

# Graph Neural Networks

## Introduction - Part 2

Andrei Nicolicioiu

Iulia Duta



Bitdefender<sup>®</sup>

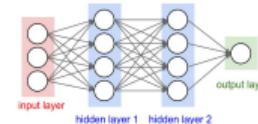
May 2021

# Choose your model

UNSTRUCTURED



Petal length	Petal width	Petal length	Petal width	Species
5.1	3.5	1.4	0.2	Iris setosa
4.9	3.0	1.4	0.2	Iris setosa
7.0	3.2	4.7	1.4	Iris versicolor
6.4	3.2	4.5	1.5	Iris versicolor
6.5	3.3	4.9	2.5	Iris virginica
5.8	3.3	4.9	2.5	Iris virginica



MLP

SEQUENTIAL

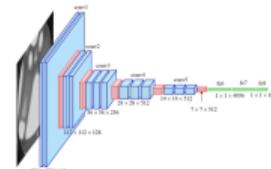
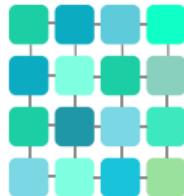


Have a nice day! :)



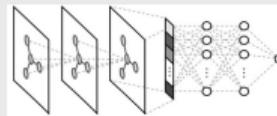
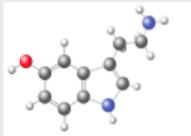
RNN

GRID



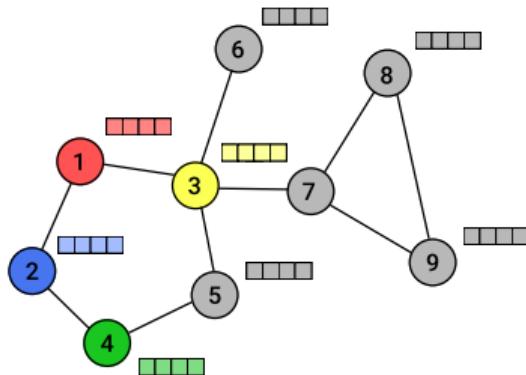
CNN

RELATIONAL STRUCTURE

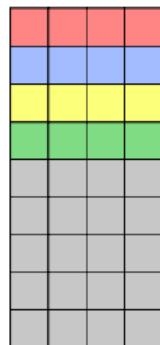


GNN

# Graph Data

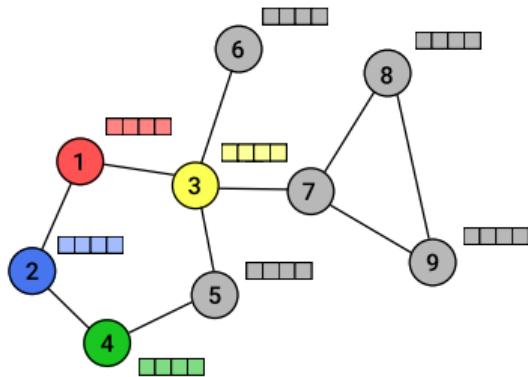


$$X \in \mathbb{R}^{N \times D}$$

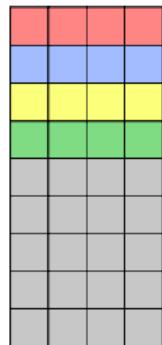


- all the nodes  $x_i \in \mathbb{R}^D$  are stacked into a matrix  $X \in \mathbb{R}^{N \times D}$
- each row corresponds to a node  $x_i \in \mathbb{R}^D$

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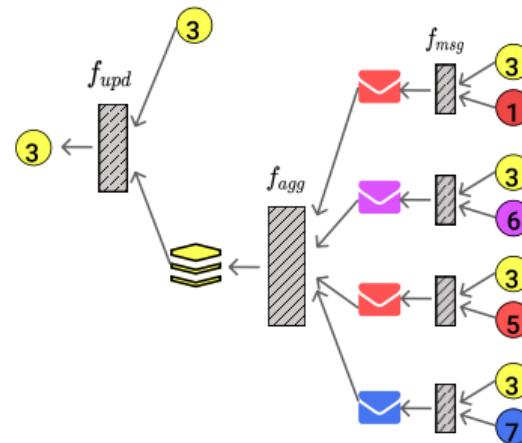
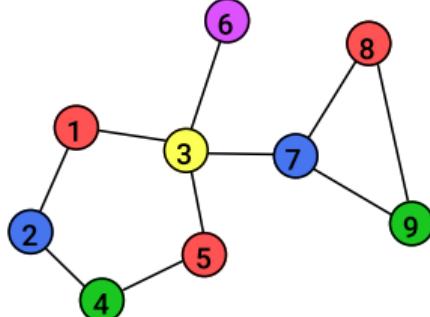


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$$f\left(\begin{array}{c} \text{graph} \\ \text{structure} \end{array}\right) = Y$$

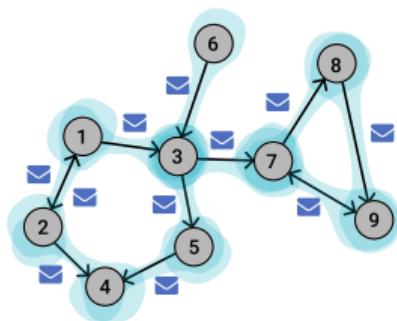
# Learning

- the output of a GNN for a node  $i$  is obtained by applying a **sequence of operations** on the initial nodes
- all the operations along the sequence should be **differentiable**



# GNNs: Message Passing Framework - Send

- $f_{msg}$  is a learnable function (e.g. an MLP)
  - its parameters are shared between each pair of nodes



$$m_{ij} = \overbrace{f_{msg}(x_i, x_j)}^{\text{Learnable function}} \in \mathbb{R}^C \quad \forall (i, j) \in \mathcal{E}$$

$$m_{3,6} = f_{msg}(\text{_____}, \text{_____})$$

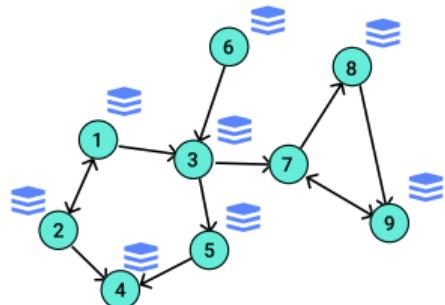
$$m_{3,1} = \overline{f}_{msg}(\textcolor{red}{\square\square\square}, \textcolor{blue}{\square\square\square}) \xleftarrow{\text{Same parameters}}$$

• • •

$$m_{4,2} = f_{msg}(\textcolor{red}{\texttt{████}}, \textcolor{blue}{\texttt{███}})$$

# GNNs: Message Passing Framework - Aggregation

- aggregate incoming messages with the function  $f_{agg}$ :
  - usually not learnable: e.g. sum, mean, max, min
  - learnable: e.g. LSTM
- it should be **invariant to the order** of the nodes and should **allow a variable number** of messages



**operator**

$$h_i = \widehat{f_{agg}} (\{m_{ij} | \forall j \in \mathcal{N}_i\}) \in \mathbb{R}^C$$

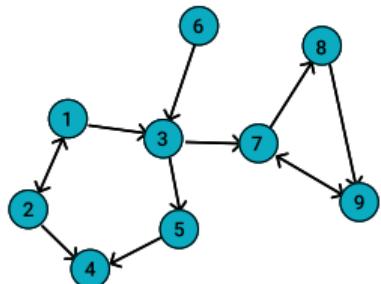
$$h_3 = f_{agg}(\{\text{✉}, \text{✉}\})$$

...

$$h_1 = f_{agg}(\{\text{✉}\})$$

# GNNs: Message Passing Framework - Update

- $f_{upd}$  is a learnable function (e.g. an MLP)
- its parameters are shared between all the nodes



Learnable function

$$\tilde{x}_i = \overbrace{f_{upd}(x_i, h_i)} \in \mathbb{R}^C$$

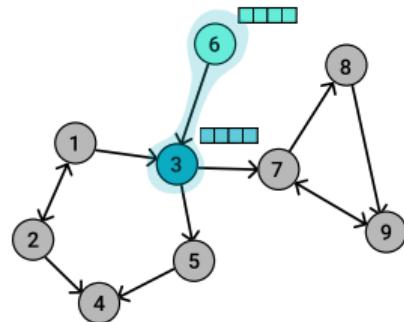
$$\begin{aligned}\tilde{x}_3 &= f_{upd}(\text{[hidden state]}, \text{[node features]}) \\ &\dots \\ \tilde{x}_2 &= f_{upd}(\text{[hidden state]}, \text{[node features]})\end{aligned}$$

Same parameters

# GNNs - Overview

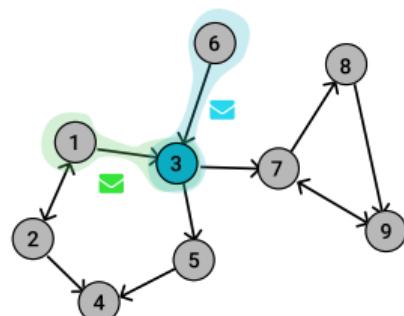
## 1. Send

$$m_{ij} = f_{msg}(x_i, x_j)$$



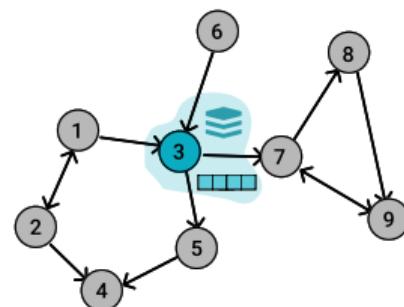
## 2. Aggregate

$$H_i = f_{agg}(\{m_{ij} | \forall j \in \mathcal{N}_i\})$$



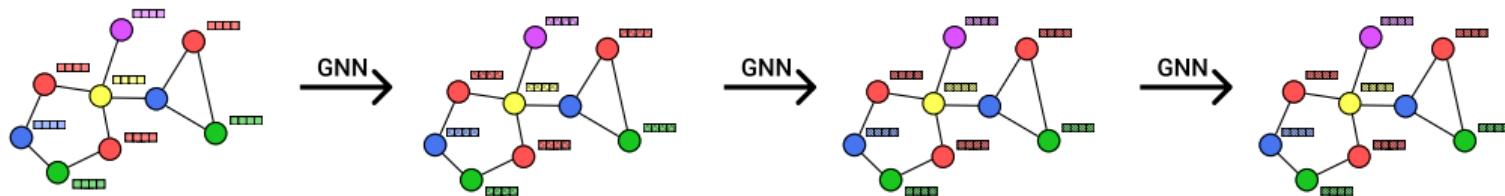
## 3. Update

$$\tilde{x}_i = f_{upd}(x_i, H_i)$$



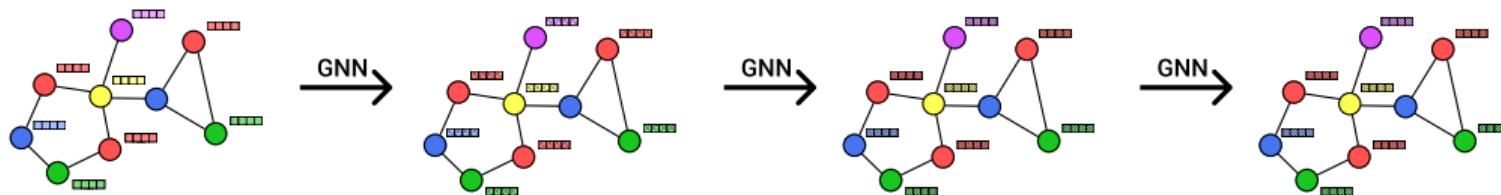
# Multiple Layers

- for a more powerful representation, we can stack multiple layers



# Multiple Layers

- for a more powerful representation, we can stack multiple layers
- each layer increases the receptive field of each node



RECEPTIVE FIELD:



# Application: Fake News Identification

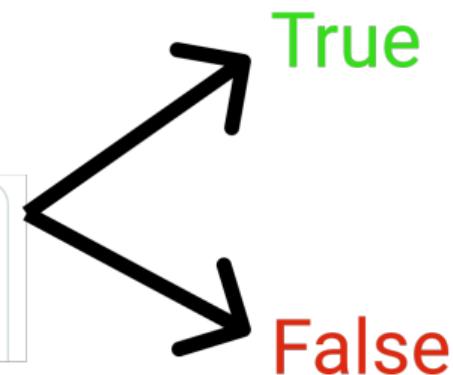
Goal: determine if a Tweet links to a fake news article

New paper from Brain Zurich and Berlin!

We try a conv and attention free vision architecture: MLP-Mixer  
[arxiv.org/abs/2105.01601](https://arxiv.org/abs/2105.01601)

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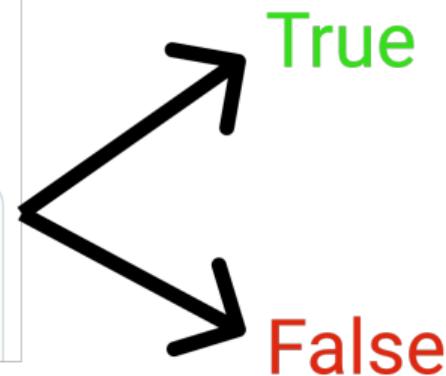
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Well, not *\*actually\** conv free.  
1st layer: "Per-patch fully-connected" == "conv layer with 16x16 kernels and 16x16 stride"  
other layers: "MLP-Mixer" == "conv layer with 1x1 kernels"

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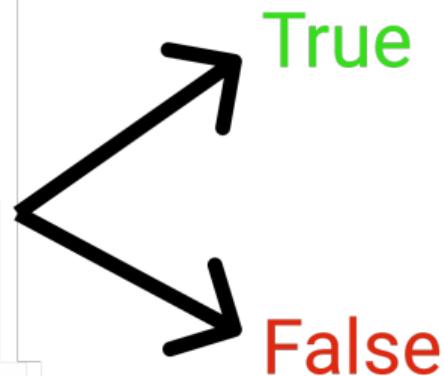
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Replying to @ [redacted] May 7

An MLP is a conv net with a Kernel that is the size of the input and no stride

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# Application: Fake News Identification

## Challenges:

- understanding news requires knowledge of political / social context
- often written in bad faith to appear real
- highly nuanced

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<sup>1</sup>[1]: Vosoughi et. al. The spread of true and false news online. Science (2018).

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### News Spread

"Falsehood diffused significantly *farther, faster, deeper, and more broadly* than the truth"<sup>1</sup>

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## Idea

- Analyse the news diffusion patterns with GNNs.

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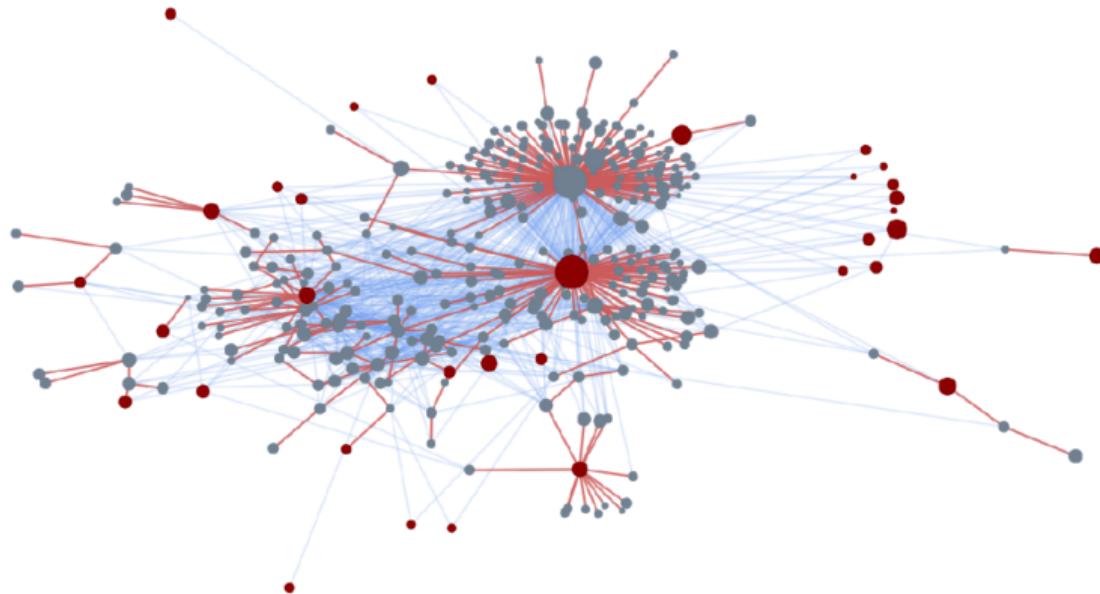
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# Application: Fake News Identification

- gather stories classified by fact-checking orgs like Snopes, PolitiFact
- for each story form a graph of all the tweets and retweets mentioning it
- edges are follow relations or retweet relations

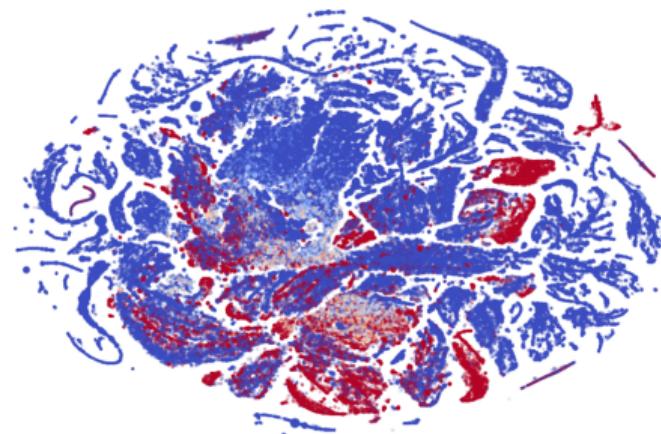
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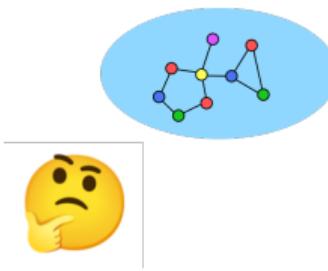
# Application: Fake News Identification

- apply standard GNN model
- node features:
  - User profile (geolocalization, language, embedding of self-description, date of account creation)
  - Network and spreading (No. of followers, timestamps, No. of replies, quotes, favorites and retweets for the source tweet)
  - Content (embeddings of tweet text).
    - Surprising: not that relevant!

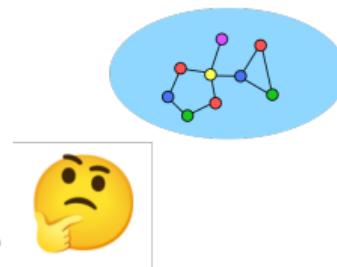


# When to use GNNs?

- What are the cases where it is beneficial to use GNNs?



# When to use GNNs?



- What are the cases where it is beneficial to use GNNs? 
- What design choices should be made for a specific task?
  - Do we want sum or max in the aggregation?
  - Should we share parameters between layers?
  - Should we use distances or order information when we have them?

# When to use GNNs?

Usual deep Learning approach:

- learn end-to-end  $f(X)$  from data with the specific model  $f$  (MLP, CNN, RNN etc.)
- each model is appropriate in certain cases

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## Inductive biases

An inductive bias allows a learning algorithm to prioritize one solution over another, independent of the observed data.

	UNSTRUCTURED	SEQUENTIAL	GRID	COMPLEX RELATIONAL STRUCTURE
Structure				
Model	MLP	RNN	CNN	GNN
Inductive Bias	Weak	Sequentiality	Locality	Strong relational bias

# Relational Reasoning

## Relational Reasoning

Manipulating *structured* data, that consists in multiple **entities** that establish various **relations** between them.

# Relational Reasoning

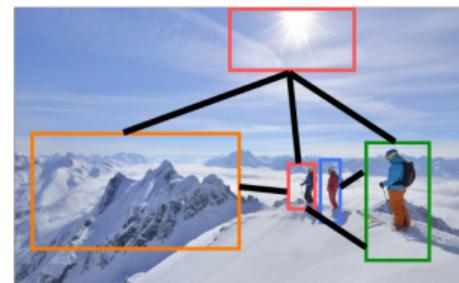
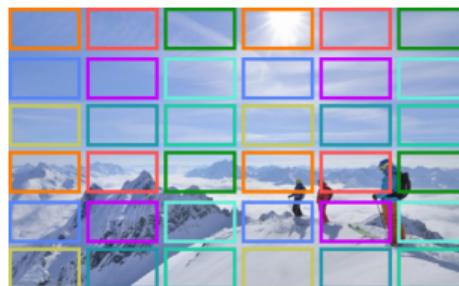
## Relational Reasoning

Manipulating *structured* data, that consists in multiple **entities** that establish various **relations** between them.

From some perspective, relational reasoning could be appealing.

For example, a visual scene could be seen as:

- an image / a grid of points
- a set of objects with multiple relations between them



# Relational Inductive Biases

## Inductive biases

An inductive bias allows a learning algorithm to prioritize one solution over another, independent of the observed data.

Relational inductive biases in GNN:

- explicit factorisation into nodes, each corresponding to an entity
- explicit modeling of pairwise relations between nodes
- flexibility in establishing different connectivity
- order invariant

# When to use a GNN?

GNNs could be appropriate if:

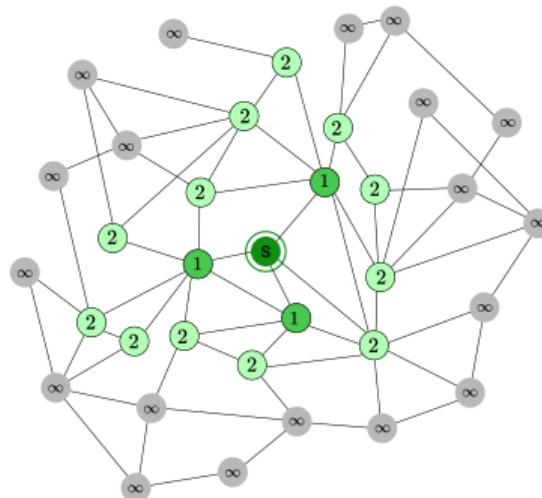
- there exist entities and relations in the data
  - explicit: social networks, molecules
  - implicit: visual scenes, environments...
- the relational processing is beneficial

# Shortest path Problem

- Lets analyse a purely reasoning problem of finding the shortest path in a graph.
  - Can GNNs solve this problem and how sample efficient are they?

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## GNN Method

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```
for layer k in 1 .. K do
    for node i in  $\mathcal{V}$  do
         $x_i^k = f_{upd}\{x_i^{k-1}, \max_{\forall j \in \mathcal{N}_i} \{f_{msg}(x_i^{k-1}, x_j^{k-1})\}$ 
    end for
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## Bellman-Ford Algorithm

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# Task Alignment

- if the GNN learns to simulate the update step in the Dynamic Problem, it will solve the problem
- if the operation is easy to learn, then the GNN can easily solve the problem
- if both are true, we say that the GNN is *well aligned* with the task

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## Alignment

We say that a model is **aligned** with a task, if by replacing some parts of the model with some ideal operations we would solve the task. If the parts can *easily learn* the ideal operations, we say that it is **well aligned** with the task.

Generally:

- If a model is well aligned with a task, it will learn it easily  
(it has low sample complexity).

# Task Alignment

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- What decision can we take to have the GNN "more aligned"?

# Task Alignment

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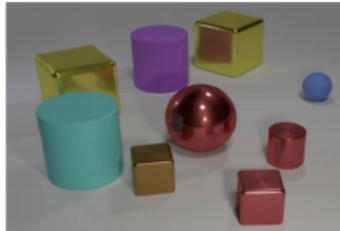
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```

---

- What decision can we take to have the GNN "more aligned"?
  - use **min** as an aggregator function
  - **share** the parameters between layers
- Is  $\tilde{x}_i = \text{MLP}([x_1, x_2, \dots, x_N])$  well aligned?
  - it is less aligned than the GNN functions
  - it has to learn to create node pairs and then it has to select the minimum between

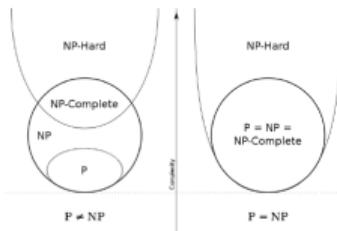
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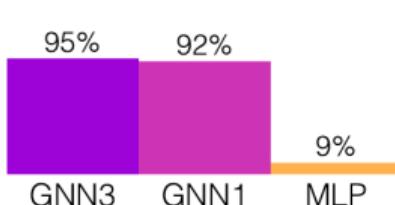
*Relational argmax*  
What are the colors of the furthest pair of objects?



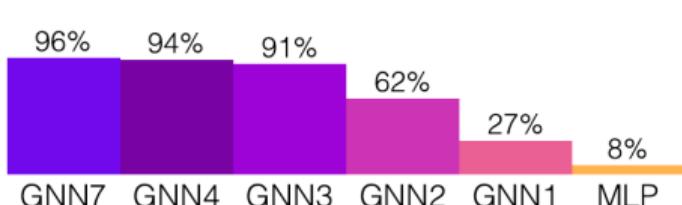
*Dynamic programming*  
What is the cost to defeat monster X by following the optimal path?



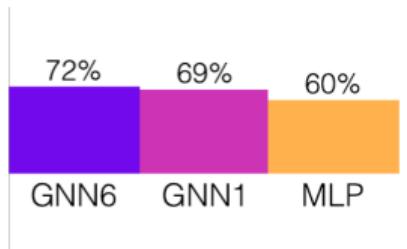
*NP-hard problem*  
Subset sum: Is there a subset that sums to 0?



Relational argmax

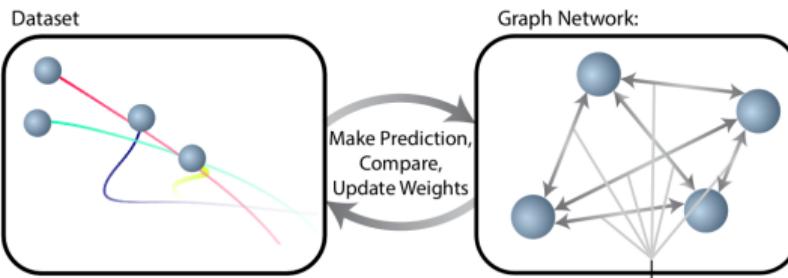


Dynamic Programming



NP - hard problem  
(random = 50%)

# Alignment: Physical Particles



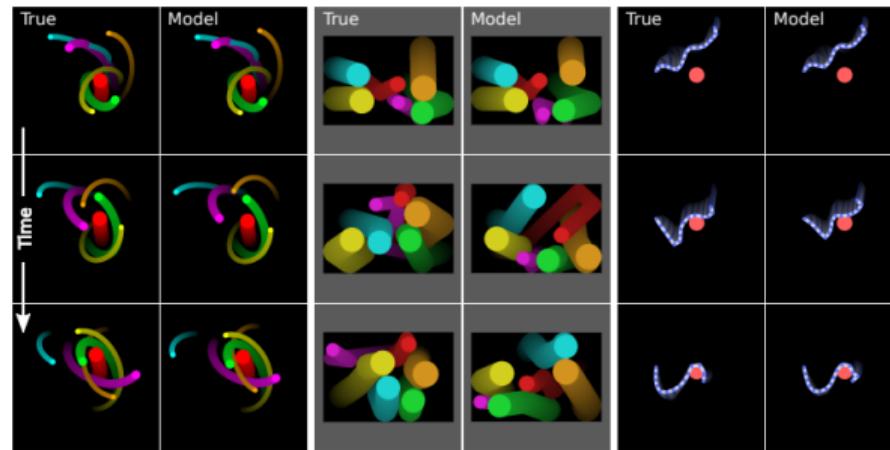
GNNs	Analogy to Newtonian Mechanics
Nodes	Particles
Pair of nodes	Two interacting particles (i,j)
Send Function: $f_{msg}$	Compute force $F_{ij}$
Aggregate Function: $f_{msg}$	Sum into net force $F_{net,i}$
Update Function: $f_{msg}$	Compute acceleration $a_i = F_{net,i}/m_i$

# When to use a GNN?

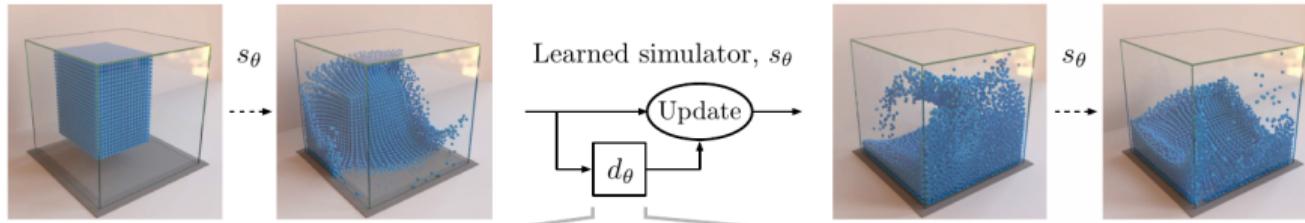
- Apply GNNs on tasks that are well aligned with this model
  - dynamic programming
  - relational reasoning
- Apply GNNs when relational processing is beneficial
  - explicit entities and relations: social networks, molecules
  - implicit entities and relations: visual scenes, environments...
- Try to design your GNN to be as aligned as possible to your problem

# Application: Physical Particle Interactions

[6] Battaglia et. al. NeurIPS 2016

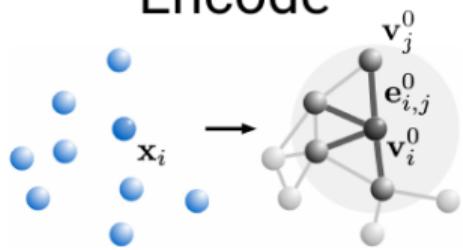


[7] Gonzalez et al. ICML 2020

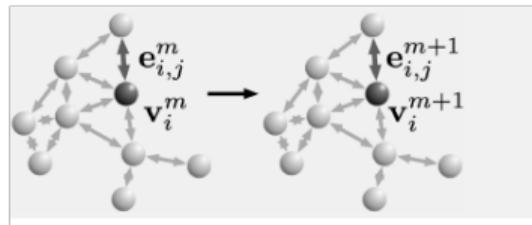


# Application: Physical Particle Interactions

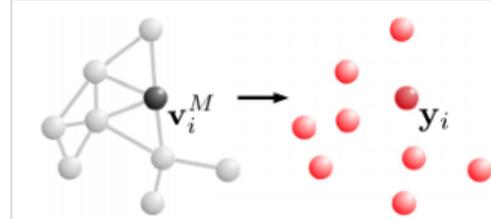
Encode



Process



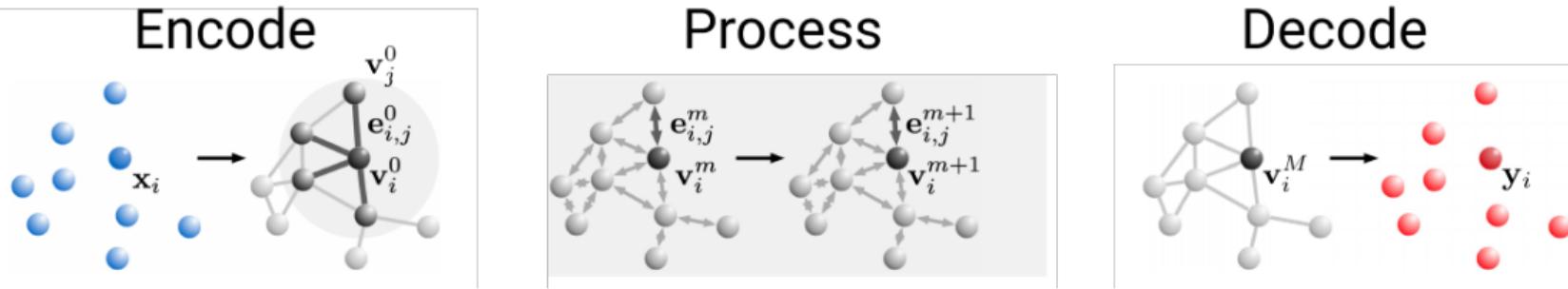
Decode



## Encoder:

- each node corresponds to a particle
- link top-k nearest neighbors
- Node features:
  - position and velocity
  - particle type

# Application: Physical Particle Interactions



## Process:

- use 10 GNN layers
- local propagation based on neighbourhood

## Decoder:

- predict next step attributes
- train based on node level loss

[7] Gonzalez et al. ICML 2020

# Application: Physical Particle Interactions

Observations:

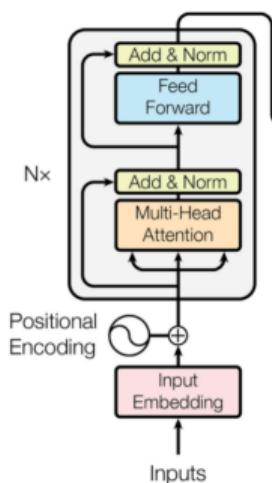
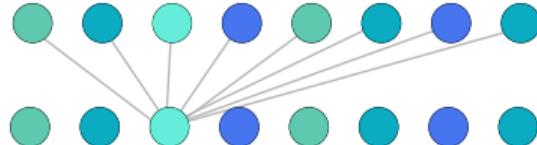
- the method is trained for next step predictions but at test time is unrolled for thousand of steps
- GNN method could generalise to 34 times more nodes at test time
  - because the interactions to nearest neighbours
- relative positions are better than global positions
  - underlying physical processes are invariant to spatial position,

Overall:

- GNN is aligned to the task
- the GNN has built in good relational biases
  - use local interactions
  - relative position for built in spatial invariance

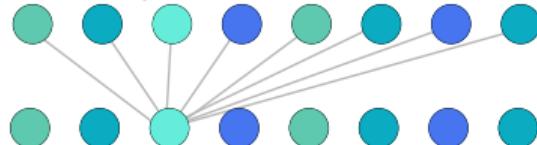
# Transformer

Task: analyse a sequence of words.  $X = x_1, x_2, \dots, x_N$ .



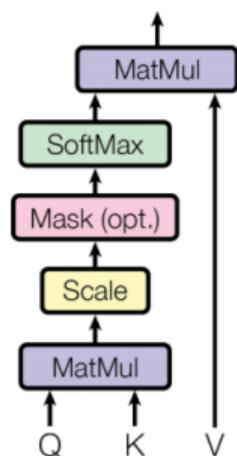
# Transformer

Task: analyse a sequence of words.  $X = x_1, x_2, \dots, x_N$ .



## Self - Attention

Scaled Dot-Product Attention



- Process a sequence in multiple layers
- Each element attends to all other elements in the previous layer

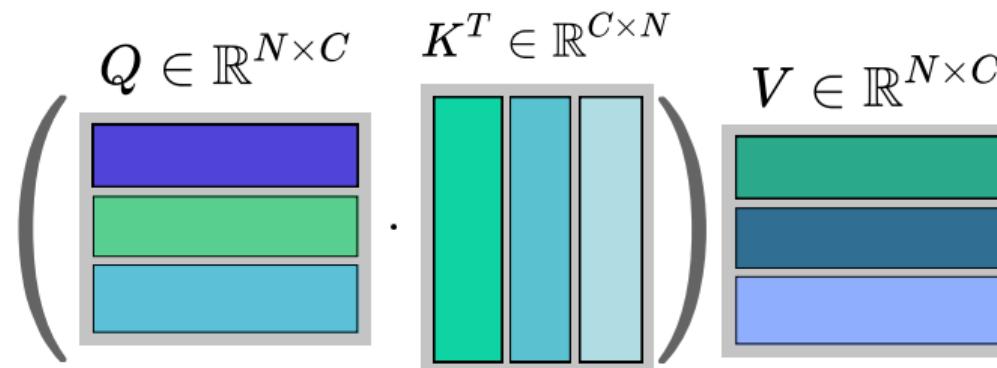
$$Y = \text{softmax}\left(\frac{QK^T}{\sqrt{d}}\right)V$$

- where  $Q = XW_q$ ,  $K = XW_k$ ,  $V = XW_v$

## Self-attention

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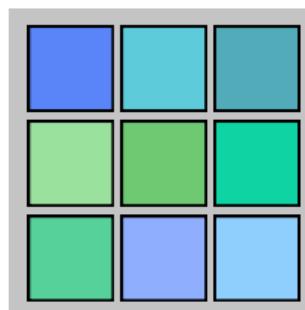
# Transformer

## Self-attention

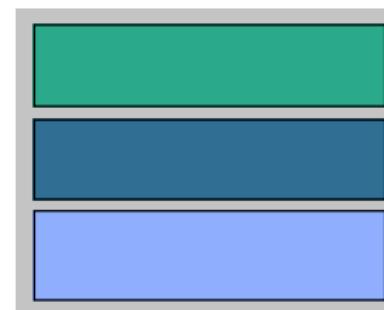
$$Y = \text{softmax}\left(\frac{QK^T}{\sqrt{d}}\right)V$$

where  $Q = XW_q$ ,  $K = XW_k$ ,  $V = XW_v$

$$A \in \mathbb{R}^{N \times N}$$



$$V \in \mathbb{R}^{N \times C}$$



.

Self-attention

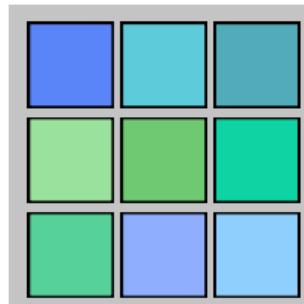
$$Y = \underbrace{\text{softmax}\left(\frac{QK^T}{\sqrt{d}}\right)V}_{A}$$

GCN

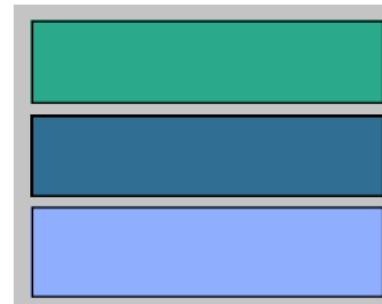
$$Y = \sigma(A X W)$$

where  $Q = XW_q$ ,  $K = XW_k$ ,  $V = XW_v$

$$A \in \mathbb{R}^{N \times N}$$

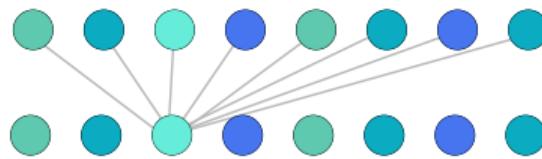


$$V \in \mathbb{R}^{N \times C}$$



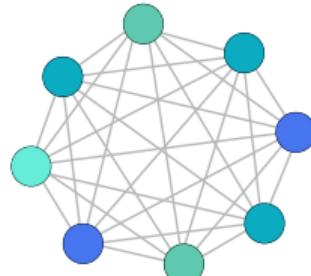
.

# Transformer



$$Y = \frac{QK^T}{\sqrt{d}}V$$

$$y_i = \sum_{\forall j} \underbrace{\frac{1}{\sqrt{d}}}_{\alpha(x_i, x_j)} \underbrace{(x_i W_q)}_{\text{Query}} \underbrace{(x_j W_k)^T}_{\text{Key}} \underbrace{(x_j W_v)}_{\text{Value}}$$



$$y_i = f_{upd}(x_i, \sum_{\forall j \in \mathcal{N}_i} \{\alpha(x_i, x_j) \phi(x_j)\})$$

$$\alpha(x_i, x_j) = \frac{1}{\sqrt{d}} (x_i W_q)^T (x_j W_k)$$

$$\phi(x_j) = x_j W_j$$

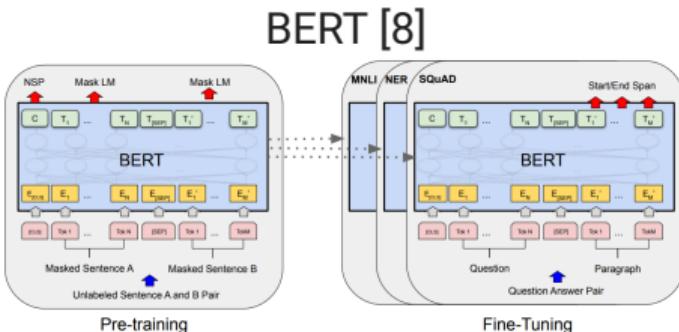
## Transformers vs GNNs

Transformer is a special case of Graph Neural Networks where

- all the nodes are connected
- pairwise messages are weighted by dot product attention

# Transformer - NLP

Transformers are now the standard model in NLP.



## GPT-3 [9]

### Zero-shot

The model predicts the answer given only a natural language description of the task. No gradient updates are performed.



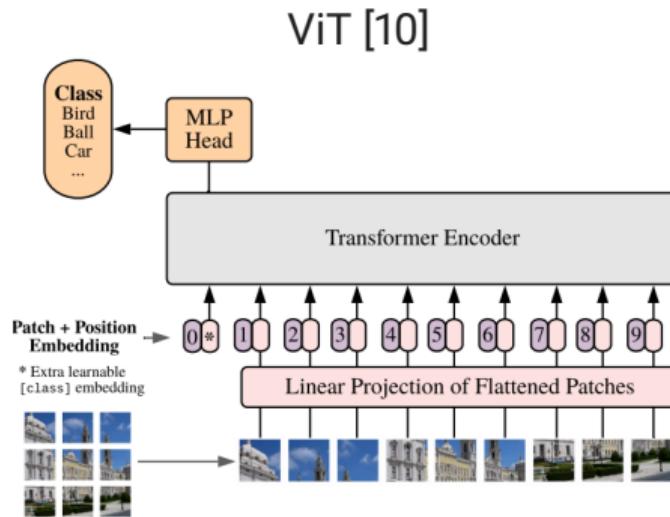
### One-shot

In addition to the task description, the model sees a single example of the task. No gradient updates are performed.



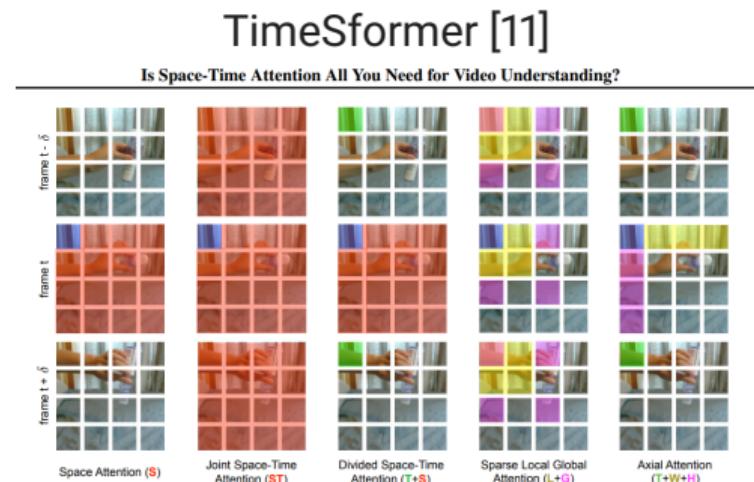
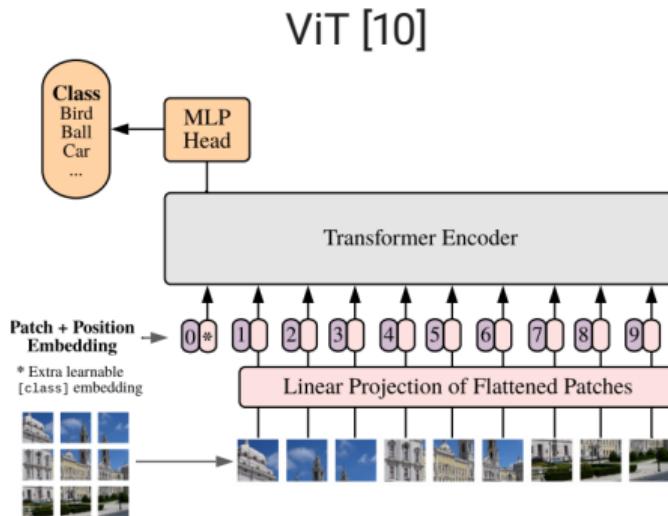
# Transformer - Vision

Transformers are becoming popular in CV.



# Transformer - Vision

Transformers are becoming popular in CV.



# GNN - Challenges: Scalability

*Context:*

- ML methods work with mini-batches where each element is independent
- in many *node level* graph tasks, the entire dataset forms a large graph where each node is connected to many other ones.

**Problem:**

- the whole graph is too big to fit into memory.
  - process independently the neighbourhood of each node
  - the neighbourhood could still grow exponentially:

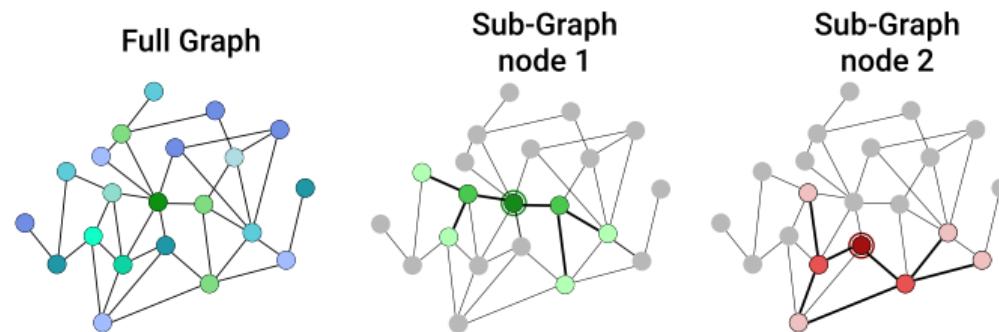
# Challenges: Scalability

## Solution:

- sample [12],[13] the nodes, forming sub-graphs and apply the GNN over them

## Benefits:

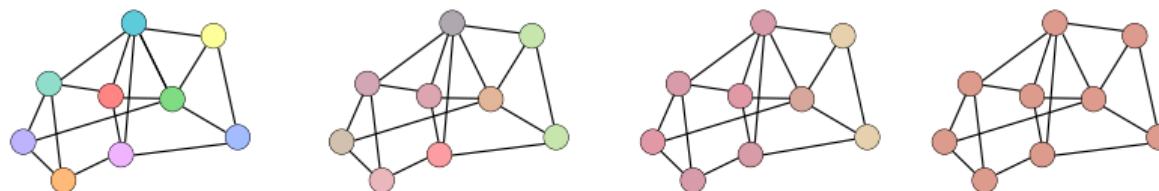
- can work with very large graphs
- the sampling acts as a regularizer, similar to dropout



# Challenges: Oversmoothing

If we want node information from a K-order neighbourhood

- use K layers of Graph propagation
- usual problems
  - harder to optimize due to vanishing / exploding gradients
  - overfitting due to large number of parameters
- graph propagation problem: **oversmoothing**
  - graph propagation can be seen as "smoothing" the a node according to its neighbourhood
  - if we do many propagations, different nodes would become almost *indistinguishable*, hurting *node-level* tasks



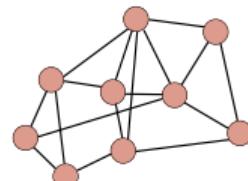
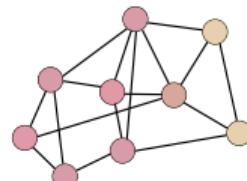
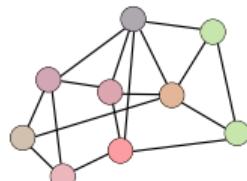
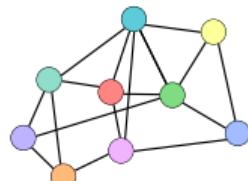
# Challenges: Oversmoothing

## Oversmoothing

Nodes with similar structure in their neighbourhoods would end up indistinguishable, regardless of their initial features.

More often:

- when the graph is dense
- when using self-loop in the update function

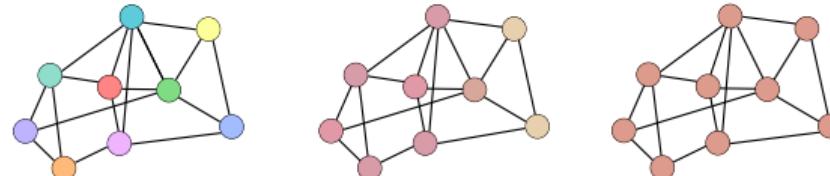


# Oversmoothing: Solutions

Solutions:

- **residual Connections** [14, 15]:

- skip one or more layers
- add the representations of a node from different layers  $h_i^{k+1} \leftarrow h_i^{k+1} + h_i^k$
- takes more into account the identity of each node

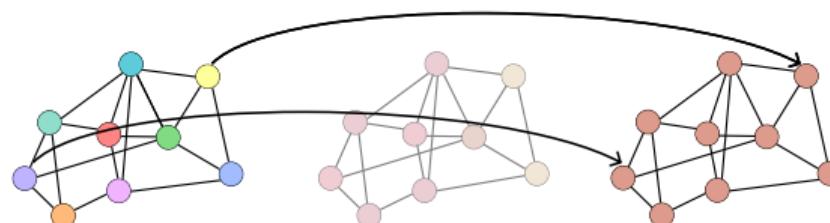


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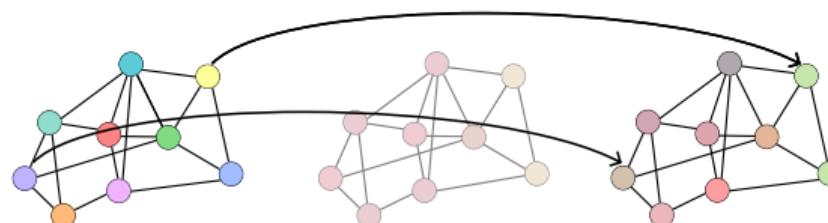


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# Oversmoothing: Solutions

Solutions:

- make the graph more **sparse**: e.g apply dropout on edges [16]
- PairNorm[17]: add a **normalisation** term that encourages  $h_i^{t+1}$  and  $h_i^t$  to remain close while neighbouring nodes maximise their similarity and distant nodes minimise their similarity

# Connections to PageRank

Long range are obtained by stacking multiple layers:  $A\sigma(A..\sigma(AXW_1)..W_{n-1})W_n$

# Connections to PageRank

Long range are obtained by stacking multiple layers:  $A\sigma(A..\sigma(AXW_1)..W_{n-1})W_n$

## Random Walk

- start in a node and randomly move to adjacency nodes.
- $W = I$  and  $X \in \mathbb{R}^N$  a vector containing the probability of being in each node and  $A$  is the transition probability
- this arrives at the PageRank algorithm  $X^{t+1} = AX^t$

# Connections to Personalised PageRank

- PageRank converges to an  $Y$  that does not depend of the initial  $X$
  - this is related to the oversmoothing problem in the GNN
  - in Personalised PageRank the initial starting point count more
    - at each step there is a chance  $\alpha$  to go back to the initial state
- $$X^{t+1} = (1 - \alpha)AX^t + \alpha X^0$$

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- in Personalised PageRank the initial starting point count more
  - at each step there is a chance  $\alpha$  to go back to the initial state
$$X^{t+1} = (1 - \alpha)AX^t + \alpha X^0$$
- we can use a similar formulation in our graph propagation to alleviate the oversmoothing
  - the residual connection could be seen as a non-probabilistic variant

# Connections to Personalised PageRank

How can it be used in GNNs?

- make a prediction independently at each node and propagate the answer [18]

$$X^1 = X^0 W$$

$$X^{t+1} = (1 - \alpha) A X^t + \alpha X^0$$

# Connections to Personalised PageRank

How can it be used in GNNs?

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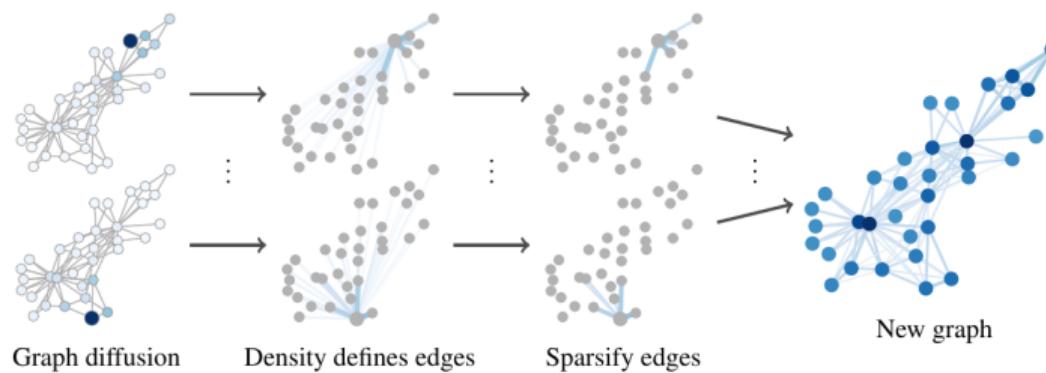
- this is somehow related to label propagation [19]

# Connections to Personalised PageRank

Alternatively:

- compute Personalized Page Rank diffusion matrix  $S$  [20][21]
- sparsify the diffusion matrix
- and use it in a GCN

$$Y = \sigma(SXW)$$



# Overview

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  - alignment
- application: simulating particles
- Transformers are GNNs
- challenges: oversmoothing
- connections to PageRank

# Graph Neural Networks - Resources

For a more in depth understanding of Graph Neural Networks and other related areas, please take a look:

- Michael Bronstein, *Geometric deep learning, from Euclid to drug design* [▶ Link](#)
- Petar Veličković, *Theoretical Foundations of Graph Neural Networks* [▶ Link](#)
- Jure Leskovec, *CS224W: Machine Learning with Graphs* [▶ Link](#)
- William L. Hamilton, *Graph Representation Learning Book* [▶ Link](#)
- Razvan Pascanu, *GraphNets - Lecture at TMLSS (Transylvanian Machine Learning Summer School)*
- Xavier Bresson, *Convolutional Neural Networks on Graphs* [▶ Link](#)
- Michael Bronstein, *Graph Deep Learning Blog* [▶ Link](#)

# Thank You!

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May 2021

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