Nanjing Explained

Sunday, April 11, 2021 8:55 PM

Before we begin, there are a few variables that I would like to define:

If taking a wall sample to test, the width is always the same regardless of how the base and lengths vary.

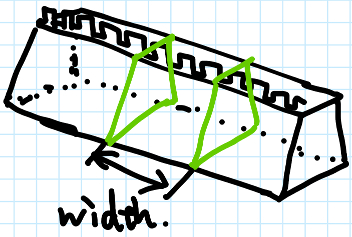


Figure 1

Densities:
Nanjing city wall is mainly made by three sections—the bricks, ramparts and bricks again.

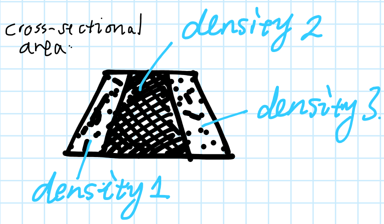


Figure 2

Height:
For Nanjing city wall (only), the height of all three sections is the same.

for convience, I will somtimes use "h" in this explanation

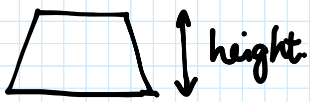


Figure 3

Lengths:

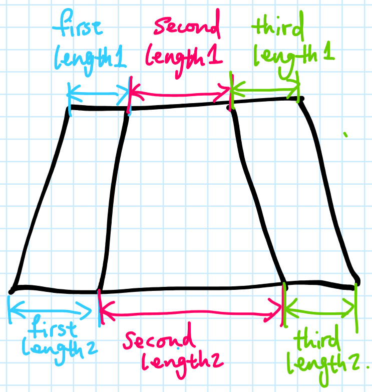


Figure 4

Mass:

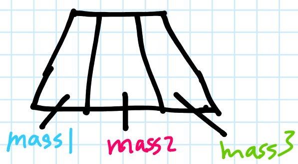


Figure 5

Use the method mentioned in (2.3.2) to find the total mass (Mass).

$$\text{Mass} = \text{mass1} + \text{mass2} + \text{mass3}.$$

In our model, both the York and Nanjing, we let all the attacks from the left hand side of the wall.

Figure 6

Equation (equation 3) of centre of mass of a trapezium.

Find the centre of masses of each sections.

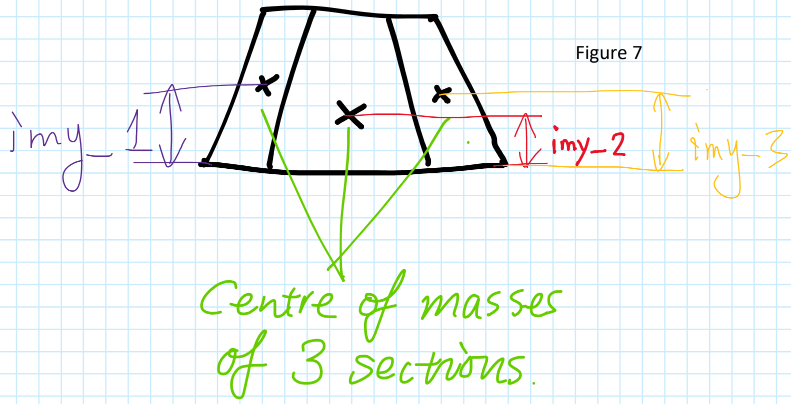


Figure 7

Equation of the average height of the centre of mass is:

$$M\bar{y}=\sum m_i y_i$$

Apply it into our model (Equation 7):

$$\text{Height of the centre of mass}=\bar{y}=imY=\frac{(mass1)\times(imy\_1)+(mass2)\times(imy\_2)+(mass3)\times(imy\_3)}{Mass}$$

iGPE (initial Gravitational Potential Energy):

$$iGPE=Mass\times imY\times 9.81$$

where 9.81 = average gravitational field strength on earth,

Define variables:

A = total cross-sectional area of the wall.

Figure 8

Length 1 and Length2 are the total top length and bottom length of the trapezium respectively.

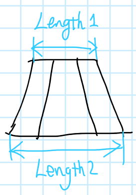


Figure 9

B is the area of the back triangle.
L is the length of the base of this triangle.
Theta is the angle of this triangle made with the ground.

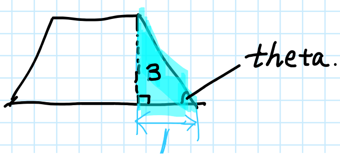
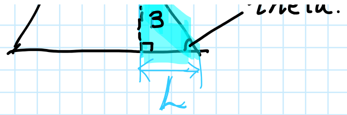
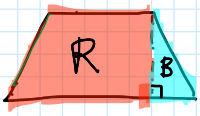


Figure 10



R is the rest of the area of the triangle, R = A - B



$R = A - B$

Figure 11

J is the area of the 45 degrees triangle formed by the collapsed wall.
J_h is the triangle J with height “h”.

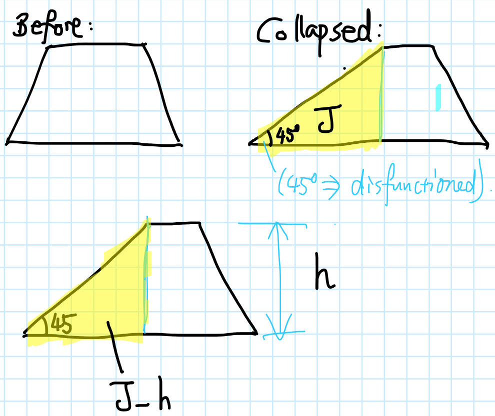


Figure 12

fmY = height of the final (disfunctioned) centre of mass.

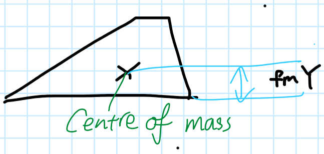


Figure 13

Calculate Area:

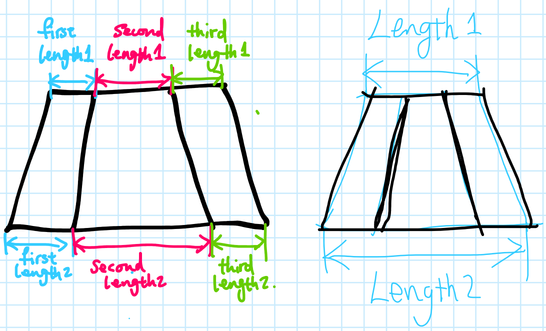
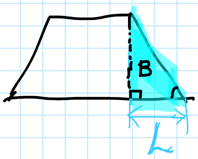


Figure 14

Length1 = first_length1 + second_length1 + third_length1
Length2 = first_length1 + second_length2 + third_length2

Calculate B and therefore R:



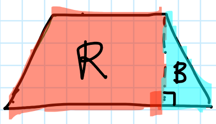
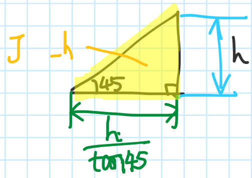


Figure 15

$$B = \frac{L \times height}{2}$$

$$R = A - B$$

Calculate J_h:



$$J\_h = \frac{\left(\frac{h}{\tan 45}\right)\times(h)}{2} = \frac{h^2}{2\tan 45^\circ}$$

Figure 16

There are two possible scenarios when the Nanjing wall collapses:
(This depends on the length, height, structure of the wall)

case1:

The area of the back(B) (right hind side) triangle stays the same, the front wall (R) does have sufficient area to collapse into a triangle that has 45 degrees to the ground.
The height does NOT change.

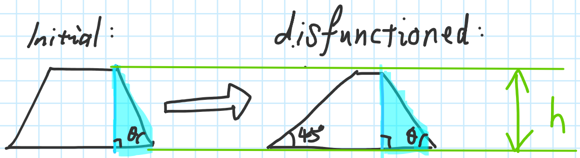


Figure 17

case2:

The area of the back (B) (right hind side) triangle becomes smaller, to provide more area to form a 45 degrees triangle (J_h), because the front wall (R) does NOT have sufficient area to collapse into a triangle that has 45 degrees to the ground.
The height DOES change.

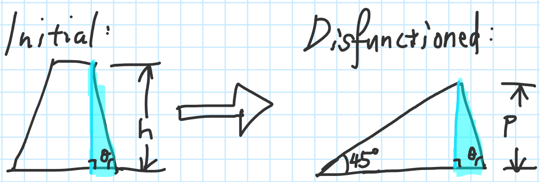
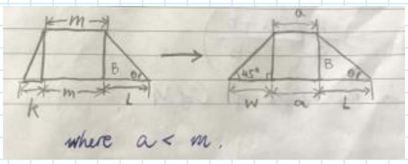


Figure 18

$$h > P$$

If R > or = J_h:
It's case 1.



L, m, k, θ are known.
a is unknown.

Deduce:

$$w = \frac{h}{\tan 45^\circ}$$

Calculate the area R:

$$R = \frac{[a+(a+w)]\times h}{2}$$

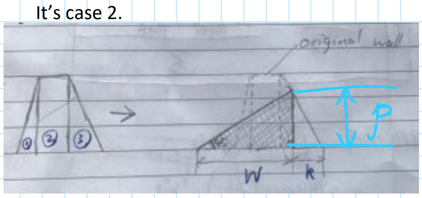
$$R = \frac{\left(2a + \frac{h}{\tan 45^\circ}\right)\times h}{2}$$

Rearrange a to the subject.

$$a = \frac{\left(\frac{2R}{h} - \frac{h}{\tan 45^\circ}\right)}{2}$$

Use the method mentioned in (2.3.1) (equation 3) to find the centre of mass.

If R < or = J_h:
It's case 2.



Sections 1 and 2 collapses to make a slop with an angle 45 degrees with the ground triangle (J with a new shorter height, not height h), section 3 also uses part of its area to fill this triangle.

unknown value: p.

Deduce:

$$w = \frac{p}{\tan 45^\circ}, \quad k = \frac{p}{\tan\theta}, \quad \theta = \tan^{-1}\left(\frac{\text{height}}{L}\right)$$

Total area of the cross-section(triangle).

$$A = \frac{(w+k)\cdot p}{2}$$

Rearrange p to the subject.

$$p = \sqrt{\frac{2A}{\frac{1}{\tan 45^\circ} + \frac{1}{\tan\theta}}}$$

$$fmY = \overline{y} = \left(\frac{h}{3}\right) \times \left(\frac{2a + (w + a + L)}{a + (w + a + L)}\right)$$

Centre of mass of a triangle is 1/3 of its height, so the final height of the centre of mass is:

substitute a and w,
simplify it further:

$$\boxed{fmY = \frac{h}{3} + \left(\frac{\frac{R}{3} - \frac{h^2}{6\tan 45°}}{\frac{2R}{h} + L}\right)}$$

$$\frac{p}{3} = \boxed{fmY = \frac{\sqrt{\frac{2A}{\frac{1}{\tan 45°} + \frac{1}{\tan\theta}}}}{3}}$$

fGPE (Final Gravitational Potential Energy):

$$\boxed{fGPE = Mass \times fmY \times 9.81}$$

Change_GPE (Change in the Gravitational Potential Energy) = iGPE - fGPE