.4704866 -5.2743876
```

graficamos ambos scores

```
par(mfrow=c(2,1))
```

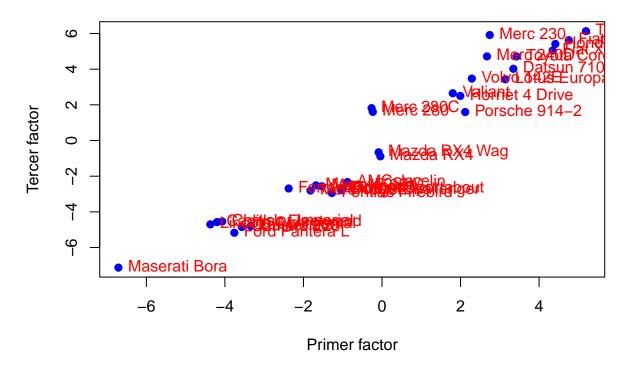
Factor I y II

scores con factor I y II con PCFA



Factor I y III

scores con factor I y III con PCFA



Factor II y III

scores con factor II y III con PCFA

