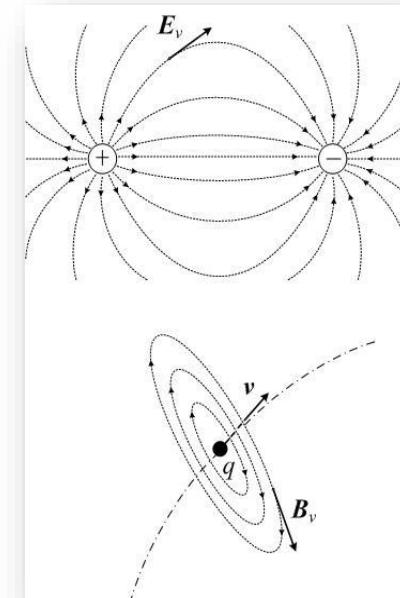


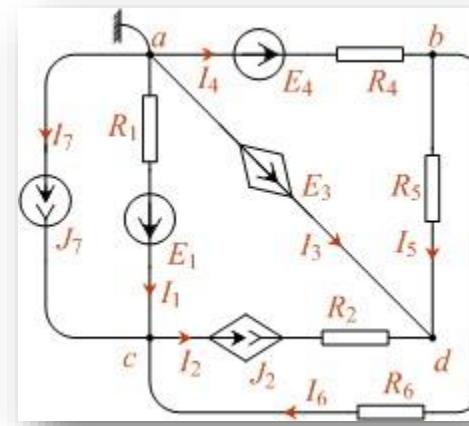
BAZELE ELECTROTEHNICII

- note de curs -

- Teoria câmpului electromagnetic



- Teoria circuitelor electrice



I. Introducere

- Sarcina electrică
- Intensitatea curentului electric
- Tensiunea electrică
- Elemente de topologie

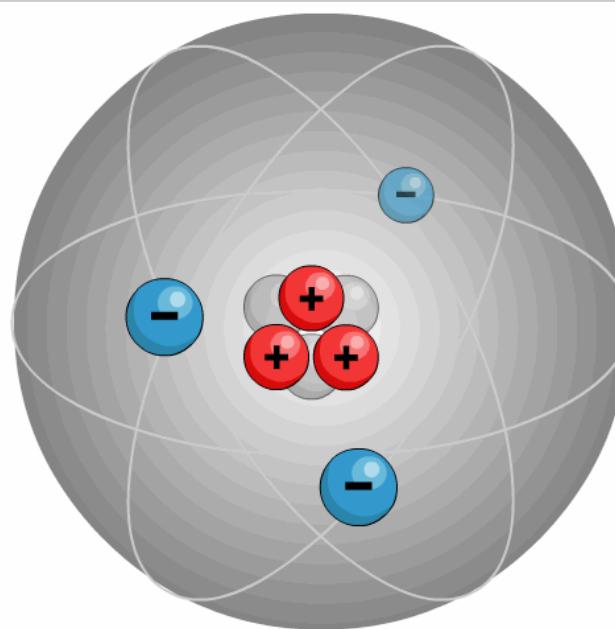
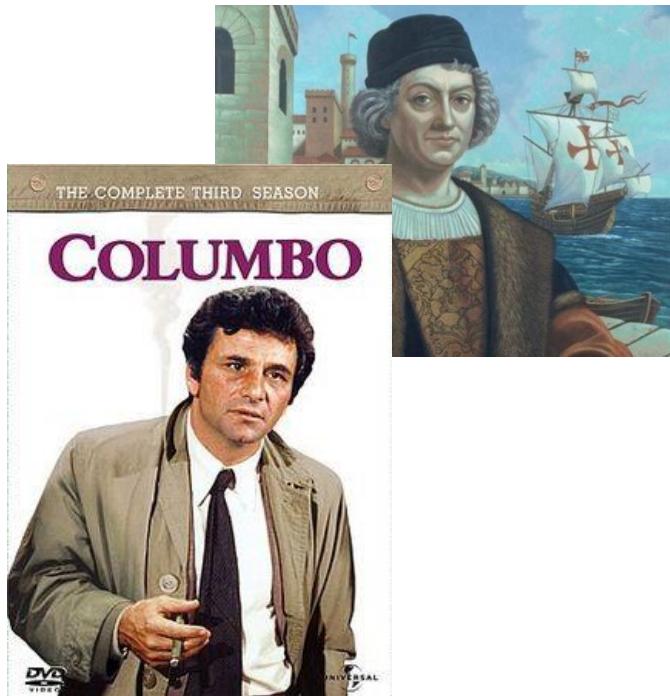
I. Introducere

- Sarcina electrică
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Tabelul periodic - Sistemul periodic al elementelor																																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18																
IA	IIA	IIIB	IVB	VIB	VIIB	VIIIB	VIIIB	VIIIB	VIIIB	IB	IIIB	IIIA	IVA	V A	VI A	VIIA	VIII A																
1 H Hidrogen 1.0079	2 He Heliu 4.0026	3 Li Litu 6.941	4 Be Beriliu 9.0122	5 Na Sodiu 22.9897	6 Mg Magneziu 24.305	7 K Potasiu 39.0983	8 Ca Calciu 40.078	9 Sc Scandiu 44.9559	10 Ti Titaniu 47.867	11 V Vanadiu 50.9415	12 Cr Crom 51.9961	13 Mn Mangan 54.9380	14 Fe Fier 55.845	15 Co Cobalt 58.9331	16 Ni Nickel 58.6934	17 Cu Cupru 63.546	18 Zn Zinc 65.38	19 Ga Galuu 69.723	20 Ge Germaniu 72.64	21 As Arsen 74.9216	22 Se Seleniu 78.96	23 Br Brom 79.904	24 Kr Kripton 83.798										
21 Sc Scandiu 44.9559	22 Ti Titaniu 47.867	23 V Vanadiu 50.9415	24 Cr Crom 51.9961	25 Mn Mangan 54.9380	26 Fe Fier 55.845	27 Co Cobalt 58.9331	28 Ni Nickel 58.6934	29 Cu Cupru 63.546	30 Zn Zinc 65.38	31 Ga Galuu 69.723	32 Ge Germaniu 72.64	33 As Arsen 74.9216	34 Se Seleniu 78.96	35 Br Brom 79.904	36 Kr Kripton 83.798	37 Rb Rubidiu 85.4678	38 Sr Stronciu 87.62	39 Y Ytriu 88.9058	40 Zr Zirconiu 91.224	41 Nb Niobiu 92.9063	42 Mo Molibden 95.96	43 Tc Technetiu [98]	44 Ru Ruteniu 101.07	45 Rh Rodium 102.9055	46 Pd Paladiu 106.42	47 Ag Argint 107.6682	48 Cd Cadmiu 112.411	49 In Indiu 114.816	50 Sn Stibiu 116.71	51 Sb Stibiu 121.76	52 Te Telur 127.6	53 I Iod 126.9044	54 Xe Xenon 131.293
55 Cs Cesiu 132.9054	56 Ba Bariu 137.327	57-71 Lantanide	72 Hf Hafniu 178.49	73 Ta Tantaliu 180.9478	74 W Wolfram 183.84	75 Re Reniu 186.207	76 Os Osmiu 190.23	77 Ir Iridiu 192.217	78 Pt Platina 195.084	79 Au Aur 196.9665	80 Hg Mercur 200.59	81 Tl Taliu 204.3833	82 Pb Plumb 207.2	83 Bi Bismut 208.9804	84 Po Poloniu [209]	85 At Astatin [210]	86 Rn Radon [222]																
87 Fr Franciu [223]	88 Ra Radiu [226]	89-103 Actinide	104 Rf Rutherfordiu [263.11]	105 Db Dubnium [268]	106 Sg Seaborgiu [271]	107 Bh Bohriu [270]	108 Hs Hassiu [269]	109 Mt Meitneriu [278]	110 Ds Darmstadiu [281]	111 Rg Roentgeniu [285]	112 Cp Copernicium [286]	113 Uut Ununtriu [286]	114 Uup Ununpentiu [289]	115 Uuh Ununquatu [289]	116 Uus Ununhexiu [293]	117 Uos Ununseptiu [294]	118 Uuo Ununoctiu [294]																
57 La Lantaniu 138.9054	58 Ce Ceriu 140.116	59 Pr Praseodiu 140.9076	60 Nd Neodiu 144.242	61 Pm Prometiu [145]	62 Sm Samariu 150.36	63 Eu Euriu 151.964	64 Gd Gadoliniu 157.25	65 Tb Terbiu 158.9253	66 Dy Dysprosiu 162.5	67 Ho Holmiu 164.9303	68 Er Erbiu 167.259	69 Tm Tuliu 168.9342	70 Yb Yterbiu 173.054	71 Lu Lutetiu 174.9668																			
89 Ac Actiniu [227]	90 Th Thoriu 232.0381	91 Pa Protactiniu 231.0358	92 U Uraniu 238.0289	93 Np Neptuniu [237]	94 Pu Plutoniu [244]	95 Am Americiu [243]	96 Cm Curiu [247]	97 Bk Berkeliu [247]	98 Cf Californiu [251]	99 Es Einsteiniu [252]	100 Fm Fermiu [257]	101 Md Mendeleviu [258]	102 No Nobeliu [259]	103 Lr Lawrenciu [262]																			

I. Introducere

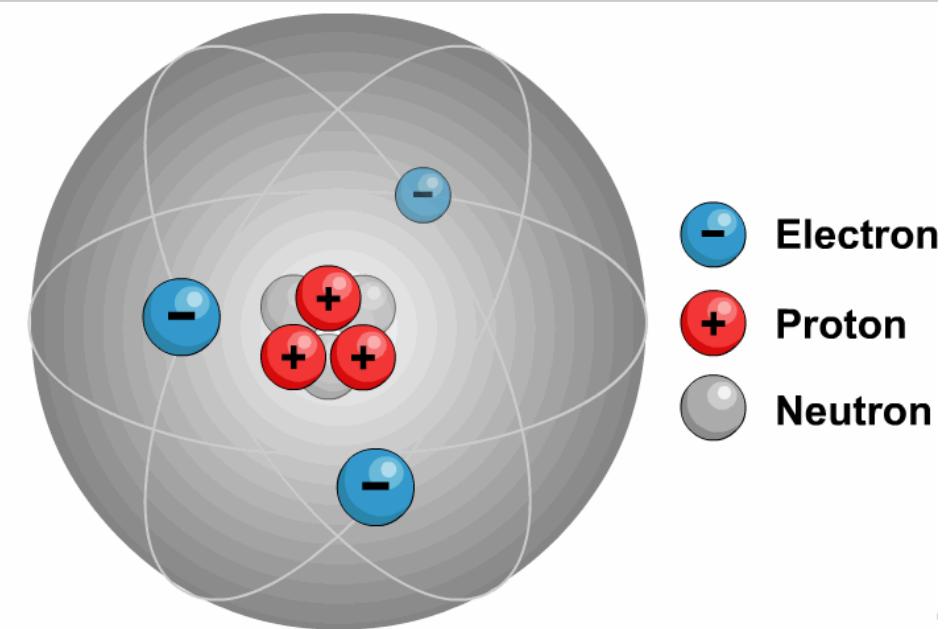
- Sarcina electrică
- Intensitatea curentului electric
- Tensiunea electrică
- Elemente de topologie



- Electron
- + Proton
- Neutral Neutron

I. Introducere

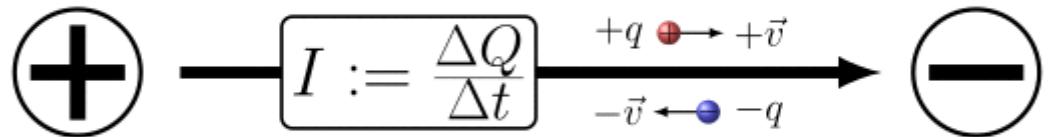
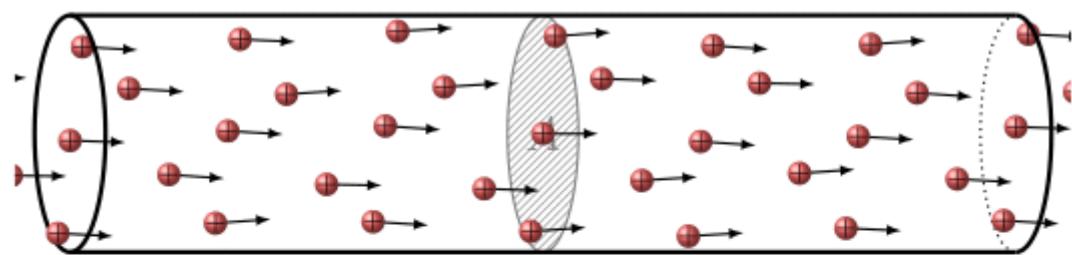
- Sarcina electrică
- Intensitatea curentului electric
- Tensiunea electrică
- Elemente de topologie



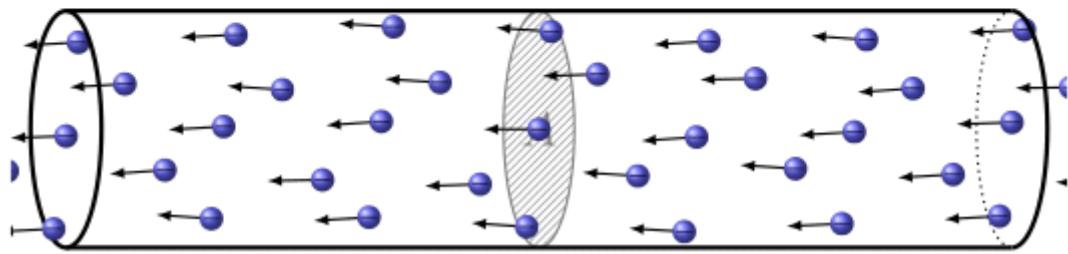
I. Introducere

- Sarcina electrică
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$$i = \lim_{\Delta t \rightarrow 0} \frac{\Delta q_{micro}}{\Delta t} = \frac{d q_{micro}}{dt}$$



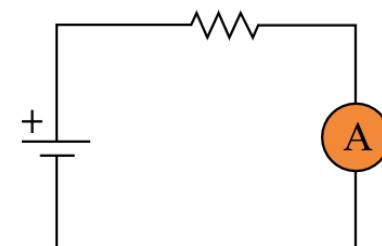
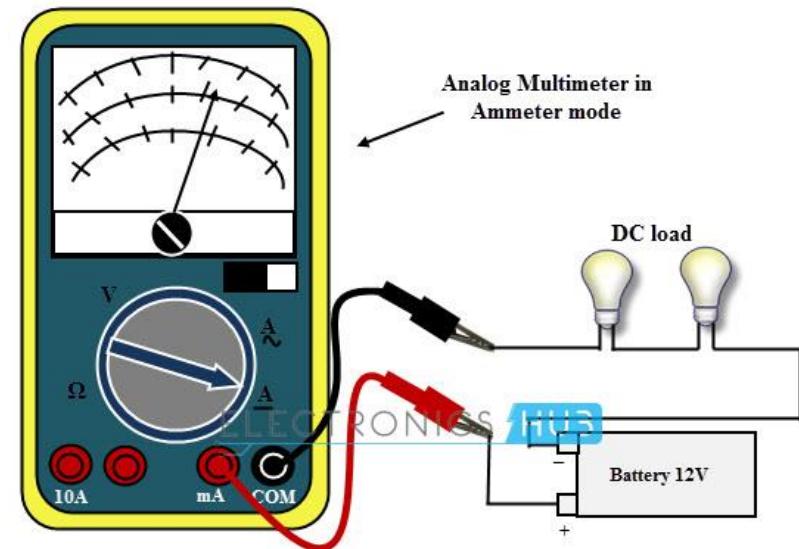
$$Q = \int_{t_1}^{t_2} i(t) dt$$



I. Introducere

- Sarcina electrică
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$$[i]_{SI} = \frac{C}{S} = A$$



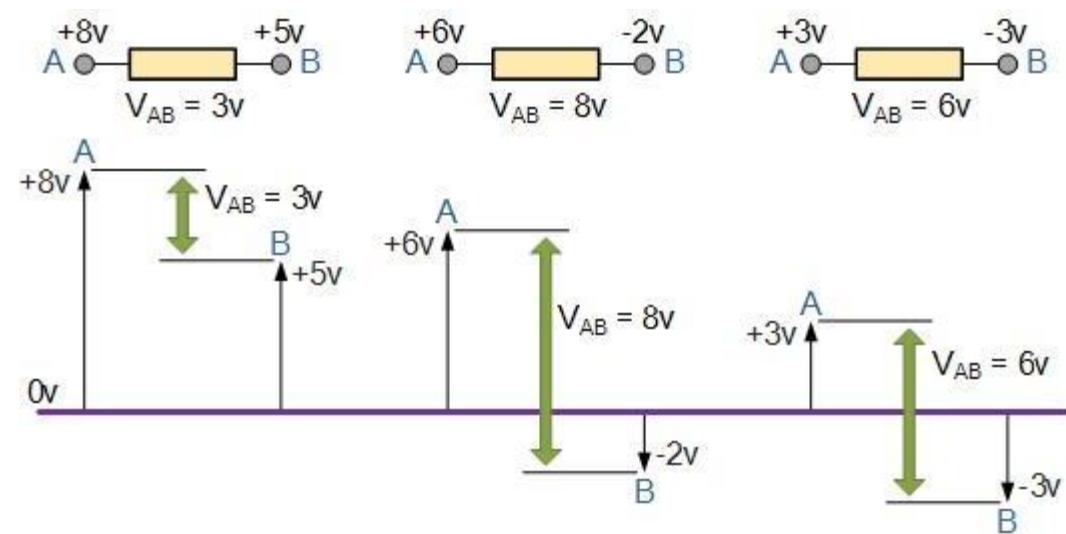
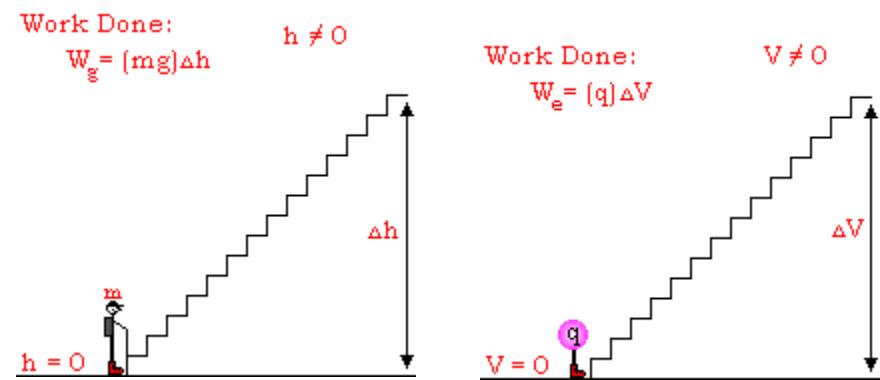
I. Introducere

- Sarcina electrică
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$$u = \frac{dw}{dq}$$

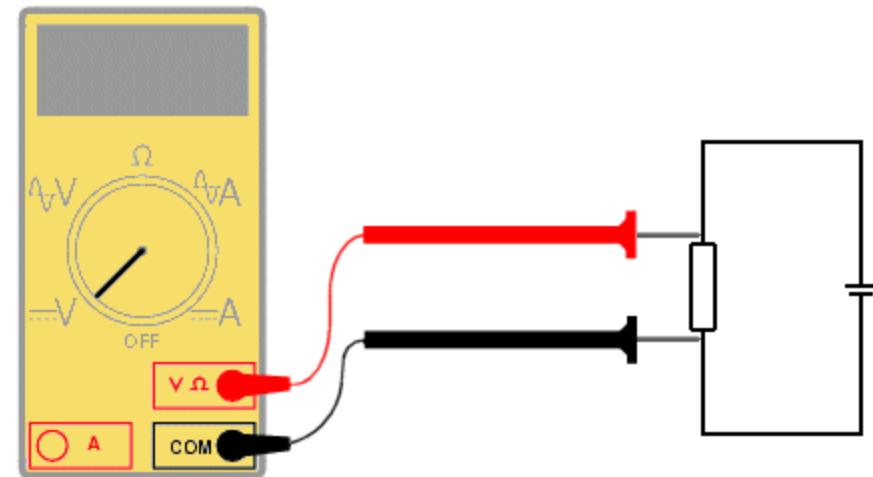
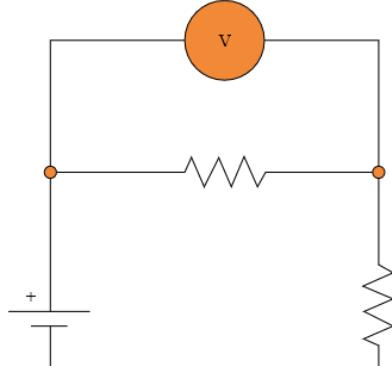
$$[u]_{SI} = \frac{J}{C} = V$$

$$u_{AB} = V_A - V_B$$



I. Introducere

- Sarcina electrică
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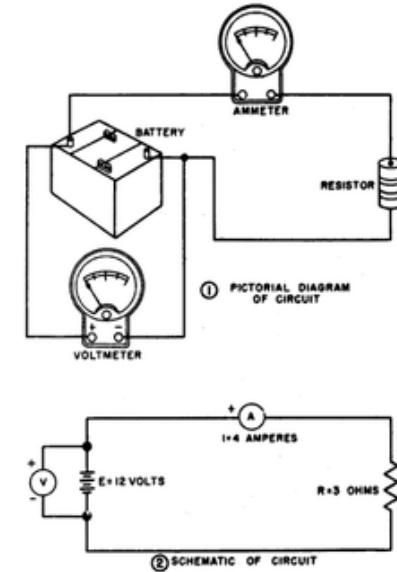


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$$p = \frac{dw}{dt} \quad [W] \qquad W = \int_{t_1}^{t_2} p(t) dt$$

I. Introducere

- Sarcina electrică
- Intensitatea curentului electric
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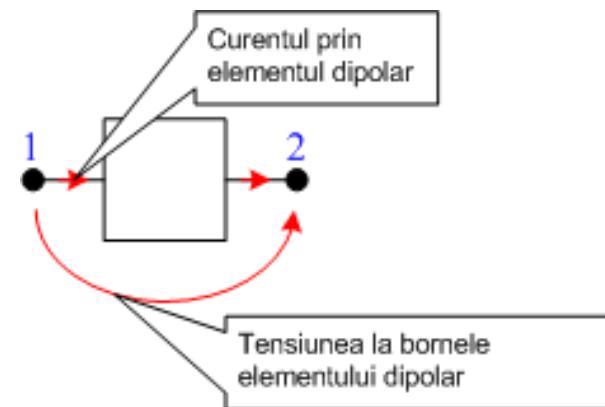
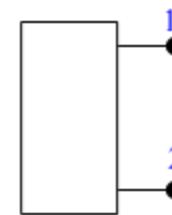
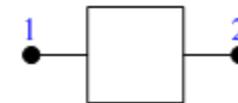
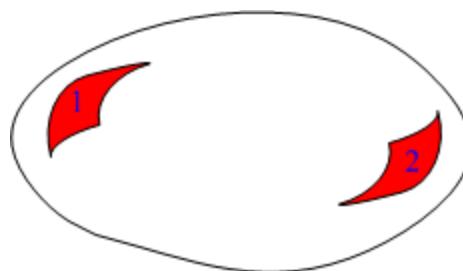
Circuit electric = un ansamblu de elemente ideale de circuit interconectate, pentru a indeplini un scop.

Element ideal de circuit = domeniu spatial a carui conexiune cu exteriorul se realizeaza prin puncte de pe suprafata sa, numite borne.

I. Introducere

- Sarcina electrică
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N=2 – element dipolar
de circuit



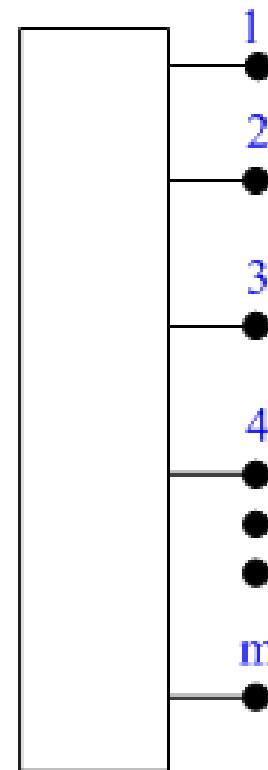
I. Introducere

- Sarcina electrică
- Intensitatea curentului electric
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N=3 – element tripolar de circuit

N=4 – element quadripolar de circuit

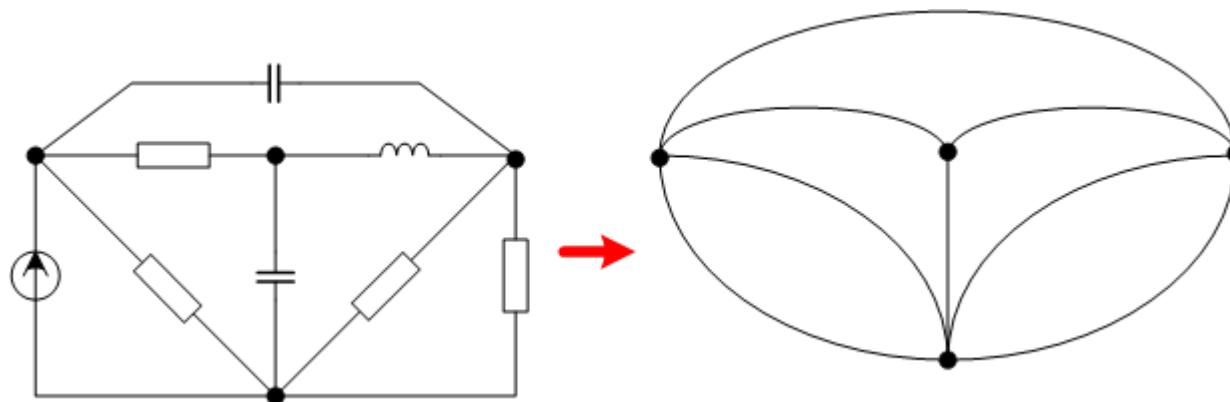
N=m – element multipolar de circuit



I. Introducere

- Sarcina electrică
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Graful unui circuit = o reprezentare a circuitului în care bornele sunt puncte, numite noduri, iar elementele de circuit sunt arce de curba, numite laturi.



I. Introducere

- Sarcina electrică
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Bucla = reprezinta o multime de laturi care alcatuiesc o curba inchisa, iar nodurile acesteia sunt parcurse o singura data.

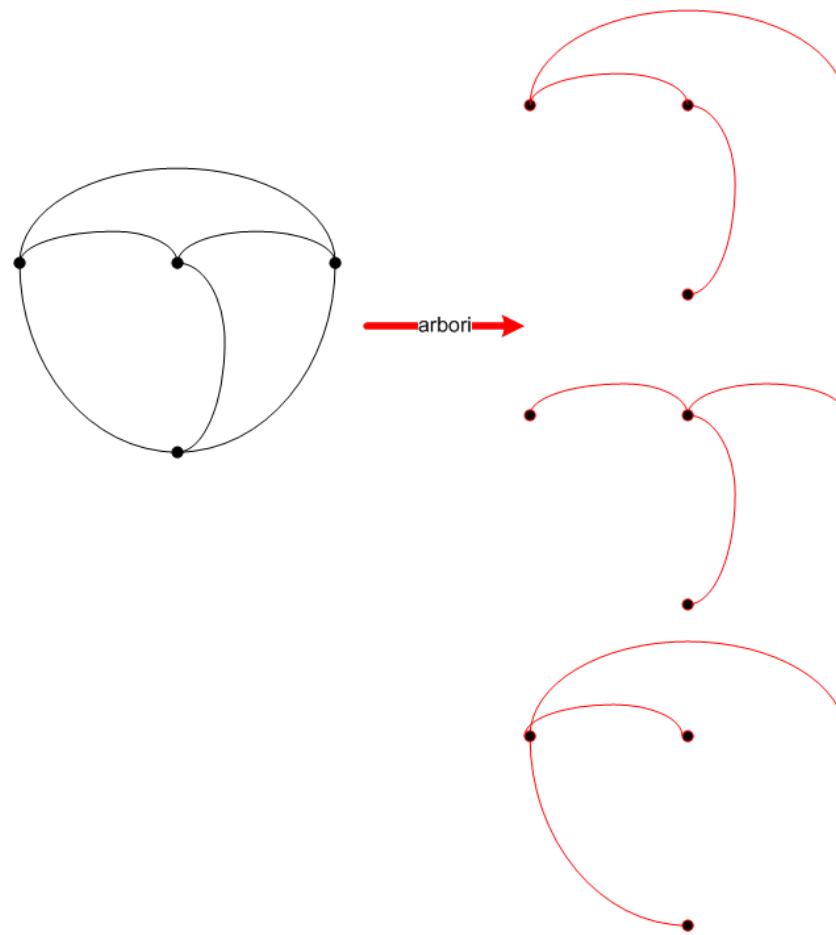
Ochi = reprezinta o bucla in interiorul careia nu exista alte bucle.

Sectiune = reprezinta o multime de laturi din graf, prin eliminarea carora graful se imparte in doua parti disjuncte.

Arborele = reprezinta o multime de laturi dintr-un graf (un subgraf) care nu formeaza bucle si contine toate nodurile.

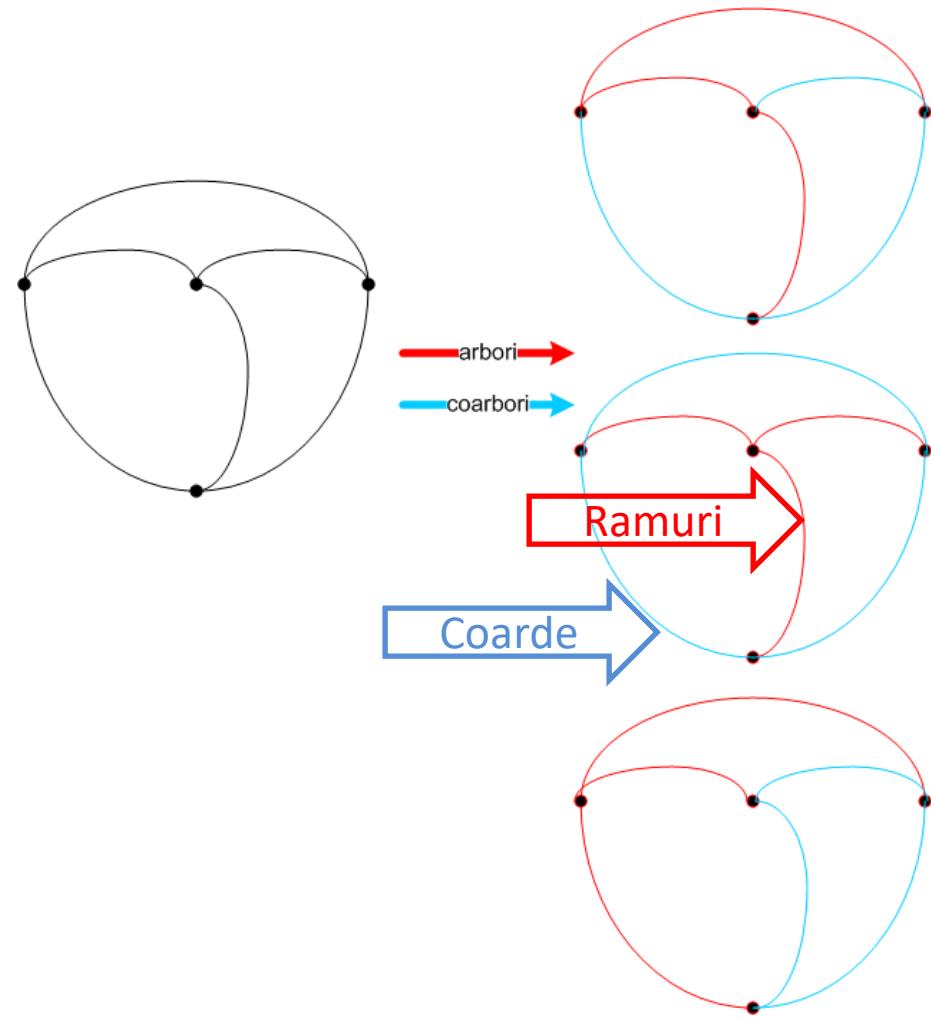
I. Introducere

- Sarcina electrică
- Intensitatea curentului electric
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I. Introducere

- Sarcina electrică
- Intensitatea curentului electric
- Tensiunea electrică
- Elemente de topologie

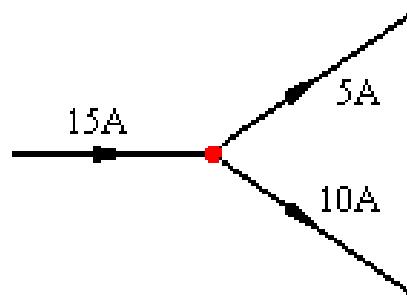


II. Teoremele lui Kirchhoff

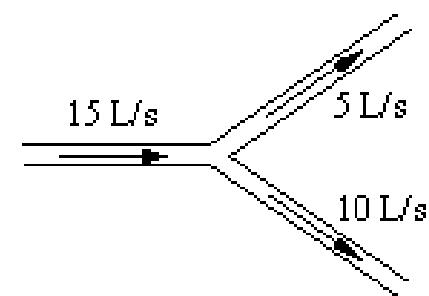
Suma algebraica a curentilor din laturile incidente oricarui nod dintr-un circuit electric este zero.

$$\sum_{k \in (n)}^{(A)} i_k = 0 \quad \forall n$$

Electricity:



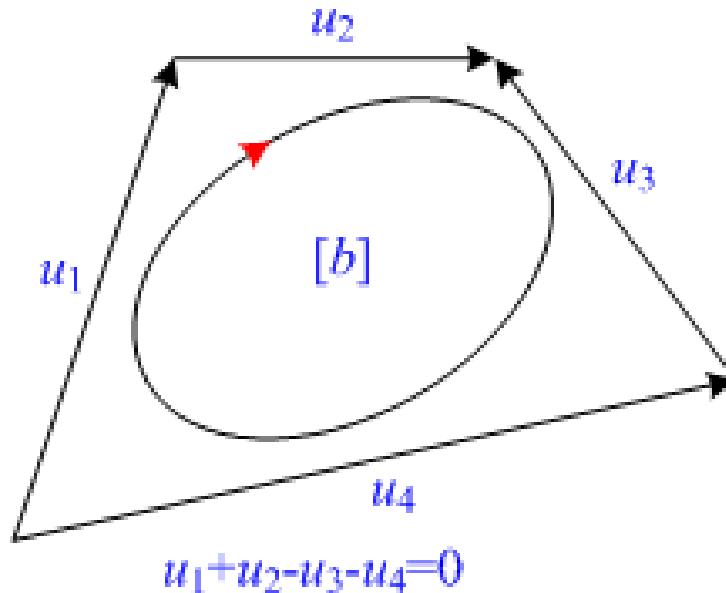
Water analogy:



II. Teoremele lui Kirchhoff

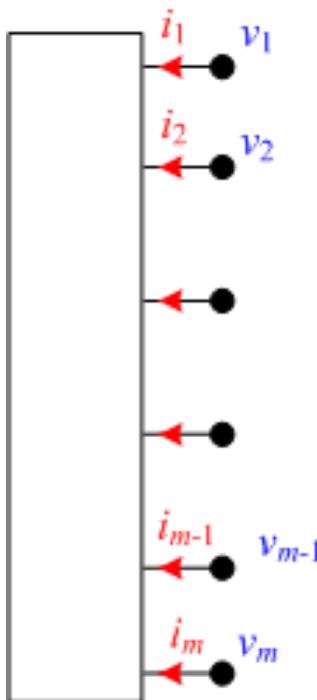
Suma algebraica a tuturor tensiunilor electrice din orice bucla a unui circuit electric este zero.

$$\sum_{k \in [b]} {}^{(A)} u_k = 0 \quad \forall b$$



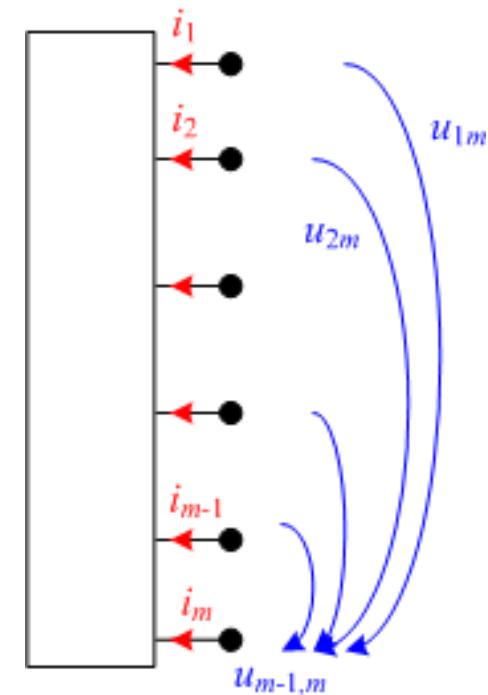
Teorema puterii transferate

Puterea transferata pe la bornele unui element multipolar de circuit este produsul dintre curentii bornelor si potențialele lor.

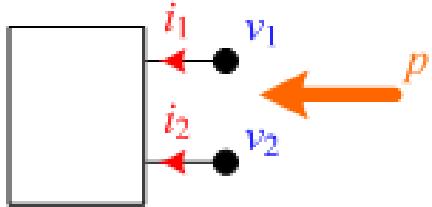


$$p = v_1 i_1 + v_2 i_2 + \dots + v_m i_m = \sum_{k=1}^m v_k i_k$$

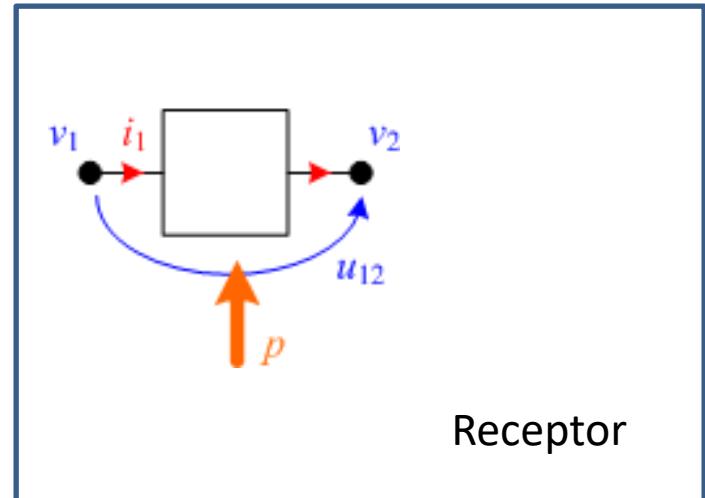
$$p = u_{1m} i_1 + u_{2m} i_2 + \dots + u_{m-1,m} i_m = \sum_{k=1}^{m-1} u_{k,m} i_k$$



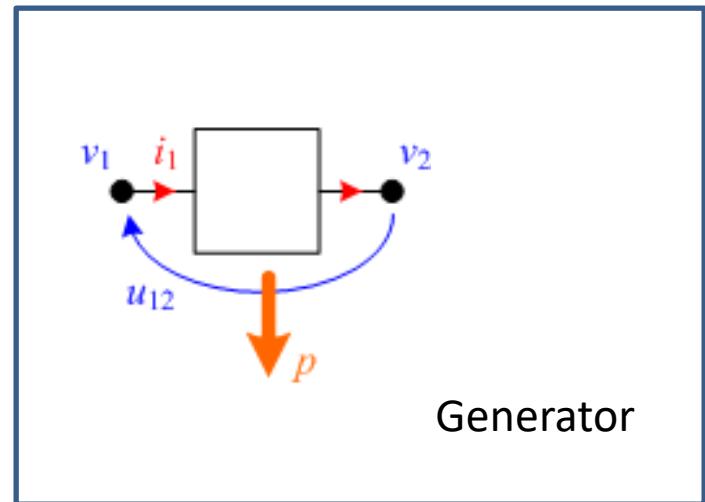
Pentru un element dipolar de circuit:



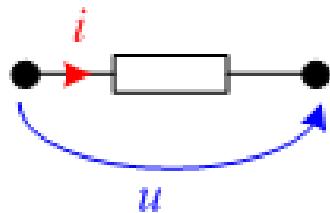
$$p = u_{12} i_1$$



$$p = u_{12} i_1$$

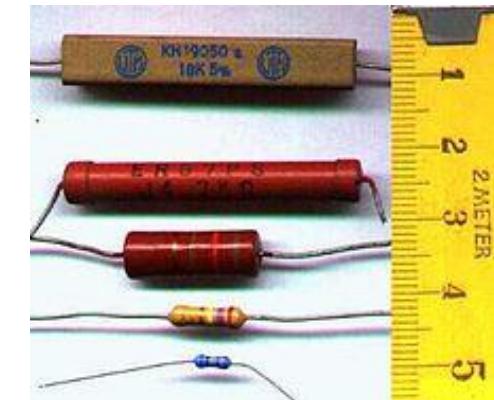


III. Elemente ideale de circuit - Rezistorul ideal liniar



$$u \sim i$$

Element de circuit pentru care tensiunea la bornele lui este proporțională cu curentul ce-l străbate.



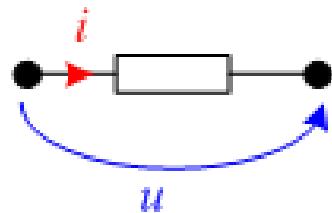
Parametri:

R rezistenta $[R]=\Omega$ (Ohm)

G conductanta $[G]=S$ (Siemens)

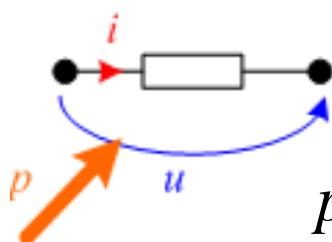


III. Elemente ideale de circuit - Rezistorul ideal liniar



$$u(t) = R \cdot i(t)$$

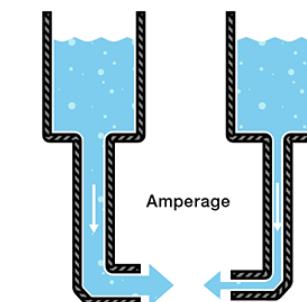
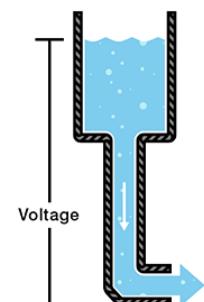
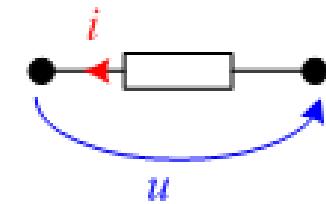
$$i(t) = G \cdot u(t)$$



$$p = u \cdot i = R i^2 = G u^2$$

$$u(t) = -R \cdot i(t)$$

$$i(t) = -G \cdot u(t)$$



Resistance

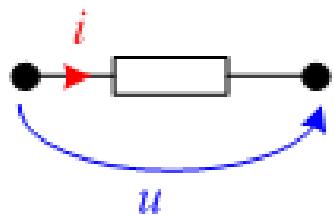
Less resistance



More resistance

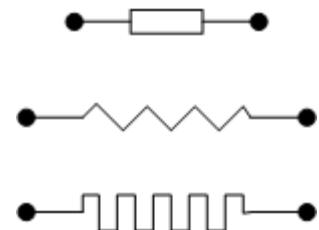
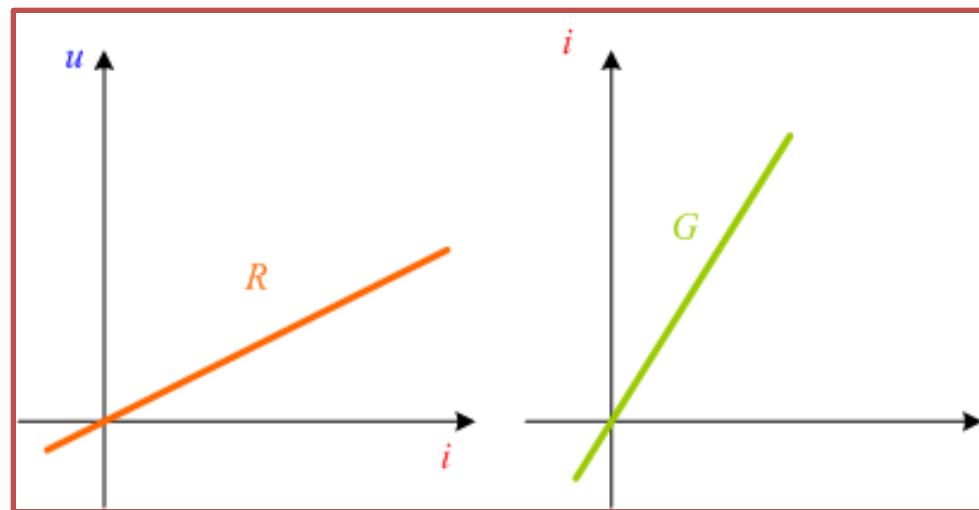
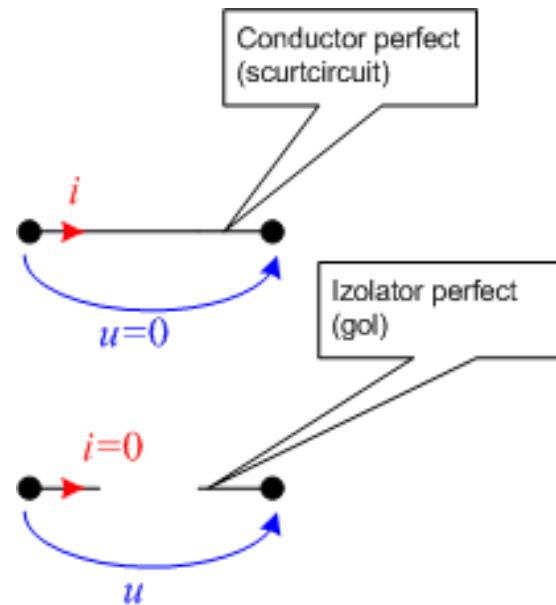


III. Elemente ideale de circuit - Rezistorul ideal liniar



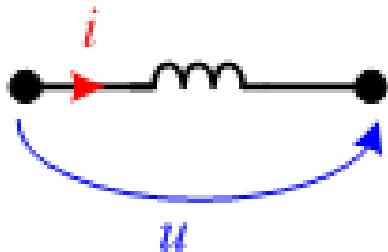
$$R = 0 \Rightarrow u = 0$$

$$G = 0 \Rightarrow i = 0$$



III. Elemente ideale de circuit

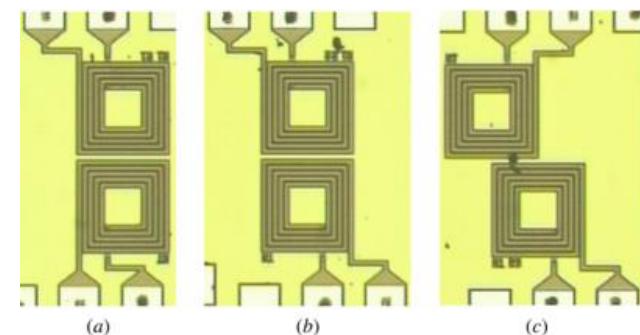
- Bobina ideală liniară



$$u \sim \frac{di}{dt}$$

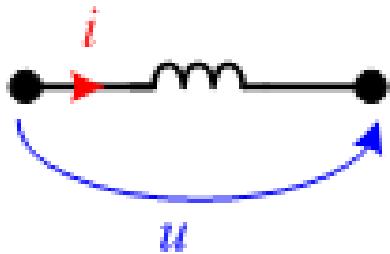
Element de circuit pentru care tensiunea la borne este proporțională cu derivata curentului ce o străbate.

Parametru:
L inductivitate [L]=H (Henry)



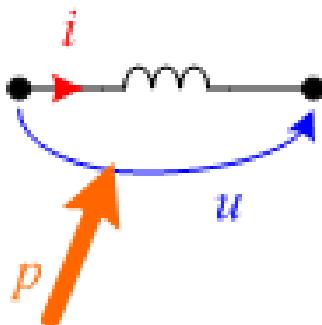
III. Elemente ideale de circuit

- Bobina ideală liniară



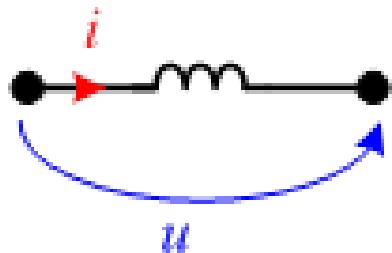
$$u(t) = L \frac{di(t)}{dt}$$

$$i(t) = i(0) + \frac{1}{L} \int_0^t u(t') dt'$$



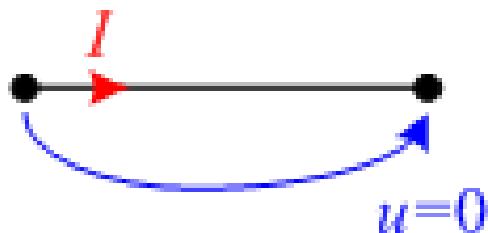
$$p(t) = u \cdot i = i \cdot L \frac{di(t)}{dt} = \frac{d}{dt} \left(\frac{Li^2}{2} \right) = \frac{dW_m}{dt}$$

III. Elemente ideale de circuit - Bobina ideală liniară

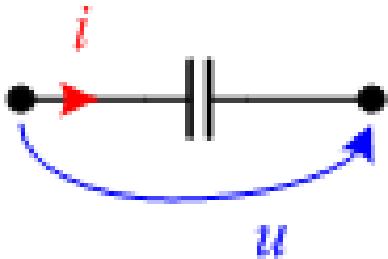


$$u(t) = L \frac{di(t)}{dt}$$

$$i(t) = \text{const} \Rightarrow i(t) = I \Rightarrow \frac{di}{dt} = 0 \Rightarrow u = 0$$



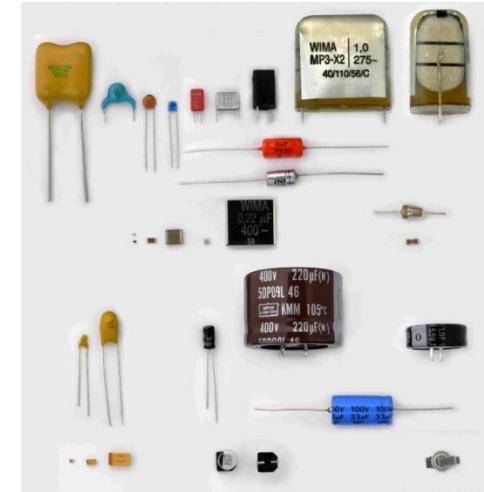
III. Elemente ideale de circuit - Condensatorul ideal liniar



$$i \sim \frac{du}{dt}$$

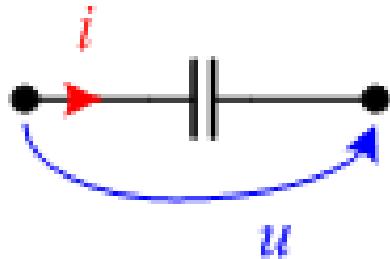
Element de circuit pentru care
currentul ce-l strabate este
proportional cu derivata
tensiunii de la bornele acestuia.

Parametru:
C capacitate [C] = F (Farad)



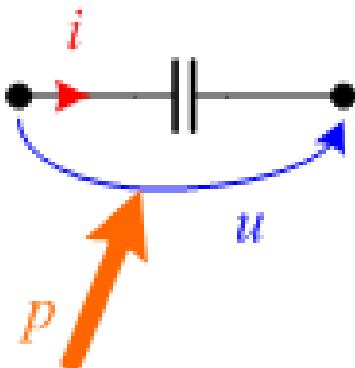
III. Elemente ideale de circuit

- Condensatorul ideal liniar



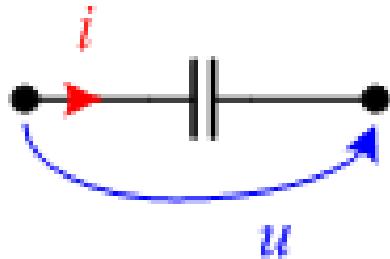
$$i(t) = C \frac{du(t)}{dt}$$

$$u(t) = u(0) + \frac{1}{C} \int_0^t i(t') dt'$$



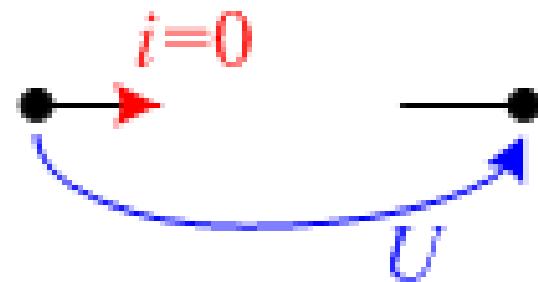
$$p(t) = u \cdot i = u \cdot C \frac{du(t)}{dt} = \frac{d}{dt} \left(\frac{Cu^2}{2} \right) = \frac{dW_e}{dt}$$

III. Elemente ideale de circuit - Condensatorul ideal liniar

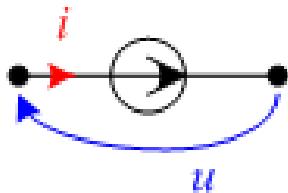


$$i(t) = C \frac{du(t)}{dt}$$

$$u(t) = \text{const} \Rightarrow u(t) = U \Rightarrow \frac{du}{dt} = 0 \Rightarrow i = 0$$



III. Elemente ideale de circuit - Sursa ideală de tensiune

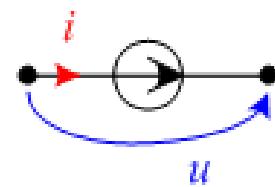


$$u(t) = e(t) \quad \forall i(t)$$

Element de circuit pentru care tensiunea la borne este egală cu t.e.m. a sursei, independent de valoarea curentului ce o strabate.

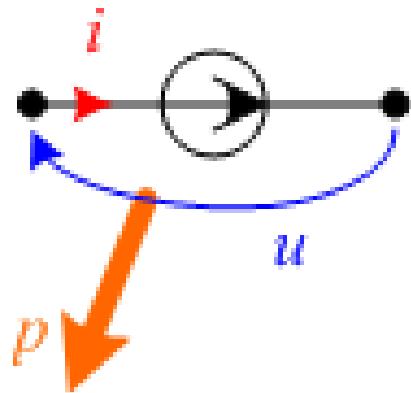
Parametru:

e tensiune electromotoare [e] = V (Volt)

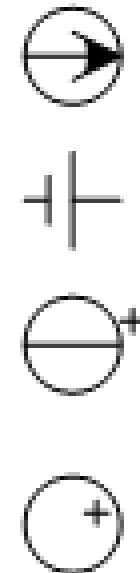
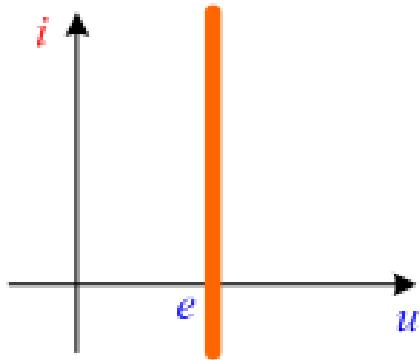


$$u(t) = -e(t) \quad \forall i(t)$$

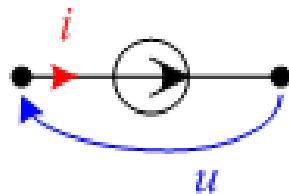
III. Elemente ideale de circuit - Sursa ideală de tensiune



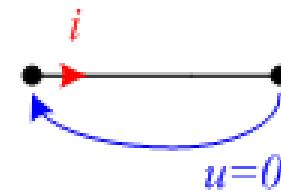
$$p(t) = u(t) \cdot i(t)$$



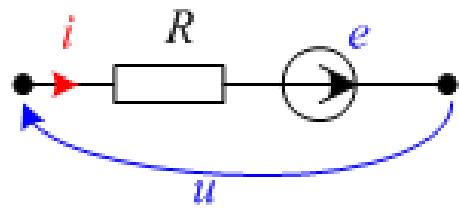
III. Elemente ideale de circuit - Sursa ideală de tensiune



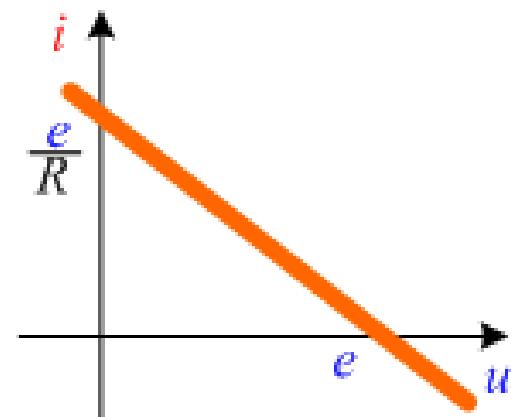
$$e = 0 \Rightarrow u = 0$$



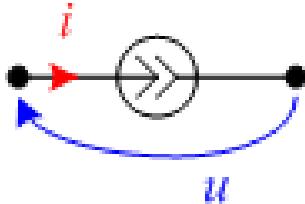
- Sursa reală de tensiune



$$u = e - R \cdot i$$



III. Elemente ideale de circuit - Sursa ideală de curent

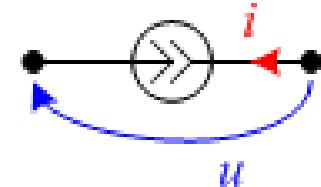


$$i(t) = j(t) \quad \forall u(t)$$

Element de circuit pentru care curentul ce o strabate este egal cu valoarea sursei, independent de valoarea tensiunii de la bornele acesteia.

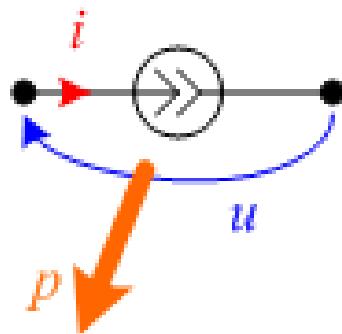
Parametru:

j curent electromotor [j] = A (amper)

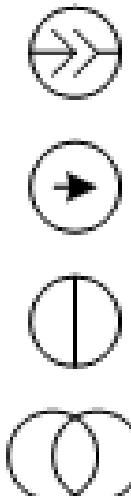
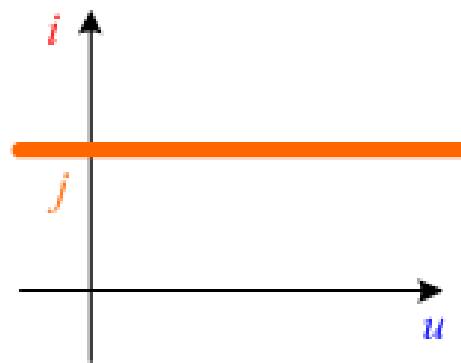


$$i(t) = -j(t) \quad \forall u(t)$$

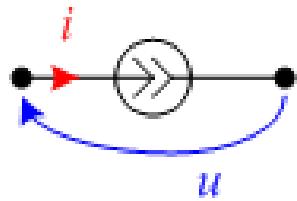
III. Elemente ideale de circuit - Sursa ideală de curent



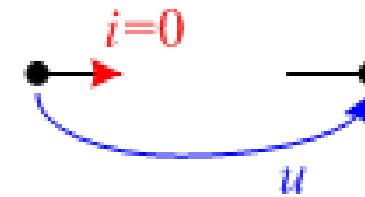
$$p(t) = u(t) \cdot i(t)$$



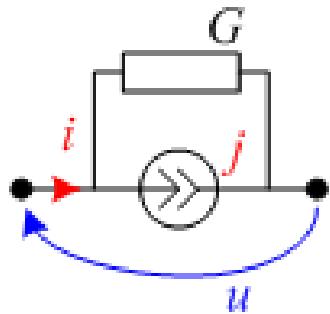
III. Elemente ideale de circuit - Sursa ideală de curent



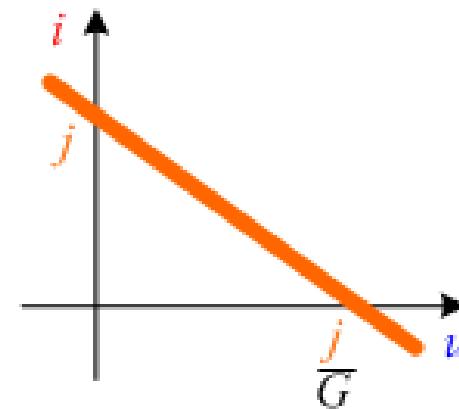
$$j = 0 \Rightarrow i = 0$$



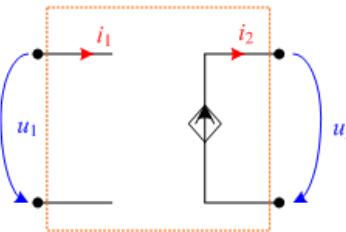
- Sursa reală de curent



$$i = j - G \cdot u$$

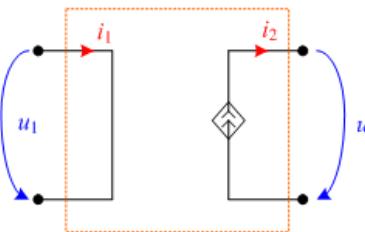


III. Elemente ideale de circuit - Surse comandate



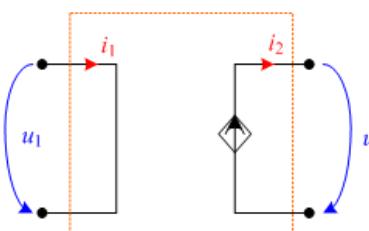
STCT

$$\begin{cases} i_1 = 0 \\ u_2 = \alpha \cdot u_1 \end{cases}$$



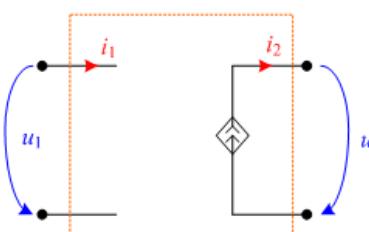
SCCC

$$\begin{cases} u_1 = 0 \\ i_2 = \beta \cdot i_1 \end{cases}$$



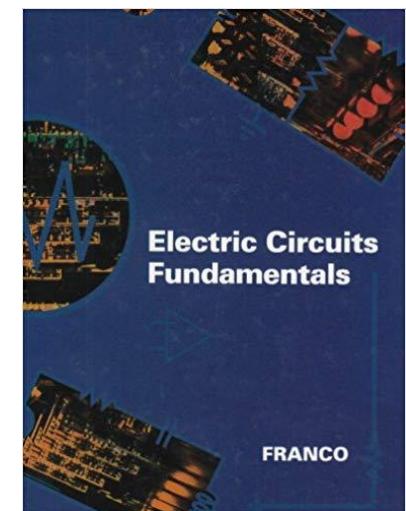
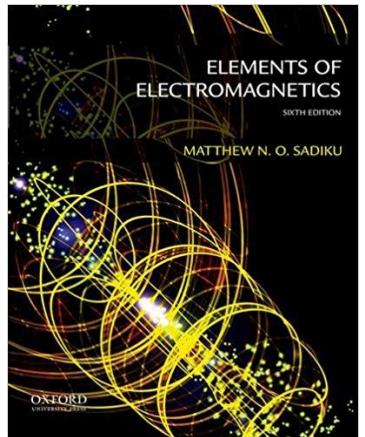
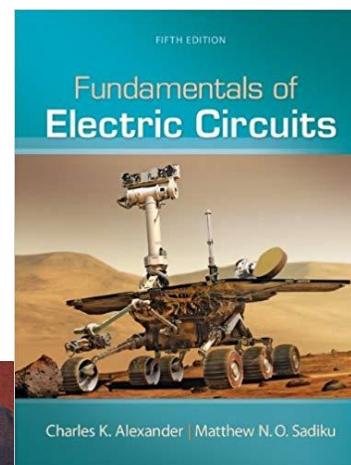
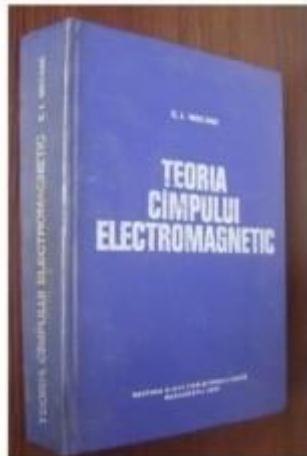
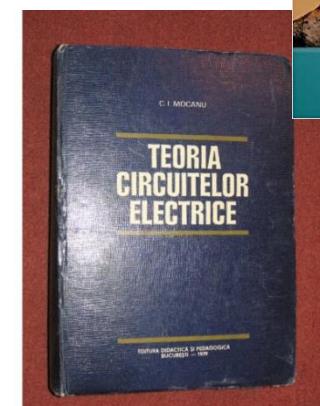
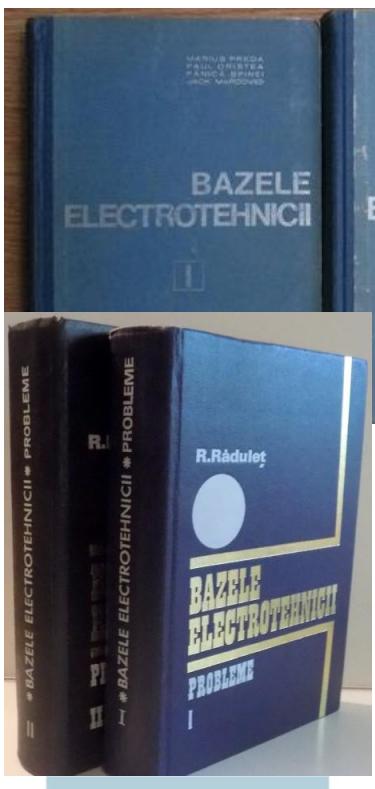
STCC

$$\begin{cases} u_1 = 0 \\ u_2 = \rho \cdot i_1 \end{cases}$$



SCCT

$$\begin{cases} i_1 = 0 \\ i_2 = \gamma \cdot u_1 \end{cases}$$



BAZELE ELECTROTEHNICII

- note de curs -

- Regimurile de functionare ale circuitelor electrice:

- **regim de curent continuu**
- **regim variabil (periodic, tranzitoriu, etc.)**

- Metode de analiza a circuitelor electrice

Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff***
- Metoda curentilor ciclici***
- Metoda potențialelor nodurilor***

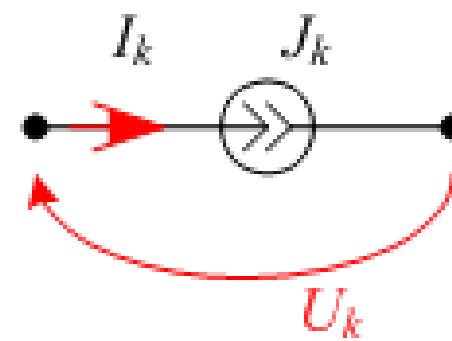
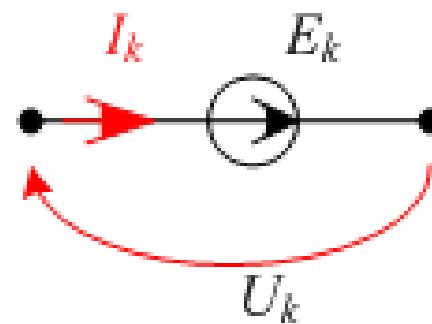
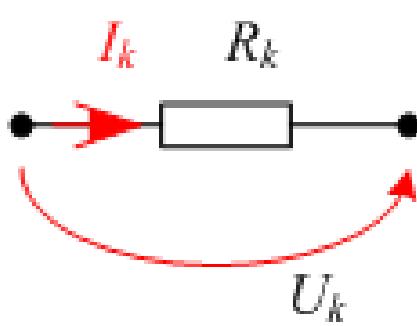
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Necunoscute



$$\begin{bmatrix} I_k \\ U_k \end{bmatrix}$$

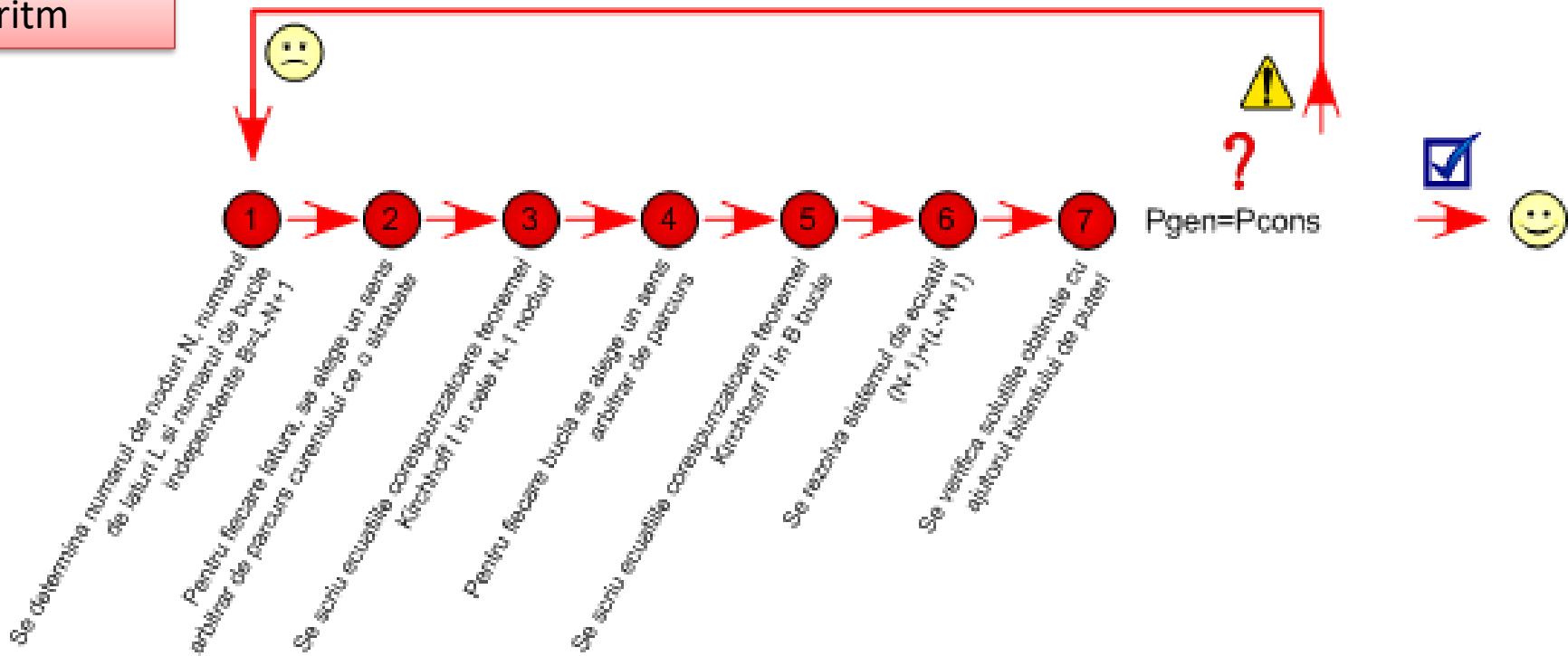
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

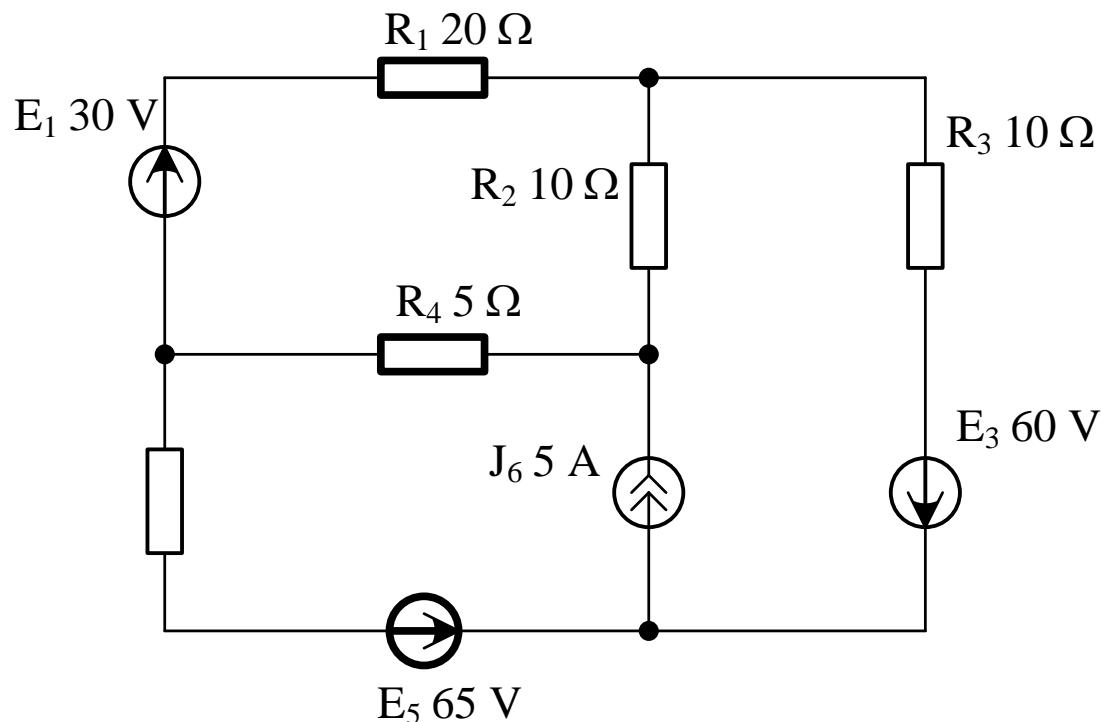
Algoritm



Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff
- Metoda curentilor ciclici
- Metoda potențialelor nodurilor

Algoritm



P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I ($N-1$)

P4 – Ecuatii TK II ($L-N+1$)

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

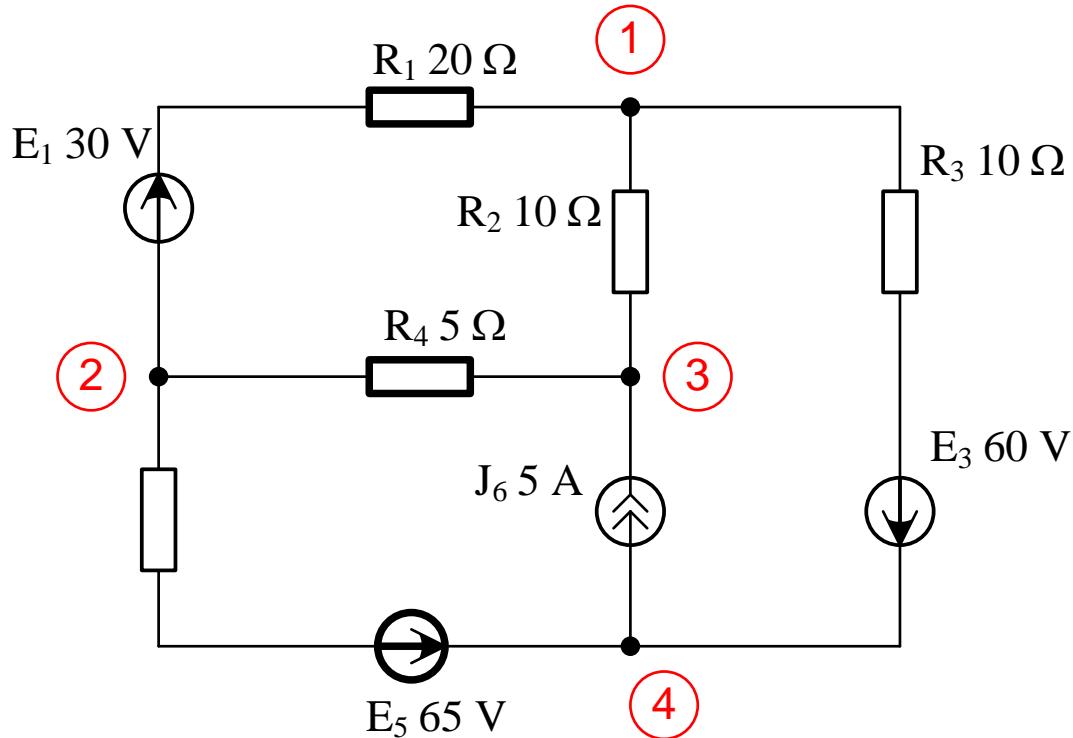
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Algoritm



$$\begin{aligned}N &= 4 \\L &= 6 \\B &= 3\end{aligned}$$

P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I ($N-1$)

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P7 – Verificare bilant puteri

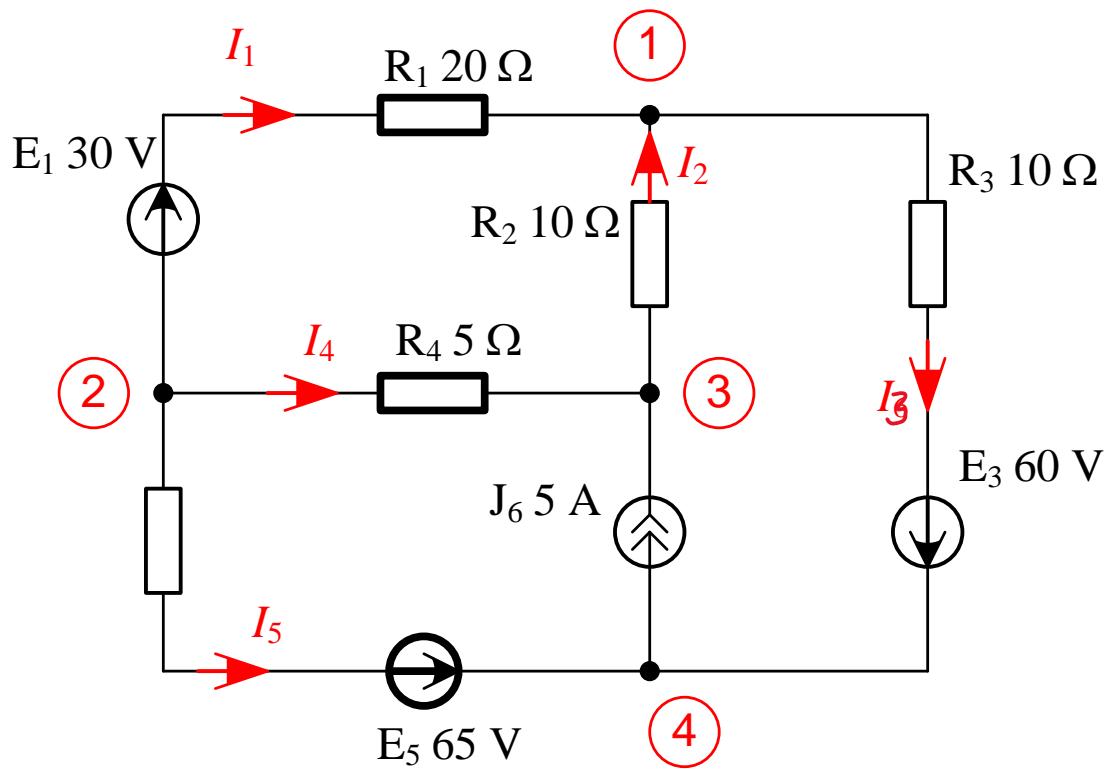
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Algoritm



$$\begin{aligned} N &= 4 \\ L &= 6 \\ B &= 3 \end{aligned}$$

P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

**P2 – Sens arbitrar curenti,
sens arbitrar bucle**

P3 – Ecuatii TK I ($N-1$)

P4 – Ecuatii TK II ($L-N+1$)

P5 – completare system cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

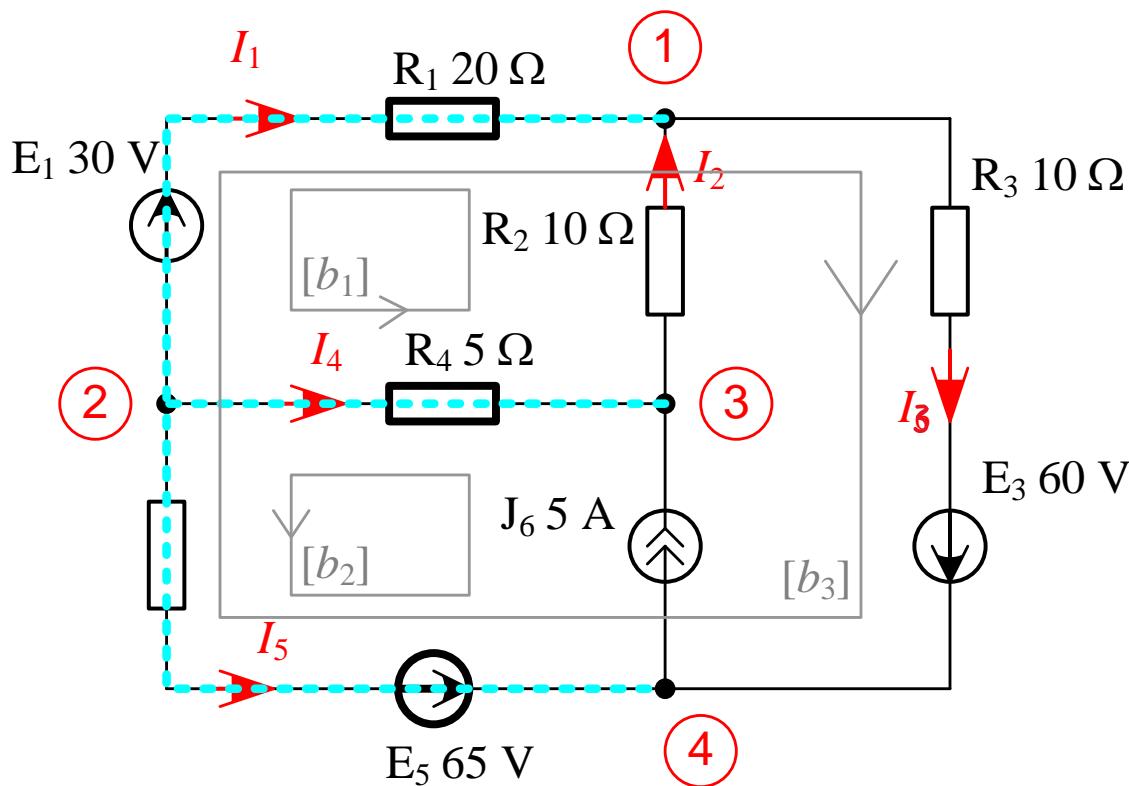
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Algoritm



P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I ($N-1$)

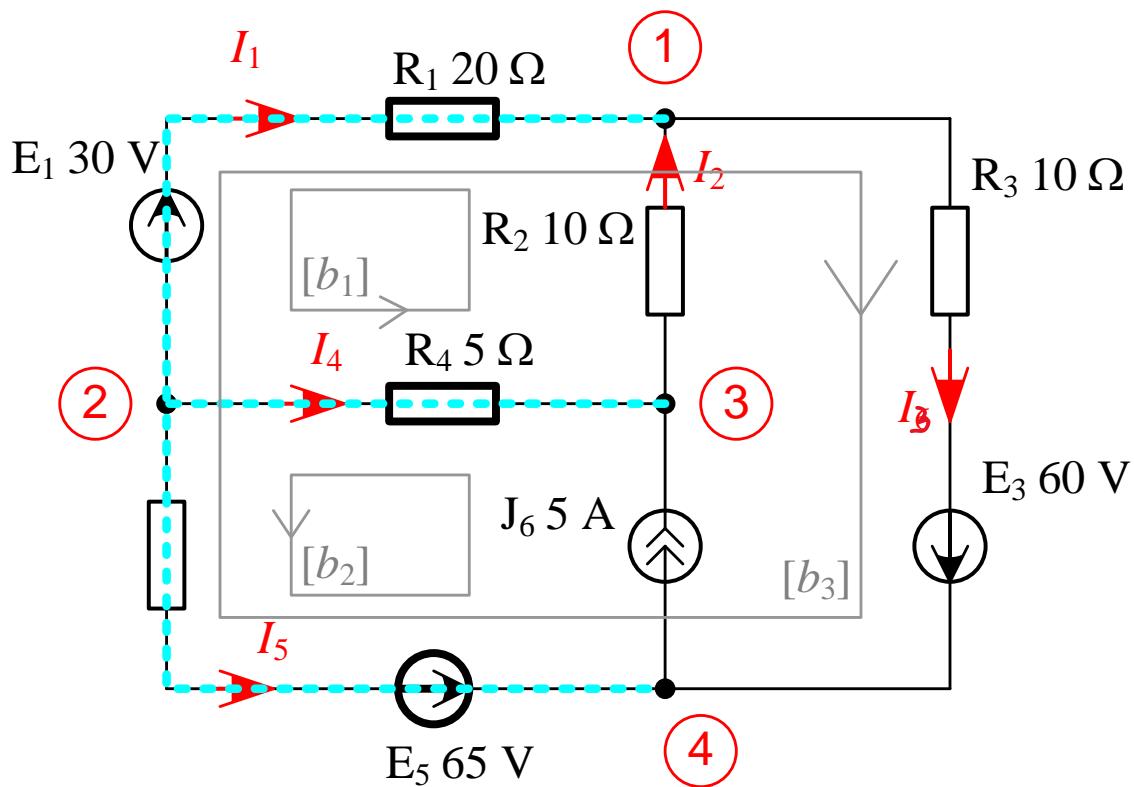
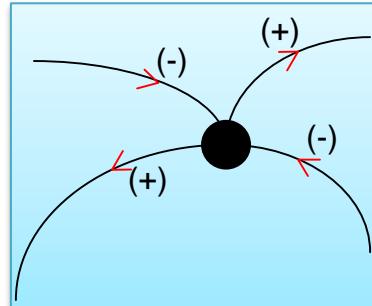
P4 – Ecuatii TK II ($L-N+1$)

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

$$\left\{ \begin{array}{l} -I_1 - I_2 + I_3 = 0 \quad \leftarrow n1 \end{array} \right.$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I (N-1)

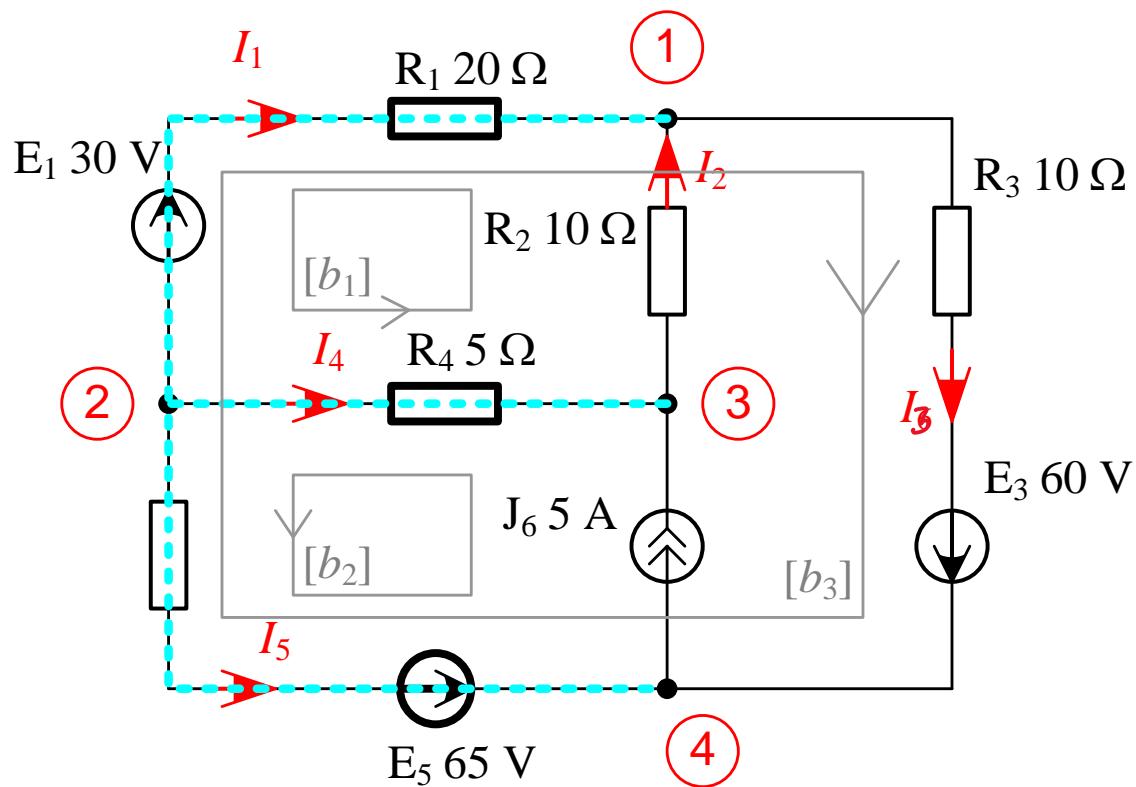
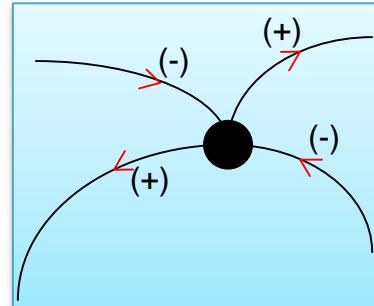
P4 – Ecuatii TK II ($L-N+1$)

P5 – completare system cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

$$\begin{cases} -I_1 - I_2 + I_3 = 0 & \leftarrow n1 \\ I_1 + I_4 + I_5 = 0 & \leftarrow n2 \end{cases}$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I (N-1)

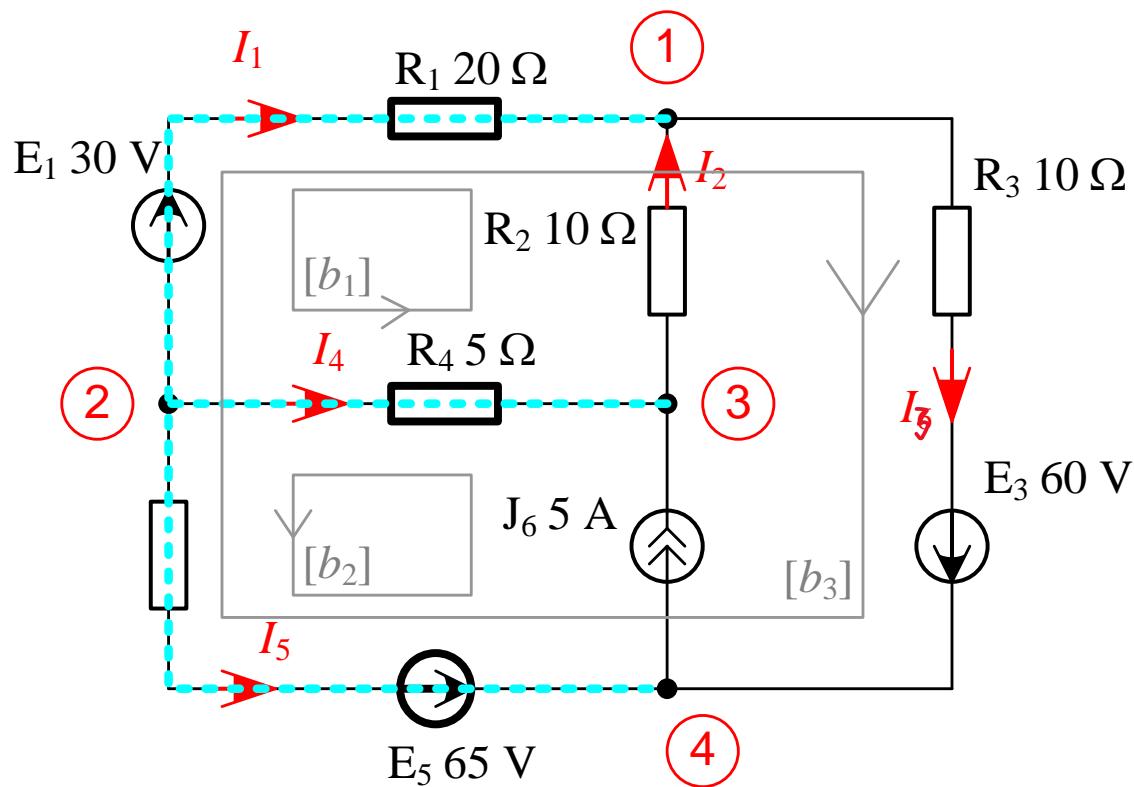
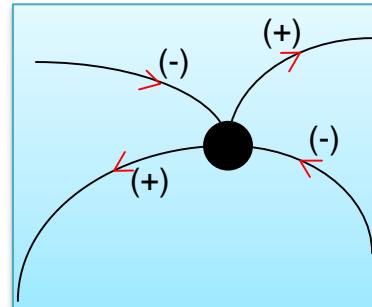
P4 – Ecuatii TK II (L-N+1)

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

$$\begin{cases} -I_1 - I_2 + I_3 = 0 & \leftarrow n1 \\ I_1 + I_4 + I_5 = 0 & \leftarrow n2 \\ I_2 - I_4 - I_6 = 0 & \leftarrow n3 \end{cases}$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I (N-1)

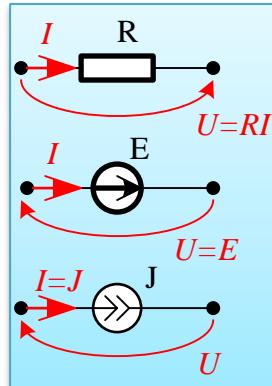
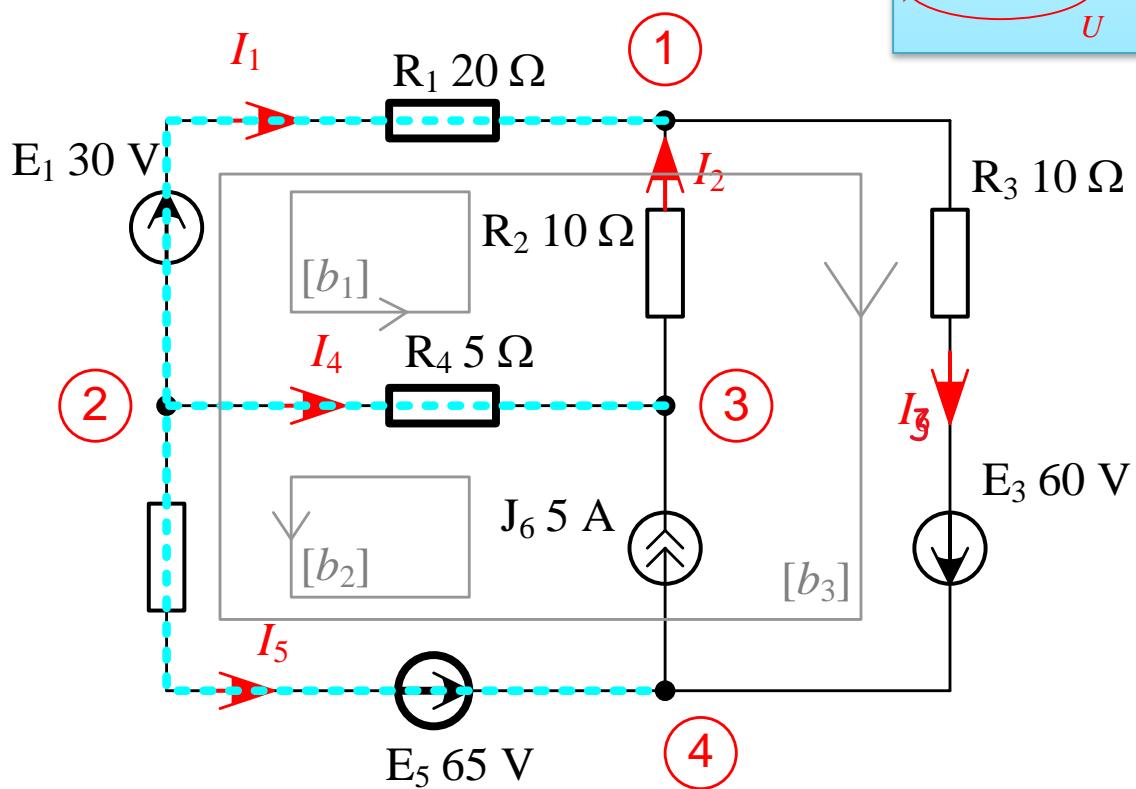
P4 – Ecuatii TK II (L-N+1)

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

$$\left\{ R_4 I_4 + R_2 I_2 - R_1 I_1 + E_1 = 0 \quad \leftarrow b1 \right.$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I (N-1)

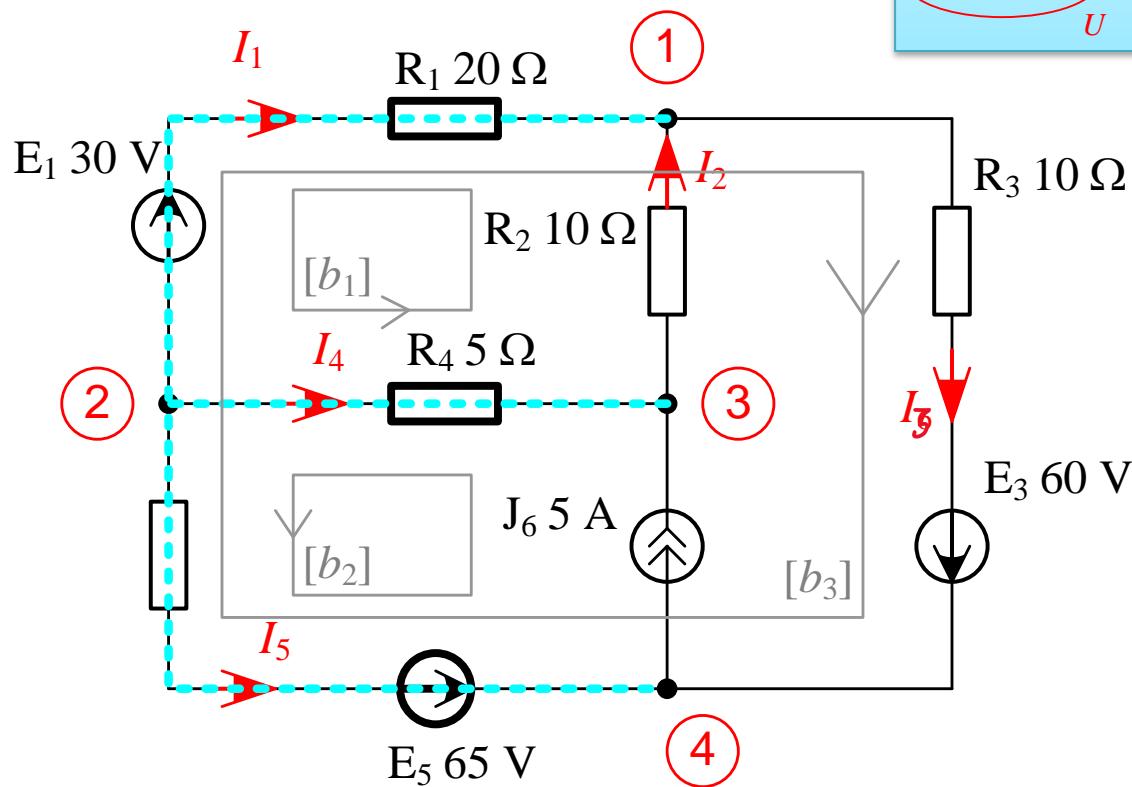
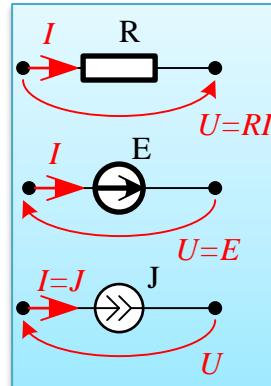
P4 – Ecuatii TK II (L-N+1)

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

$$\begin{cases} R_4 I_4 + R_2 I_2 - R_1 I_1 + E_1 = 0 & \leftarrow b1 \\ R_5 I_5 - E_5 - U_6 - R_4 I_4 = 0 & \leftarrow b2 \end{cases}$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I (N-1)

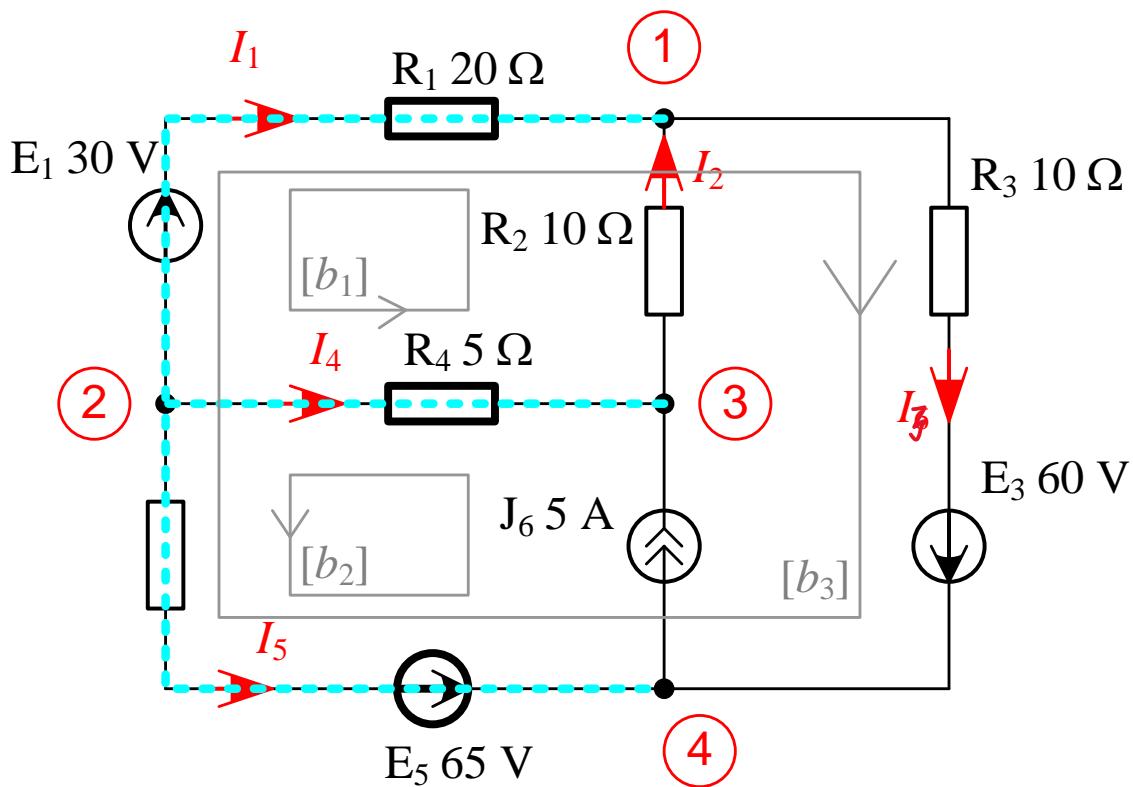
P4 – Ecuatii TK II (L-N+1)

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

$$\begin{cases} R_4 I_4 + R_2 I_2 - R_1 I_1 + E_1 = 0 & \leftarrow b1 \\ R_5 I_5 - E_5 - U_6 - R_4 I_4 = 0 & \leftarrow b2 \\ R_3 I_3 - E_3 + E_5 - R_5 I_5 - E_1 + R_1 I_1 = 0 & \leftarrow b3 \end{cases}$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I ($N-1$)

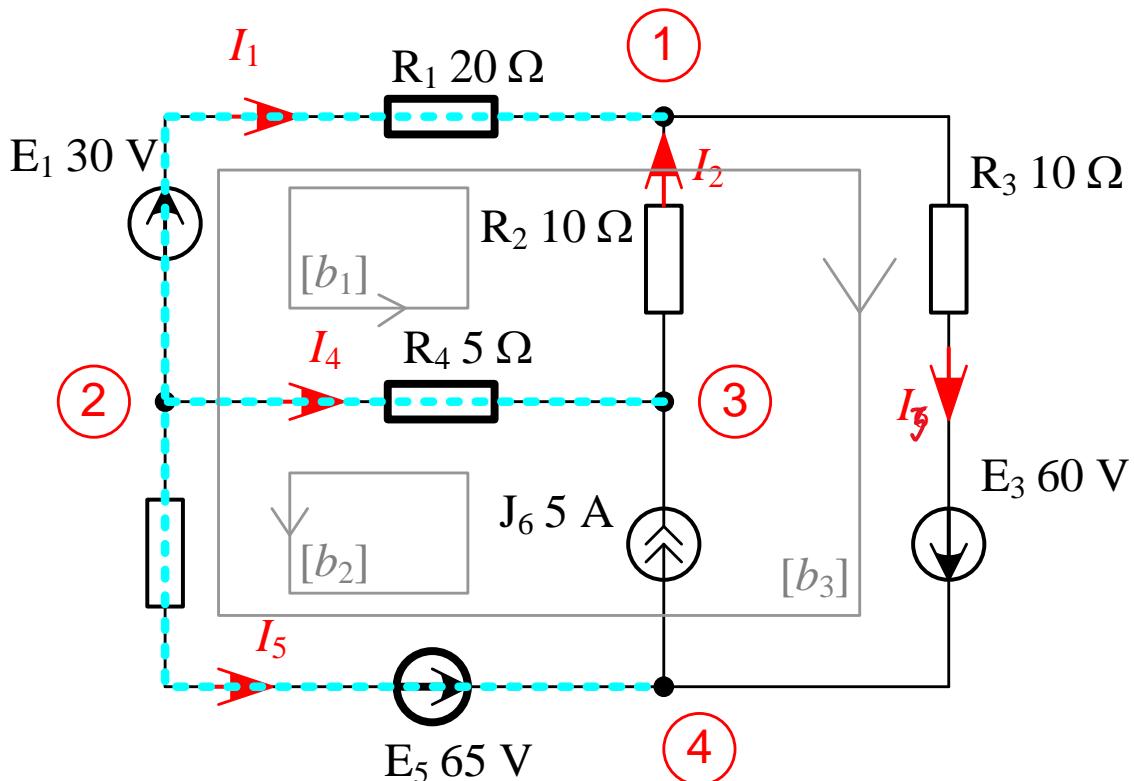
P4 – Ecuatii TK II ($L-N+1$)

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

$$\left\{ \begin{array}{l} -I_1 - I_2 + I_3 = 0 \\ I_1 + I_4 + I_5 = 0 \\ I_2 - I_4 - J_6 = 0 \\ R_4 I_4 + R_2 I_2 - R_1 I_1 + E_1 = 0 \\ R_5 I_5 - E_5 - U_6 - R_4 I_4 = 0 \\ R_3 I_3 - E_3 + E_5 - R_5 I_5 - E_1 + R_1 I_1 = 0 \end{array} \right.$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I (N-1)

P4 – Ecuatii TK II (L-N+1)

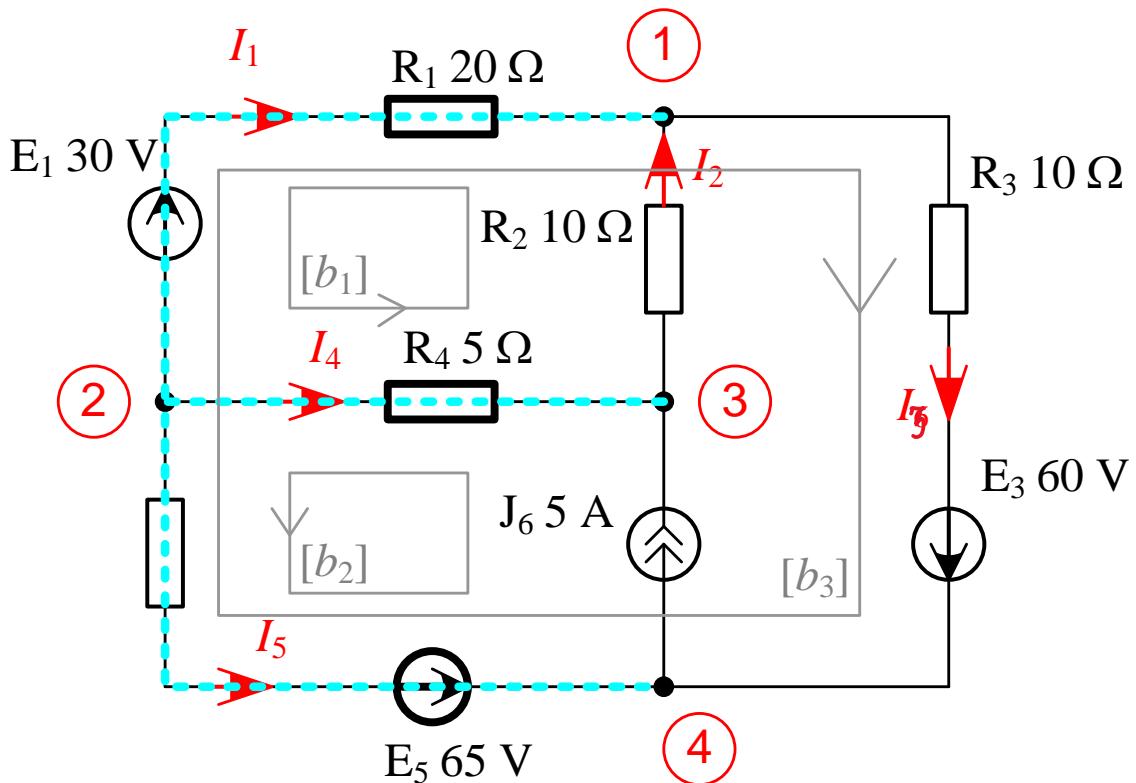
**P5 – completare sistem cu
expresiile surselor
comandate**

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

$$\left\{ \begin{array}{l} -I_1 - I_2 + I_3 = 0 \\ I_1 + I_4 + I_5 = 0 \\ I_2 - I_4 - 5 = 0 \\ 5I_4 + 10I_2 - 20I_1 + 30 = 0 \\ 5I_5 - 65 - U_6 - 5I_4 = 0 \\ 10I_3 - 60 + E_5 - 5I_5 - 30 + 20I_1 = 0 \end{array} \right.$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -4 \\ 3 \\ -30 \end{bmatrix}$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I ($N-1$)

P4 – Ecuatii TK II ($L-N+1$)

P5 – completare sistem cu
expresiile surselor
comandate

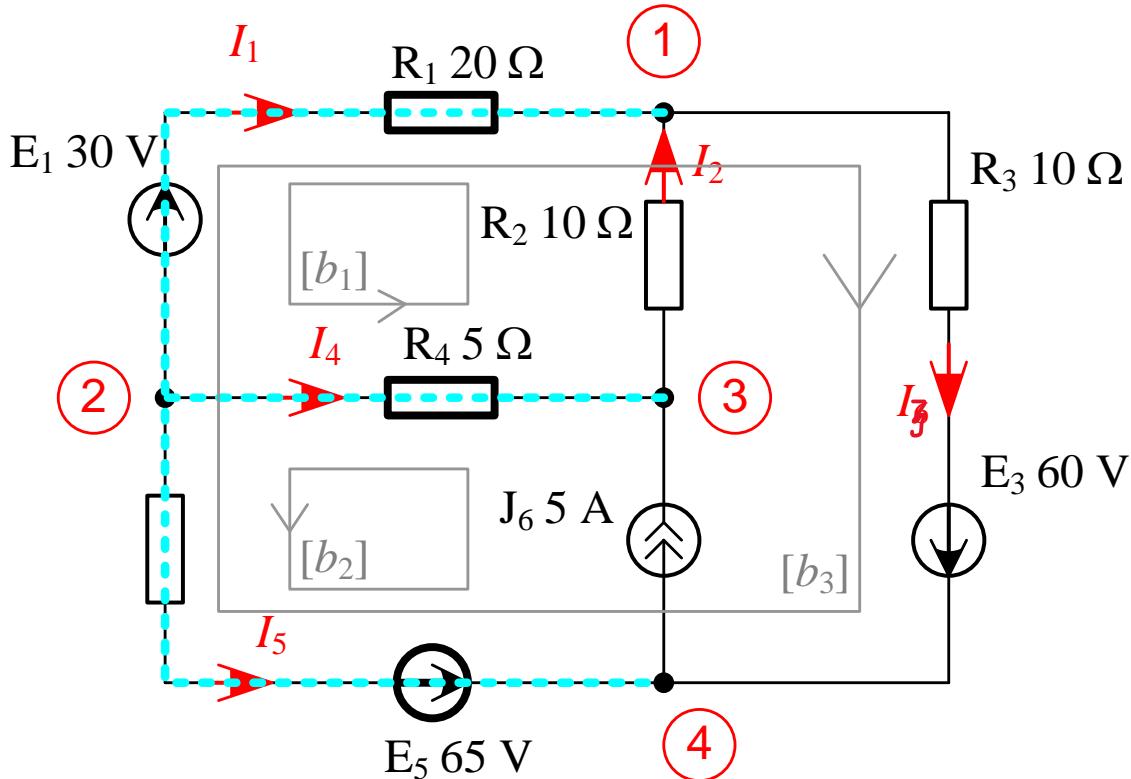
**P6 – Rezolvare numerica a
sistemului**

P7 – Verificare bilant puteri

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -4 \\ 3 \\ -30 \end{bmatrix}$$

$$P_{gen} = E_1 I_1 + E_3 I_3 + E_5 I_5 + U_6 J_6 = 195W$$

$$P_{cons} = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 + R_4 I_4^2 + R_5 I_5^2 = 195W$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle

P3 – Ecuatii TK I (N-1)

P4 – Ecuatii TK II (L-N+1)

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

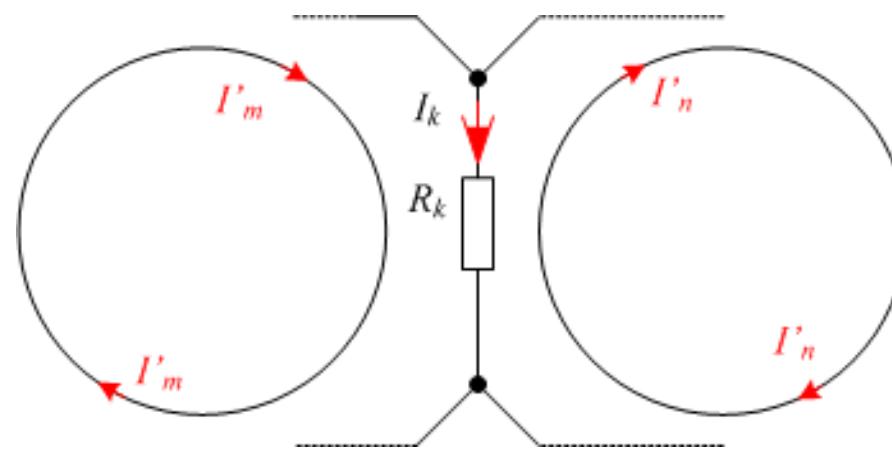
Metode de analiza a circuitelor electrice:

-*Metoda teoremelor lui Kirchhoff*

-**Metoda curentilor ciclici**

-*Metoda potențialelor nodurilor*

Necunoscute



$$I_k = \sum_{p \supset k} (A) I'_p$$

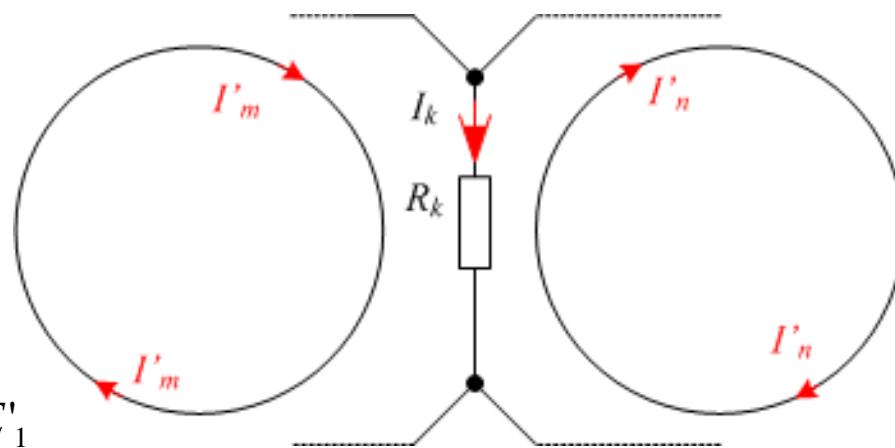
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Necunoscute



$$\begin{cases} R_{11}I'_1 + R_{12}I'_2 + \dots + R_{1B}I'_B = E'_1 \\ R_{21}I'_1 + R_{22}I'_2 + \dots + R_{2B}I'_B = E'_2 \\ \dots \\ R_{B1}I'_1 + R_{B2}I'_2 + \dots + R_{BB}I'_B = E'_B \end{cases}$$

$$I_k = \sum_{p \supset k} (A) I'_p$$

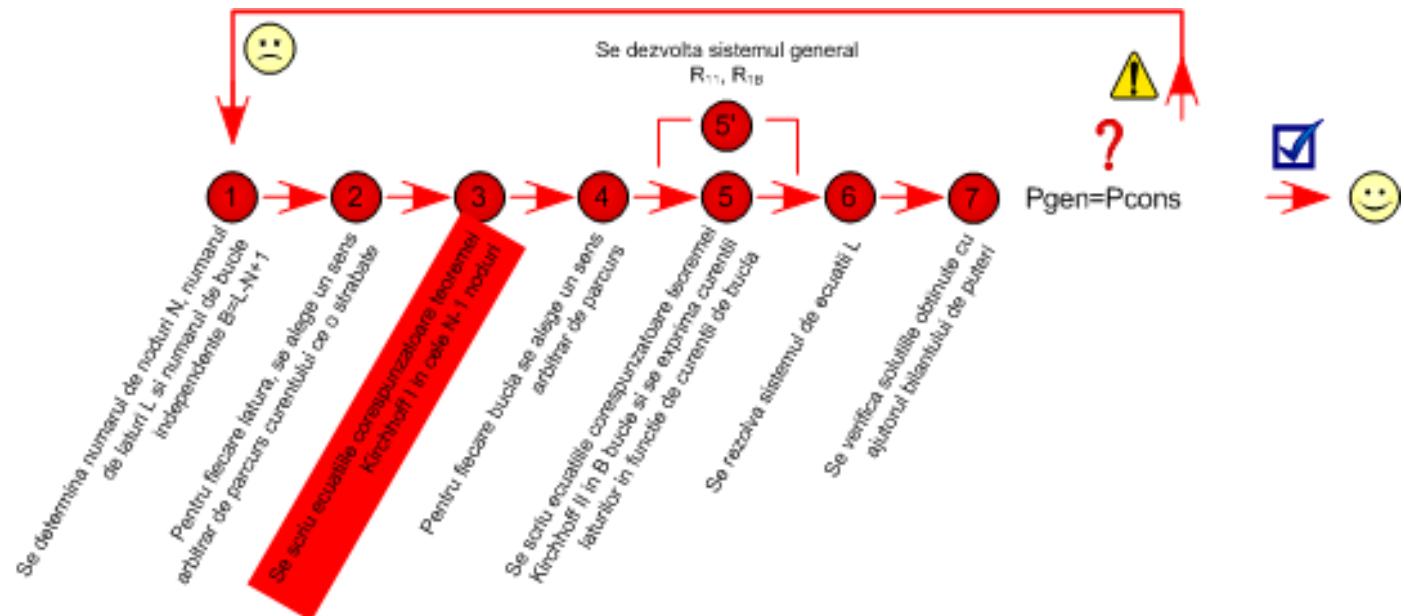
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Algoritm



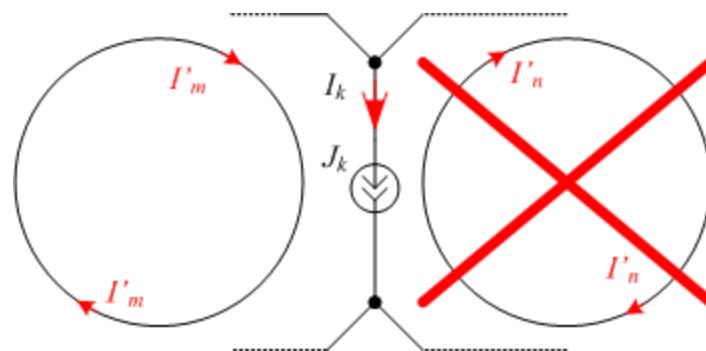
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Caz special

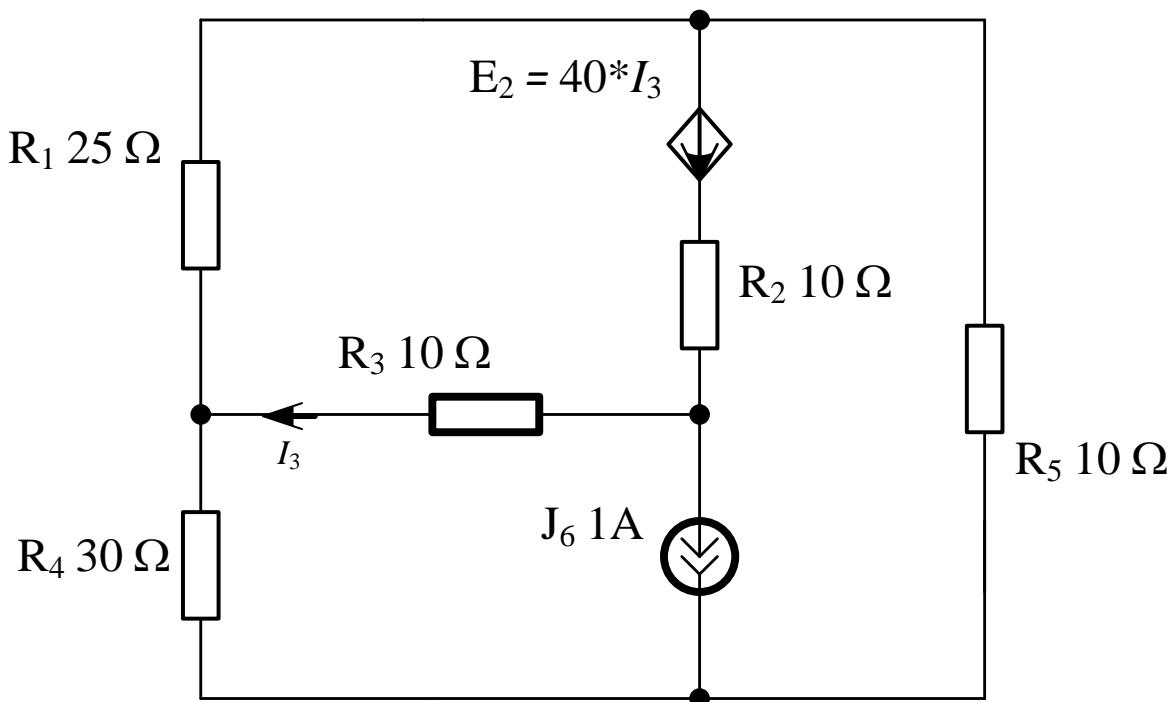


$$I'm = J_k$$

Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff
- Metoda curentilor ciclici
- Metoda potențialelor nodurilor

Algoritm



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle (curenti
ciclici)

P3 – Ecuatii TK II (bucle fara
surse de curent)

P4 – Expresiile curentilor
functie de curentii ciclici

P5 – completare sistem cu
expresiile surselor
comandate

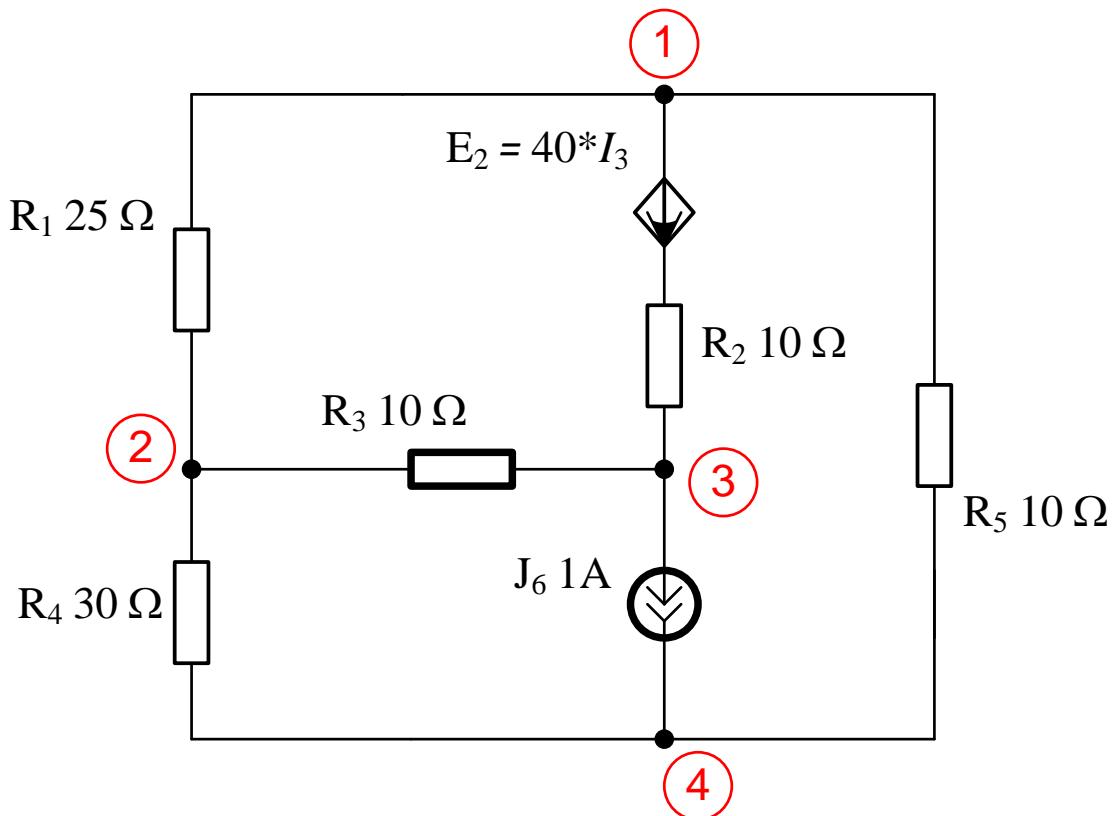
P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff
- Metoda curentilor ciclici
- Metoda potențialelor nodurilor

Algoritm



$$\begin{aligned} N &= 4 \\ L &= 6 \\ B &= 3 \end{aligned}$$

P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle (curenti
ciclici)

P3 – Ecuatii TK II (bucle fara
surse de curent)

P4 – Expresiile curentilor
functie de curentii ciclici

P5 – completare sistem cu
expresiile surselor
comandate

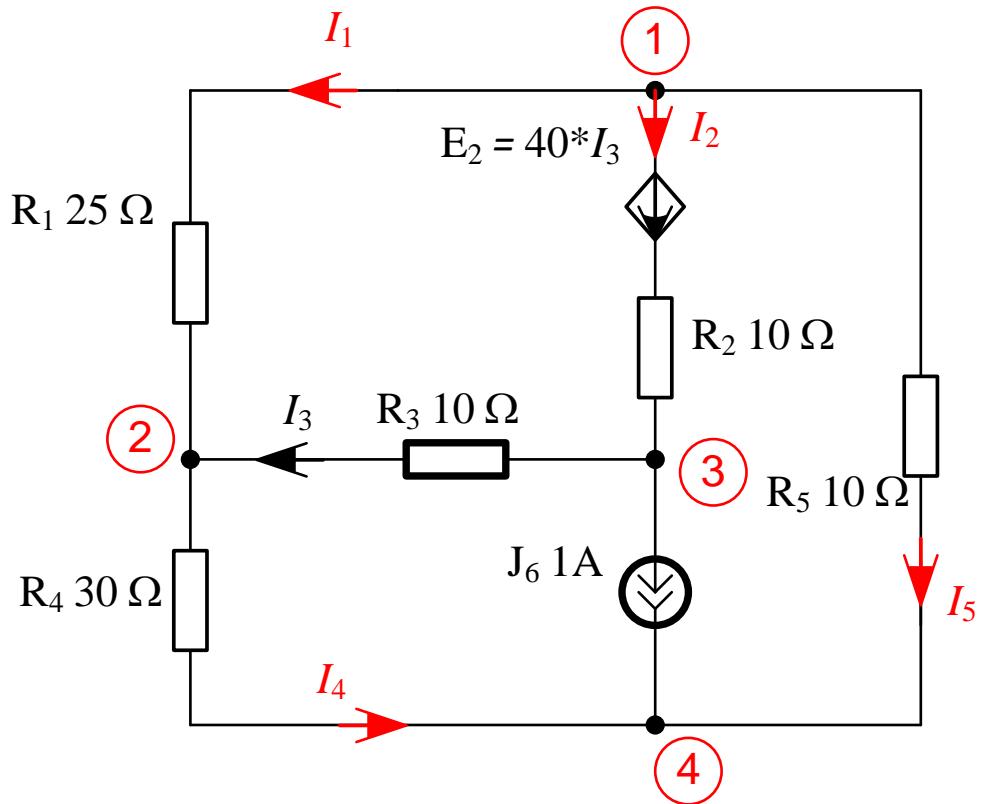
P6 – Rezolvare numerica a
sistemului

P7 – Verificare bilant puteri

Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff
- Metoda curentilor ciclici
- Metoda potențialelor nodurilor

Algoritm



$$\begin{aligned} N &= 4 \\ L &= 6 \\ B &= 3 \end{aligned}$$

P1 – numar de noduri N,
numar de laturi L, numar de bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle (currenti ciclici)

P3 – Ecuatii TK II (bucle fara surse de curent)

P4 – Expresiile curentilor
functie de curentii ciclici

P5 – completare sistem cu
expresiile surselor
comandate

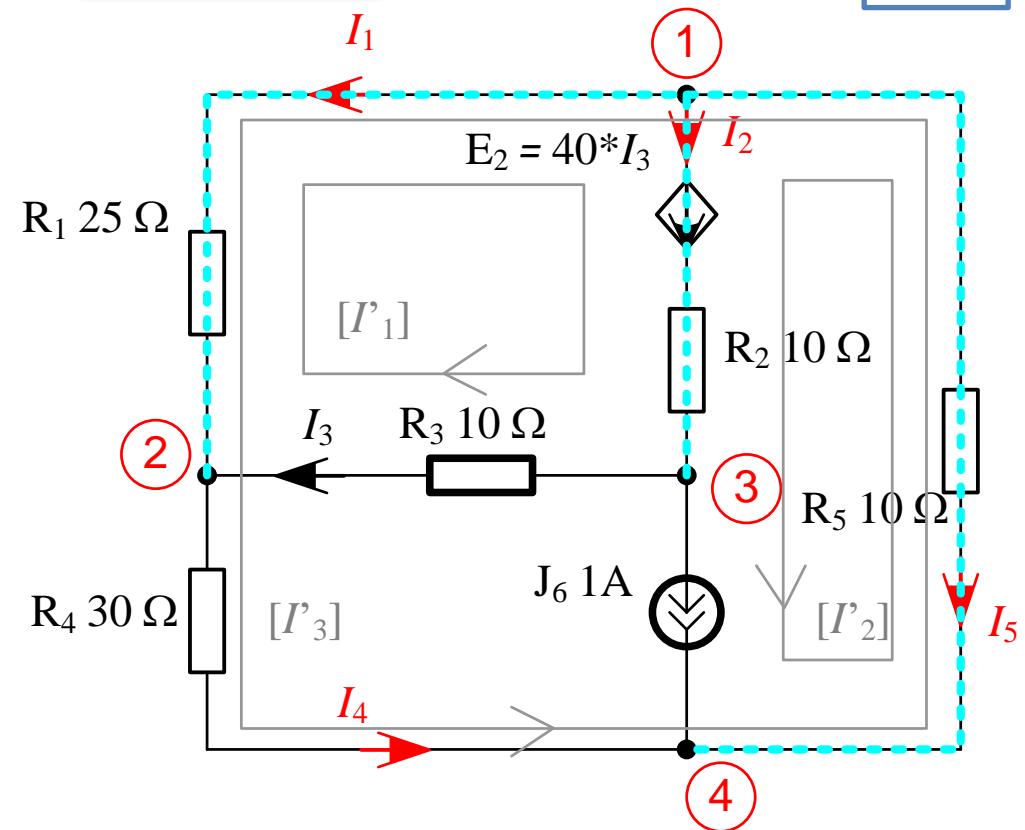
P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff
- Metoda curentilor ciclici
- Metoda potențialelor nodurilor

Algoritm



$$\begin{aligned} N &= 4 \\ L &= 6 \\ B &= 3 \end{aligned}$$

P1 – numar de noduri N,
numar de laturi L, numar de bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle (currenti ciclici)

P3 – Ecuatii TK II (bucle fara surse de curent)

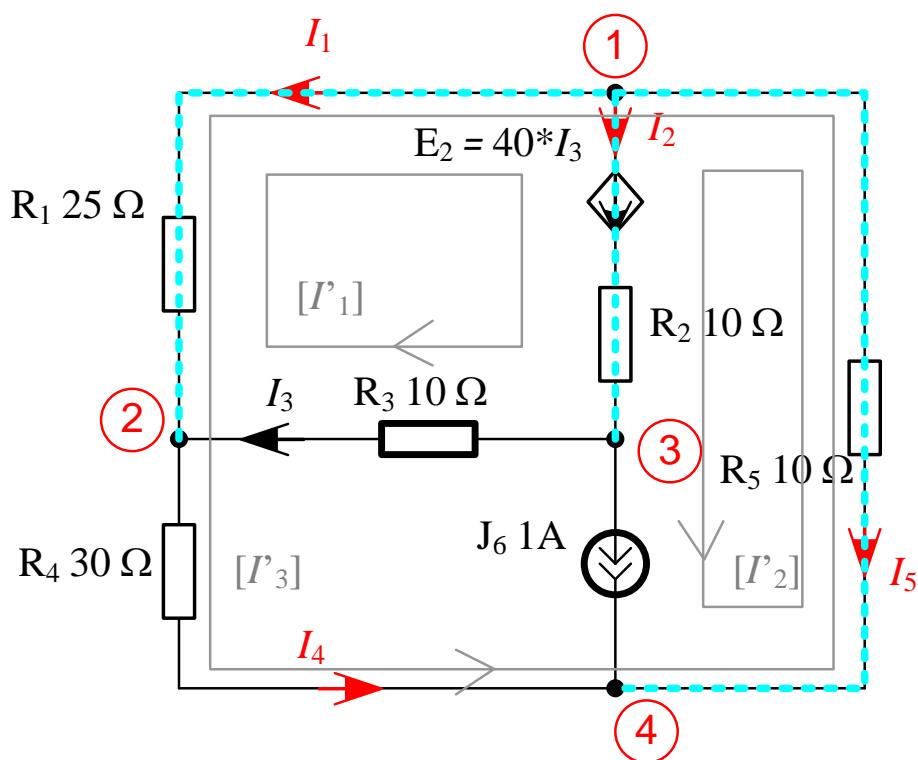
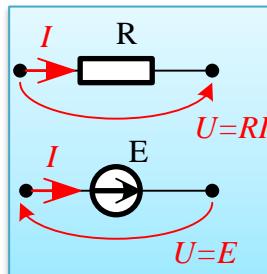
P4 – Expresiile curentilor functie de curentii ciclici

P5 – completare sistem cu expresiile surselor comandate

P6 – Rezolvare numérica a sistemului

P7 – Verificare bilant puteri

$$\begin{cases} R_2 I_2 + R_3 I_3 - R_1 I_1 - E_2 = 0 & \leftarrow b1 \\ R_4 I_4 - R_5 I_5 + R_1 I_1 = 0 & \leftarrow b3 \end{cases}$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle (curenti
ciclici)

**P3 – Ecuatii TK II (bucle fara
surse de curent)**

P4 – Expresiile curentilor
functie de curentii ciclici

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemului

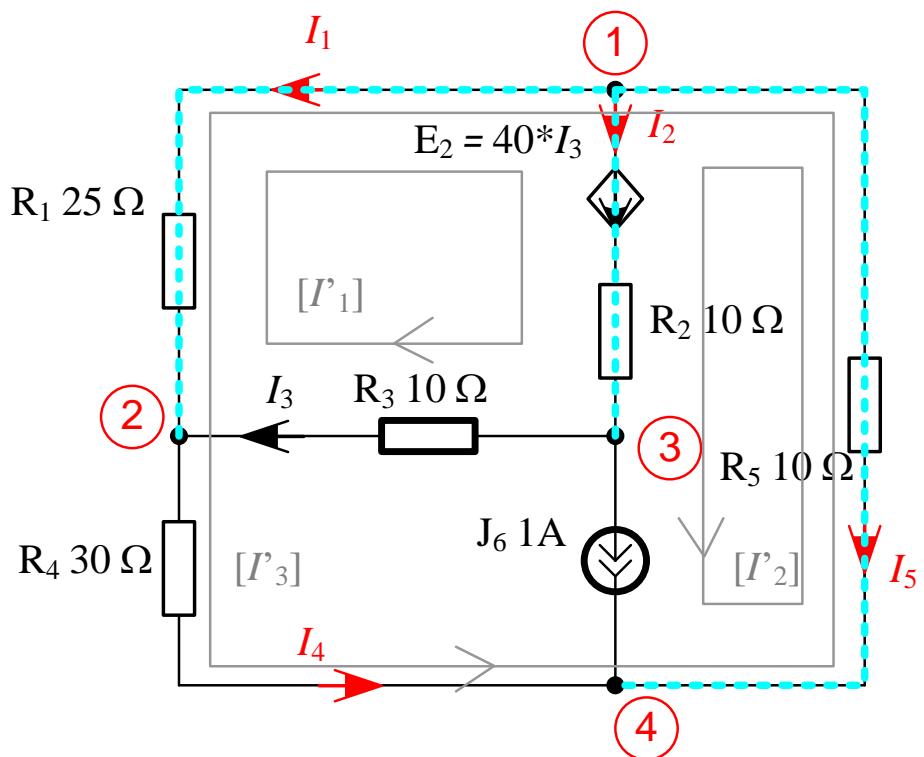
P7 – Verificare bilant puteri

$$\begin{cases} R_2 I_2 + R_3 I_3 - R_1 I_1 - E_2 = 0 & \leftarrow b1 \\ R_4 I_4 - R_5 I_5 + R_1 I_1 = 0 & \leftarrow b3 \end{cases}$$

$$\begin{cases} R_2(I'_1 + I'_2) + R_3(I'_1) - R_1(-I'_1 + I'_3) - E_2 = 0 \\ R_4(I'_3) - R_5(-I'_2 - I'_3) + R_1(-I'_1 + I'_3) = 0 \end{cases}$$

$$I'_2 = J_6$$

$$\begin{aligned} I_1 &= -I'_1 + I'_3 \\ I_2 &= I'_1 + I'_2 \\ I_3 &= I'_1 \\ I_4 &= I'_3 \\ I_5 &= -I'_2 - I'_3 \end{aligned}$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle (curenti
ciclici)

P3 – Ecuatii TK II (bucle fara
surse de curent)

**P4 – Expresiile curentilor
functie de curentii ciclici**

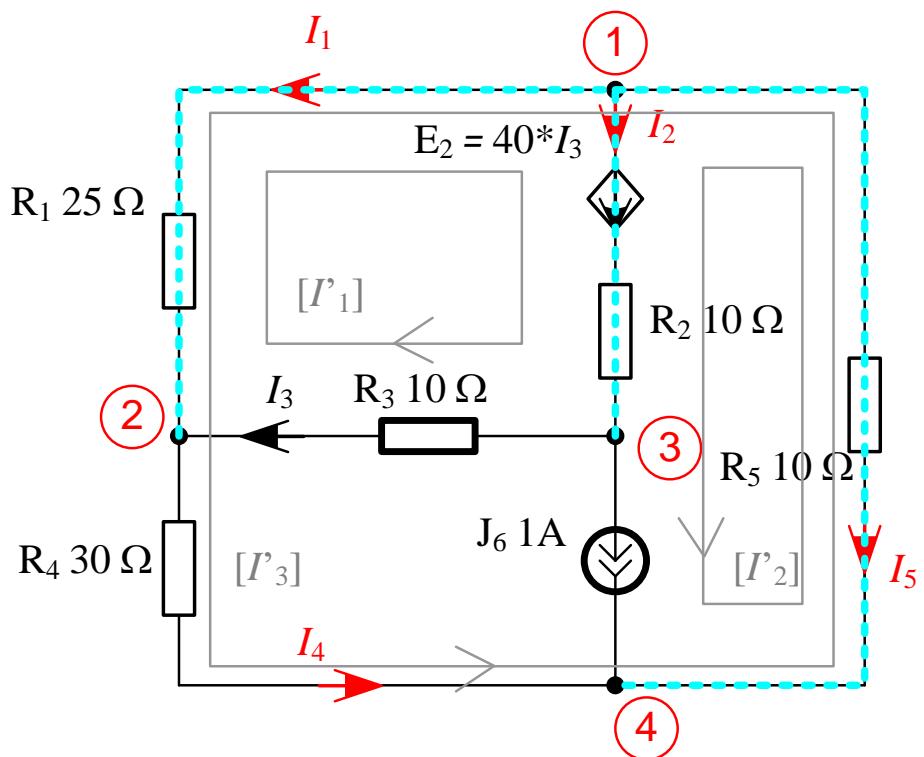
P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numerica a
sistemu

P7 – Verificare bilant puteri

$$\begin{cases} R_2(I'_1 + I'_2) + R_3(I'_1) - R_1(-I'_1 + I'_3) - E_2 = 0 \\ R_4(I'_3) - R_5(-I'_2 - I'_3) + R_1(-I'_1 + I'_3) = 0 \\ I'_2 = J_6, \quad E_2 = 40 \cdot I_3 = 40(I'_1) \end{cases}$$

$$\begin{aligned} I_1 &= -I'_1 + I'_3 \\ I_2 &= I'_1 + I'_2 \\ I_3 &= I'_1 \\ I_4 &= I'_3 \\ I_5 &= -I'_2 - I'_3 \end{aligned}$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle (curenti
ciclici)

P3 – Ecuatii TK II (bucle fara
surse de curent)

P4 – Expresiile curentilor
functie de curentii ciclici

P5 – completare sistem cu
expresiile surselor
comandate

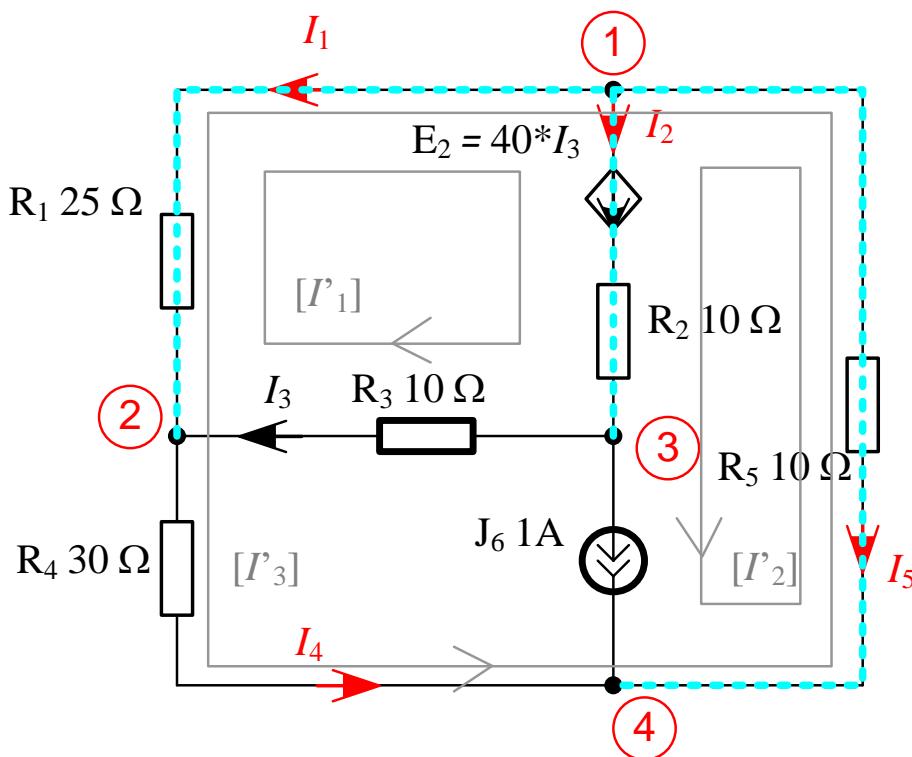
P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

$$\begin{bmatrix} I'_1 \\ I'_2 \\ I'_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 3 \\ 1 \\ -2 \end{bmatrix}$$

$$\begin{aligned} I_1 &= -I'_1 + I'_3 \\ I_2 &= I'_1 + I'_2 \\ I_3 &= I'_1 \\ I_4 &= I'_3 \\ I_5 &= -I'_2 - I'_3 \end{aligned}$$

$$U_6 = -R_4 I_4 - R_3 I_3 = -60V$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle $B=L-N+1$

P2 – Sens arbitrar curenti,
sens arbitrar bucle (curenti
ciclici)

P3 – Ecuatii TK II (bucle fara
surse de curent)

P4 – Expresiile curentilor
functie de curentii ciclici

P5 – completare sistem cu
expresiile surselor
comandate

**P6 – Rezolvare numerica a
sistemu**

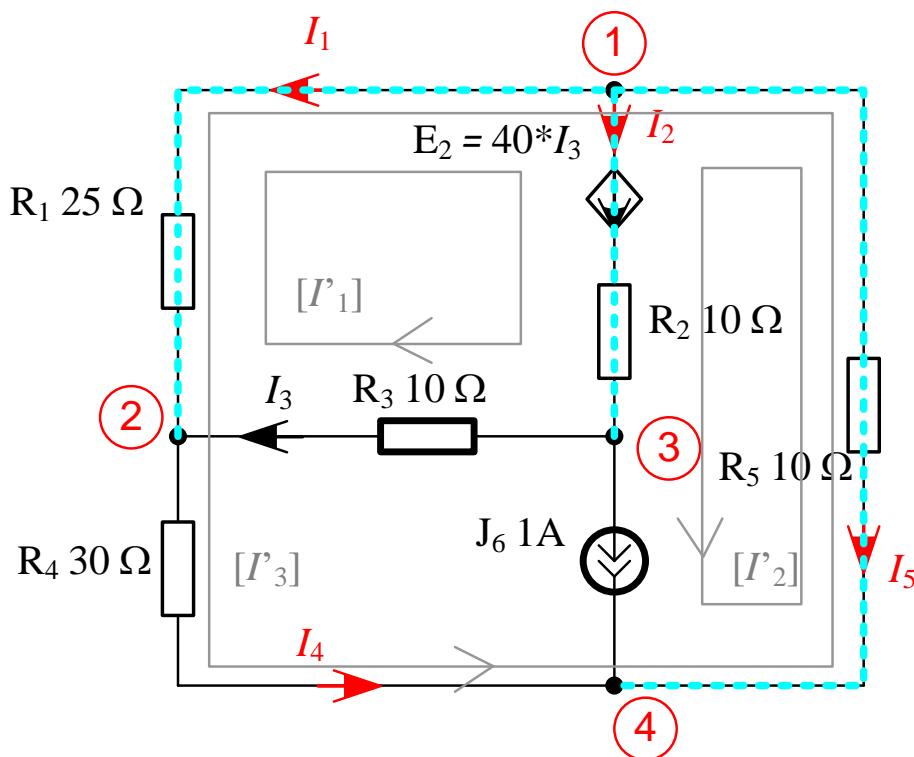
P7 – Verificare bilant puteri

$$\begin{bmatrix} I'_1 \\ I'_2 \\ I'_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 3 \\ 1 \\ -2 \end{bmatrix}$$

$$U_6 = -R_4 I_4 - R_3 I_3 = -60V$$

$$P_{gen} = E_2 I_2 + U_6 J_6 = 420W$$

$$P_{cons} = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 + R_4 I_4^2 + R_5 I_5^2 = 420W$$



P1 – numar de noduri N,
numar de laturi L, numar de
bucle B=L-N+1

P2 – Sens arbitrar curenti,
sens arbitrar bucle (curenti
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expresiile surselor
comandate

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

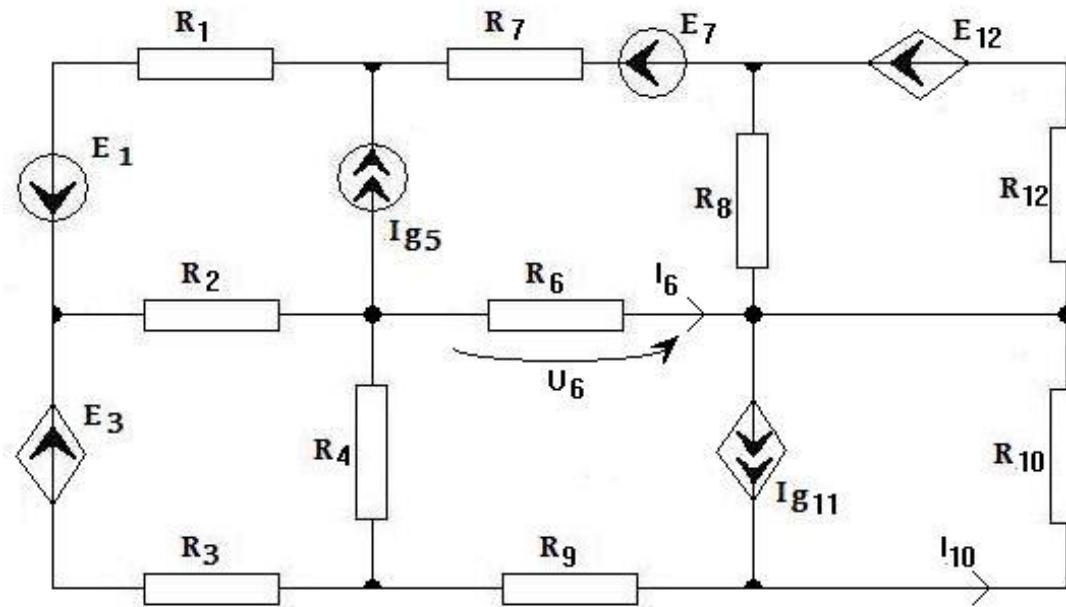
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

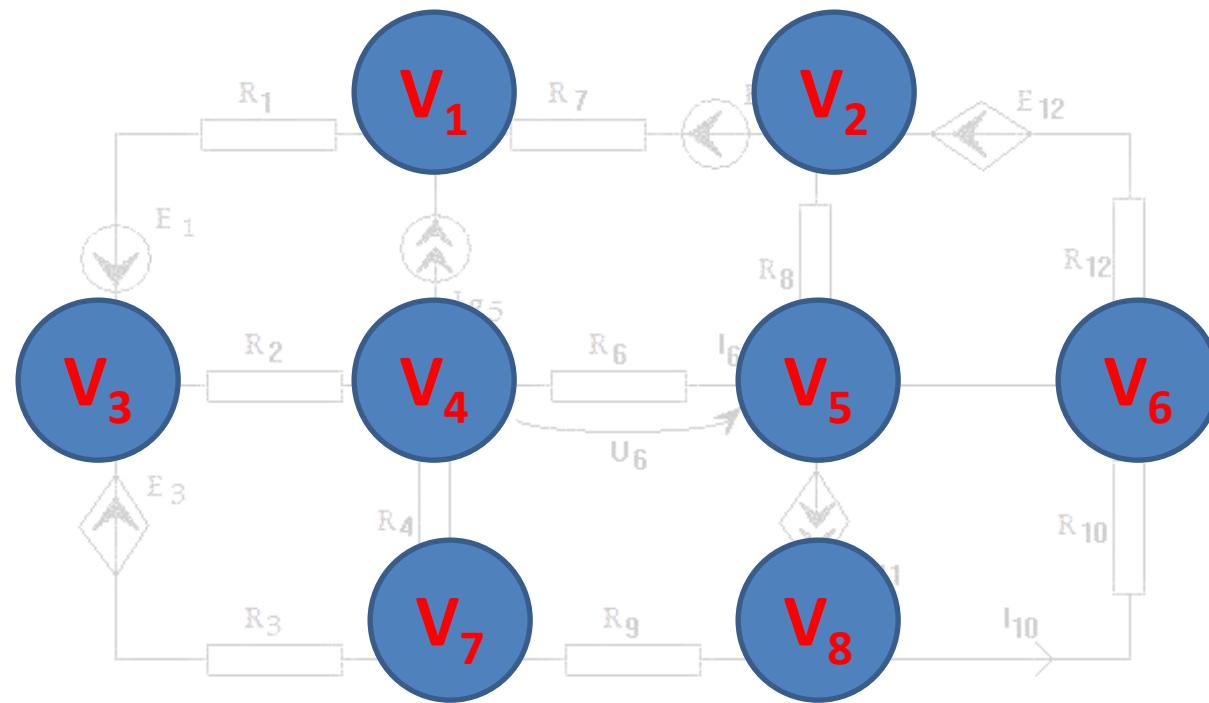
Necunoscute



Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff
- Metoda curentilor ciclici
- Metoda potențialelor nodurilor

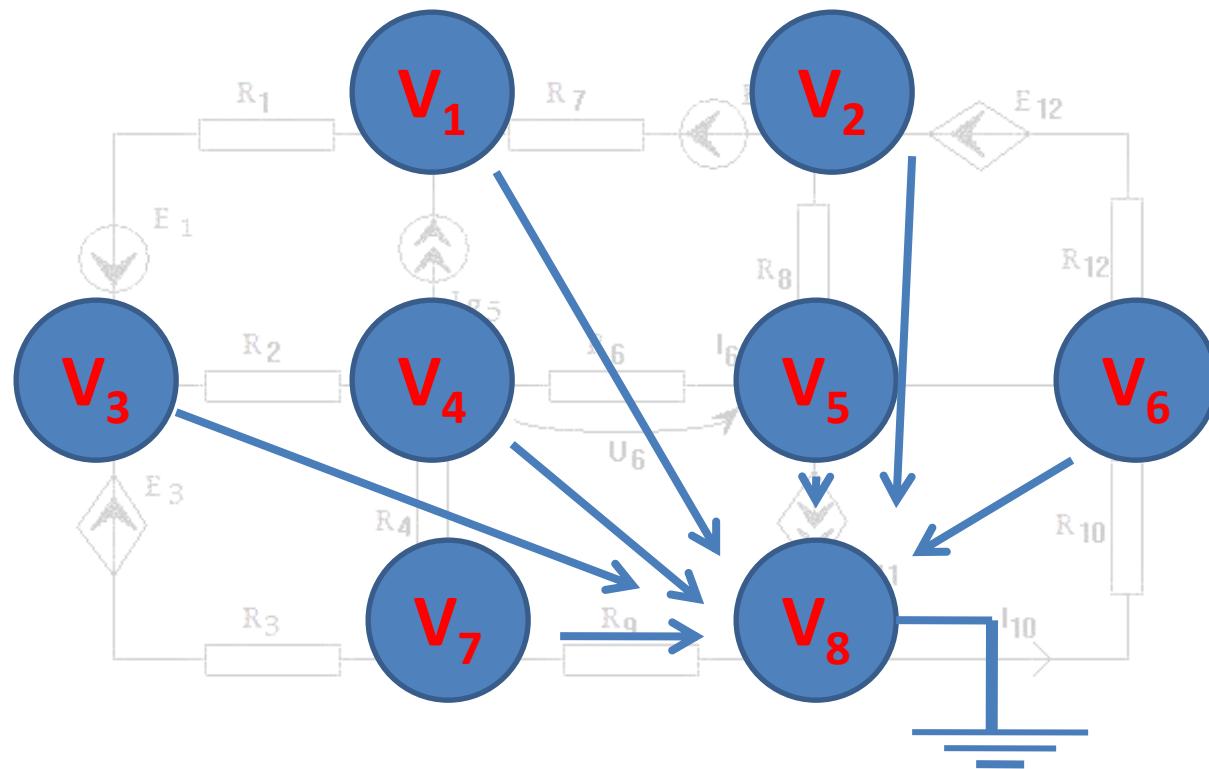
Necunoscute



Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff
- Metoda curentilor ciclici
- Metoda potențialelor nodurilor

Necunoscute



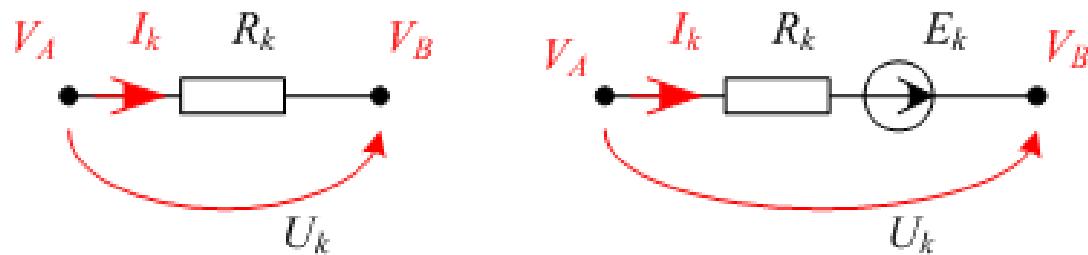
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Necunoscute



$$I_k = \frac{V_A - V_B}{R_k}$$

$$I_k = \frac{V_A - V_B + E_k}{R_k}$$

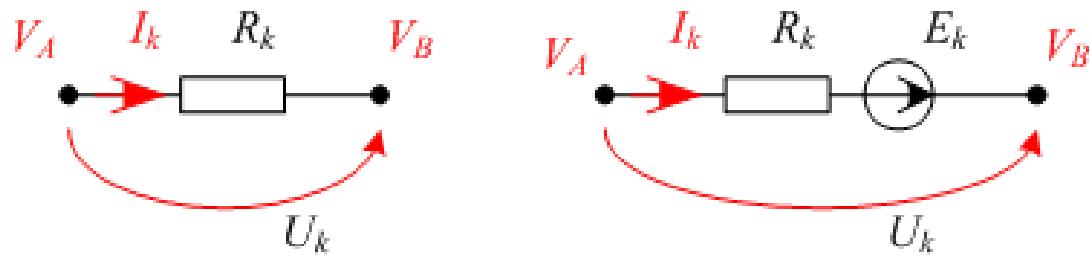
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Necunoscute



$$\begin{cases} G_{11}V_1 + G_{12}V_2 + \dots + G_{1N-1}V_{N-1} = Isc_1 \\ G_{21}V_1 + G_{22}V_2 + \dots + G_{2N-1}V_{N-1} = Isc_2 \\ \dots \\ G_{N-1,1}V_1 + G_{N-1,2}V_2 + \dots + G_{N-1,N-1}V_{N-1} = Isc_{N-1} \end{cases}$$

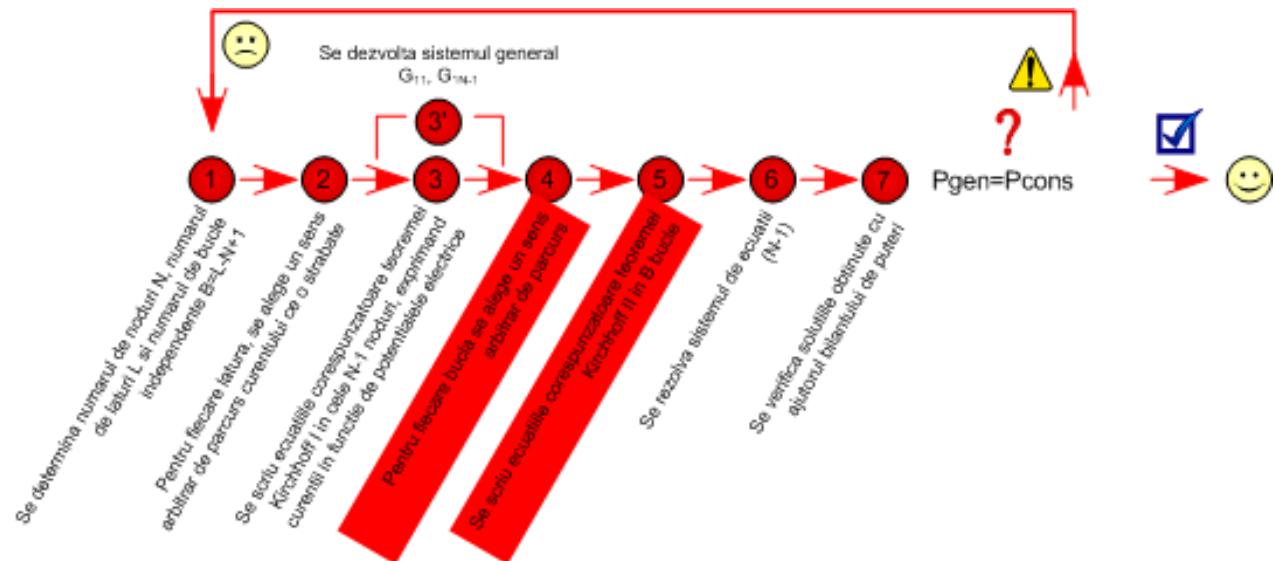
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Algoritm



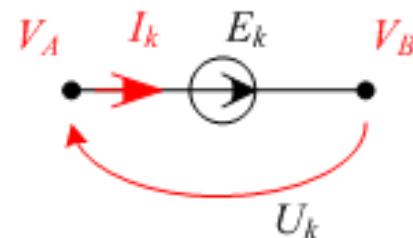
Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

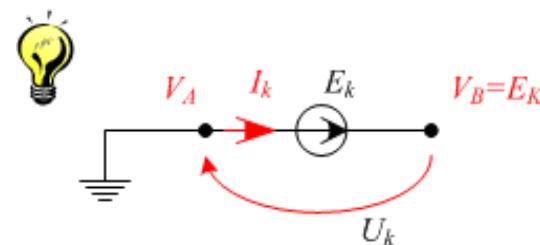
-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Caz special



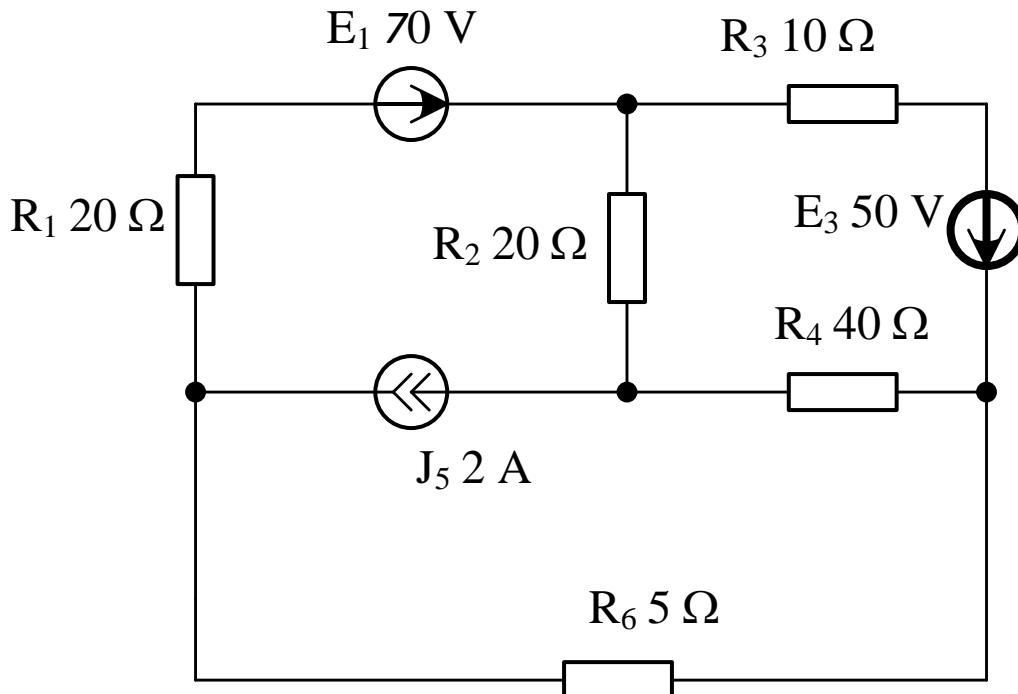
$$U_k = E_k = V_B - V_A$$



Metode de analiza a circuitelor electrice:

- Metoda teoremelor lui Kirchhoff
- Metoda curentilor ciclici
- Metoda potențialelor nodurilor

Algoritm



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

P4 – Expresiile curentilor
functie de potențialele
electrice

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

Metode de analiza a circuitelor electrice:

-Metoda teoremelor lui Kirchhoff

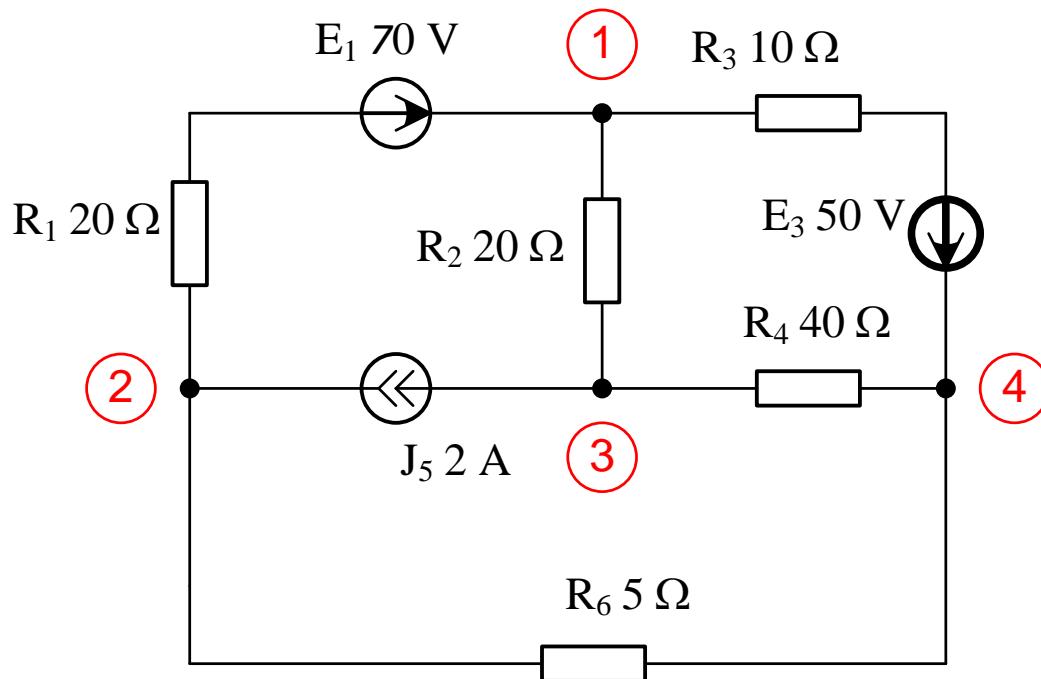
-Metoda curentilor ciclici

-Metoda potențialelor nodurilor

Algoritm

$$N = 4$$

$$L = 6$$



P1 – numar de noduri N,
numar de laturi L

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potential de referinta

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comandate

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sistemului

P7 – Verificare bilant puteri

Metode de analiza a circuitelor electrice:

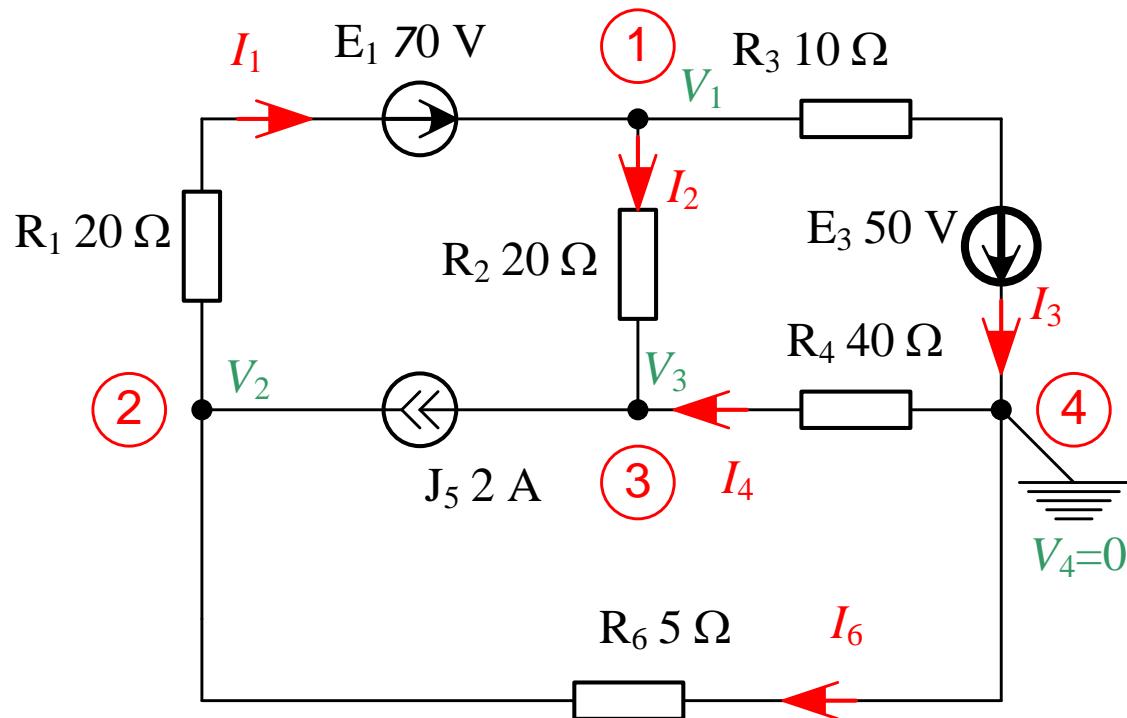
-Metoda teoremelor lui Kirchhoff

-Metoda curentilor ciclici

-Metoda potentialelor nodurilor

Algoritm

$$\begin{aligned} N &= 4 \\ L &= 6 \end{aligned}$$



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

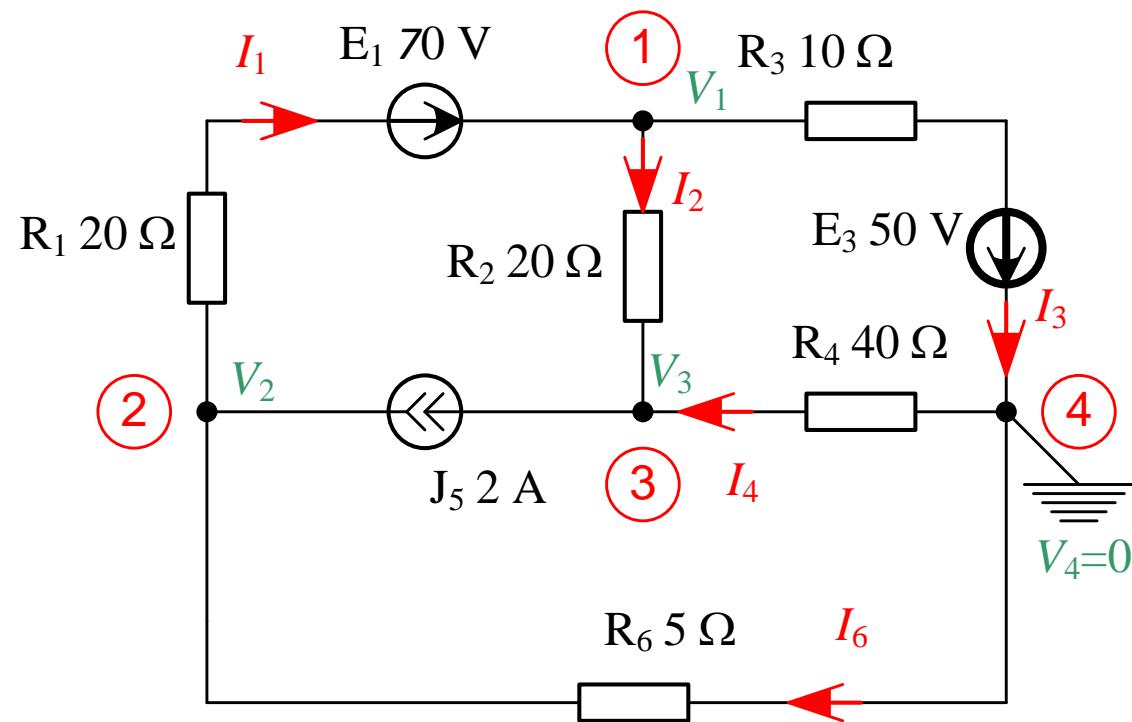
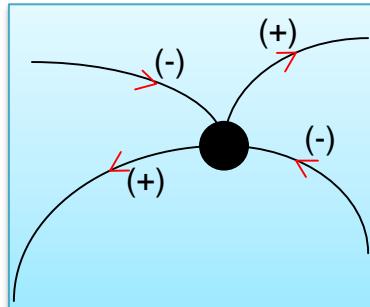
P4 – Expresiile curentilor
functie de potențialele
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P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

$$\left\{ -I_1 + I_2 + I_3 = 0 \quad \leftarrow n1 \right.$$



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

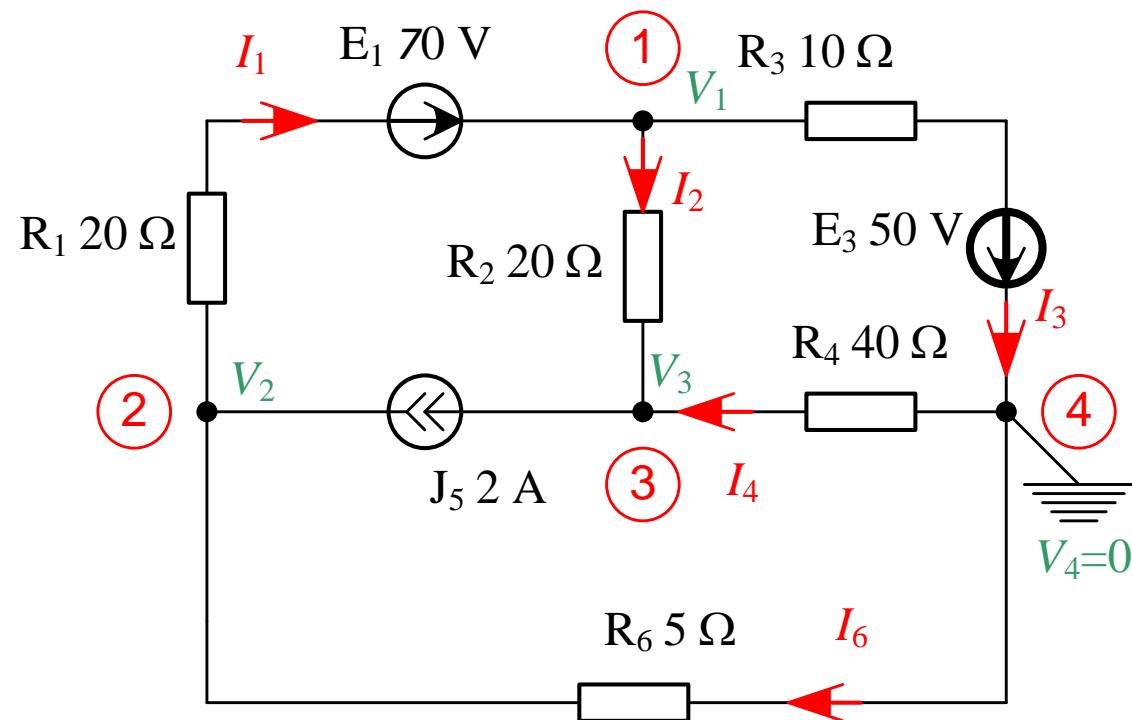
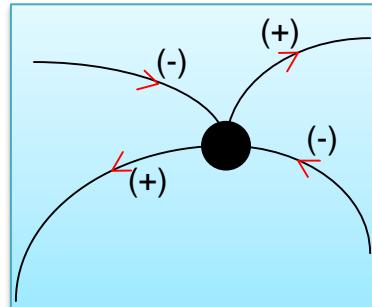
P4 – Expresiile curentilor
functie de potentiialele
electrice

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

$$\begin{cases} -I_1 + I_2 + I_3 = 0 & \leftarrow n1 \\ I_1 - J_5 - I_6 = 0 & \leftarrow n2 \\ -I_2 + J_5 - I_4 = 0 & \leftarrow n3 \end{cases}$$



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

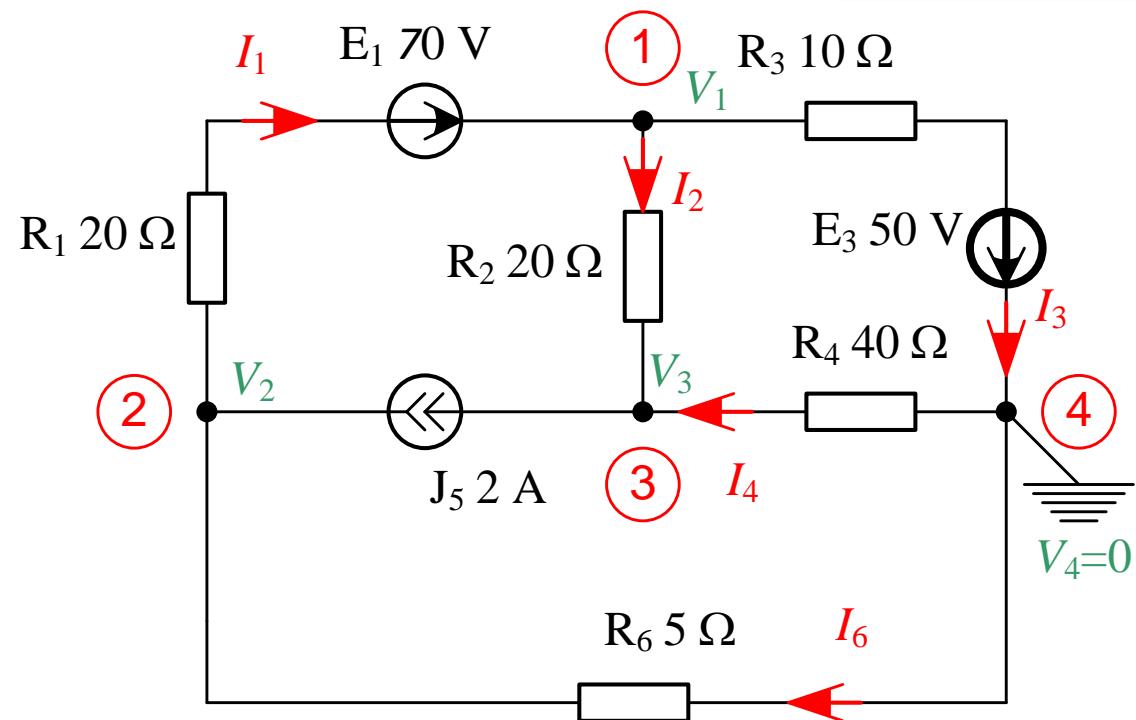
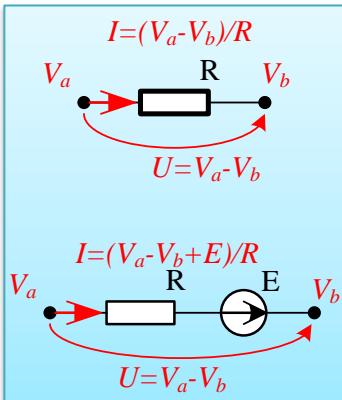
P4 – Expresiile curentilor
functie de potentiialele
electrice

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

$$\begin{cases} -I_1 + I_2 + I_3 = 0 \\ I_1 - J_5 - I_6 = 0 \\ -I_2 + J_5 - I_4 = 0 \end{cases} \quad \left\{ \begin{array}{l} -\frac{V_2 - V_1 + E_1}{R_1} + \frac{V_1 - V_3}{R_2} + \frac{V_1 + E_3}{R_3} = 0 \\ \frac{V_2 - V_1 + E_1}{R_1} - J_5 - \frac{-V_2}{R_6} = 0 \\ -\frac{V_1 - V_3}{R_2} - \frac{-V_3}{R_4} + J_5 = 0 \end{array} \right.$$



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

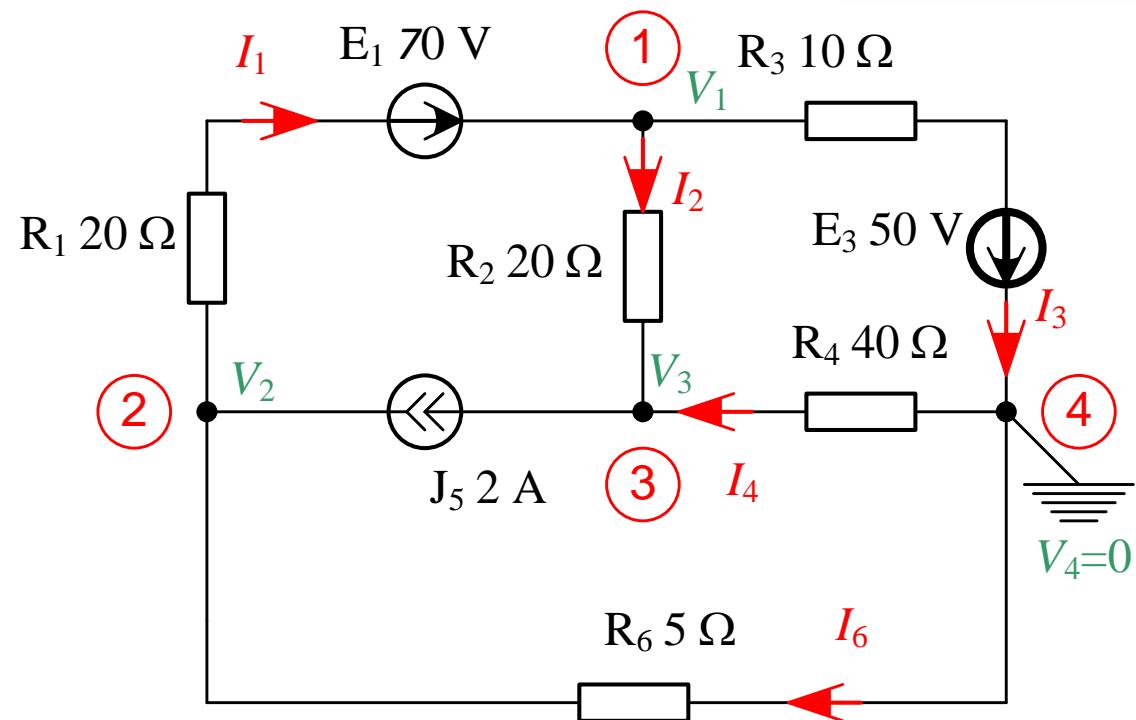
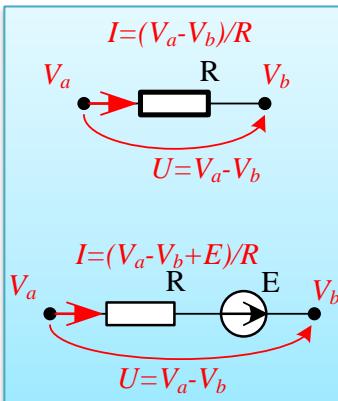
P4 – **Expresiile curentilor
functie de potentiile
electrice**

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

$$\begin{cases} -I_1 + I_2 + I_3 = 0 \\ I_1 - J_5 - I_6 = 0 \\ -I_2 + J_5 - I_4 = 0 \end{cases} \quad \left\{ \begin{array}{l} -\frac{V_2 - V_1 + E_1}{R_1} + \frac{V_1 - V_3}{R_2} + \frac{V_1 + E_3}{R_3} = 0 \\ \frac{V_2 - V_1 + E_1}{R_1} - J_5 - \frac{-V_2}{R_6} = 0 \\ -\frac{V_1 - V_3}{R_2} - \frac{-V_3}{R_4} + J_5 = 0 \end{array} \right.$$



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

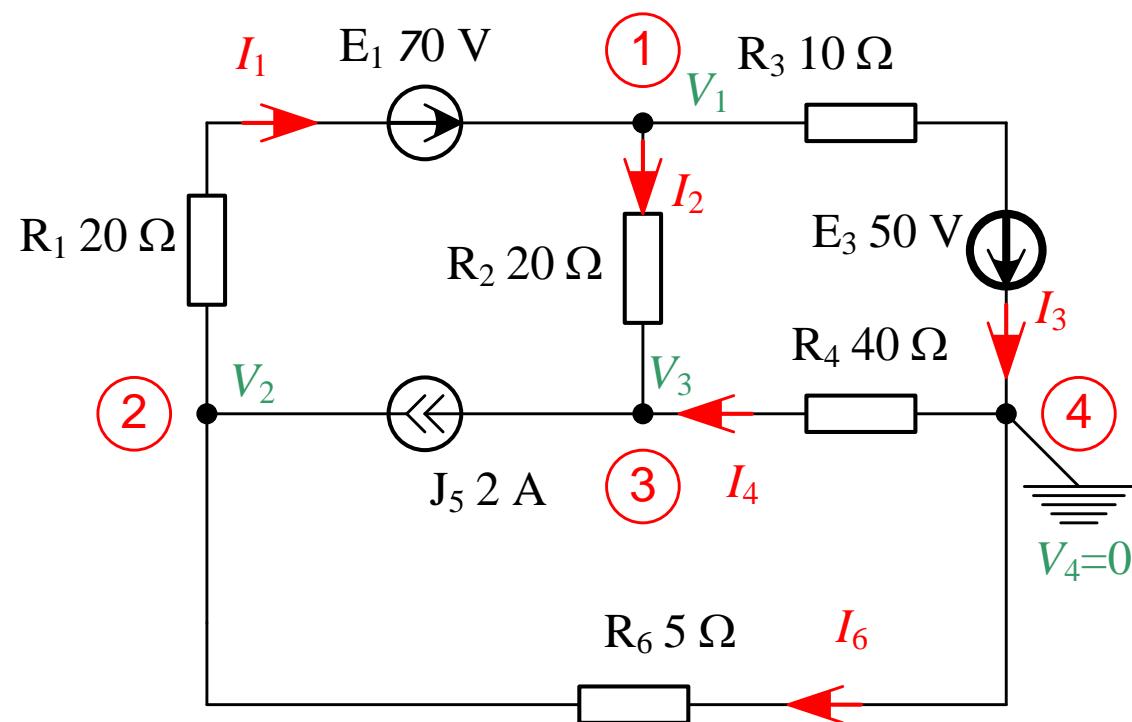
P4 – Expresiile curentilor
functie de potentiialele
electrice

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

$$\begin{cases} -\frac{V_2 - V_1 + 70}{20} + \frac{V_1 - V_3}{20} + \frac{V_1 + 50}{10} = 0 \\ \frac{V_2 - V_1 + 70}{20} - 2 - \frac{-V_2}{5} = 0 \\ -\frac{V_1 - V_3}{20} - \frac{-V_3}{40} + 2 = 0 \end{cases}$$



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

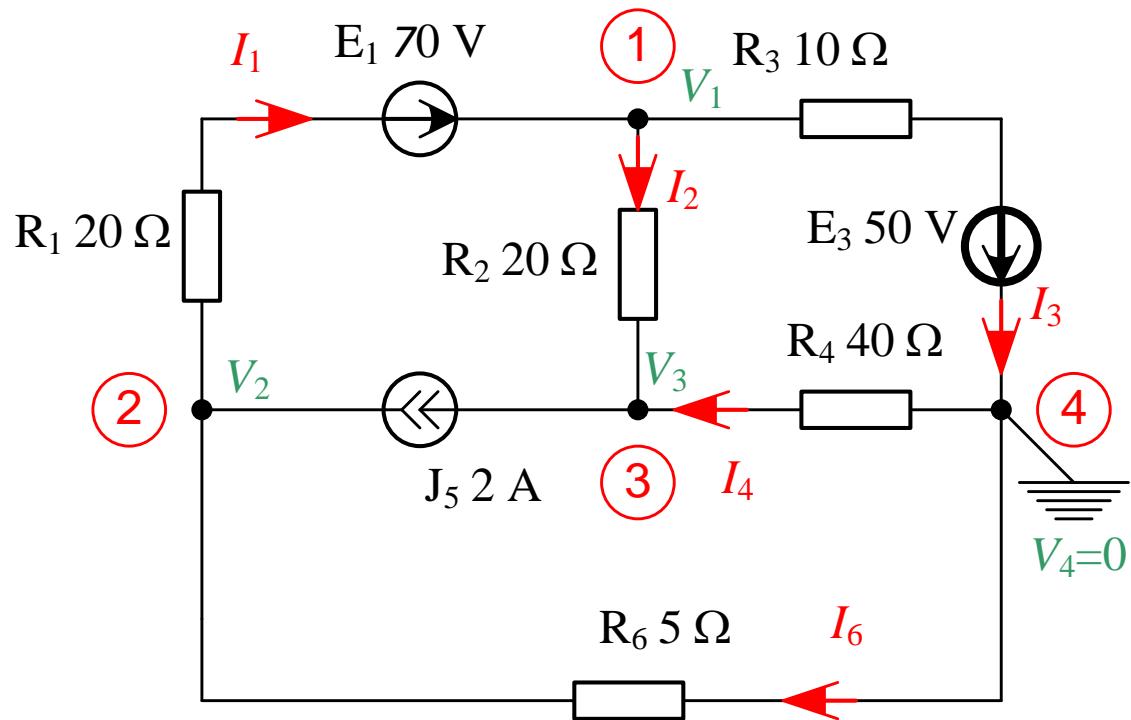
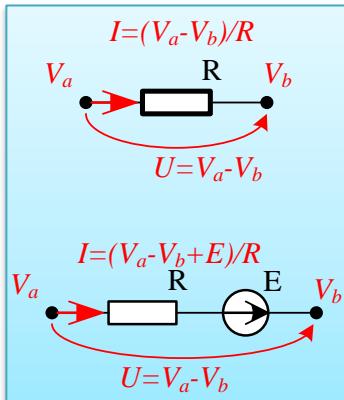
P4 – Expresiile curentilor
functie de potentiialele
electrice

P5 – completare sistem cu
expresiile surselor
comandate

**P6 – Rezolvare numérica a
sistemului**

P7 – Verificare bilant puteri

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -20 \\ -10 \\ -40 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ U_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 3 \\ 1 \\ 30 \\ 2 \end{bmatrix}$$



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

P4 – Expresiile curentilor
functie de potentiile
electrice

P5 – completare sistem cu
expresiile surselor
comandate

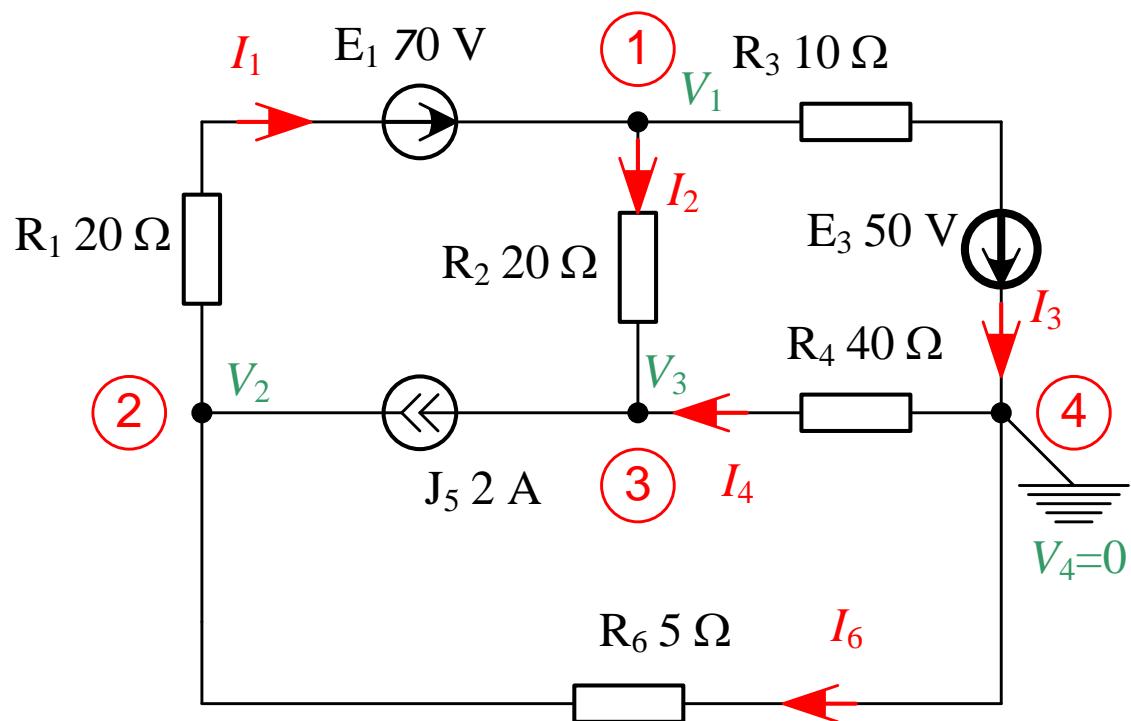
**P6 – Rezolvare numerica a
sistemului**

P7 – Verificare bilant puteri

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -20 \\ -10 \\ -40 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ U_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 3 \\ 1 \\ 30 \\ 2 \end{bmatrix}$$

$$P_{gen} = E_1 I_1 + E_3 I_3 + U_5 J_5 = 490W$$

$$P_{cons} = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 + R_4 I_4^2 + R_6 I_6^2 = 490W$$



P1 – numar de noduri N,
numar de laturi L

P2 – Sens arbitrar curenti,
potential de referinta

P3 – Ecuatii TK I

P4 – Expresiile curentilor
functie de potentiialele
electrice

P5 – completare sistem cu
expresiile surselor
comandate

P6 – Rezolvare numérica a
sistemului

P7 – Verificare bilant puteri

BAZELE ELECTROTEHNICII

- note de curs -

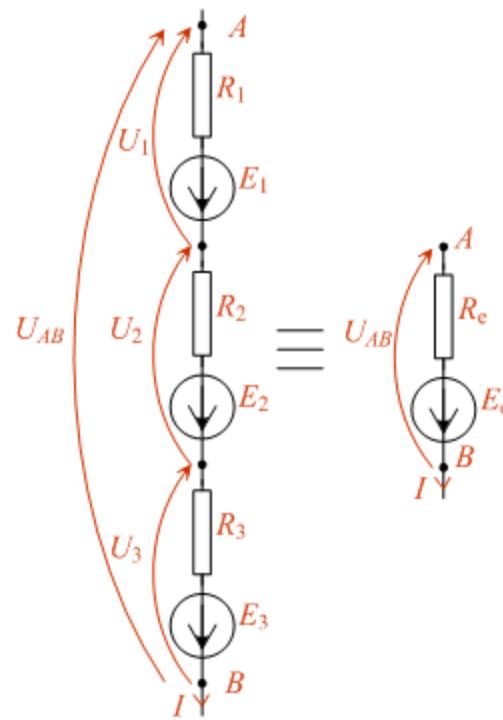
Teoreme ale circuitelor electrice de curent continuu

-Teoremele surseilor reale de tensiune echivalente

Surse reale de tensiune conectate in serie

$$E_e = \sum_{k=1}^n (A) E_k$$

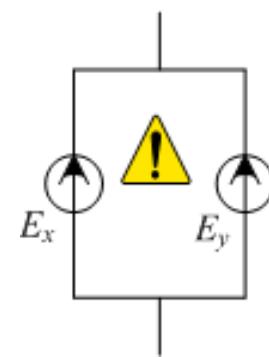
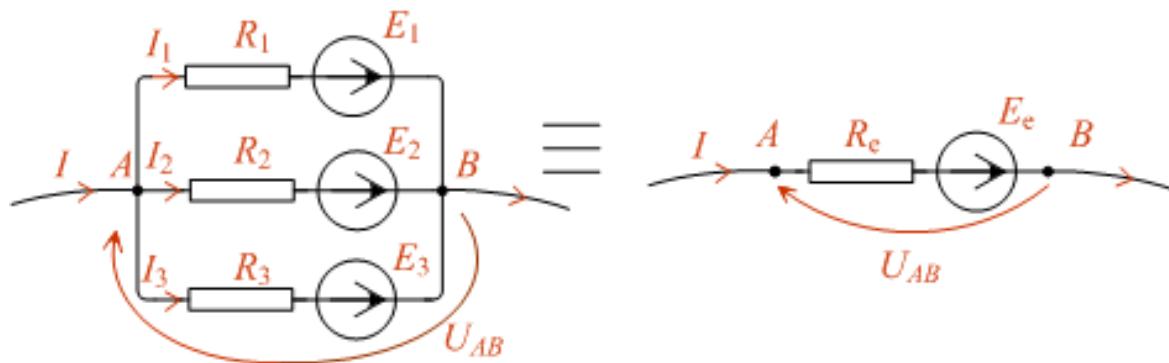
$$R_e = \sum_{k=1}^n R_k$$



Teoreme ale circuitelor electrice de curent continuu

-Teoremele sursei reale de tensiune echivalente

Surse reale de tensiune conectate in paralel



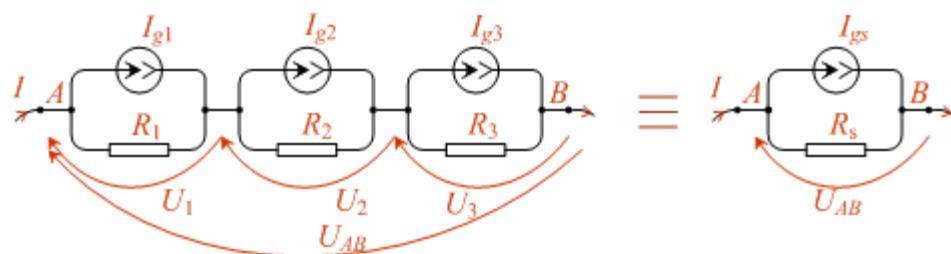
$$E_e = \frac{\sum_{k=1}^n \frac{E_k}{R_k}}{\sum_{k=1}^n \frac{1}{R_k}}$$

$$R_e = \frac{1}{\sum_{k=1}^n \frac{1}{R_k}}$$

Teoreme ale circuitelor electrice de curent continuu

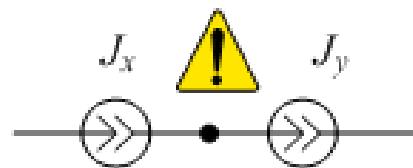
-Teoremele sursei reale de curent echivalente

Surse reale de curent conectate in serie



$$I_{gs} = \frac{\sum_{k=1}^n (A) I_{gk} R_k}{\sum_{k=1}^n R_k}$$

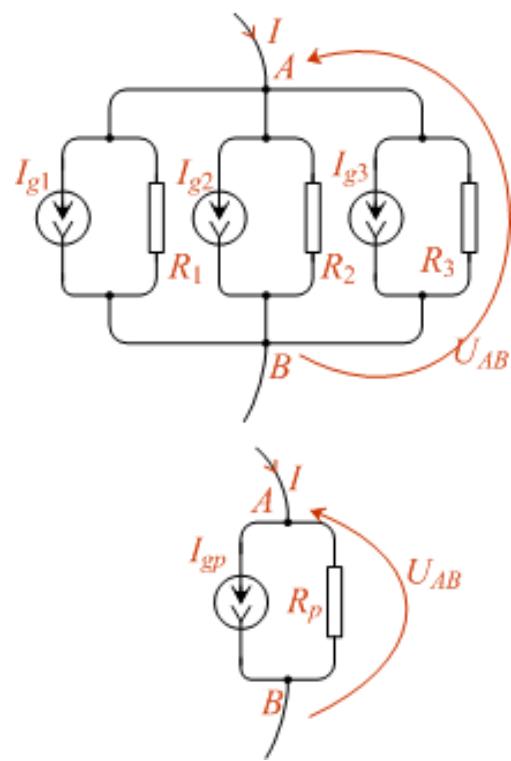
$$R_s = \sum_{k=1}^n R_k$$



Teoreme ale circuitelor electrice de curent continuu

-Teoremele sursei reale de curent echivalente

Surse reale de curent conectate in paralel

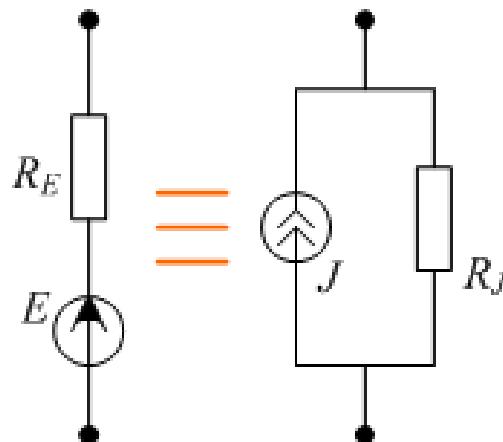


$$I_{gp} = \sum_{k=1}^n (A) I_{gk}$$

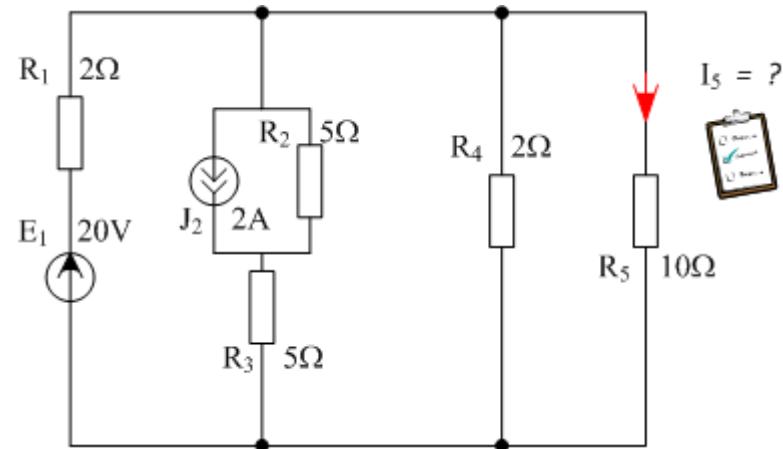
$$R_p = \frac{1}{\sum_{k=1}^n \frac{1}{R_k}}$$

Teoreme ale circuitelor electrice de curent continuu

Sursa reală de curent – Sursa reală de tensiune

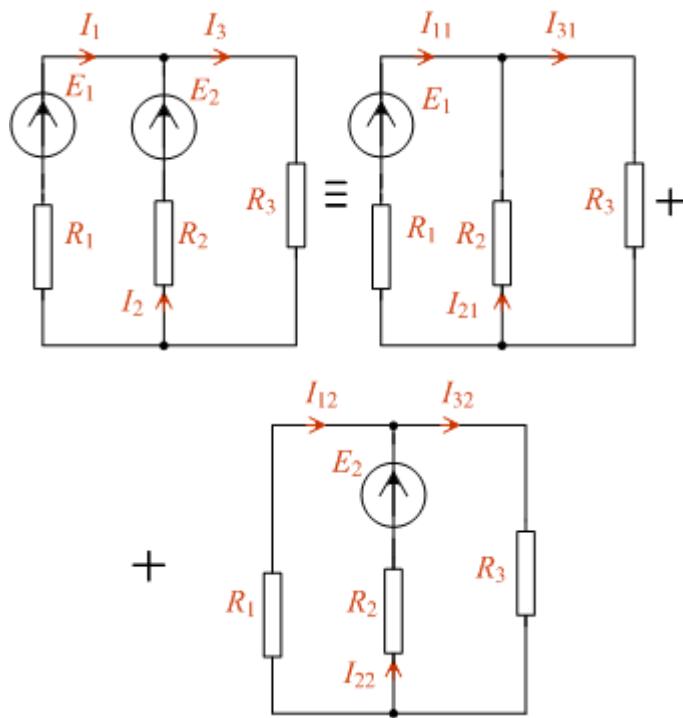


$$\left\{ \begin{array}{l} R_E = R_J = R \\ E = J \cdot R \\ J = \frac{E}{R} \end{array} \right.$$



Teoreme ale circuitelor electrice de curent continuu

-Teorema superpoziției



Intensitatea curentului electric printr-o latură a unui circuit electric este suma algebrică a intensităților curenților determinați prin latura respectivă de fiecare sursă în parte, în absența celorlalte surse de energie

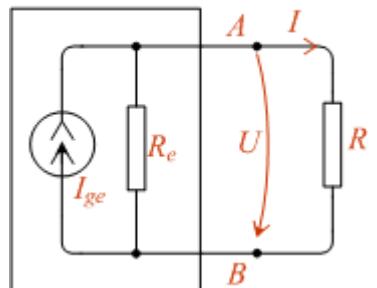
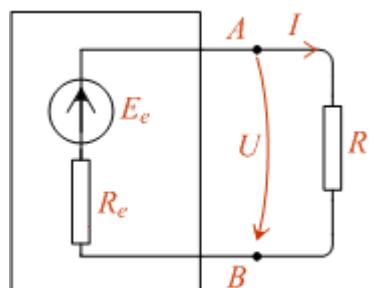
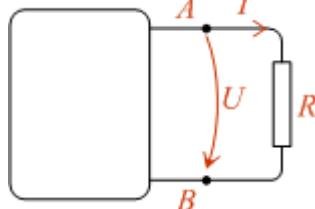
$$I_3 = I_{31} + I_{32}$$

Teoreme ale circuitelor electrice de curent continuu

-Teoremele generatoarelor echivalente

Teorema lui Thevenin

Teorema lui Norton



$$\begin{cases} E_e = U_{AB0} \\ R_e = R_{AB0} \end{cases}$$

$$I_{AB} = \frac{U_{AB0}}{R_{AB0} + R}$$

$$\begin{cases} I_{ge} = I_{ABsc} \\ R_e = R_{AB0} \end{cases}$$

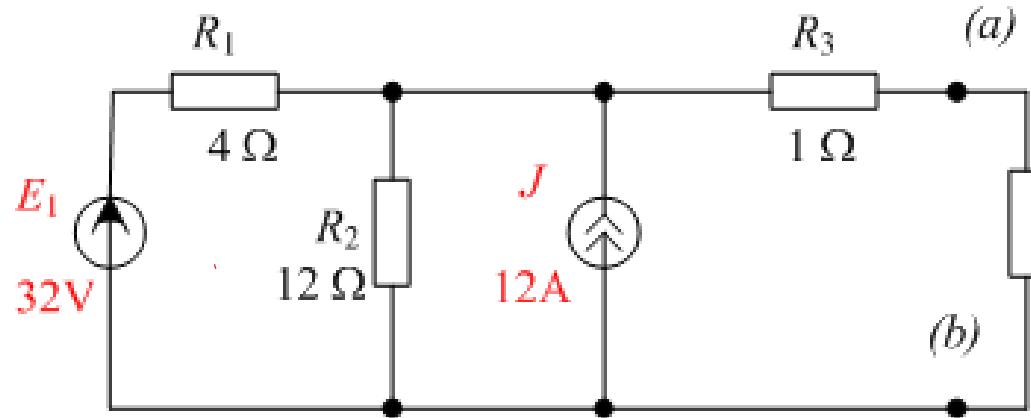
$$U_{AB} = \frac{I_{ABsc}}{G_{AB0} + G}$$

Teoreme ale circuitelor electrice de curent continuu

-Teoremele generatoarelor echivalente

Teorema lui Thevenin

Teorema lui Norton



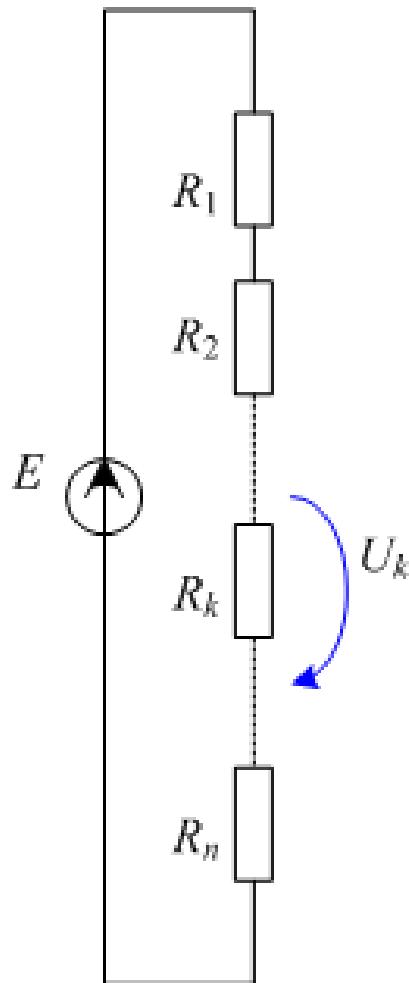
$$\begin{cases} E_e = U_{AB0} \\ R_e = R_{AB0} \end{cases}$$

$$\begin{cases} I_{ge} = I_{ABsc} \\ R_e = R_{AB0} \end{cases}$$

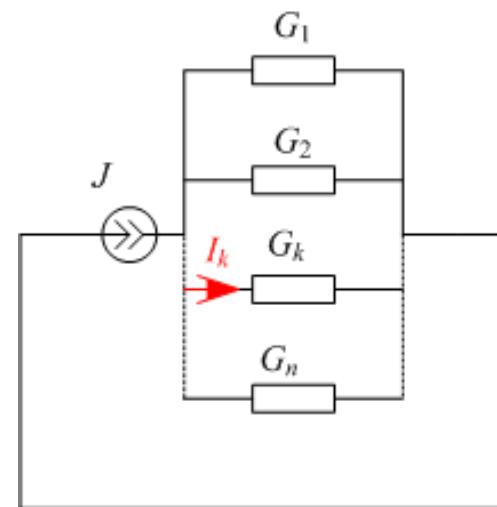


Teoreme ale circuitelor electrice de curent continuu

- **Teorema divizorului de tensiune**
- **Teorema divizorului de curent**

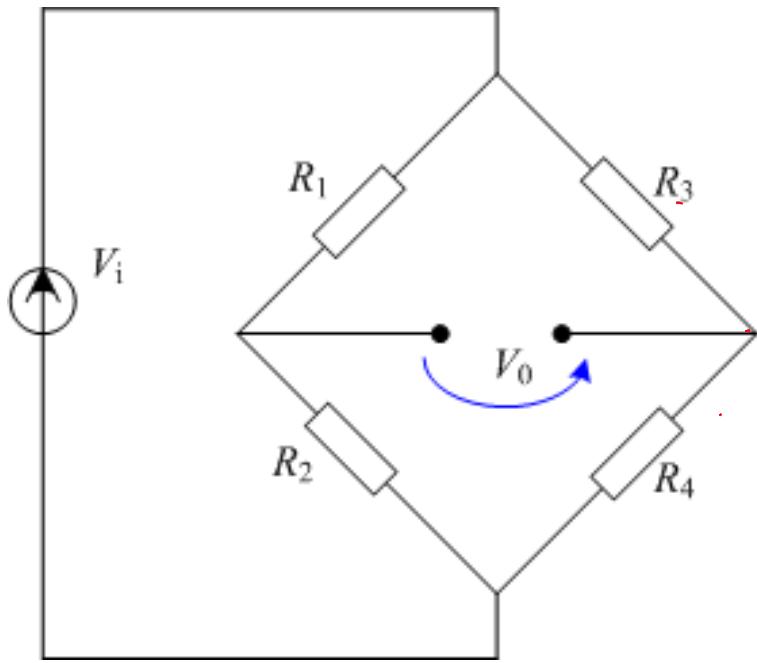


$$U_k = E \cdot \frac{R_k}{\sum_{j=1}^n R_j}$$



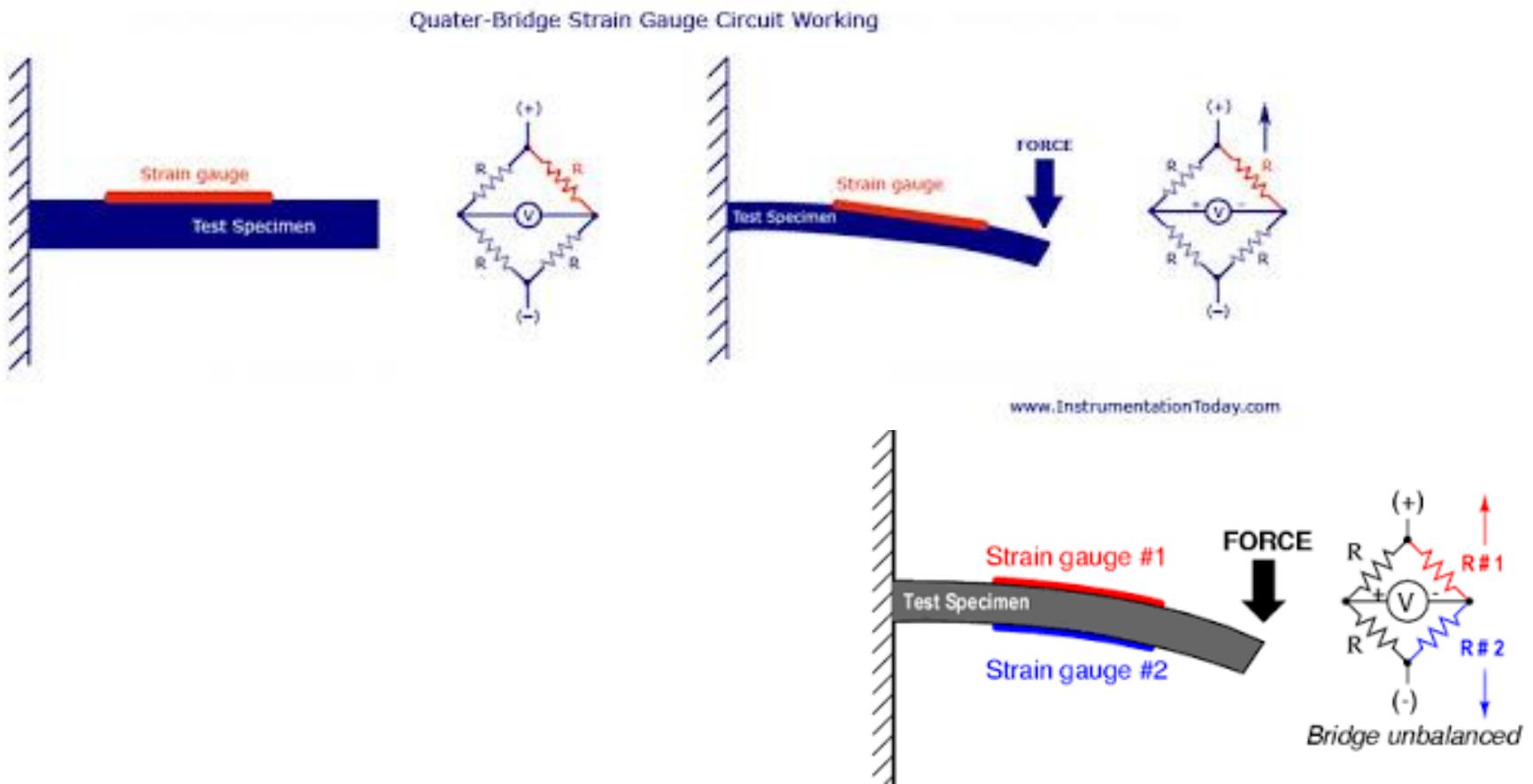
$$I_k = J \cdot \frac{G_k}{\sum_{j=1}^n G_j}$$

Teoreme ale circuitelor electrice de curent continuu



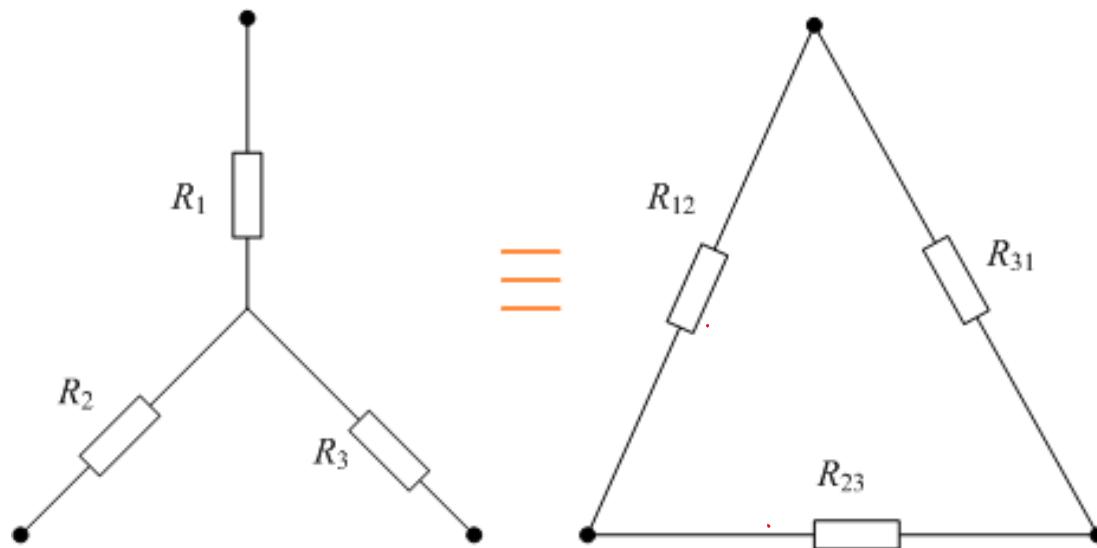
$$V_0 = V_i \cdot \left(\frac{1}{1 + \frac{R_1}{R_2}} - \frac{1}{1 + \frac{R_3}{R_4}} \right)$$

Teoreme ale circuitelor electrice de curent continuu



Teoreme ale circuitelor electrice de curent continuu

-Echivalenta stea-triunghi (Y-Δ)

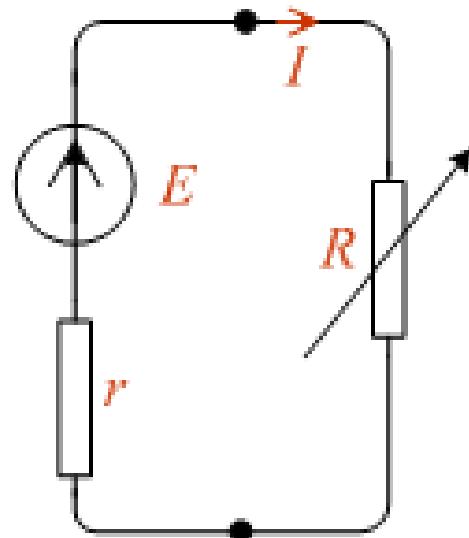


$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

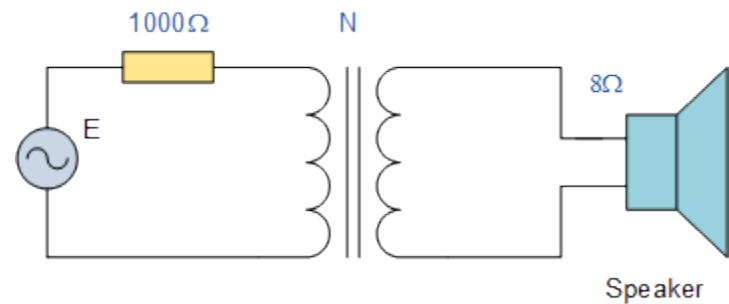
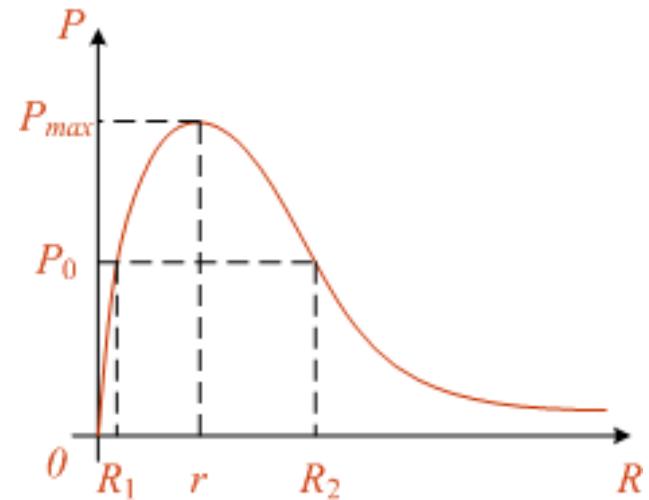
$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Teoreme ale circuitelor electrice de curent continuu

-Transfer maxim de putere



$$P = RI^2 = R \frac{E^2}{(R + r)^2} = P(R).$$

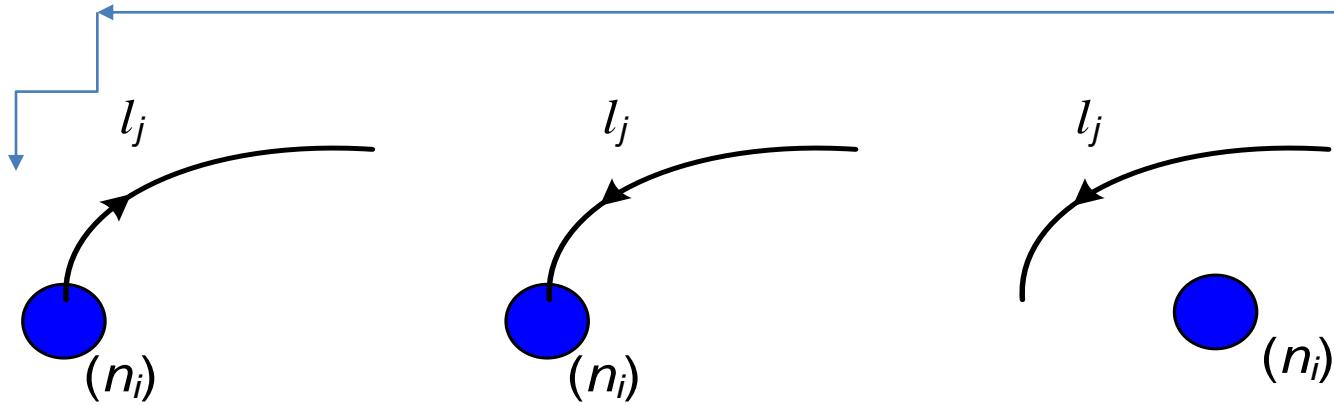


Forma matriceală a teoremelor lui Kirchhoff

$$\mathbf{A}' = [a_{i,j}]$$

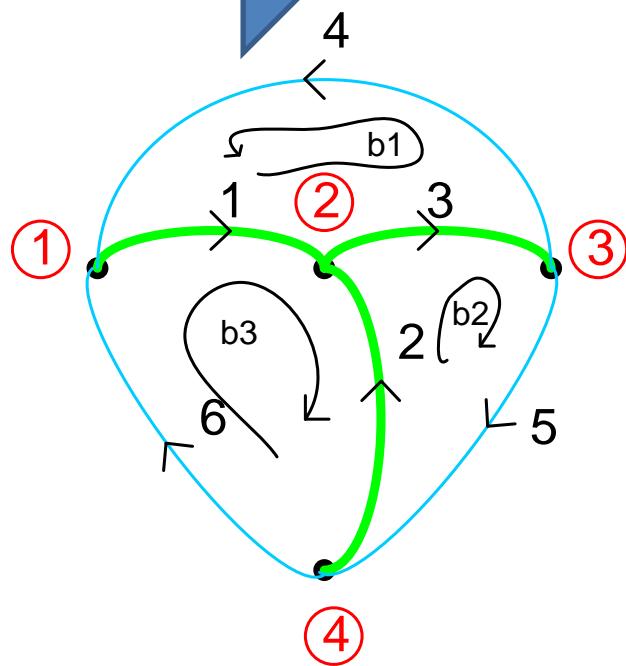
Matricea incidentelor
laturi-noduri

$$a_{i,j} = \begin{cases} 1 & \text{daca } l_j \in n_i \text{ si are sensul pozitiv} \\ -1 & \text{daca } l_j \in n_i \text{ si are sensul negativ} \\ 0 & \text{daca } l_j \notin n_i \end{cases}$$



Forma matriceală a teoremelor lui Kirchhoff

Matricea incidentelor
laturi-noduri

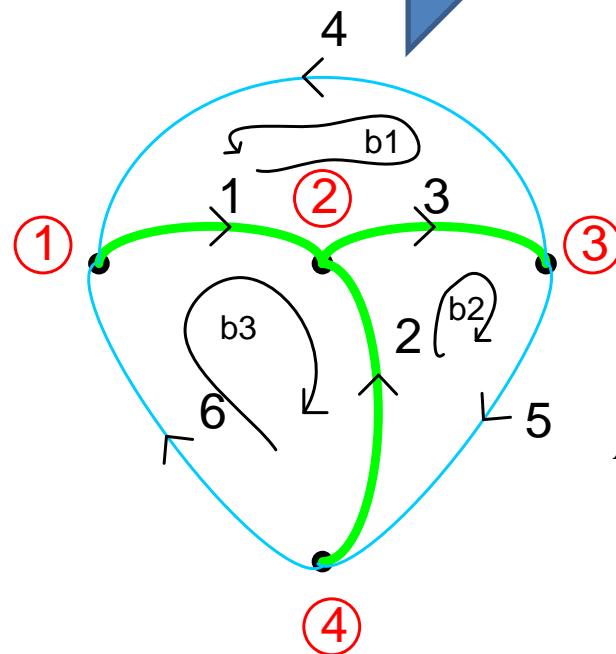


$$A' = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{bmatrix}$$

Forma matriceală a teoremelor lui Kirchhoff

Matricea incidentelor
laturi-noduri



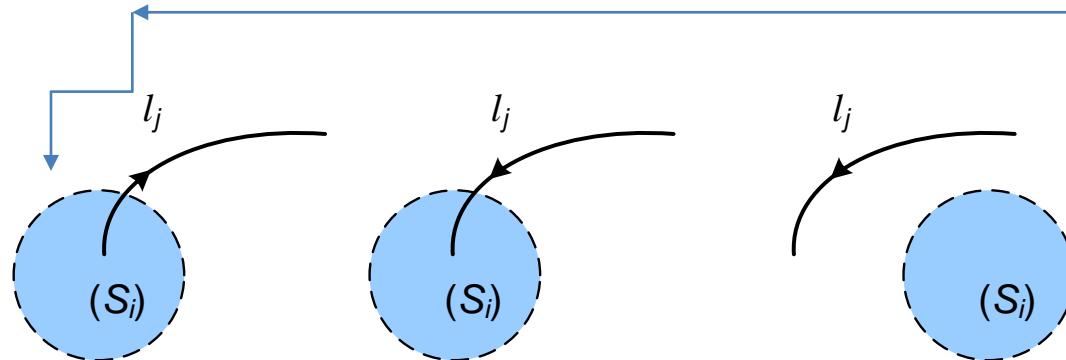
$$A * I = 0 \Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{bmatrix} * \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Forma matriceală a teoremelor lui Kirchhoff

$$\mathbf{C}' = [c_{i,j}]$$

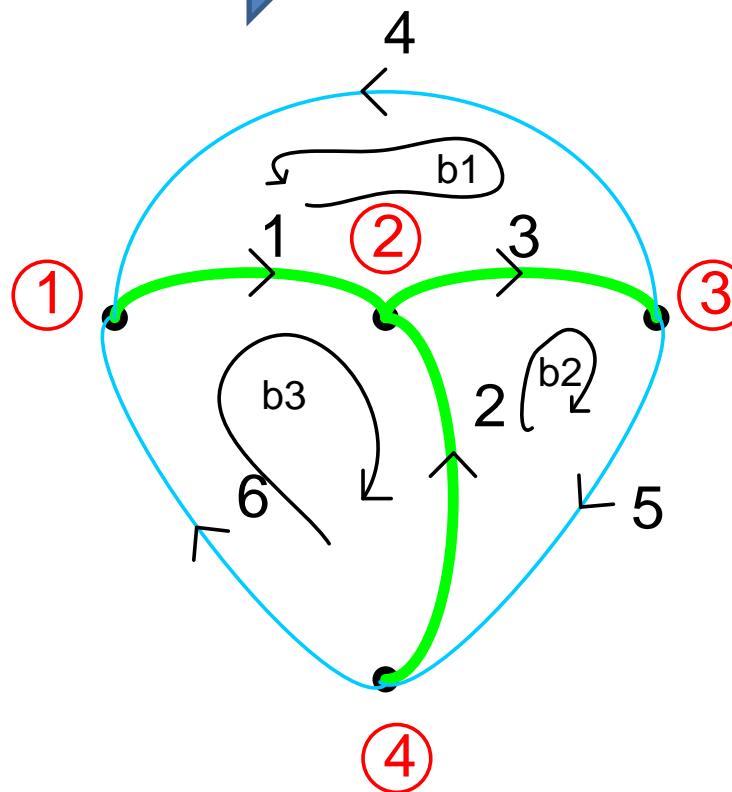
Matricea apartenențelor
laturi-sectiuni

$$c_{i,j} = \begin{cases} 1 & \text{daca } l_j \in S_i \text{ si are sensul pozitiv} \\ -1 & \text{daca } l_j \in S_i \text{ si are sensul negativ} \\ 0 & \text{daca } l_j \notin S_i \end{cases}$$



Forma matriceală a teoremelor lui Kirchhoff

Matricea apartenentelor
laturi-sectiuni



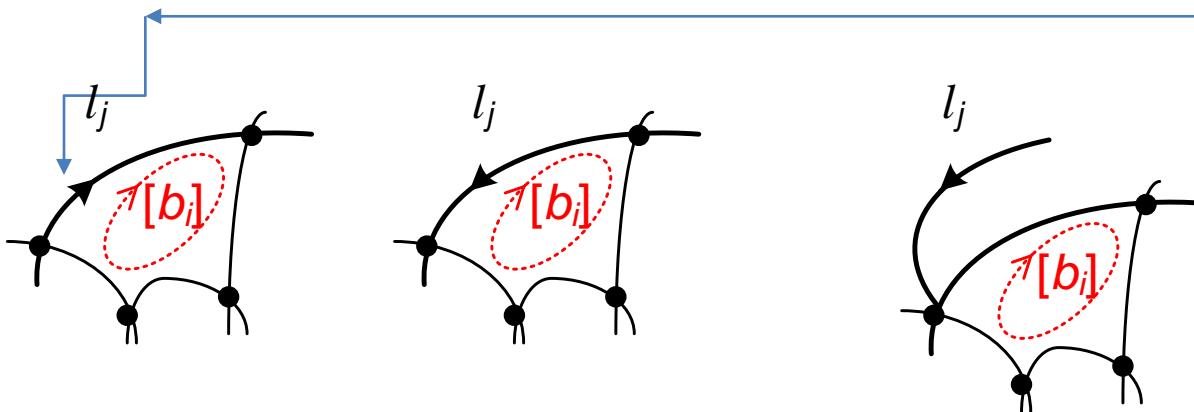
$$C =$$

Forma matriceală a teoremelor lui Kirchhoff

$$\mathbf{B} = [b_{i,j}]$$

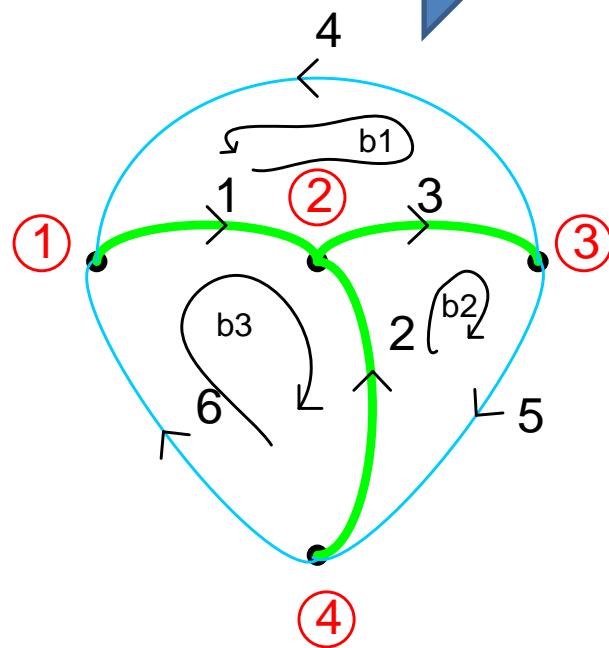
Matricea apartenențelor
laturi-bucle

$$b_{i,j} = \begin{cases} 1 & \text{daca } l_j \in \text{buclei}_i \text{ si are sensul acesteia} \\ -1 & \text{daca } l_j \in \text{buclei}_i \text{ si are sensul opus} \\ 0 & \text{daca } l_j \notin \text{buclei}_i \end{cases}$$



Forma matriceală a teoremelor lui Kirchhoff

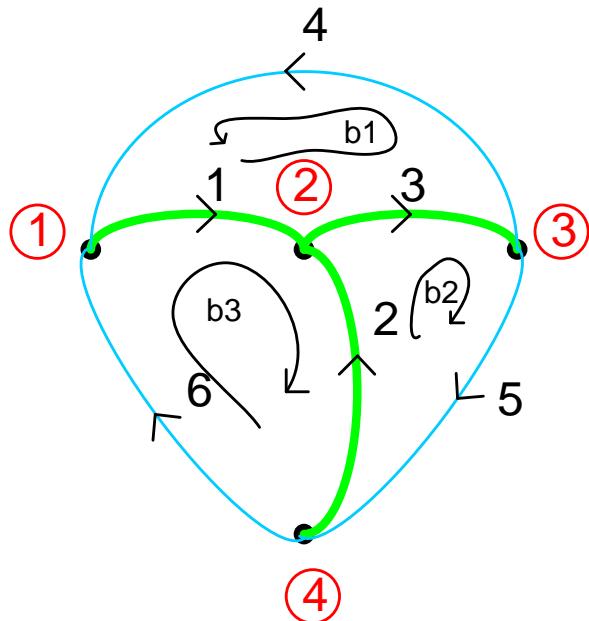
Matricea apartenentelor
laturi-bucle



$$U = A^T V \Leftrightarrow \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$B * U = 0 \Leftrightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Forma matriceală a teoremelor lui Kirchhoff



$$B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \quad \begin{array}{l} \text{ramuri} \\ \text{coarde} \end{array} = [D \quad 1]$$

$$C = \begin{bmatrix} 1 & -D^T \end{bmatrix}$$

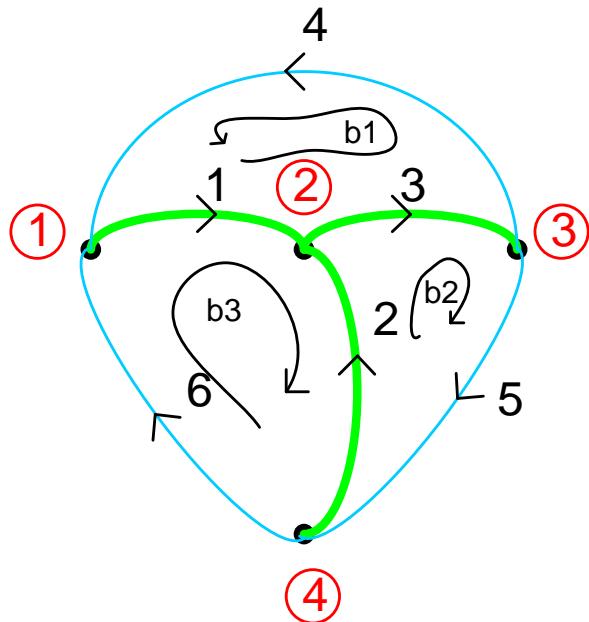
$$C * I = 0 \Leftrightarrow \begin{bmatrix} 1 & -D^T \end{bmatrix} \begin{bmatrix} I_a \\ I_c \end{bmatrix} = I_a - D^T I_c = 0 \Rightarrow I_a = D^T I_c$$

$$I = \begin{bmatrix} I_a \\ I_c \end{bmatrix} = \begin{bmatrix} D^T I_c \\ I_c \end{bmatrix} = \begin{bmatrix} D^T \\ 1 \end{bmatrix} I_c = B^T I_c$$

$$B * U = 0 \Leftrightarrow [D \quad 1] \begin{bmatrix} U_a \\ U_c \end{bmatrix} = DU_a + U_c = 0 \Rightarrow U_c = -DU_a$$

$$U = \begin{bmatrix} U_a \\ U_c \end{bmatrix} = \begin{bmatrix} U_a \\ -DU_a \end{bmatrix} = \begin{bmatrix} 1 \\ -D \end{bmatrix} U_a = C^T U_a$$

Forma matriceală a teoremelor lui Kirchhoff



TK I

$$A \cdot I = 0$$

$$I = B^T I_c$$

$$C \cdot I = 0$$

$$I_a = D^T I_c$$

TK II

$$U = A^T \cdot V$$

$$B \cdot U = 0$$

$$U = C^T \cdot U_a$$

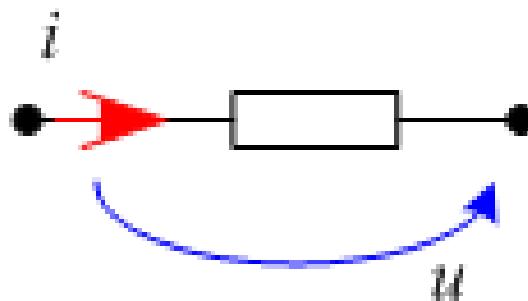
$$U_c = -D \cdot U_a$$

BAZELE ELECTROTEHNICII

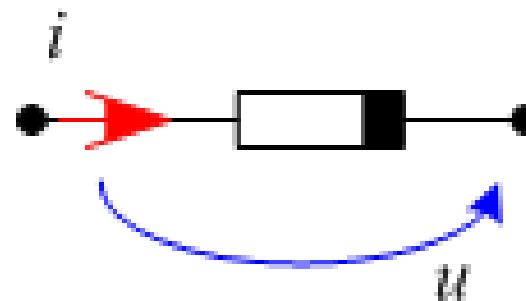
- note de curs -

Circuite electrice neliniare, de curent continuu

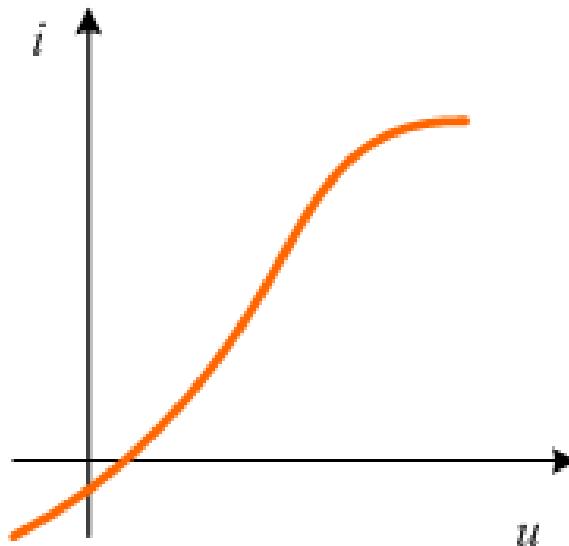
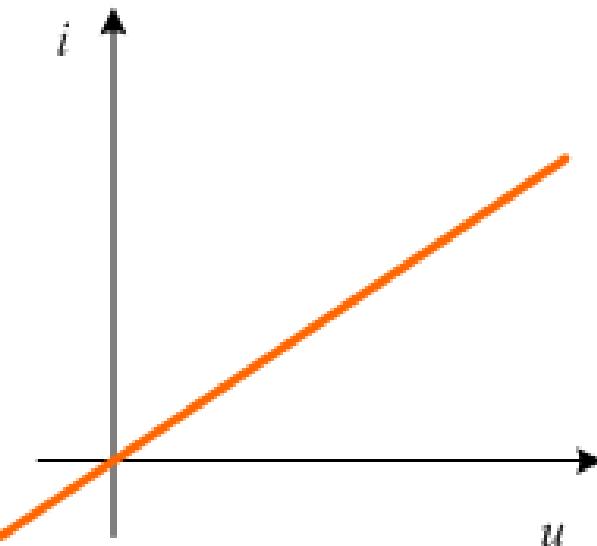
- Rezistorul liniar / neliniar



$$u = R \cdot i$$

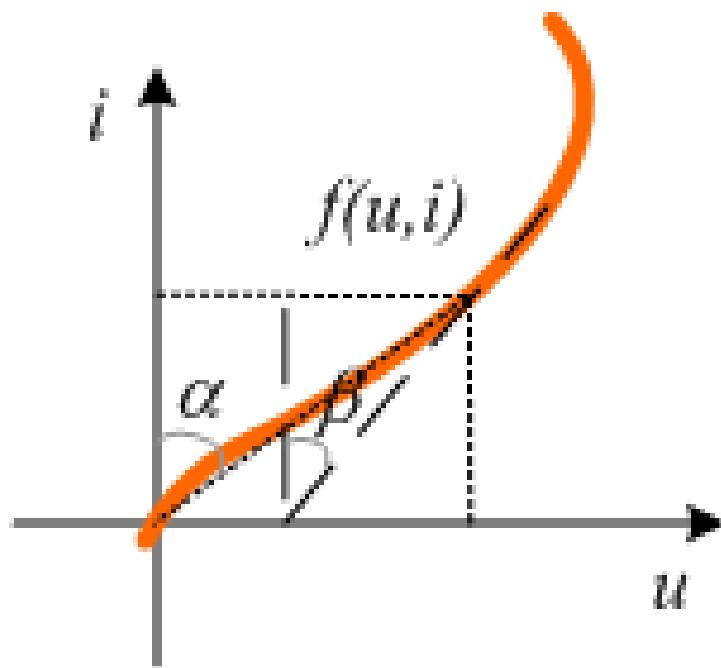


$$f(u, i) = 0$$



Circuite electrice neliniare, de curent continuu

- Rezistorul neliniar

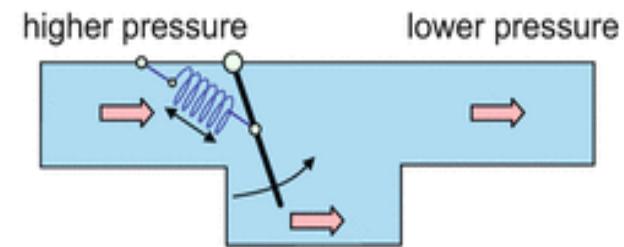
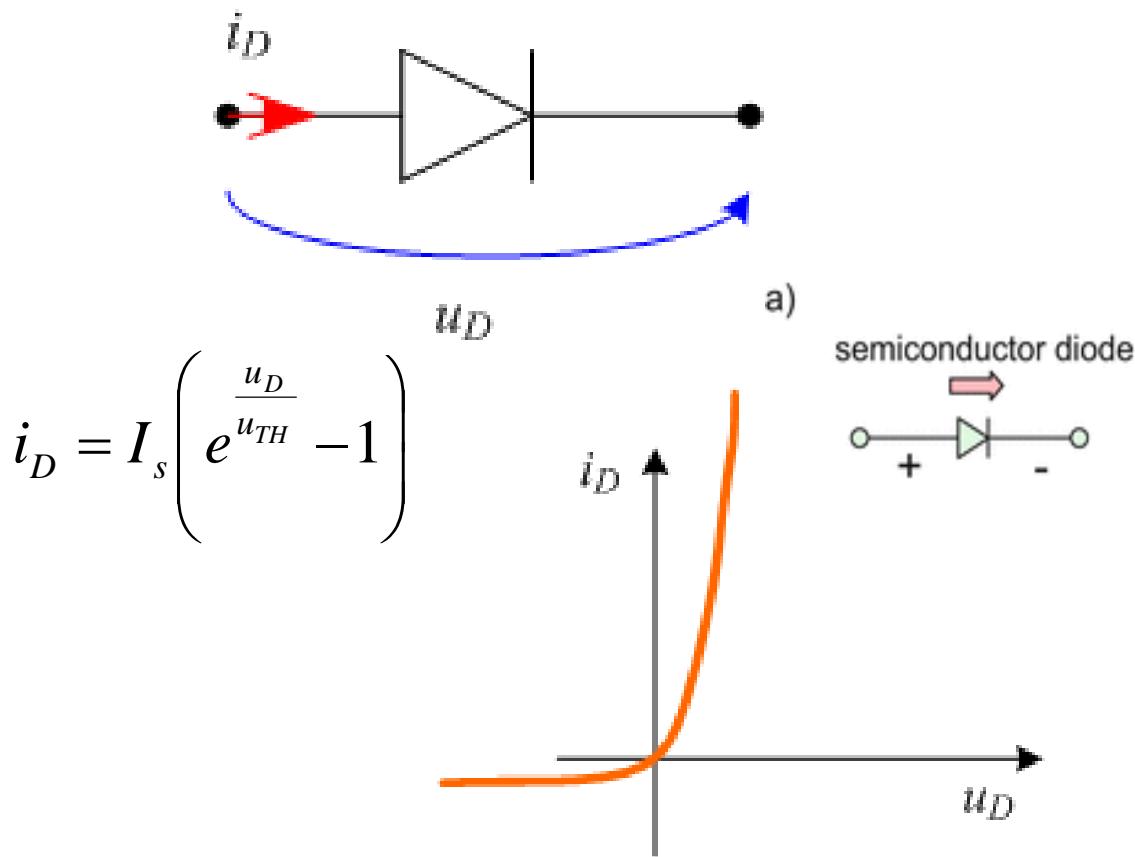


$$R_s = \frac{U}{I}$$

$$R_d = \frac{dU}{dI}$$

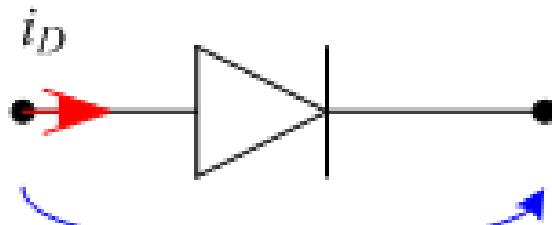
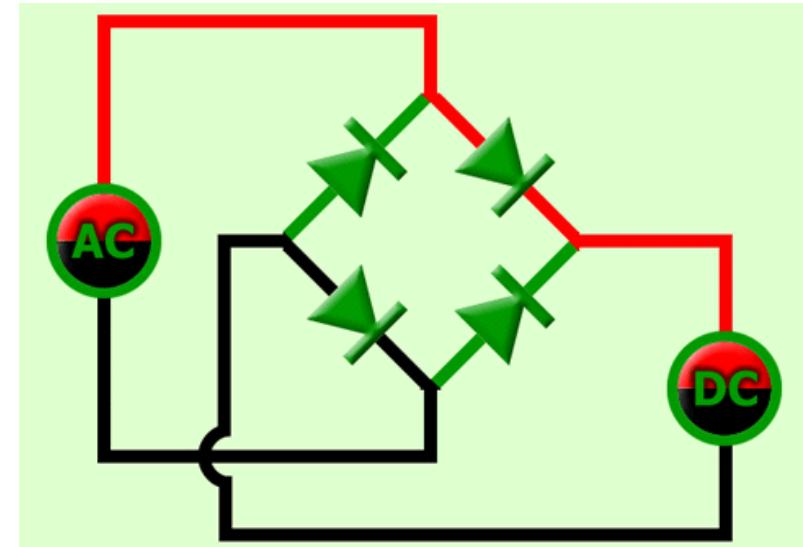
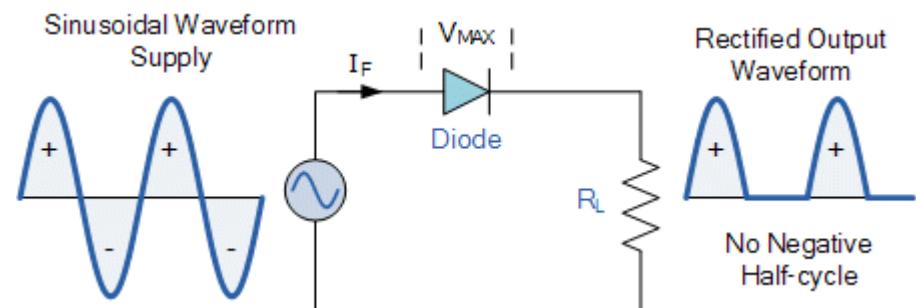
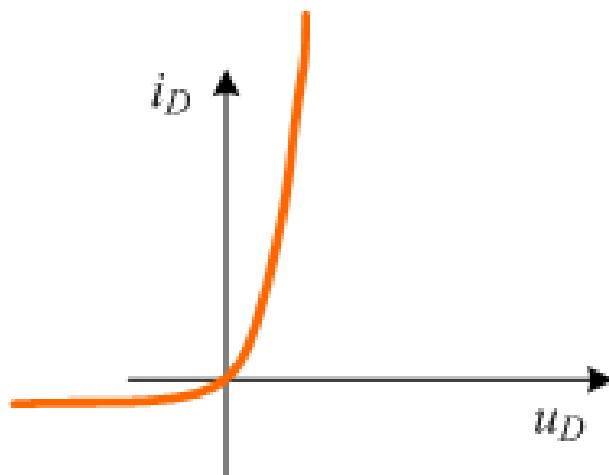
Circuite electrice neliniare, de curent continuu

- Dioda semiconductoare



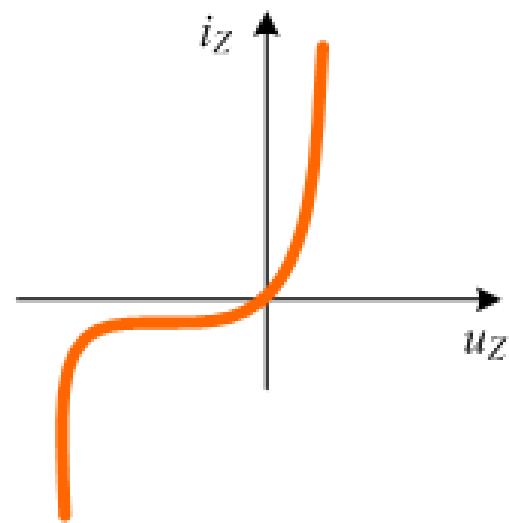
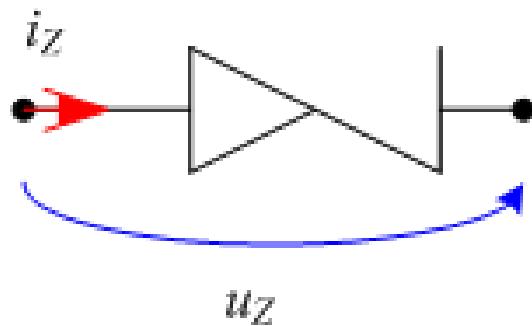
Circuite electrice neliniare, de curent continuu

- Dioda semiconductoare

 u_D 

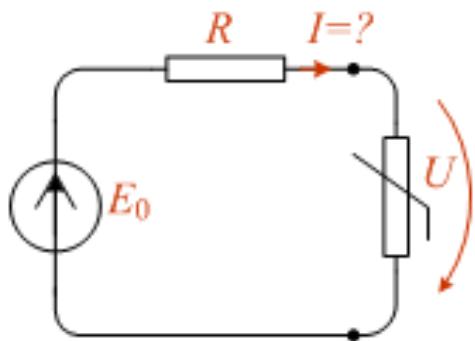
Circuite electrice neliniare, de curent continuu

- Dioda Zener

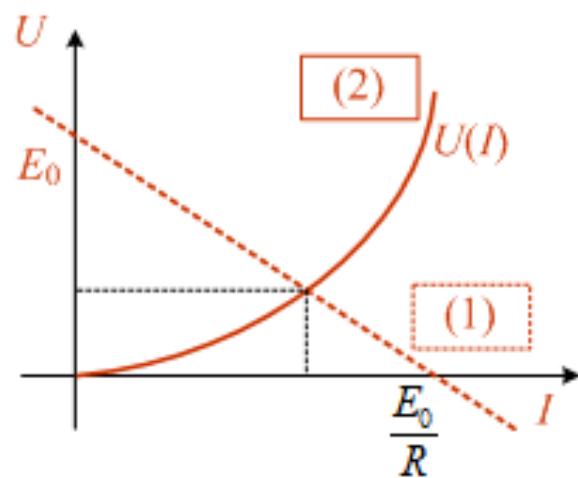


Circuite electrice nelineare, de curent continuu

- Cu un singur element neliniar - metoda dreptei de sarcina

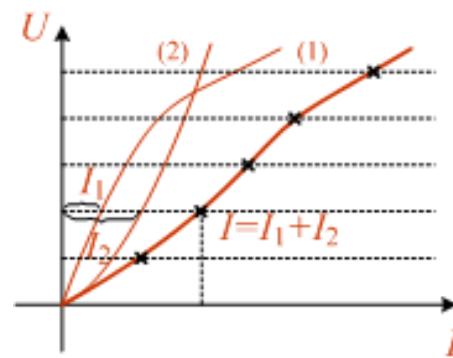
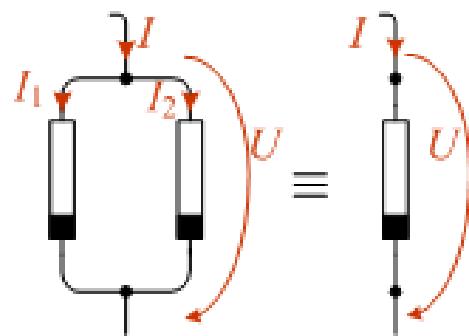
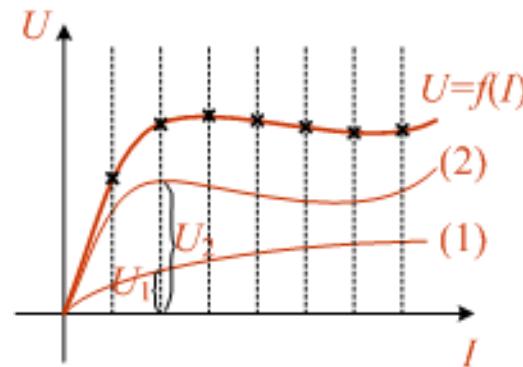
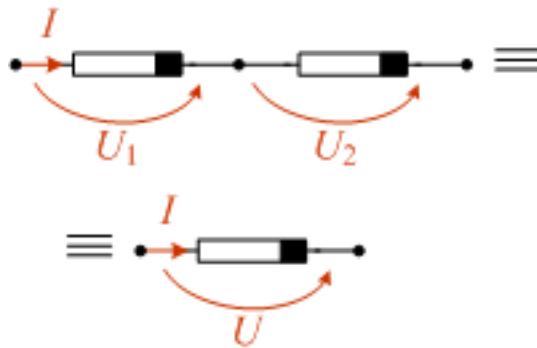


$$RI + U = E_0$$



Circuite electrice nelineare, de curent continuu

- Transformarea schemelor circuitelor electrice nelineare



Circuite electrice de curent alternativ

- *Regim permanent sinusoidal*

valoare instantanee

$$i(t), \quad u(t)$$

marime periodica

$$u(t) = u(t + kT)$$

frecventa / pulsatie

$$f = \frac{1}{T}, \omega = 2\pi \cdot f = \frac{2\pi}{T}$$

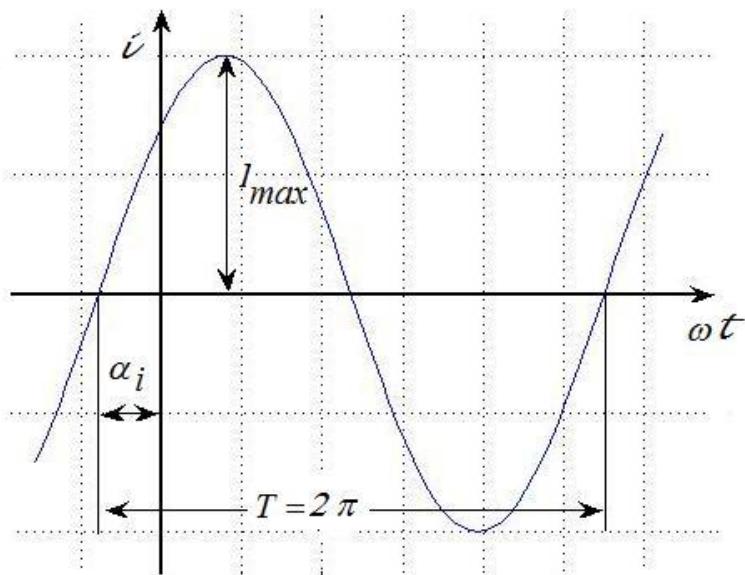
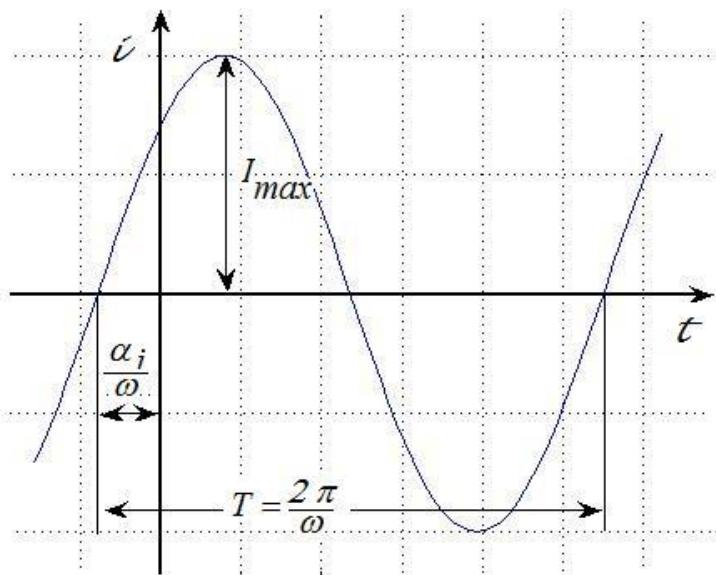
valoare de varf

$$I_{\max}, \quad \hat{I}$$

Circuite electrică de curent alternativ

- Regim permanent sinusoidal

$$i(t) = I_{\max} \sin(\omega \cdot t + \alpha_i)$$



$$I_{med} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i(t) dt$$

$$I_{ef} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} i(t)^2 dt}$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

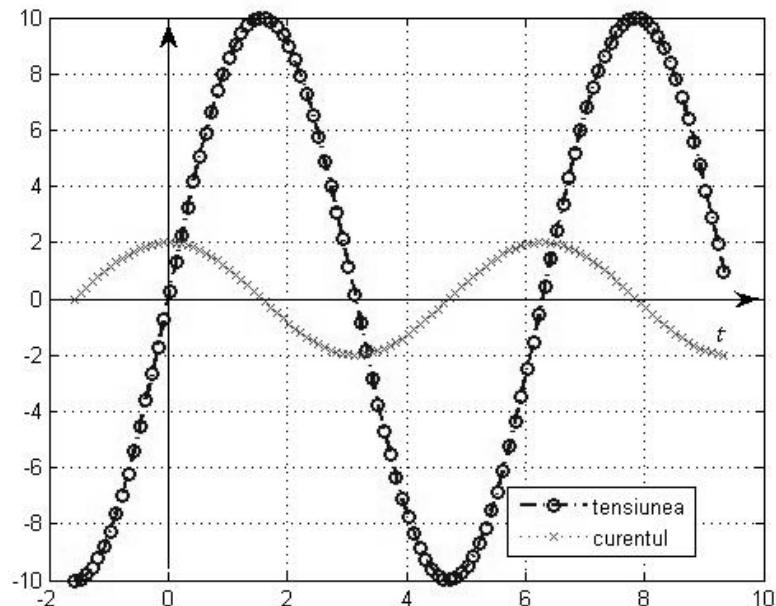
$$i_1(t) = I_1 \sqrt{2} \sin(\omega t + \gamma_1),$$

$$i_2(t) = I_2 \sqrt{2} \sin(\omega t + \gamma_2)$$

defazaj

$$\varphi_{12} = (\omega t + \gamma_1) - (\omega t + \gamma_2) = \gamma_1 - \gamma_2$$

$$-\pi < \varphi_{12} \leq \pi$$



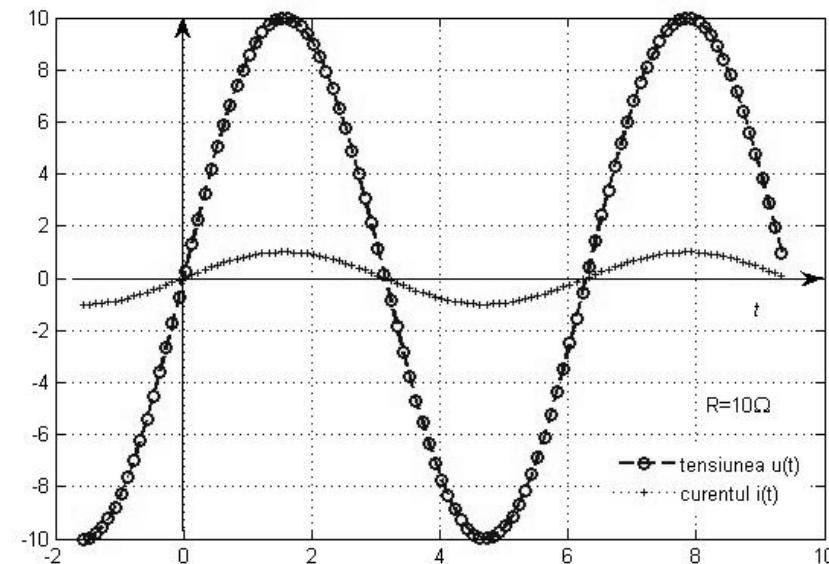
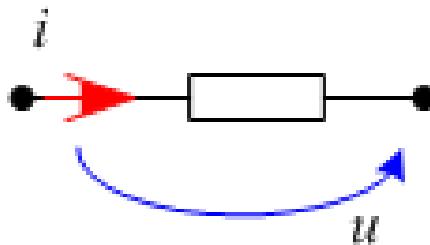
Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Rezistorul liniar

$$u(t) = U \sqrt{2} \sin(\omega \cdot t)$$

$$i(t) = \frac{U}{R} \sqrt{2} \sin(\omega \cdot t)$$



Circuite electrice de curent alternativ

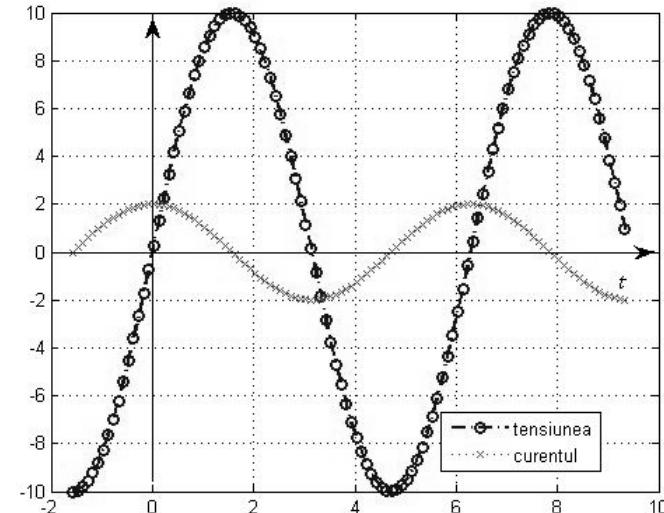
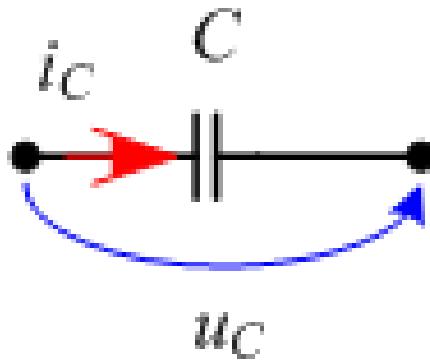
- Regim permanent sinusoidal

Condensatorul liniar

$$u(t) = U\sqrt{2} \sin(\omega \cdot t)$$

$$i_C = C \frac{du_C}{dt}$$

$$i_C(t) = \omega C U \sqrt{2} \cos(\omega \cdot t) = \omega C U \sqrt{2} \sin\left(\omega t + \frac{\pi}{2}\right)$$



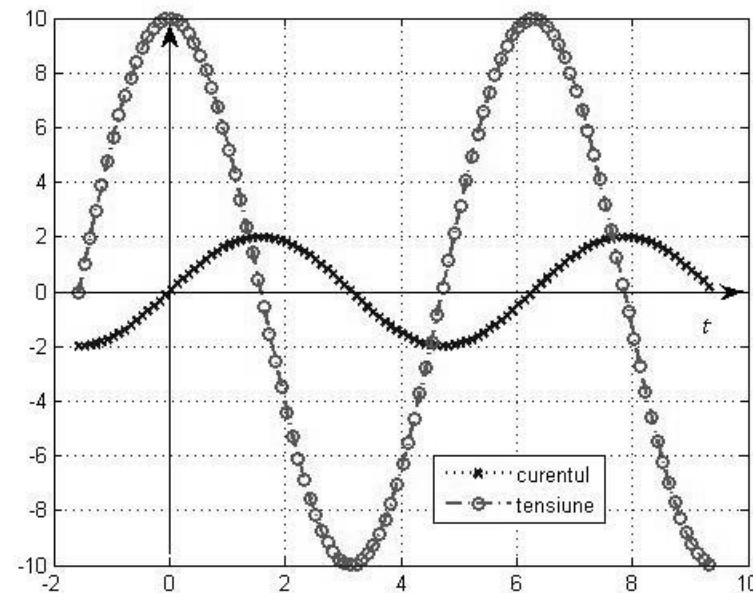
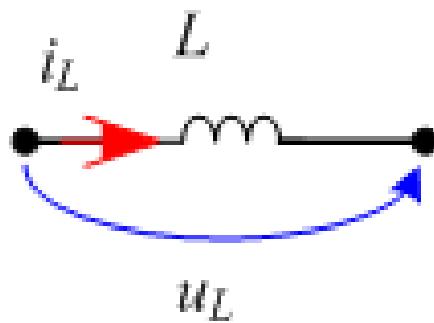
Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Bobina liniara

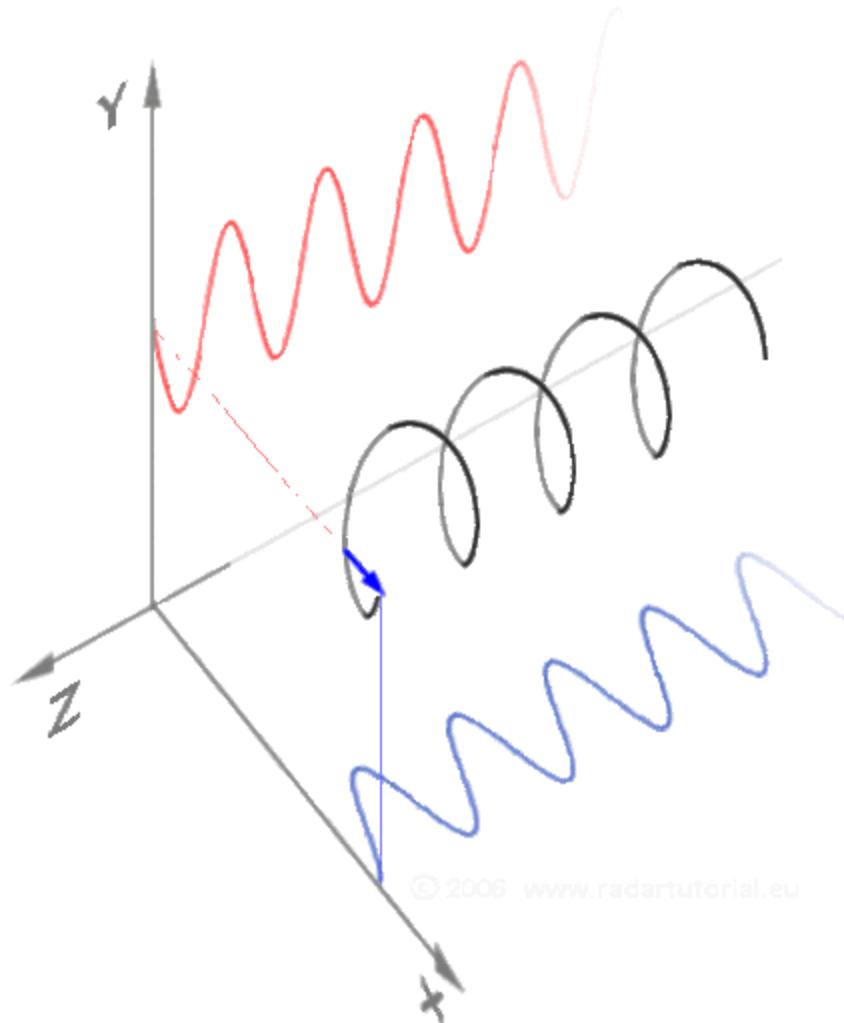
$$i(t) = I\sqrt{2} \sin(\omega \cdot t)$$

$$u_L(t) = \omega L I \sqrt{2} \cos(\omega \cdot t) = \omega L I \sqrt{2} \sin\left(\omega t + \frac{\pi}{2}\right)$$



Circuite electrice de curent alternativ

- Regim permanent sinusoidal

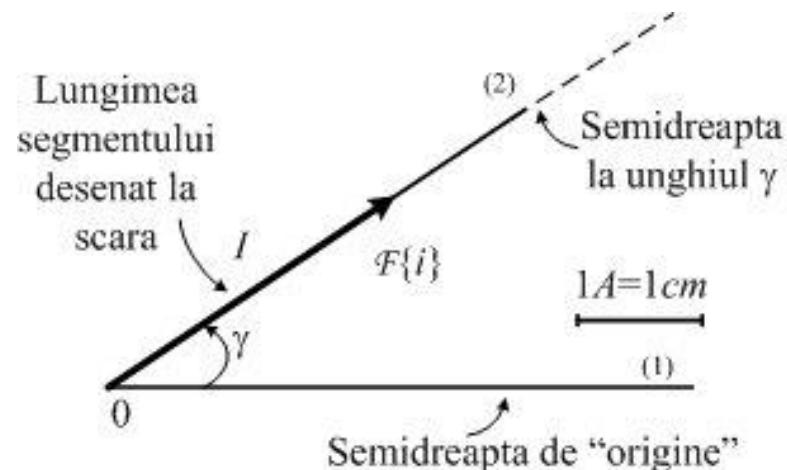


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Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Reprezentare fazoriala

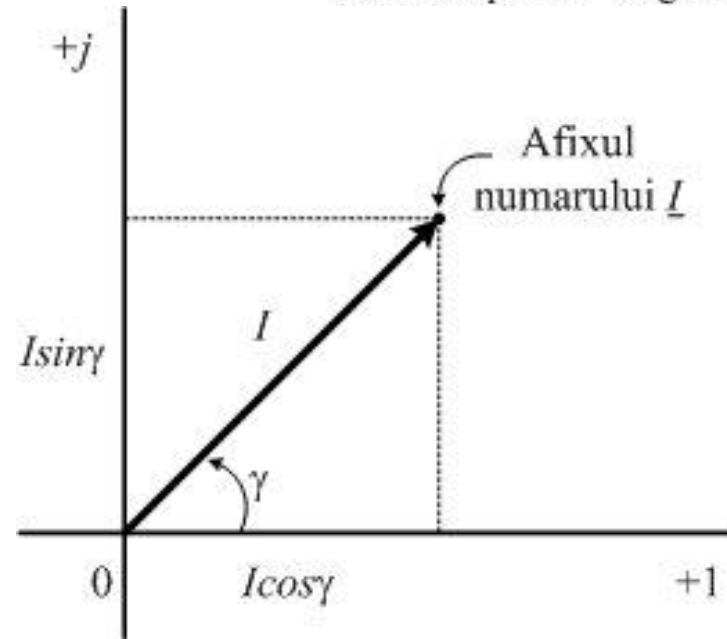


Reprezentare analitica (in complex)

$$i = I\sqrt{2} \sin(\omega t + \gamma),$$

$$C\{i\} = \underline{I} = Ie^{j\gamma} = I \cos \gamma + jI \sin \gamma$$

$$j = \sqrt{-1}$$



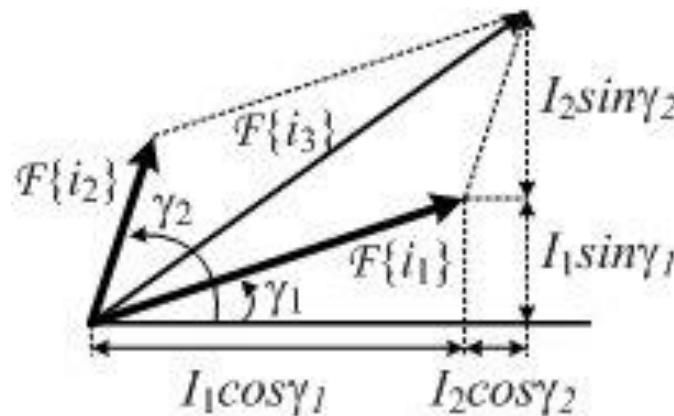
Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Adunarea

$$i_1 = I_1 \sqrt{2} \sin(\omega t + \gamma_1)$$

$$i_2 = I_2 \sqrt{2} \sin(\omega t + \gamma_2)$$



$$I_1 e^{j\gamma_1} + I_2 e^{j\gamma_2} = I_3 e^{j\gamma_3}$$

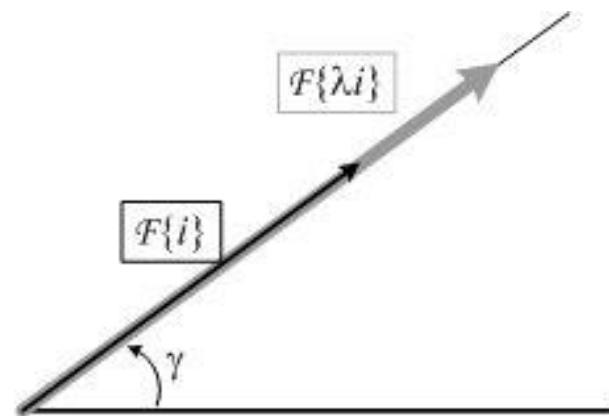
$$(I_1 \cos \gamma_1 + I_2 \cos \gamma_2) + j(I_1 \sin \gamma_1 + I_2 \sin \gamma_2) = \\ = I_3 \cos \gamma_3 + jI_3 \sin \gamma_3$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Inmultirea cu un scalar

$$\lambda i(t) = \lambda I \sqrt{2} \sin(\omega t + \gamma)$$



$$C\{\lambda i\} = \lambda I e^{j\gamma} = \lambda C\{i\}$$

Circuite electrice de curent alternativ

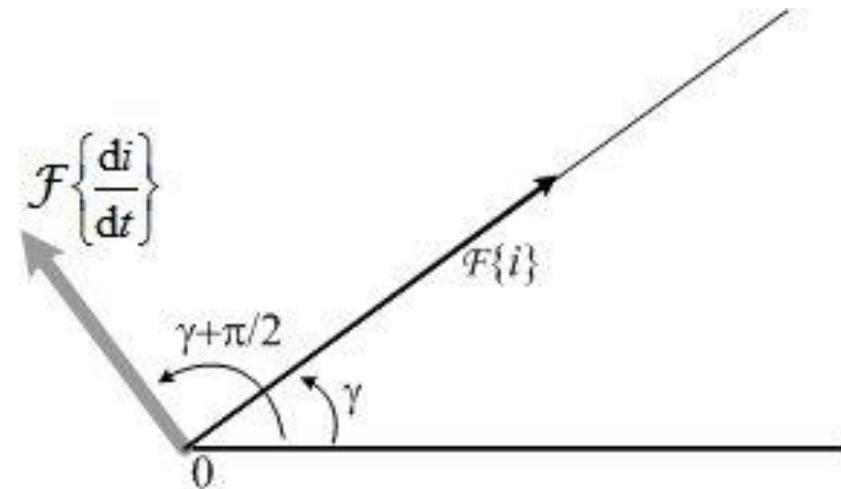
- Regim permanent sinusoidal

Derivarea

$$\frac{di}{dt} = \omega I \sqrt{2} \cos(\omega t + \gamma) = \omega I \sqrt{2} \sin(\omega t + \gamma + \frac{\pi}{2}),$$

$$\mathcal{F}\left\{\frac{di}{dt}\right\} = \left\{\omega I, \gamma + \frac{\pi}{2}\right\},$$

$$\mathcal{C}\left\{\frac{di}{dt}\right\} = \omega I e^{j(\gamma + \frac{\pi}{2})} = j\omega C\{i(t)\}.$$



Circuite electrice de curent alternativ

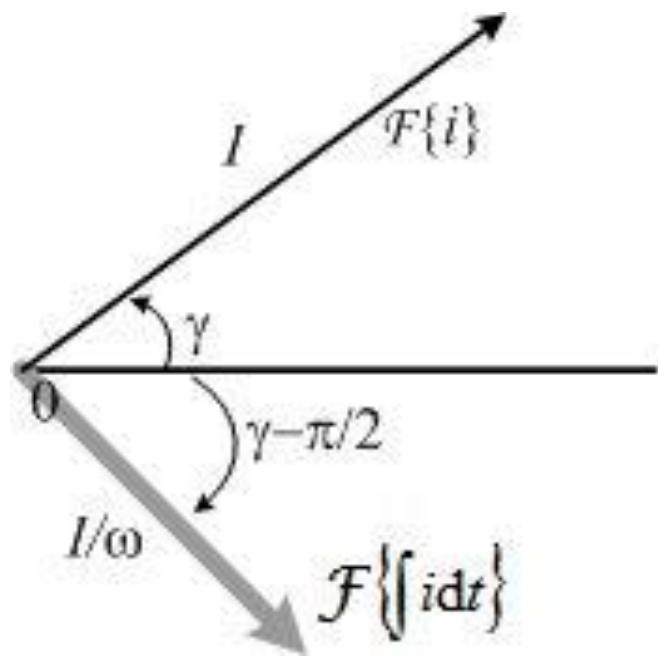
- Regim permanent sinusoidal

Integrarea

$$\int i dt = -\frac{I}{\omega} \sqrt{2} \cos(\omega t + \gamma) = \frac{I}{\omega} \sqrt{2} \sin(\omega t + \gamma - \frac{\pi}{2}),$$

$$F\left\{ \int i dt \right\} = \left\{ \frac{I}{\omega}, \gamma - \frac{\pi}{2} \right\};$$

$$C\left\{ \int i dt \right\} = \frac{I}{\omega} e^{j(\gamma - \frac{\pi}{2})} = \frac{1}{\omega} (I e^{j\gamma}) e^{-j\frac{\pi}{2}} = \frac{I}{j\omega}$$



Circuite electrică de curent alternativ

- Regim permanent sinusoidal

Impedanță / admitanță complexă

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U e^{j\gamma_u}}{I e^{j\gamma_i}} = \frac{U}{I} e^{j(\gamma_u - \gamma_i)} = \\ = Z e^{j\varphi} = Z \cos \varphi + j \sin \varphi = R_e + j X_e$$

$$|\underline{Z}| = Z [\Omega], \\ \arg\{\underline{Z}\} = \varphi [\text{rad}], \\ \operatorname{Re}\{\underline{Z}\} = R_e [\Omega], \\ \operatorname{Im}\{\underline{Z}\} = X_e [\Omega.]$$

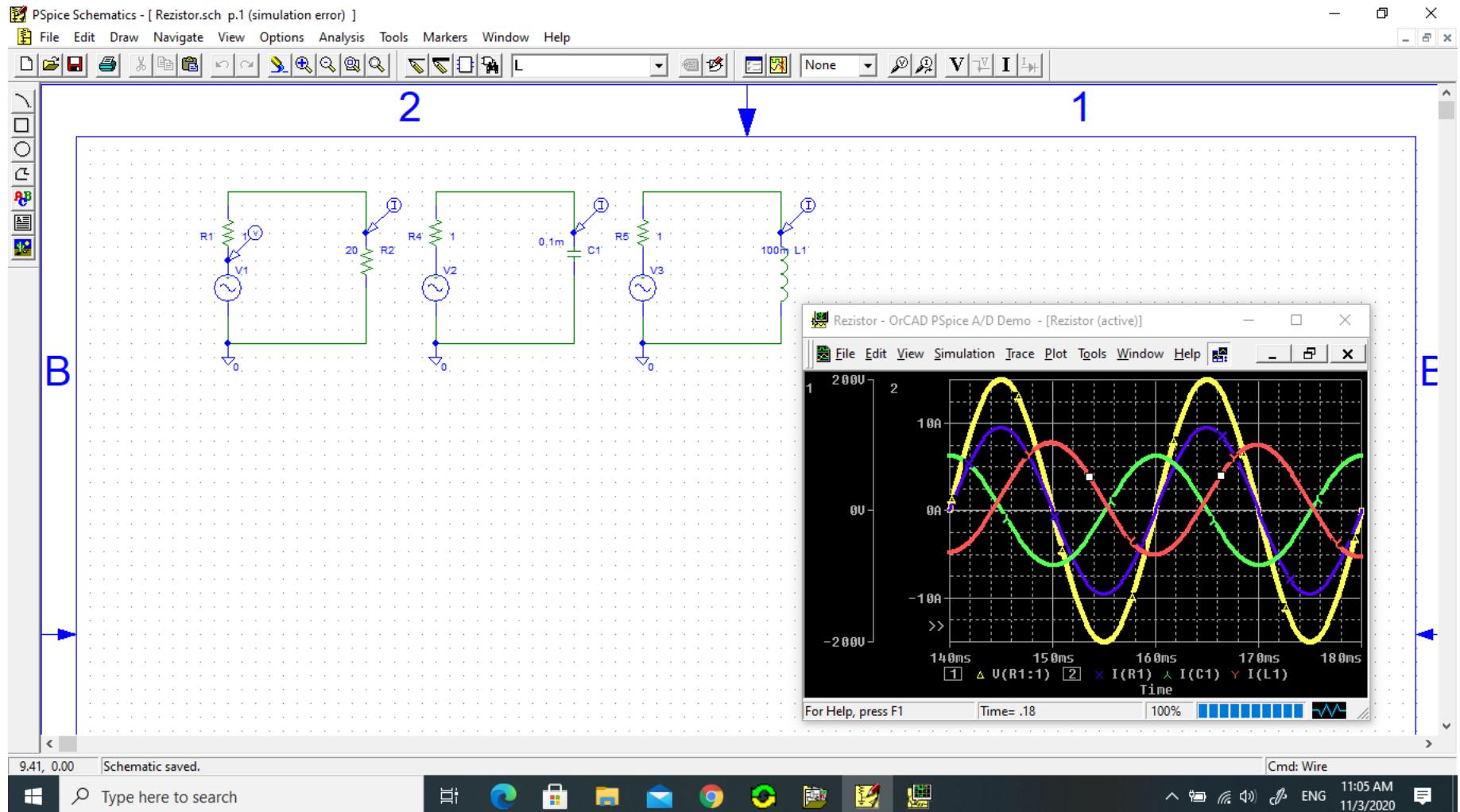
$$\underline{Y} = \frac{\underline{I}}{\underline{U}} = \frac{I e^{j\gamma_i}}{U e^{j\gamma_u}} = \frac{I}{U} e^{-j\varphi} = \frac{I \cos \varphi}{U} + j \frac{-I \sin \varphi}{U} \\ \underline{Y} = G e + j B e$$

BAZELE ELECTROTEHNICII

- note de curs -

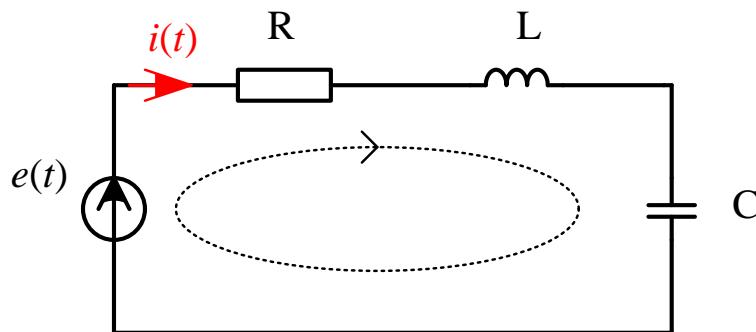
Circuite electrică de curent alternativ

- Regim permanent sinusoidal



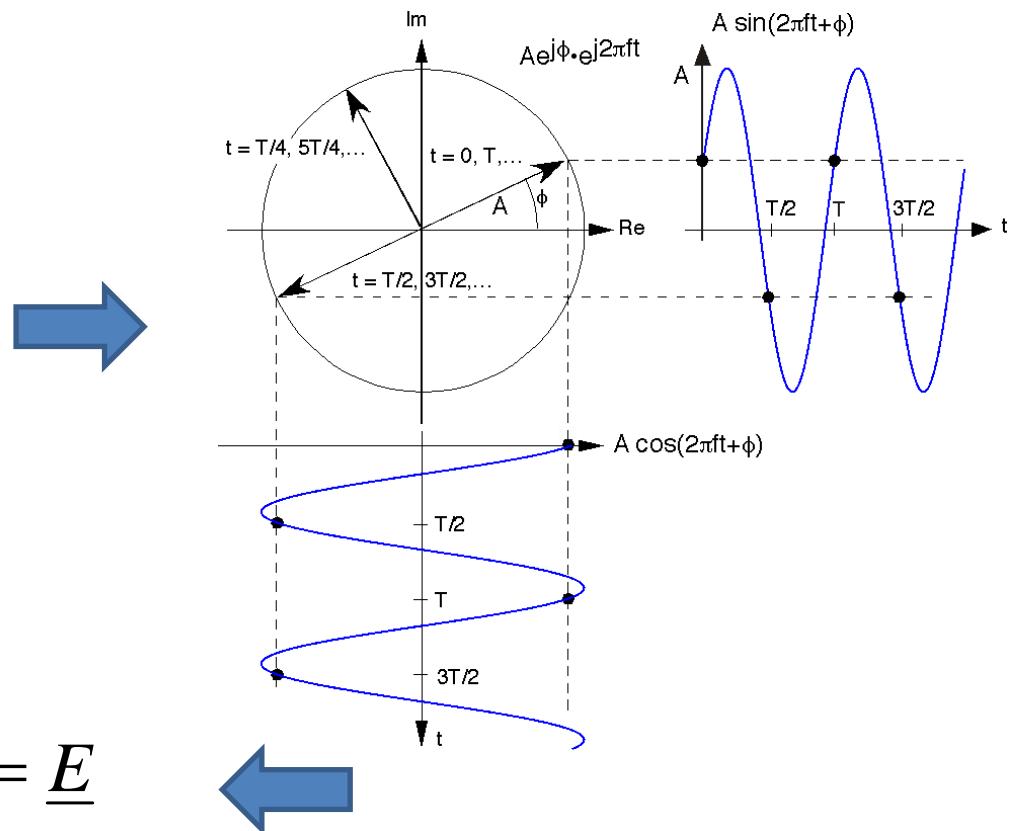
Circuite electrică de curent alternativ

- Regim permanent sinusoidal



$$R \cdot i + L \frac{di}{dt} + \frac{1}{C} \int i dt = e$$

$$R \cdot \underline{I} + j\omega L \cdot \underline{I} + \frac{1}{j\omega C} \cdot \underline{I} = \underline{E}$$



Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Impedanta / admitanta complexa

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U e^{j\gamma_u}}{I e^{j\gamma_i}} = \frac{U}{I} e^{j(\gamma_u - \gamma_i)} = Z e^{j\varphi} = Z \cos \varphi + j \sin \varphi = R_e + jX_e$$

Reactanță

Rezistență

$$\underline{Y} = \frac{\underline{I}}{\underline{U}} = \frac{I e^{j\gamma_i}}{U e^{j\gamma_u}} = \frac{I}{U} e^{-j(\gamma_u - \gamma_i)} = \frac{I \cos \varphi}{U} + j \frac{-I \sin \varphi}{U}$$

$$\underline{Y} = G_e + jB_e$$

Conductanță

Susceptanță

$$|\underline{Z}| = Z [\Omega],$$

$$\arg\{\underline{Z}\} = \varphi [\text{rad}],$$

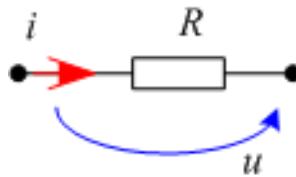
$$\operatorname{Re}\{\underline{Z}\} = R_e [\Omega],$$

$$\operatorname{Im}\{\underline{Z}\} = X_e [\Omega]$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

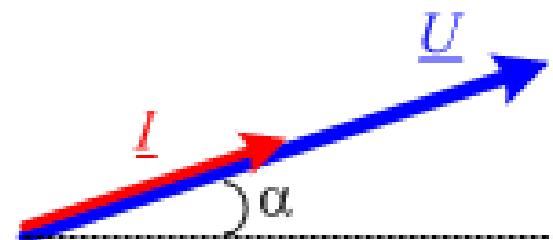
Rezistorul liniar



$$u(t) = U \sqrt{2} \sin(\omega t + \alpha)$$

$$i(t) = \frac{u(t)}{R}$$

$$i(t) = \frac{U}{R} \sqrt{2} \sin(\omega t + \alpha)$$



$$\underline{U} = U \cdot e^{j\alpha}$$

$$\underline{I} = I \cdot e^{j\alpha}$$

$$\underline{Z}_R = \frac{\underline{U}}{\underline{I}} = \frac{U}{I} = R$$

$$\underline{Z}_R = R$$

$$X = 0$$

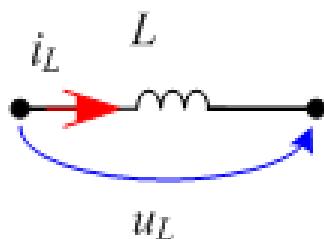
$$\varphi = 0$$



Circuite electrice de curent alternativ

- Regim permanent sinusoidal

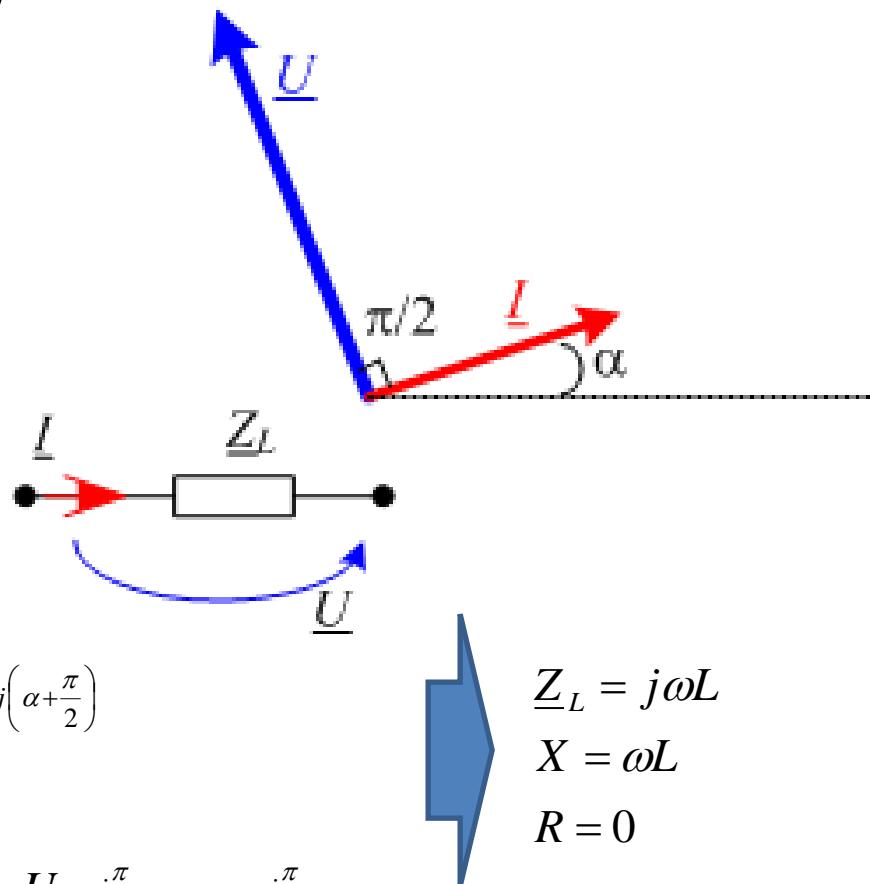
Bobina liniara



$$i(t) = I \sqrt{2} \sin(\omega t + \alpha)$$

$$u(t) = L \frac{di}{dt}$$

$$u(t) = \omega L I \sqrt{2} \sin(\omega t + \alpha + \frac{\pi}{2})$$



$$\underline{U} = U \cdot e^{j\left(\alpha + \frac{\pi}{2}\right)}$$

$$\underline{I} = I \cdot e^{j\alpha}$$

$$\underline{Z}_L = \frac{\underline{U}}{\underline{I}} = \frac{U}{I} e^{j\frac{\pi}{2}} = \omega L e^{j\frac{\pi}{2}} = j\omega L$$

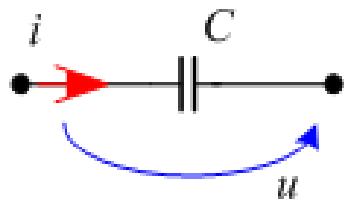
$$\begin{aligned} Z_L &= j\omega L \\ X &= \omega L \\ R &= 0 \end{aligned}$$

$$\varphi = \frac{\pi}{2}$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Condensatorul liniar



$$u(t) = U \sqrt{2} \sin(\omega t + \alpha)$$

$$i(t) = C \frac{du}{dt}$$

$$i(t) = \omega C U \sqrt{2} \sin(\omega t + \alpha + \frac{\pi}{2})$$



$$\underline{U} = U \cdot e^{j\alpha}$$

$$\underline{I} = I \cdot e^{j(\alpha + \frac{\pi}{2})}$$

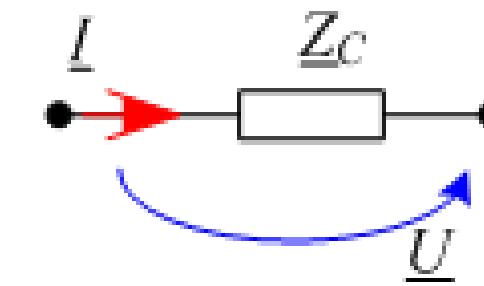
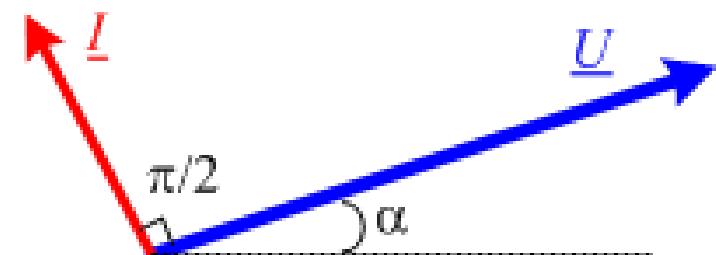
$$\underline{Z}_C = \frac{\underline{U}}{\underline{I}} = \frac{U}{I} e^{j(-\frac{\pi}{2})} = \frac{1}{\omega C} e^{j(-\frac{\pi}{2})} = -j \frac{1}{\omega C}$$

$$\underline{Z}_C = \frac{-j}{\omega C}$$

$$X = -\frac{1}{\omega C}$$

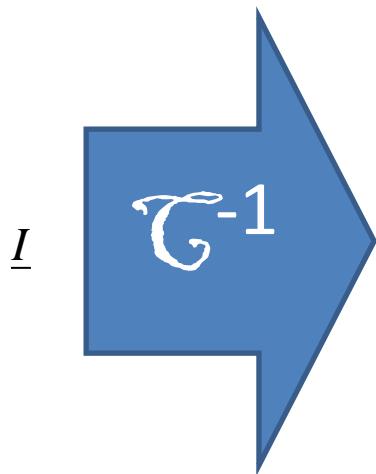
$$R = 0$$

$$\varphi = -\frac{\pi}{2}$$



Circuite electrice de curent alternativ

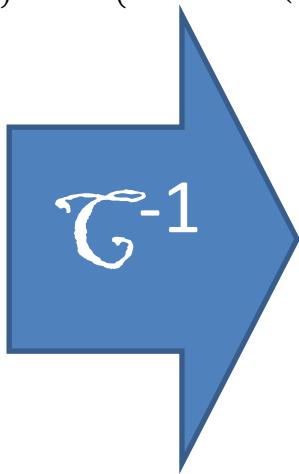
- Regim permanent sinusoidal



$$i(t) = I\sqrt{2} \sin(\omega t + \alpha)$$

$$i(t) = \text{Im}\left\{\sqrt{2}Ie^{j\omega t}\right\} = \text{Im}\left\{\sqrt{2}Ie^{j\alpha}e^{j\omega t}\right\} = \text{Im}\left\{\sqrt{2}I[\cos(\omega t + \alpha) + j \sin(\omega t + \alpha)]\right\} = I\sqrt{2} \sin(\omega t + \alpha)$$

$$I = a + j \cdot b = \sqrt{a^2 + b^2} \cdot e^{j \cdot \text{arctg}(\frac{b}{a})}$$



$$i(t) = \sqrt{2} \sqrt{a^2 + b^2} \sin(\omega t + \text{arctg} \frac{b}{a})$$

$$i(t) = I\sqrt{2} \sin(\omega t + \alpha)$$

Circuite electrice de curent alternativ

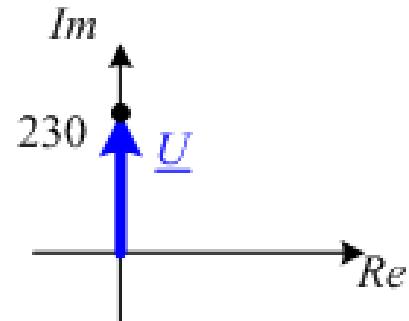
- Regim permanent sinusoidal

Exemple

$$u(t) = 230\sqrt{2} \sin(\omega t + \frac{\pi}{2})$$



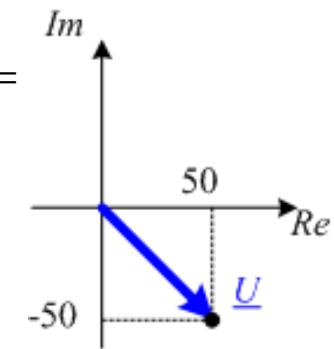
$$\begin{aligned}\underline{U} &= 230 \cdot e^{j\frac{\pi}{2}} = 230 \cdot (\cos \frac{\pi}{2} + j \sin \frac{\pi}{2}) = \\ &= 230 \cdot (0 + j \cdot 1) = 230j\end{aligned}$$



$$u(t) = 100 \sin(\omega t - \frac{\pi}{4})$$



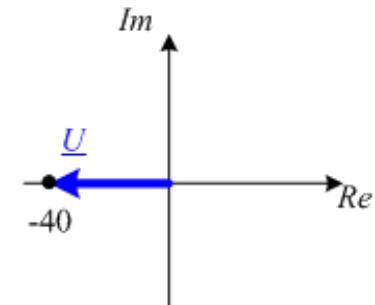
$$\begin{aligned}\underline{U} &= \frac{100}{\sqrt{2}} \cdot e^{j(-\frac{\pi}{4})} = 50\sqrt{2} \cdot (\cos \frac{\pi}{4} - j \sin \frac{\pi}{4}) = \\ &= 50\sqrt{2} \cdot (\frac{\sqrt{2}}{2} - j \cdot \frac{\sqrt{2}}{2}) = 50(1 - j)\end{aligned}$$



$$u(t) = 40\sqrt{2} \sin(\omega t + \pi)$$



$$\begin{aligned}\underline{U} &= 40 \cdot e^{j\pi} = 40 \cdot (\cos \pi + j \sin \pi) = \\ &= 40 \cdot (-1 + j \cdot 0) = -40\end{aligned}$$

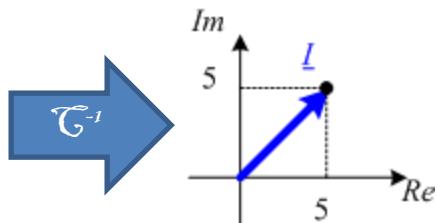


Circuite electrice de curent alternativ

- Regim permanent sinusoidal

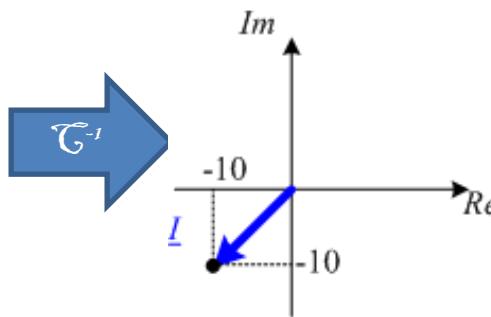
Exemple

$$\underline{I} = 5 + 5j$$



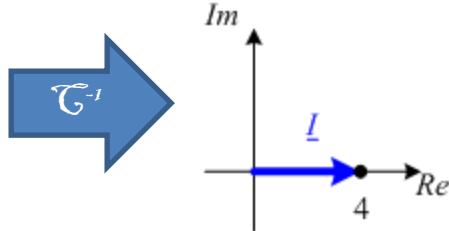
$$i(t) = \sqrt{2} \sqrt{5^2 + 5^2} \sin(\omega t + \arctg \frac{5}{5}) = 10 \sin(\omega t + \frac{\pi}{4})$$

$$\underline{I} = -10 - 10j$$



$$\begin{aligned} i(t) &= \sqrt{2} \sqrt{(-10)^2 + (-10)^2} \sin(\omega t + \arctg \frac{-10}{-10}) = \\ &= 20 \sin(\omega t - \frac{3\pi}{4}) \end{aligned}$$

$$\underline{I} = 4$$



$$i(t) = \sqrt{2} \sqrt{4^2 + 0^2} \sin(\omega t + \arctg \frac{0}{4}) = 4\sqrt{2} \sin(\omega t)$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Puteri in regim sinusoidal

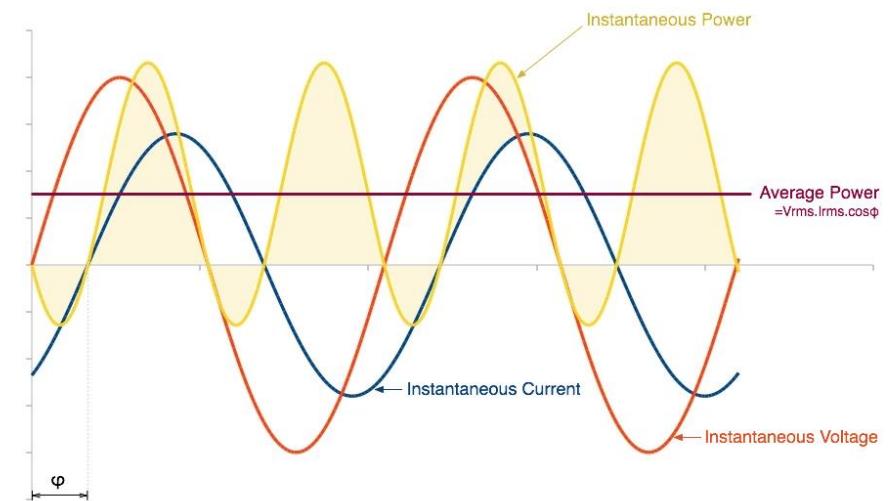
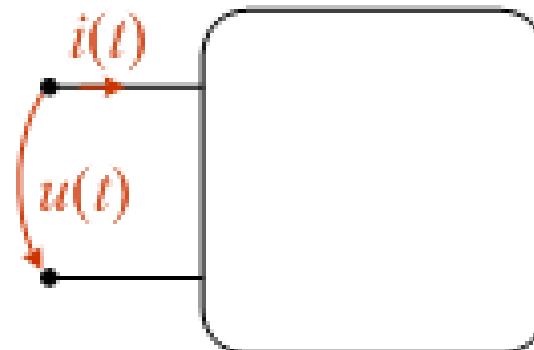
$$u(t) = U \sqrt{2} \sin(\omega t + \gamma_u),$$

$$i(t) = I \sqrt{2} \sin(\omega t + \gamma_i).$$

Puterea
instantanea

$$p = ui = 2UI \sin(\omega t + \gamma_u) \sin(\omega t + \gamma_i)$$

$$p = UI \cos \varphi - UI \cos(2\omega t + \gamma_u + \gamma_i)$$



Circuite electrice de curent alternativ

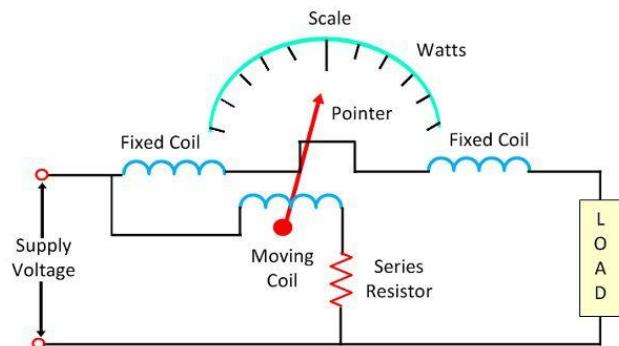
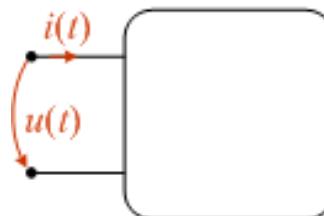
- Regim permanent sinusoidal

Puteri in regim sinusoidal

$$u(t) = U \sqrt{2} \sin(\omega t + \gamma_u),$$

$$i(t) = I \sqrt{2} \sin(\omega t + \gamma_i).$$

Puterea activă



Electrodynamometer Wattmeter

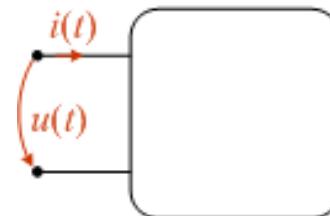
Circuit Globe



Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Puteri in regim sinusoidal



$$u(t) = U \sqrt{2} \sin(\omega t + \gamma_u),$$

$$i(t) = I \sqrt{2} \sin(\omega t + \gamma_i).$$

Puterea reactivă

$$Q = UI \sin \varphi \quad [\text{var}]$$

KDSI www.kdsimeter.com



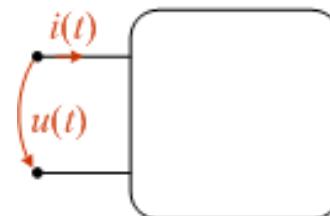
sales@kdsimeter.com



Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Puteri in regim sinusoidal



$$u(t) = U \sqrt{2} \sin(\omega t + \gamma_u),$$

$$i(t) = I \sqrt{2} \sin(\omega t + \gamma_i).$$

Puterea aparentă

$$S = UI \quad [\text{VA}]$$

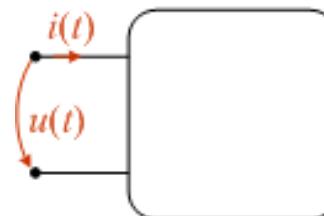
$$k_p = \frac{P}{S} = \cos \varphi > 0$$

Factor
de
putere

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Puteri in regim sinusoidal



$$u(t) = U \sqrt{2} \sin(\omega t + \gamma_u),$$

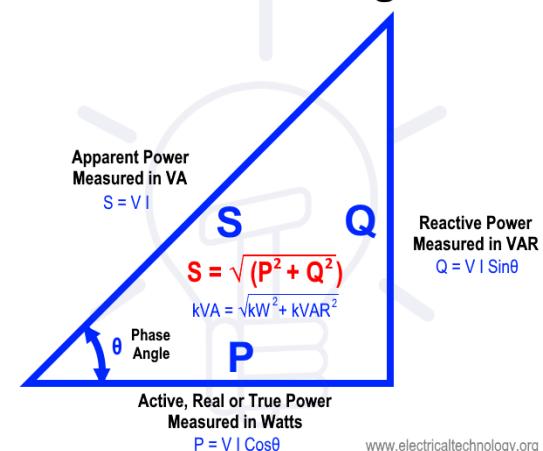
$$i(t) = I \sqrt{2} \sin(\omega t + \gamma_i).$$

Puterea aparentă complexă

$$\underline{S} = \underline{U} \cdot \underline{I}^*$$

$$\begin{aligned} \underline{S} &= U e^{j\gamma_u} \cdot I e^{-j\gamma_i} = UI e^{j\varphi} = UI \cos \varphi + jUI \sin \varphi = \\ &= P + jQ. \end{aligned}$$

Power Triangle



$$|\underline{S}| = S = UI,$$

$$\operatorname{Re}\{\underline{S}\} = UI \cos \varphi = P,$$

$$\operatorname{Im}\{\underline{S}\} = UI \sin \varphi = Q$$

$$\arg\{\underline{S}\} = \varphi$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Puteri in regim sinusoidal

Rezistorul liniar

$$\varphi = \gamma_u - \gamma_i = 0,$$

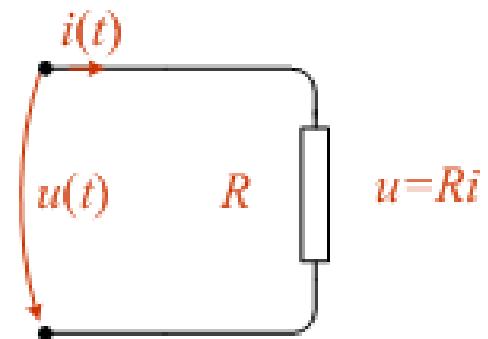


$$P = UI \cos \varphi = UI = RI^2 = \frac{U^2}{R},$$

$$Q = UI \sin \varphi = 0,$$

$$S = UI = P,$$

$$\underline{S} = \underline{U} \cdot \underline{I}^* = U \cdot I = RI^2$$



$$u(t) = U \sqrt{2} \sin(\omega t + \gamma_u),$$

$$i(t) = I \sqrt{2} \sin(\omega t + \gamma_i).$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Puteri in regim sinusoidal

Bobina liniara

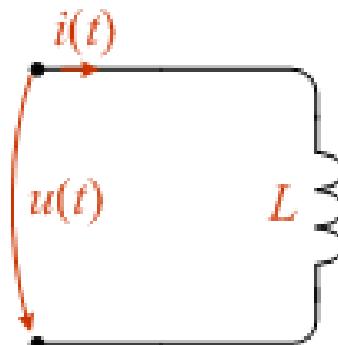
$$\varphi = \gamma_u - \gamma_i = \frac{\pi}{2},$$


$$P = UI \cos \varphi = 0$$

$$Q = UI \sin \varphi > 0,$$

$$\underline{S} = \underline{U} \cdot \underline{I}^* = j\omega L \underline{I} \cdot \underline{I}^* = j\omega L \underline{I}^2$$

$$Q = \omega L I^2$$



$$u(t) = U \sqrt{2} \sin(\omega t + \gamma_u),$$

$$i(t) = I \sqrt{2} \sin(\omega t + \gamma_i).$$

$$u = L \frac{di}{dt}; \underline{U} = j\omega L \cdot \underline{I}$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Puteri in regim sinusoidal

Condensatorul liniar

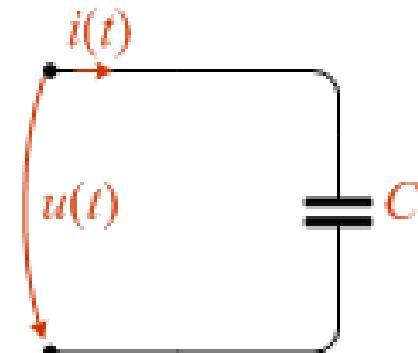
$$\varphi = \gamma_u - \gamma_i = -\frac{\pi}{2},$$


$$P = UI \cos \varphi = 0$$

$$Q = UI \sin \varphi < 0,$$

$$\underline{S} = \underline{U} \cdot \underline{I}^* = -\frac{j}{\omega C} \underline{I} \cdot \underline{I}^* = -\frac{j}{\omega C} I^2$$

$$Q = -\frac{1}{\omega C} I^2$$



$$u(t) = U \sqrt{2} \sin(\omega t + \gamma_u),$$

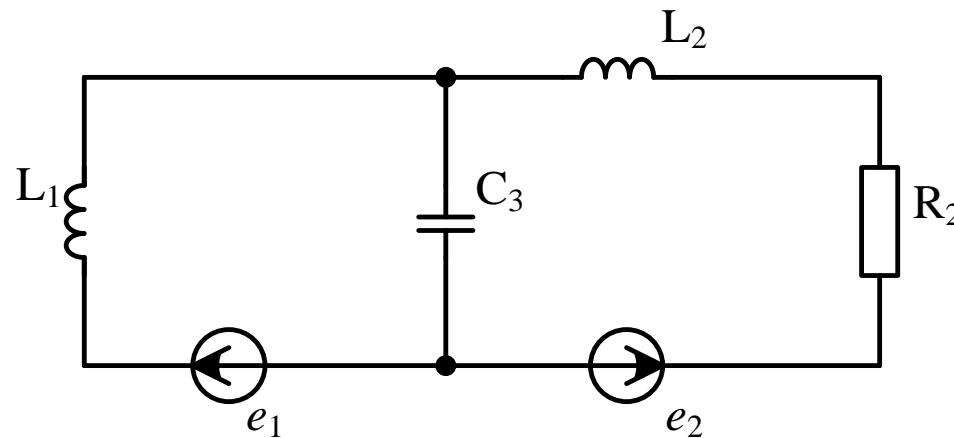
$$i(t) = I \sqrt{2} \sin(\omega t + \gamma_i).$$

$$u = \frac{1}{C} \int i dt; \underline{U} = \frac{\underline{I}}{j\omega C}$$

Circuite electrice de curent alternativ

- Regim permanent sinusoidal

Exemplu:



$$L_1 = \frac{100}{\pi} \text{ mH}, L_2 = \frac{300}{\pi} \text{ mH}, R_2 = 10 \Omega, C_3 = \frac{500}{\pi} \mu\text{F}$$

$$e_1 = 20 \sin\left(\omega t - \frac{\pi}{4}\right) \text{ V}, e_2 = 60 \sin\left(\omega t + \frac{\pi}{4}\right) \text{ V}, f = 50 \text{ Hz}$$

Circuite electrice trifazate

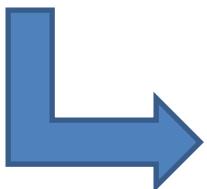
Sistem trifazat

$$\begin{cases} e_1 = E_1 \sqrt{2} \sin(\omega t + \alpha_1) \\ e_2 = E_2 \sqrt{2} \sin(\omega t + \alpha_2) \\ e_3 = E_3 \sqrt{2} \sin(\omega t + \alpha_3) \end{cases}$$



Sistem trifazat simetric

$$E_1 = E_2 = E_3 = E$$



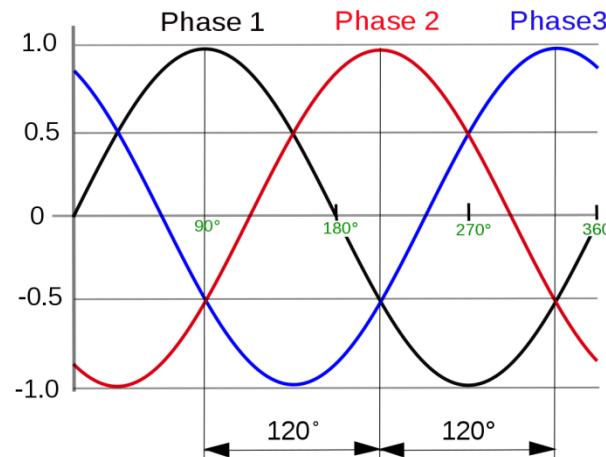
$$\alpha_2 - \alpha_1 = \alpha_3 - \alpha_2 = \alpha_1 - \alpha_3 = \frac{2\pi}{3}$$

Sistem trifazat simetric de succesiune directă

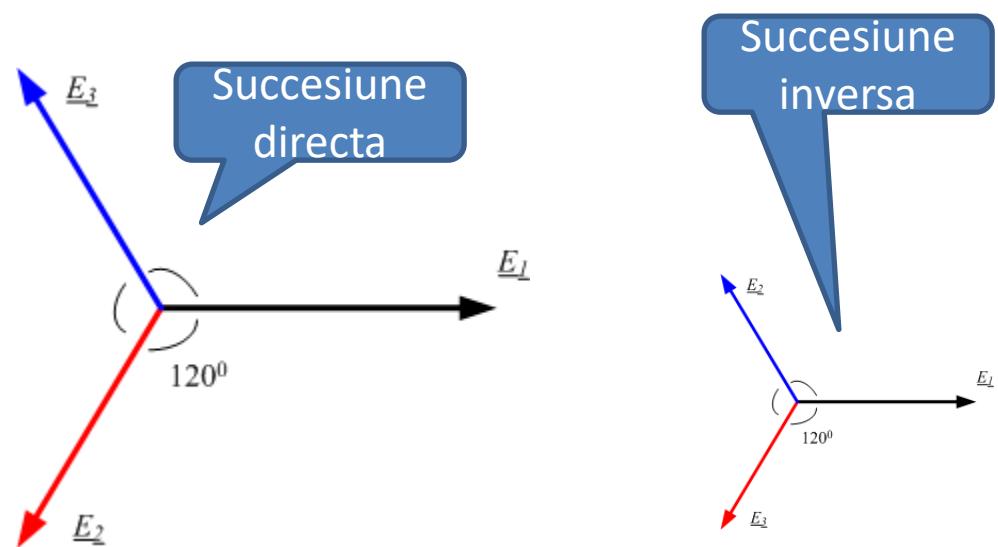


Circuite electrice trifazate

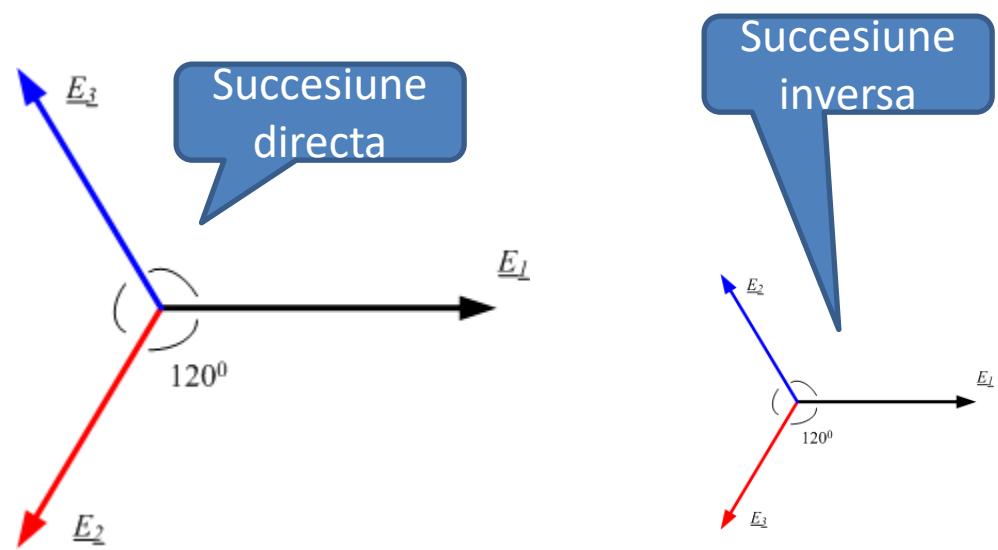
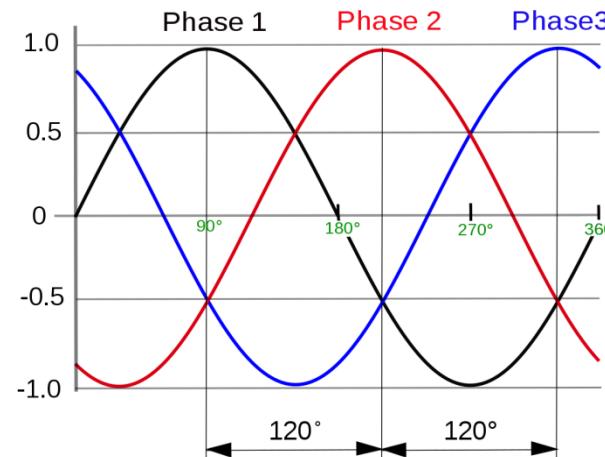
$$\begin{cases} e_1 = E\sqrt{2} \sin(\omega t) \\ e_2 = E\sqrt{2} \sin\left(\omega t - \frac{2\pi}{3}\right) \\ e_3 = E\sqrt{2} \sin\left(\omega t + \frac{2\pi}{3}\right) \end{cases}$$



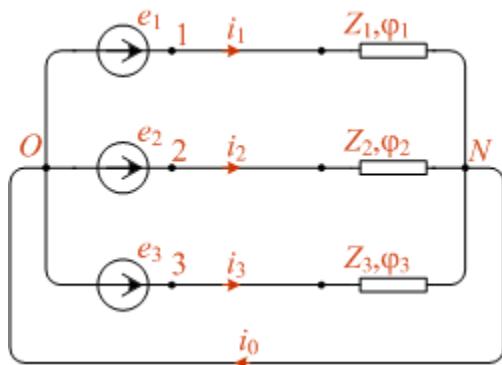
$$\begin{cases} \underline{E}_1 = E \\ \underline{E}_2 = E \cdot e^{-j\frac{2\pi}{3}} \\ \underline{E}_3 = E \cdot e^{j\frac{2\pi}{3}} \end{cases}$$



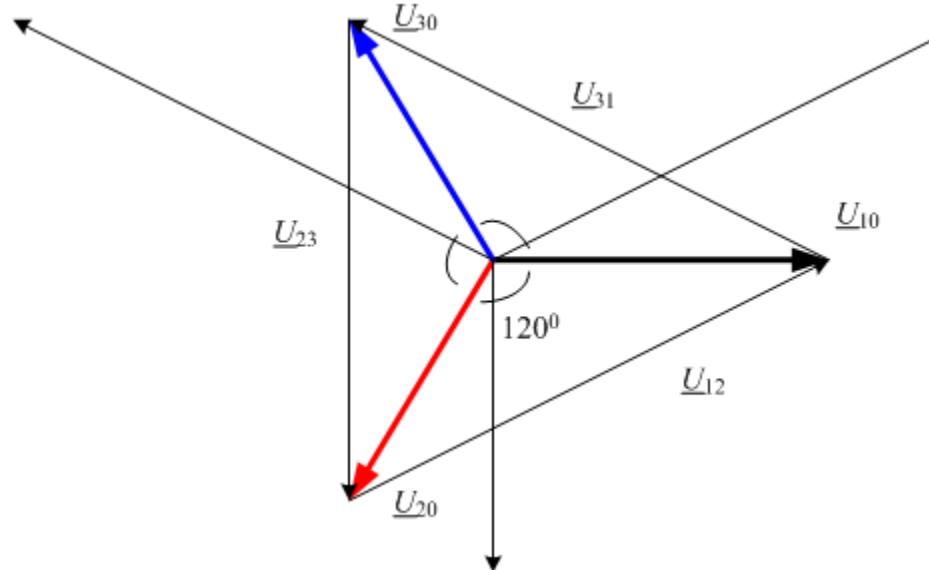
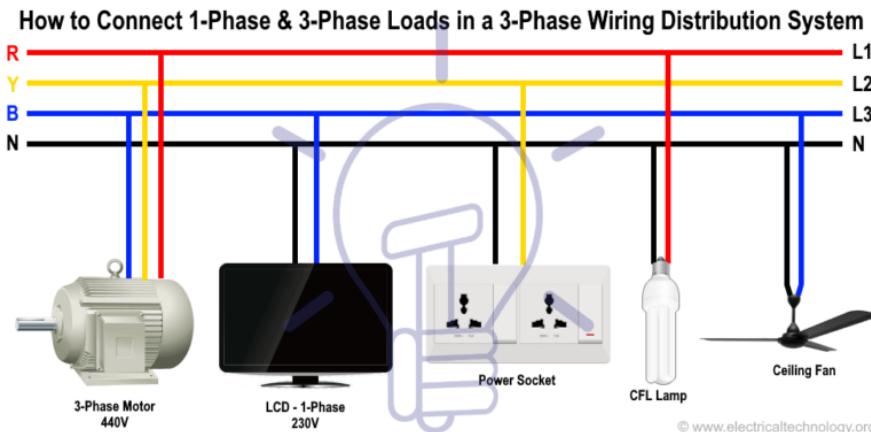
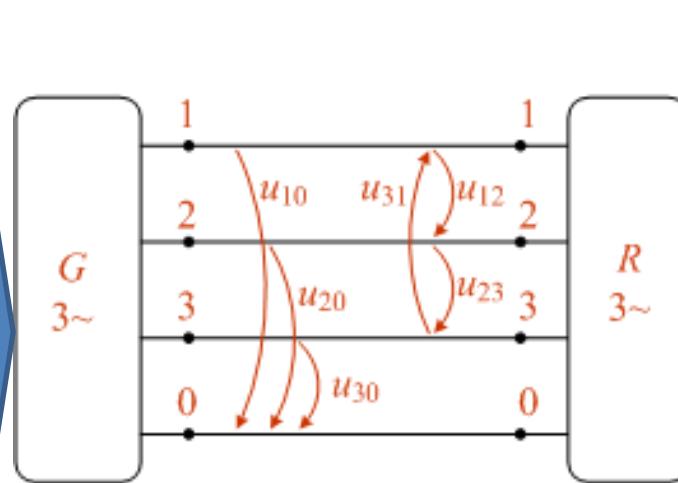
Circuite electrice trifazate



Circuite electrice trifazate



3 borne (3 faze R,S,T sau L1,L2, L3)
Si o borna de nul 0

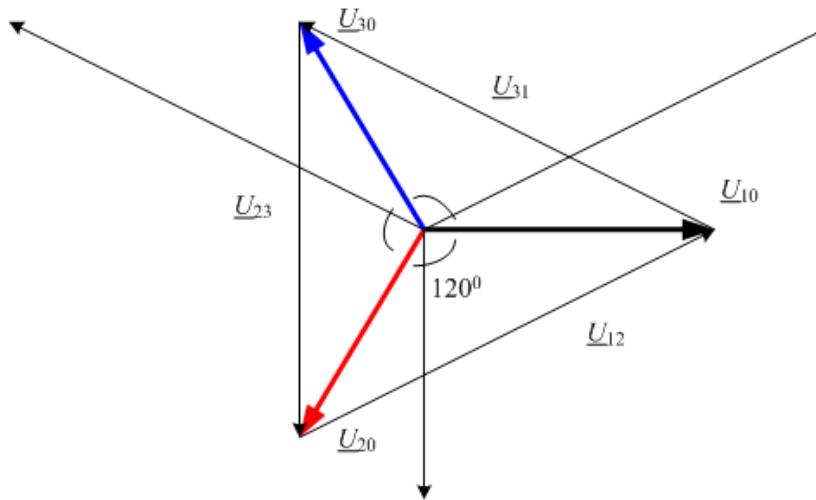
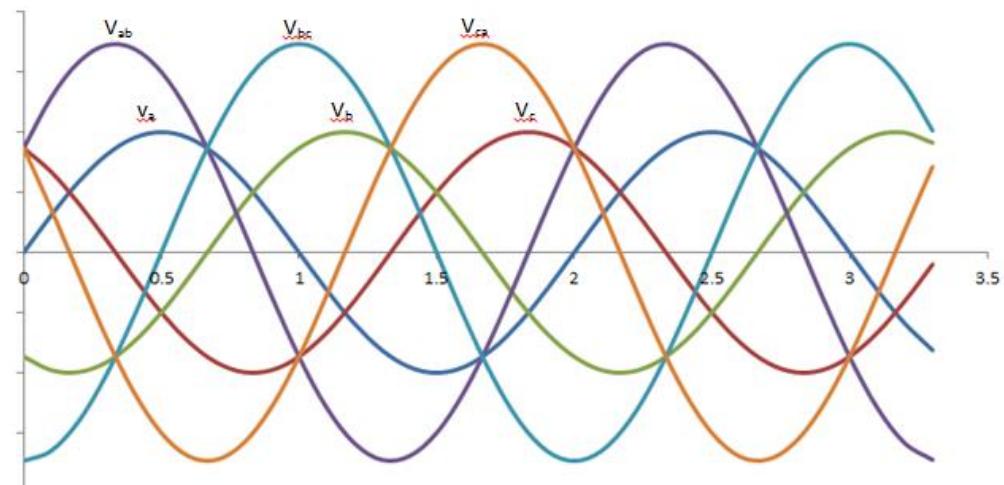


Circuite electrice trifazate

$$u_{12} = (U_f \sqrt{3}) \sqrt{2} \sin\left(\omega t + \frac{\pi}{6}\right),$$

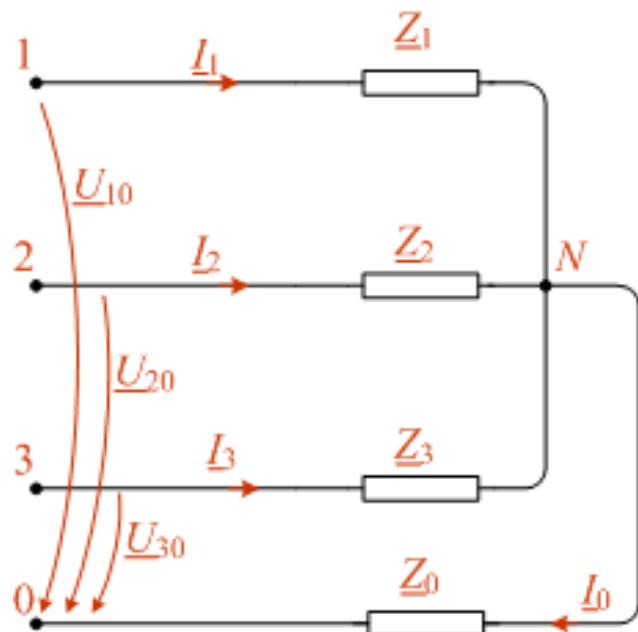
$$u_{23} = (U_f) \sqrt{2} \sin\left(\omega t + \frac{\pi}{6} - \frac{2\pi}{3}\right)$$

$$u_{31} = (U_f) \sqrt{2} \sin\left(\omega t + \frac{\pi}{6} + \frac{2\pi}{3}\right).$$



Circuite electrice trifazate

Receptor trifazat conexiune stea



$$\underline{I}_1 = \frac{\underline{V}_1 - \underline{V}_N}{\underline{Z}_1},$$

$$\underline{I}_2 = \frac{\underline{V}_2 - \underline{V}_N}{\underline{Z}_2},$$

$$\underline{I}_3 = \frac{\underline{V}_3 - \underline{V}_N}{\underline{Z}_3}.$$

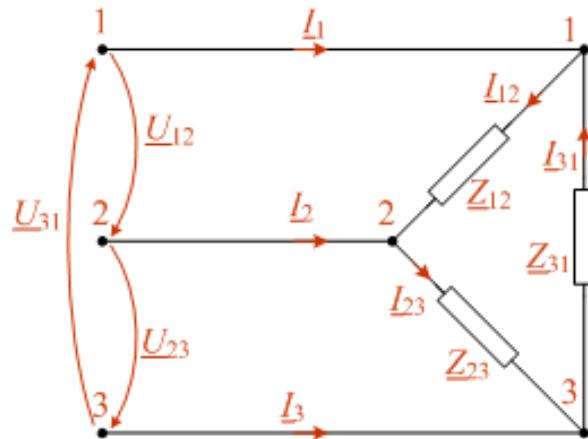
$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = \underline{I}_0$$

Formula lui Millman

$$\underline{V}_N = \frac{\underline{U}_{10} Y_1 + \underline{U}_{20} Y_2 + \underline{U}_{30} Y_3}{Y_1 + Y_2 + Y_3 + Y_0}$$

Circuite electrice trifazate

Receptor trifazat conexiune triunghi



$$\underline{I}_{12} = \frac{\underline{U}_{12}}{\underline{Z}_{12}}; \quad \underline{I}_{23} = \frac{\underline{U}_{23}}{\underline{Z}_{23}}; \quad \underline{I}_{31} = \frac{\underline{U}_{31}}{\underline{Z}_{31}}.$$



$$\underline{I}_1 = \underline{I}_{12} - \underline{I}_{31},$$

$$\underline{I}_2 = \underline{I}_{23} - \underline{I}_{12},$$

$$\underline{I}_3 = \underline{I}_{31} - \underline{I}_{23}.$$

Circuite electrice trifazate

Puteri in circuitele trifazate

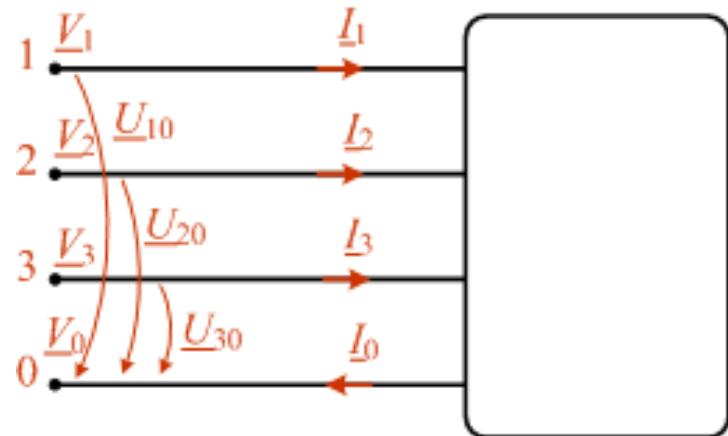
$$\underline{S}_b = \sum_{a=1,2,3,0} V_a \underline{I}_a^{ext*}$$

$$\underline{S}_b = \underline{V}_1 \underline{I}_1^* + \underline{V}_2 \underline{I}_2^* + \underline{V}_3 \underline{I}_3^* + \underline{V}_0 (-\underline{I}_0^*)$$

$$\underline{I}_0 = \underline{I}_1 + \underline{I}_2 + \underline{I}_3$$

$$\begin{aligned} \underline{S}_b &= (\underline{V}_1 - \underline{V}_0) \underline{I}_1^* + (\underline{V}_2 - \underline{V}_0) \underline{I}_2^* + (\underline{V}_3 - \underline{V}_0) \underline{I}_3^* = \\ &= \underline{U}_{10} \underline{I}_1^* + \underline{U}_{20} \underline{I}_2^* + \underline{U}_{30} \underline{I}_3^* \end{aligned}$$

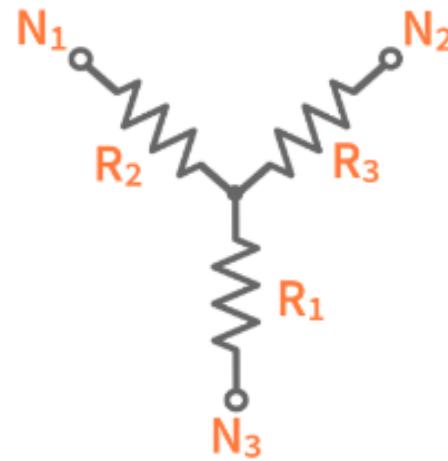
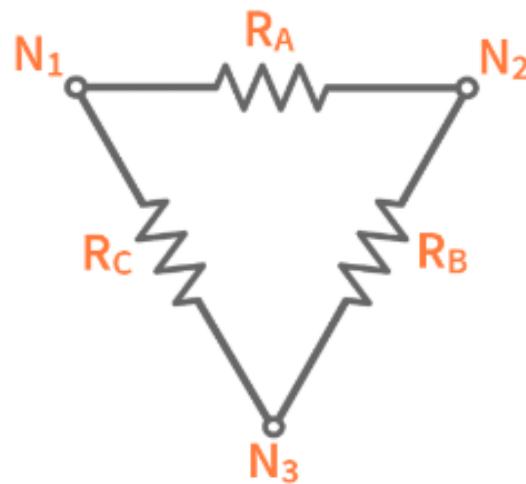
$$\underline{U} \underline{I}^* = \underline{U} \underline{I} \cos(\hat{\underline{U}}, \hat{\underline{I}}) + j \underline{U} \underline{I} \sin(\hat{\underline{U}}, \hat{\underline{I}}) = P + jQ$$



$$\begin{aligned} P_b &= U_{10} I_1 \cos(\underline{U}_{10}, \underline{I}_1) + U_{20} I_2 \cos(\underline{U}_{20}, \underline{I}_2) + \\ &\quad + U_{30} I_3 \cos(\underline{U}_{30}, \underline{I}_3), \end{aligned}$$

$$\begin{aligned} Q_b &= U_{10} I_1 \sin(\underline{U}_{10}, \underline{I}_1) + U_{20} I_2 \sin(\underline{U}_{20}, \underline{I}_2) + \\ &\quad + U_{30} I_3 \sin(\underline{U}_{30}, \underline{I}_3). \end{aligned}$$

Circuite electrice trifazate



$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_C R_A}{R_A + R_B + R_C}$$

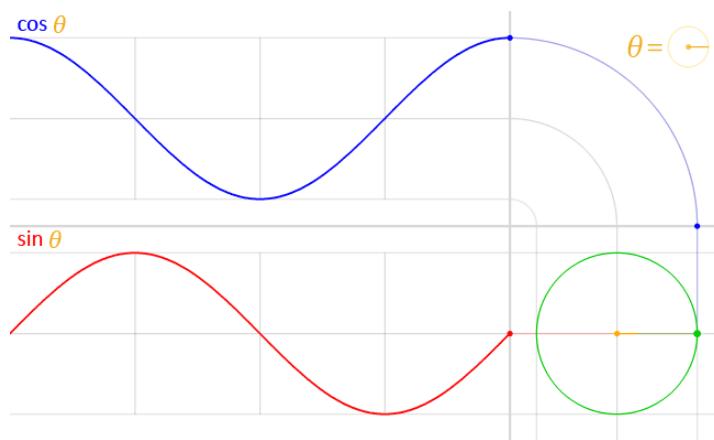
$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

BAZELE ELECTROTEHNICII

- note de curs -

Circuite electrice in regim permanent nesinusoidal

Regim sinusoidal



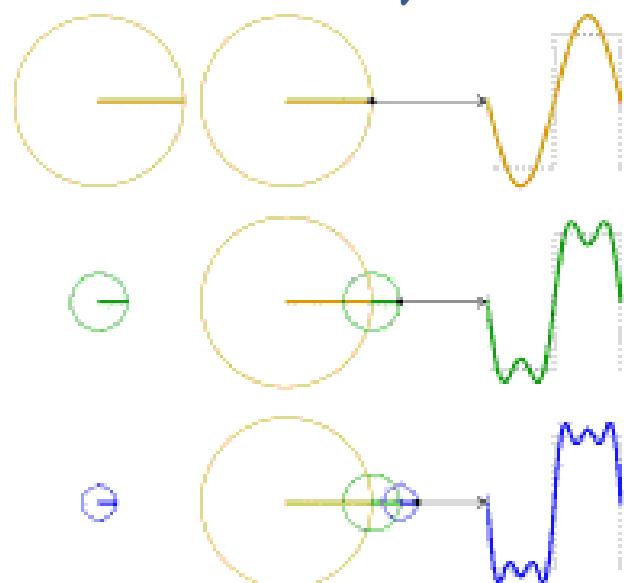
$$u(t) = \sqrt{2}U \sin(\omega t)$$

Toate tensiunile si curentii variaza cu aceeasi frecventa

Regim nesinusoidal

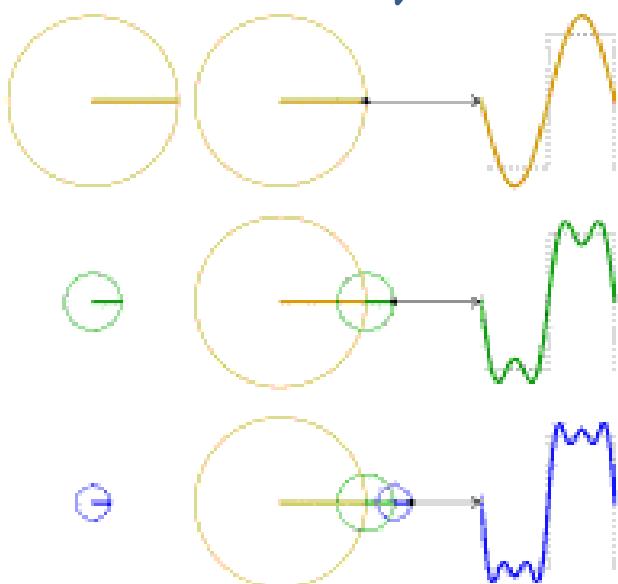
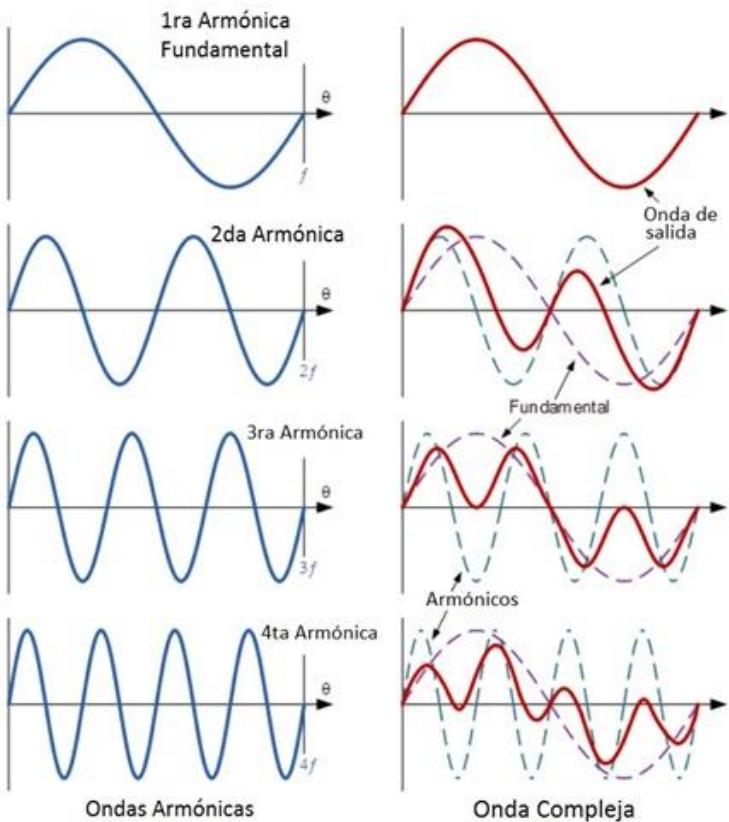
$$u(t) = \sqrt{2}U_1 \sin(\omega t) + \sqrt{2}U_2 \sin(3\omega t) + \sqrt{2}U_3 \sin(5\omega t)$$

In expresiile marimilor apar mai multe frecvențe



Circuite electrice in regim permanent nesinusoidal

Regim nesinusoidal



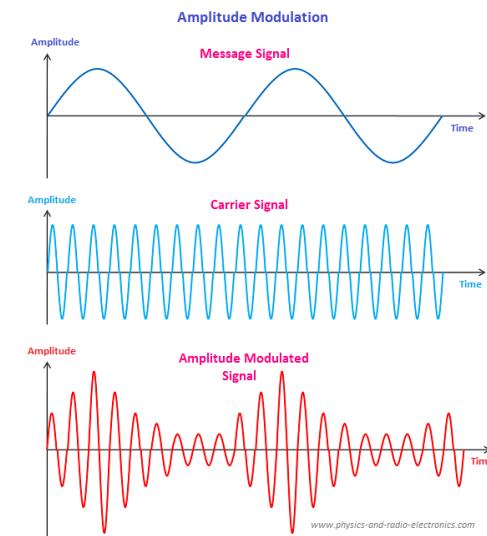
Circuite electrice in regim permanent nesinusoidal

Abaterea curbei de variație periodică în timp a unui curent sau a unei tensiuni de la forma sinusoidală se numește **deformare** sau **distorsiune**.

Cauze

Elemente neliniare, interferențe, etc.

Efecte



Circuite electrice in regim permanent nesinusoidal

Analiza circuitelor electrice în regim periodic nesinusoidal:

Descompunerea mărimilor în sume/serii de termeni sinusoidali

Funcție periodică

$$f(t) = f(t + m \cdot T), m \in \mathbb{Z}$$

Serie trigonometrică Fourier

Condiții Dirichlet

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$

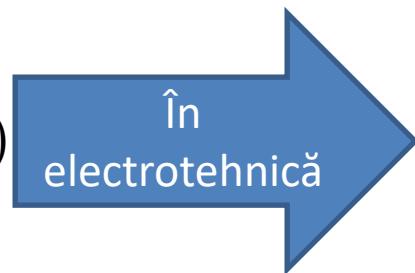
$$A_n = \frac{2}{T} \int_0^T f(t) \cos n\omega t dt$$

$$B_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t dt$$

$$\frac{A_0}{2} = \frac{1}{T} \int_0^T f(t) dt$$

Circuite electrice in regim permanent nesinusoidal

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$



$$f(t) = F_0 + \sum_{n=1}^{\infty} F_n \sqrt{2} \sin(n\omega t + \gamma_n)$$

Componenta de curent continuu

Armonici de ordinul n

$$F_n = \frac{\sqrt{A_n^2 + B_n^2}}{\sqrt{2}}$$

$$\gamma_n = \arctg \frac{B_n}{A_n}$$

Valoare efectivă

$$F = \sqrt{\frac{1}{T} \int_0^T f^2 dt} = \sqrt{F_0^2 + F_1^2 + F_2^2 + \dots + F_n^2 + \dots}$$

Circuite electrice in regim permanent nesinusoidal

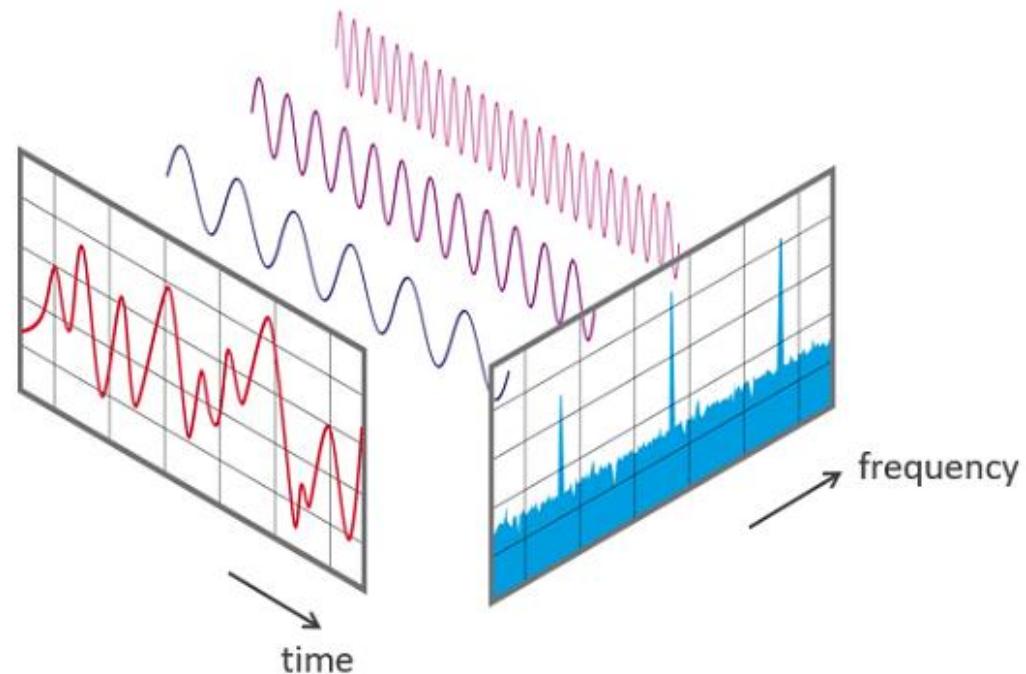
Coeficient de distorsiune

$$k_d = \frac{\sqrt{F_2^2 + F_3^2 + \dots}}{\sqrt{F_1^2 + F_2^2 + F_3^2 + \dots}}$$

$$u(t) = U_0 + \sum_{n=1}^{\infty} U_n \sqrt{2} \sin(n\omega t + \alpha_n)$$

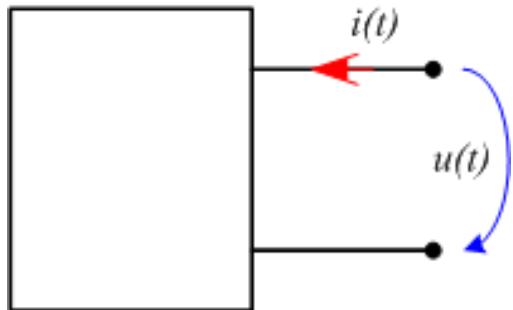
$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \sqrt{2} \sin(n\omega t + \beta_n)$$

$$f(t) = F_0 + \sum_{n=1}^{\infty} F_n \sqrt{2} \sin(n\omega t + \gamma_n)$$



Circuite electrice in regim permanent nesinusoidal

- puteri in regim nesinusoidal



$$u(t) = U_0 + \sum_{n=1}^{\infty} U_n \sqrt{2} \sin(n\omega t + \alpha_n)$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \sqrt{2} \sin(n\omega t + \beta_n)$$

Puterea activă

$$P = \frac{1}{T} \int_0^T u \cdot idt = U_0 I_0 + \sum_{n=1}^{\infty} U_n I_n \cos \varphi_n, \quad [W] \quad \varphi_n = \alpha_n - \beta_n$$

Puterea reactivă

$$Q = \sum_{n=1}^{\infty} U_n I_n \sin \varphi_n, \quad [\text{var}] \quad \varphi_n = \alpha_n - \beta_n$$

Puterea aparentă

$$S = U \cdot I = \sqrt{U_0^2 + U_1^2 + \dots} \cdot \sqrt{I_0^2 + I_1^2 + \dots}, \quad [\text{VA}]$$

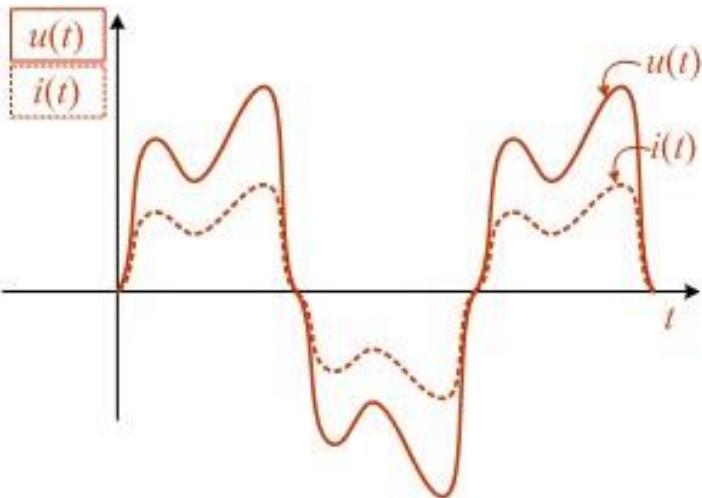
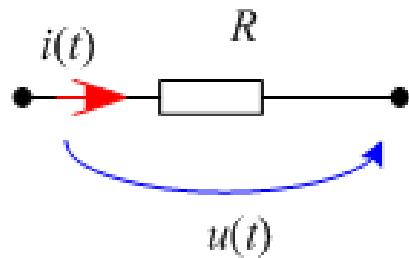
Puterea deformantă

$$D = \sqrt{S^2 - (P^2 + Q^2)}, \quad [\text{vad}]$$

Circuite electrice in regim permanent nesinusoidal

Elemente de circuit:

Rezistorul



$$u(t) = \sum_{n=1}^{\infty} U_n \sqrt{2} \sin(n\omega t + \alpha_n)$$



$$i(t) = \frac{u(t)}{R} = \sum_{n=1}^{\infty} \frac{U_n}{R} \sqrt{2} \sin(n\omega t + \alpha_n)$$

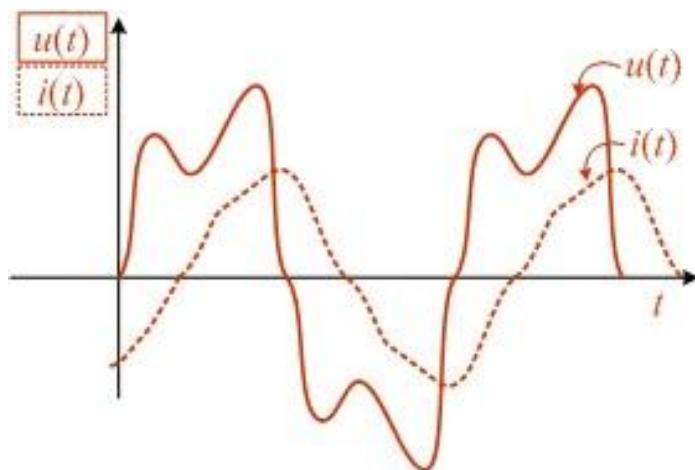
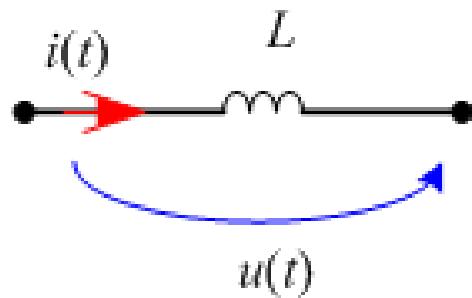


$$Z_n = \frac{U_n}{I_n} = R$$

Circuite electrice in regim permanent nesinusoidal

Elemente de circuit:

Bobina



$$u(t) = \sum_{n=1}^{\infty} U_n \sqrt{2} \sin(n\omega t + \alpha_n)$$



$$i(t) = \frac{1}{L} \int u(t) dt = \sum_{n=1}^{\infty} \frac{U_n}{n\omega L} \sqrt{2} \sin(n\omega t + \alpha_n - \frac{\pi}{2})$$

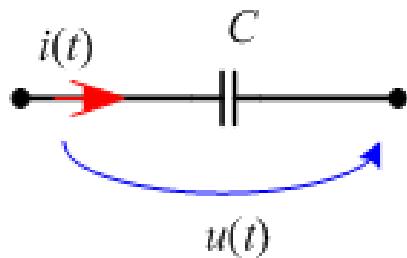


$$Z_n = \frac{U_n}{I_n} = n\omega L$$

Circuite electrice in regim permanent nesinusoidal

Elemente de circuit:

Condensatorul



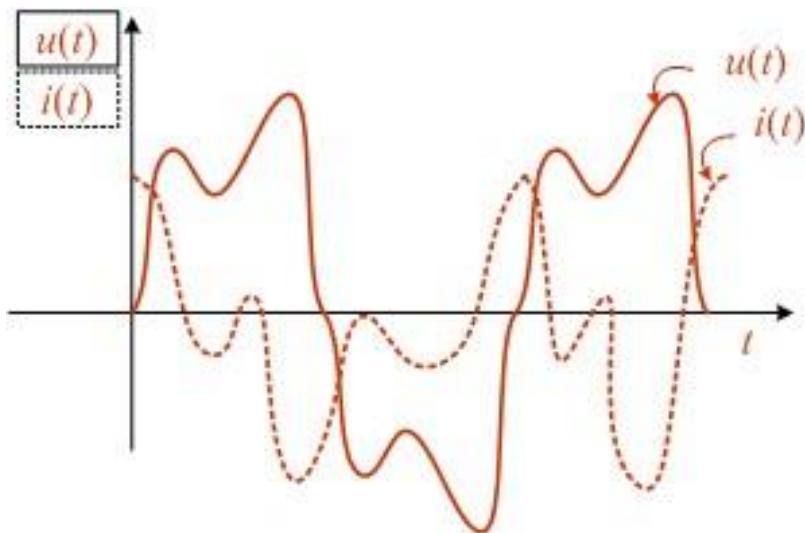
$$u(t) = \sum_{n=1}^{\infty} U_n \sqrt{2} \sin(n\omega t + \alpha_n)$$



$$i(t) = C \frac{du}{dt} = \sum_{n=1}^{\infty} n\omega C U_n \sqrt{2} \sin(n\omega t + \alpha_n + \frac{\pi}{2})$$



$$Z_n = \frac{U_n}{I_n} = \frac{1}{n\omega C}$$



Circuite electrice in regim permanent nesinusoidal

Exemplu:

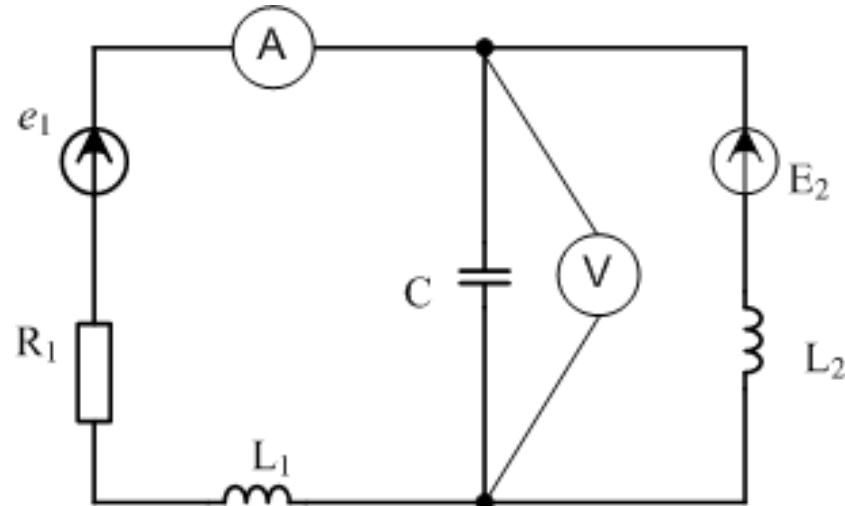
$$R_1 = 10\Omega$$

$$\omega L_1 = 2\Omega$$

$$\omega L_2 = \frac{1}{\omega C} = 6\Omega$$

$$e_1 = 20 + 30 \sin\left(\omega t - \frac{\pi}{4}\right) + 20 \sin\left(2\omega t + \frac{\pi}{4}\right) V$$

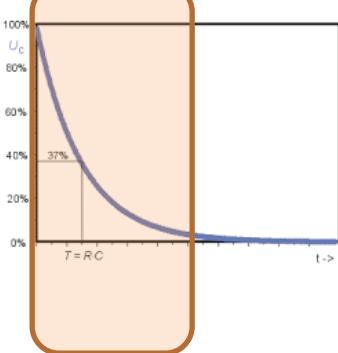
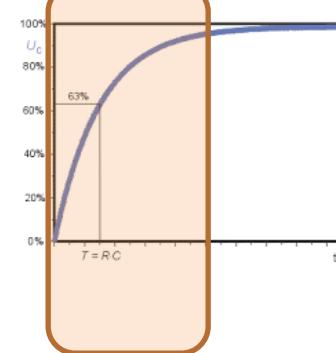
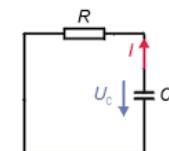
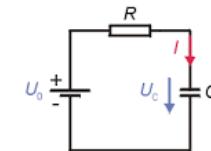
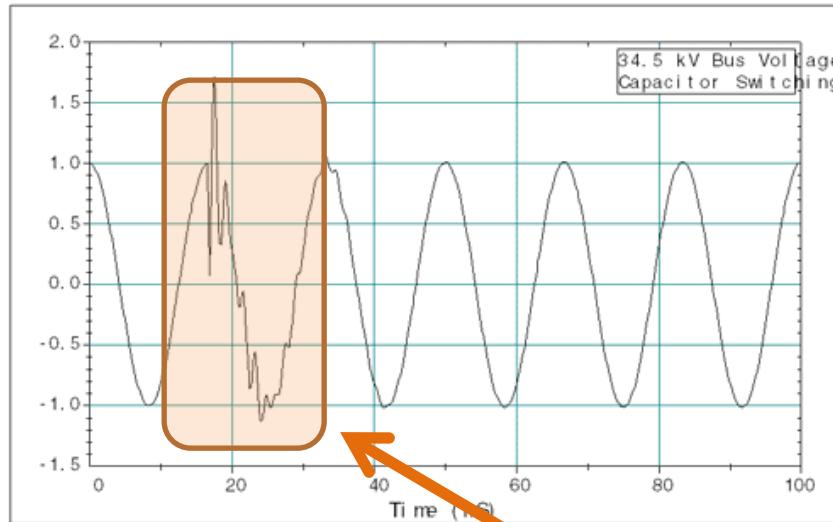
$$E_2 = 10V$$



?



Circuite electrice in regim tranzitoriu



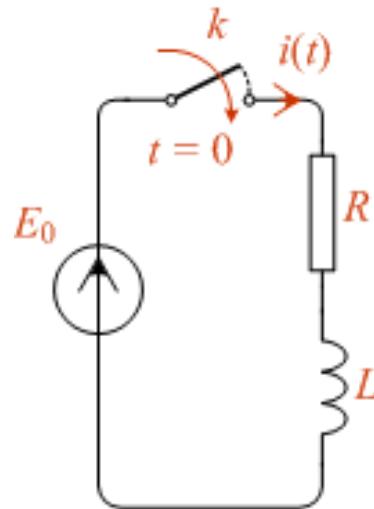
Regim
tranzitoriu

Circuite electrice in regim tranzitoriu

Metoda directă

$$L \frac{di}{dt} + Ri = E_0.$$

TKII



Solutia

$$i(t) = i_l(t) + i_f(t)$$

Solutia de
regim liber

$$L \frac{di}{dt} + Ri = 0. \quad rL + R = 0; \quad r = -\frac{R}{L}.$$

$$\left(\frac{d^n i}{dt^n} \rightarrow r^n; \frac{di}{dt} \rightarrow r^1 = r; i \rightarrow r^0 = 1 \right)$$

$$i_l(t) = K e^{rt} = K e^{-\frac{L}{R}t}$$

Solutia de
regim fortat

Circuite electrice in regim tranzitoriu

Metoda directa

$$L \frac{di}{dt} + Ri = E_0.$$

Solutia

$$i(t) = i_l(t) + i_f(t)$$

Solutia de
regim liber

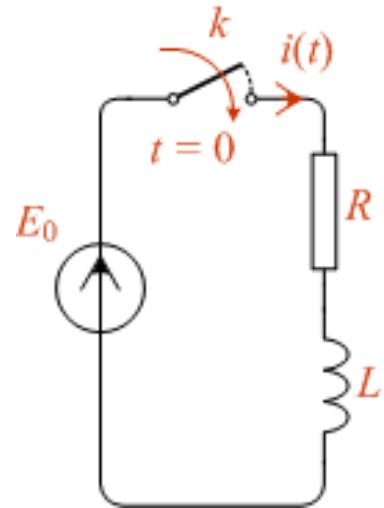
$$i_l(t) = Ke^{rt} = Ke^{-\frac{L}{R}t}$$

Solutia de
regim fortat

$$i_f(t) = I_0, \quad I_0 = \text{const.}$$

$$L \frac{di_f}{dt} + Ri_f = E_0,$$

$$RI_0 = E_0; \quad I_0 = \frac{E_0}{R}.$$



Circuite electrice in regim tranzitoriu

Metoda directa

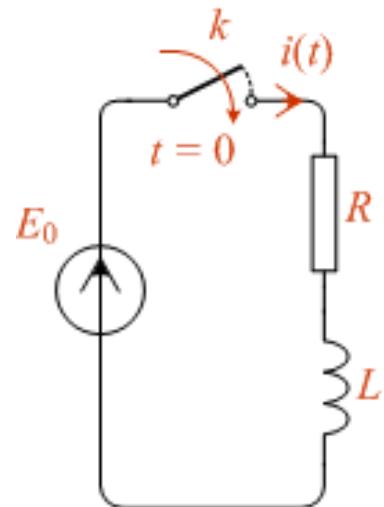
$$L \frac{di}{dt} + Ri = E_0.$$

Solutia

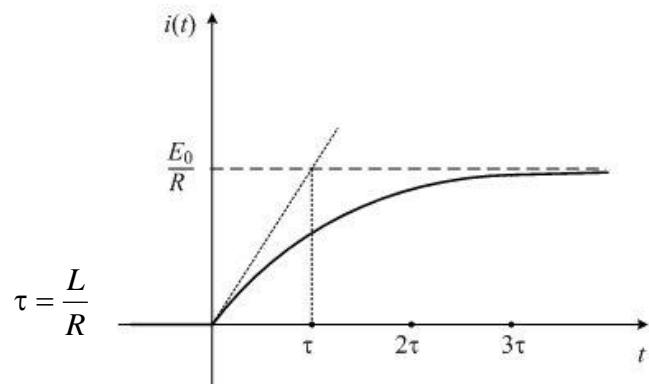
$$i(t) = Ke^{-\frac{R}{L}t} + \frac{E_0}{R}$$

$$i(0+) = K + \frac{E_0}{R} = 0,$$

$$K = -\frac{E_0}{R}$$



$$i(t) = \frac{E_0}{R} \left(1 - e^{-\frac{R}{L}t} \right)$$



Circuite electrice in regim tranzitoriu

**Metoda operatională
(Transformata Laplace)**

$$L[f(t)] = F(s) = \int_0^{\infty} f(t) \cdot e^{-st} \cdot dt$$

$$s = \sigma + j\omega$$

$$f(t) = C,$$

$$L[C] = \int_0^{\infty} C \cdot e^{-st} \cdot dt = -\frac{C}{s} e^{-st} \Big|_0^{\infty} = \frac{C}{s}.$$

$$f(t) = e^{\alpha t},$$

$$\begin{aligned} L[e^{\alpha t}] &= \int_0^{\infty} e^{\alpha t} \cdot e^{-st} \cdot dt = \frac{1}{\alpha - s} e^{(\alpha - s)t} \Big|_0^{\infty} = \\ &= \frac{1}{s - \alpha} \end{aligned}$$

$$L[\sin \omega t] = \int_0^{\infty} \sin \omega t \cdot e^{-st} \cdot dt =$$

$$= \int_0^{\infty} \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \cdot e^{-st} \cdot dt = \frac{1}{2j} \left(\frac{1}{s - j\omega} - \frac{1}{s + j\omega} \right) =$$

$$= \frac{\omega}{s^2 + \omega^2}.$$

Circuite electrice in regim tranzitoriu

**Metoda operatională
(Transformata Laplace)**

$$L[f(t)] = F(s) = \int_0^{\infty} f(t) \cdot e^{-st} \cdot dt$$

$$\begin{aligned} L[f_1 + f_2] &= \\ &= \int_0^{\infty} (f_1 + f_2) \cdot e^{-st} \cdot dt = \int_0^{\infty} f_1 \cdot e^{-st} \cdot dt + \\ &\quad + \int_0^{\infty} f_2 \cdot e^{-st} \cdot dt = L[f_1] + L[f_2]. \end{aligned}$$

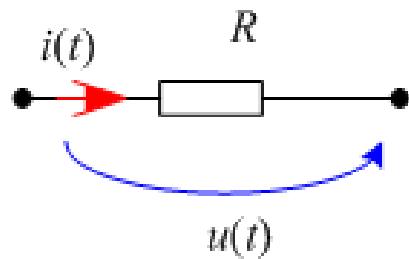
$$L\left[\frac{df}{dt}\right] = sL[f] - f(0)$$

$$L\left[\sum_{k=1}^n \lambda_k f_k(t)\right] = \sum_{k=1}^n \lambda_k L[f_k(t)]$$

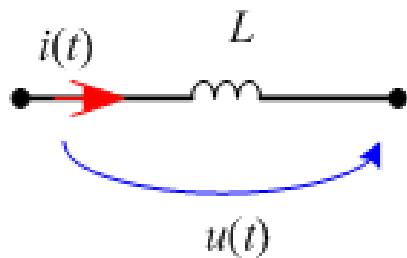
$$L\left[\int_0^t f(\tau) d\tau\right] = \frac{L[f]}{s}$$

Circuite electrice in regim tranzitoriu

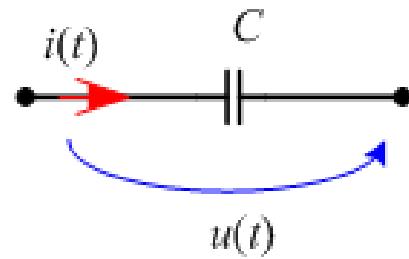
**Metoda operatională
(Transformata Laplace)**



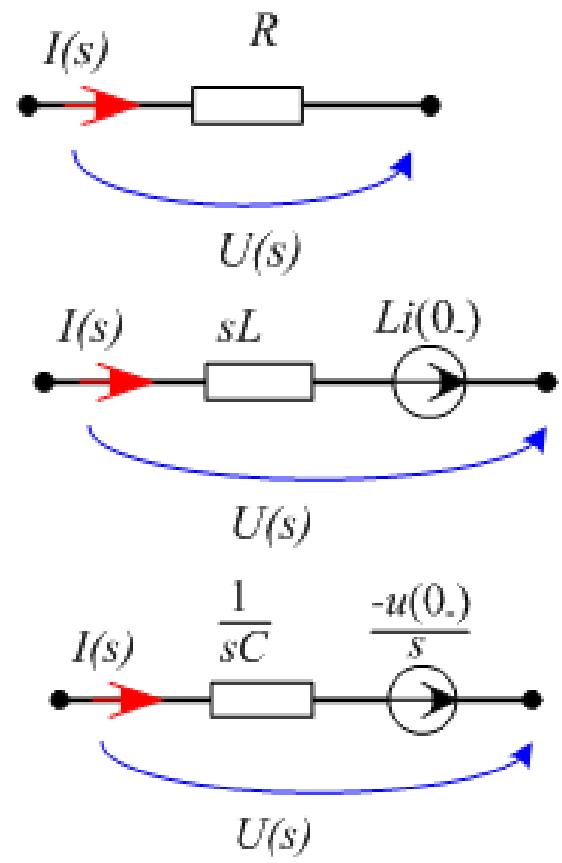
\mathcal{L}



\mathcal{L}



\mathcal{L}



Circuite electrice in regim tranzitoriu

**Metoda operatională
(Transformata Laplace)**

$$I(s) = \frac{P_1(s)}{P_2(s)}$$



$$P_2(s_k) = 0$$

$$i(t) = \sum_{k=1}^n \frac{P_1(s_k)}{P_2'(s_k)} \cdot e^{s_k t} = L^{-1} \left[\frac{P_1(s)}{P_2(s)} \right]$$

$$I(s) = \frac{P_1(s)}{s \cdot P_3(s)}$$



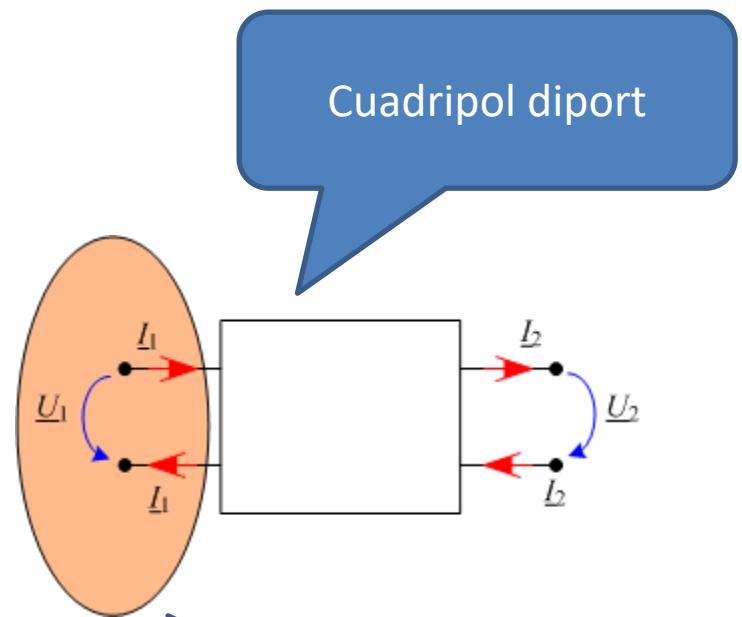
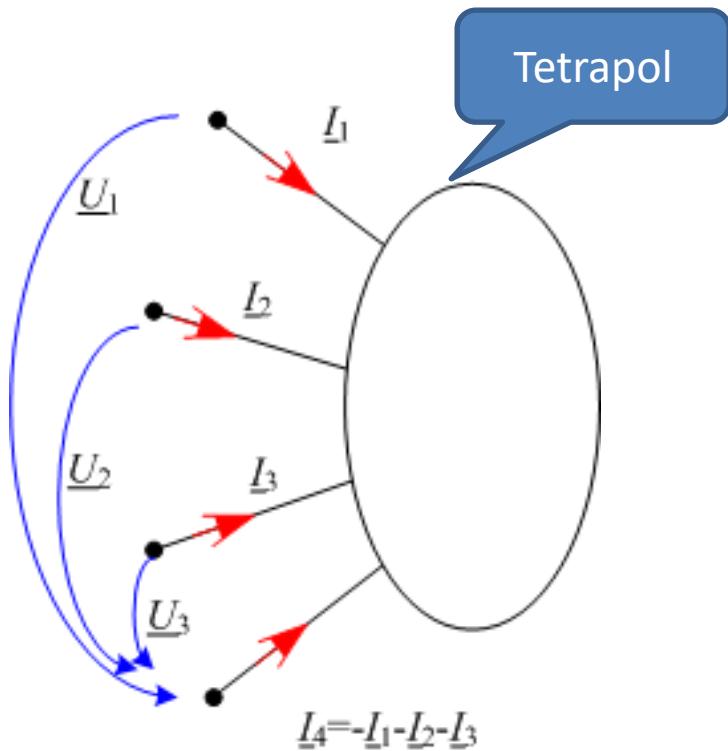
$$\begin{aligned} i(t) &= \sum_{k=1}^n \frac{P_1(s_k)}{P_3'(s_k)} \cdot e^{s_k t} = \frac{P_1(0)}{P_3(0)} + \sum_{k=1}^{n-1} \frac{P_1(s_k)}{s_k P_3'(s_k)} \cdot e^{s_k t} = \\ &= L^{-1} \left[\frac{P_1(s)}{s P_3(s)} \right] \end{aligned}$$

$$P_3(s_k) = 0$$

BAZELE ELECTROTEHNICII

- note de curs -

Cuadripoli electrici



Poarta: o grupare de borne de acces pentru care suma algebraica a curentilor este nula

- 1 – poarta de intrare
- 2 – poarta de iesire

Cuadripoli electrici

Parametri impedanță



$$\begin{cases} \underline{U}_1 = \underline{Z}_{11}\underline{I}_1 + \underline{Z}_{12}\underline{I}_2 \\ \underline{U}_2 = \underline{Z}_{21}\underline{I}_1 + \underline{Z}_{22}\underline{I}_2 \end{cases}$$

Parametri admitanță

$$\begin{cases} \underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 \\ \underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 \end{cases}$$

Parametri fundamentali

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$

Cuadripoli electrici

Parametri fundamentali

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$

$$\underline{A} = \left(\frac{\underline{U}_1}{\underline{U}_2} \right)_{\underline{I}_2=0}$$

$$\underline{B} = \left(\frac{\underline{U}_1}{\underline{I}_2} \right)_{\underline{U}_2=0}$$

$$\underline{C} = \left(\frac{\underline{I}_1}{\underline{U}_2} \right)_{\underline{I}_2=0}$$

$$\underline{D} = \left(\frac{\underline{I}_1}{\underline{I}_2} \right)_{\underline{U}_2=0}$$

Relatie de reciprocitate

$$\underline{A} \cdot \underline{D} - \underline{B} \cdot \underline{C} = 1$$



Factor de transfer
(amplificare) in tensiune

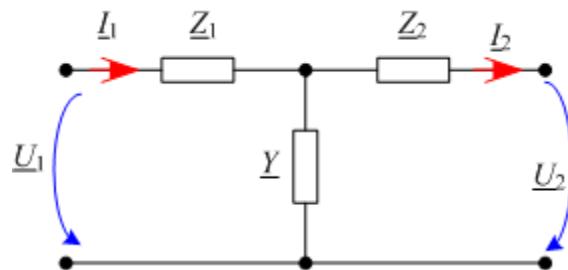
Impedanta de transfer

Admitanta de transfer

Factor de transfer
(amplificare) in curent

Cuadripoli electrici

Schema in T



$$\underline{I}_1 = \underline{I}_2 + (\underline{U}_2 + \underline{Z}_2 \underline{I}_2) \underline{Y} = \underline{Y} \underline{U}_2 + (1 + \underline{Z}_2 \underline{Y}) \underline{I}_2$$

$$\underline{U}_1 = \underline{Z}_1 \underline{I}_1 + \underline{Z}_2 \underline{I}_2 + \underline{U}_2 = (1 + \underline{Z}_1 \underline{Y}) \underline{U}_2 + (\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_1 \underline{Z}_2 \underline{Y}) \underline{I}_2$$

$$\underline{A} = 1 + \underline{Z}_1 \underline{Y}$$

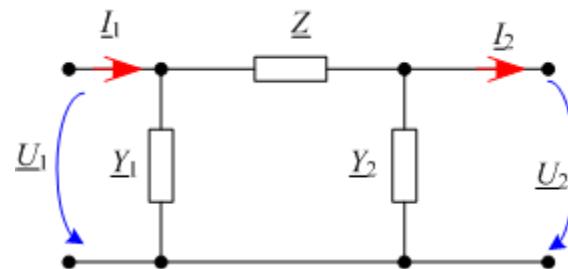
$$\underline{B} = \underline{Z}_1 + \underline{Z}_2 + \underline{Y} \underline{Z}_1 \underline{Z}_2$$

$$\underline{C} = \underline{Y}$$

$$\underline{D} = 1 + \underline{Z}_2 \underline{Y}$$

A=D
Cuadripol simetric

Schema in Π



$$\underline{A} = 1 + \underline{Y}_2 \underline{Z}$$

$$\underline{B} = \underline{Z}$$

$$\underline{C} = \underline{Y}_1 + \underline{Y}_2 + \underline{Z} \underline{Y}_1 \underline{Y}_2$$

$$\underline{D} = 1 + \underline{Y}_1 \underline{Z}$$

Cuadripoli electrici

Functionare mers in gol

$$\underline{Z}_s = \infty$$



$$\begin{cases} \underline{U}_{10} = \underline{A}\underline{U}_2 \\ \underline{I}_{10} = \underline{C}\underline{U}_2 \end{cases}$$

Functionare in scurtcircuit

$$\underline{Z}_s = 0$$



$$\begin{cases} \underline{U}_{1sc} = \underline{B}\underline{I}_2 \\ \underline{I}_{1sc} = \underline{D}\underline{I}_2 \end{cases}$$

$$\begin{cases} \underline{U}_1 = \underline{U}_{10} + \underline{U}_{1sc} \\ \underline{I}_1 = \underline{I}_{10} + \underline{I}_{1sc} \end{cases}$$

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$



Cuadripoli electrici

Impedanță caracteristică

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 = \underline{U}_2(A + \sqrt{BC}) \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 = \underline{I}_2(A + \sqrt{BC}) \end{cases}$$

$$Z_C = \frac{\underline{U}_2}{\underline{I}_2} = \frac{\underline{U}_1}{\underline{I}_1}$$

Impedanță care conectată la poarta de ieșire se regăsește la poarta de intrare

Pt un cuadripol simetric

$$Z_C = \sqrt{\underline{B}/\underline{C}}$$

Constantă de propagare

$$\gamma = \ln \frac{\underline{U}_1}{\underline{U}_2} = \ln(A + \sqrt{BC})$$

$$\gamma = \alpha + j \cdot \beta$$

Constantă de atenuare

Constantă de fază

Cuadripoli electrici – filtre electrice

Filtrele electrice – cuadripoli care transmit semnale (tensiuni și curenți) ale căror frecvențe sunt cuprinse în anumite intervale.

Frecvențele care se transmit fără atenuare alcătuiesc benzile de trecere (BT). Frecvențele atenuate puternic alcătuiesc benzile de oprire (BO).

Filtru trece jos

$$BT \in [0, f_1]$$

$$BO \in (f_1, \infty)$$

Filtru trece bandă

$$BT \in [f_1, f_2]$$

$$BO \in (0, f_1) \cup (f_2, \infty)$$

Filtru oprește bandă

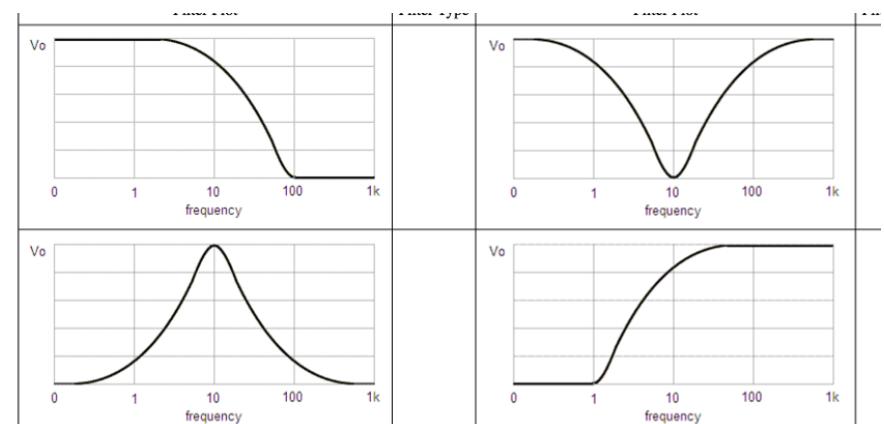
$$BO \in [f_1, f_2]$$

$$BT \in (0, f_1) \cup (f_2, \infty)$$

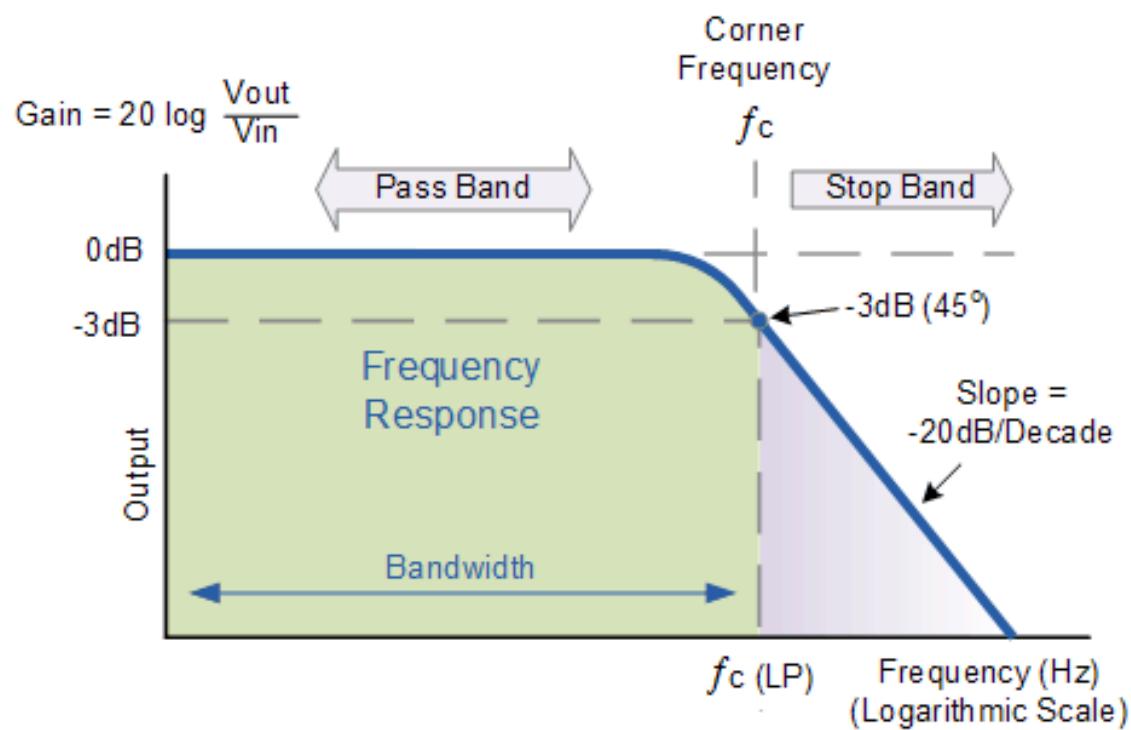
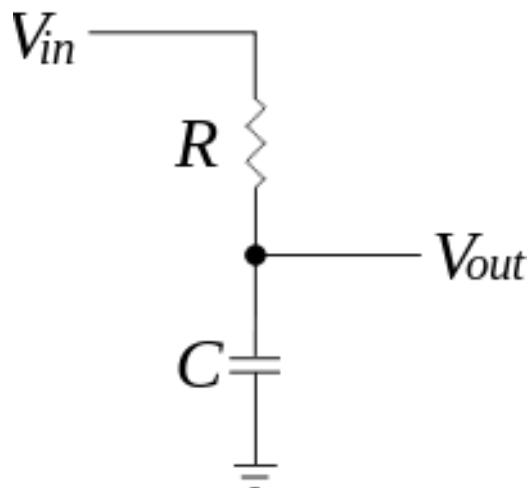
Filtru trece sus

$$BO \in [0, f_1]$$

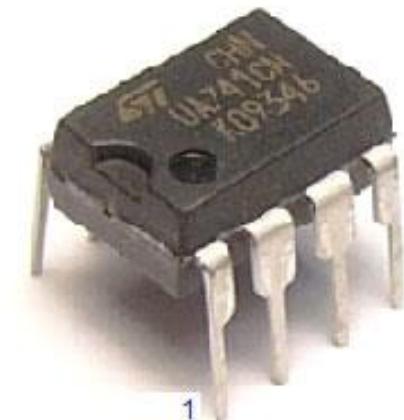
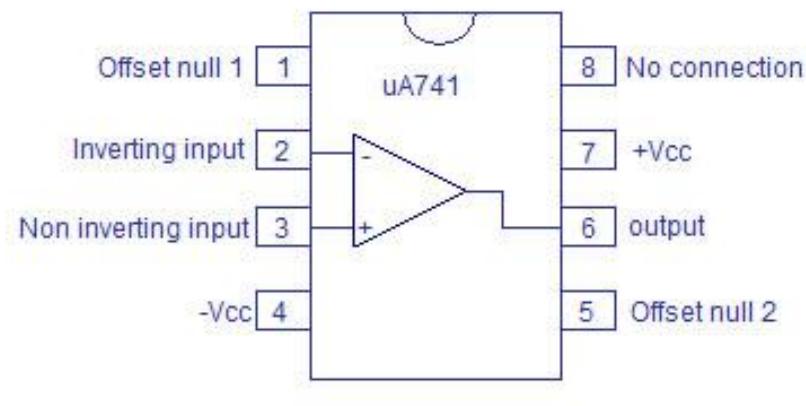
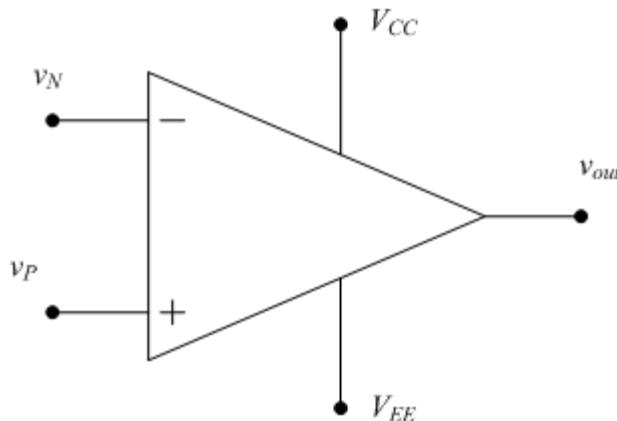
$$BT \in (f_1, \infty)$$



Cuadripoli electrici – filtre electrice



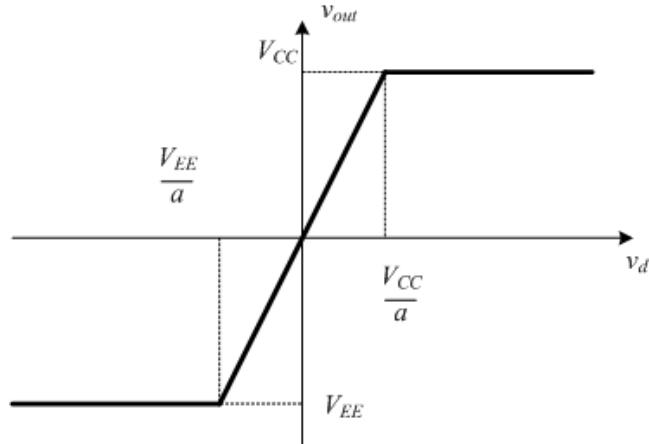
Amplificatorul operational (AO)



uA741 opamp Pinout and External appearance

Amplificatorul operational (AO)

$$v_{out} = a \cdot v_d \quad \text{daca } V_{EE} < a \cdot v_d < V_{CC}$$

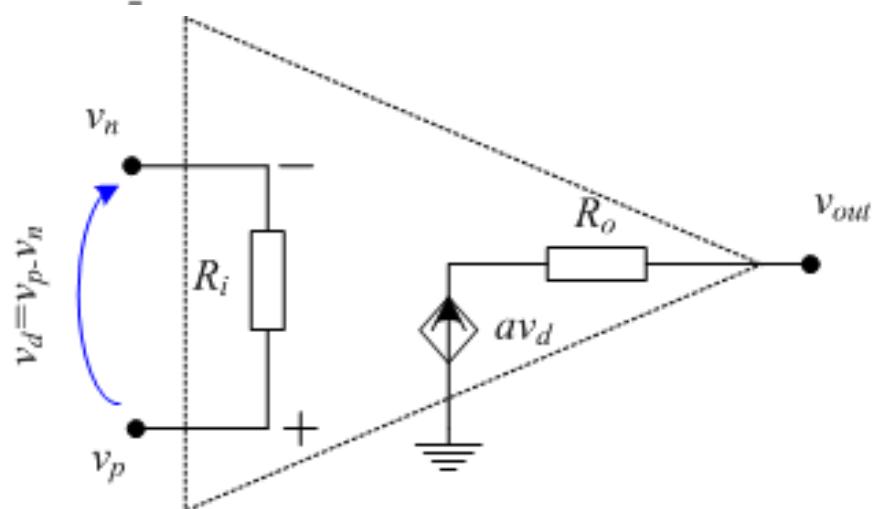
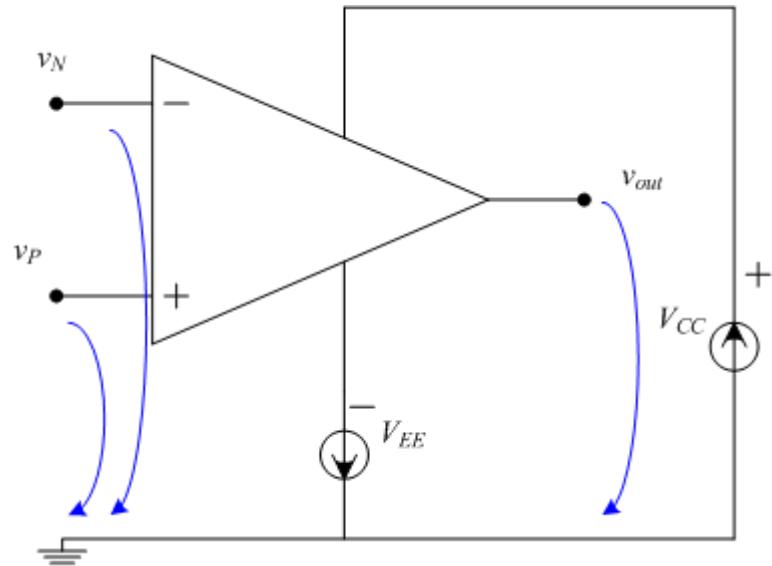


$$(R_i = 2M\Omega, R_o = 75\Omega, a = 200.000V/V, v_d \sim \mu V)$$

AO ideal

$$(R_i \rightarrow \infty, R_o = 0, a \rightarrow \infty, v_d \rightarrow 0)$$

$$(v_p = v_n = 0, i_p = i_n = 0)$$



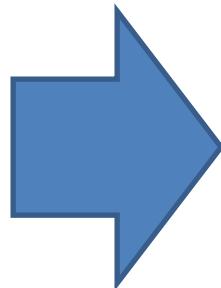
Amplificatorul operational (AO)

Inversor

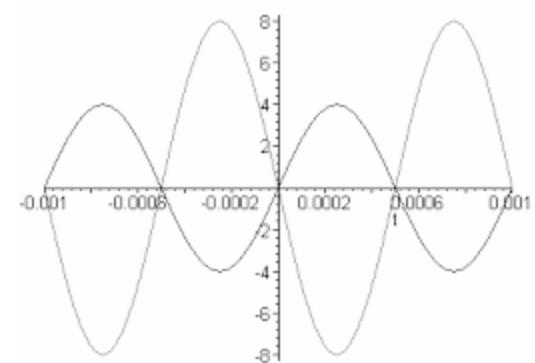
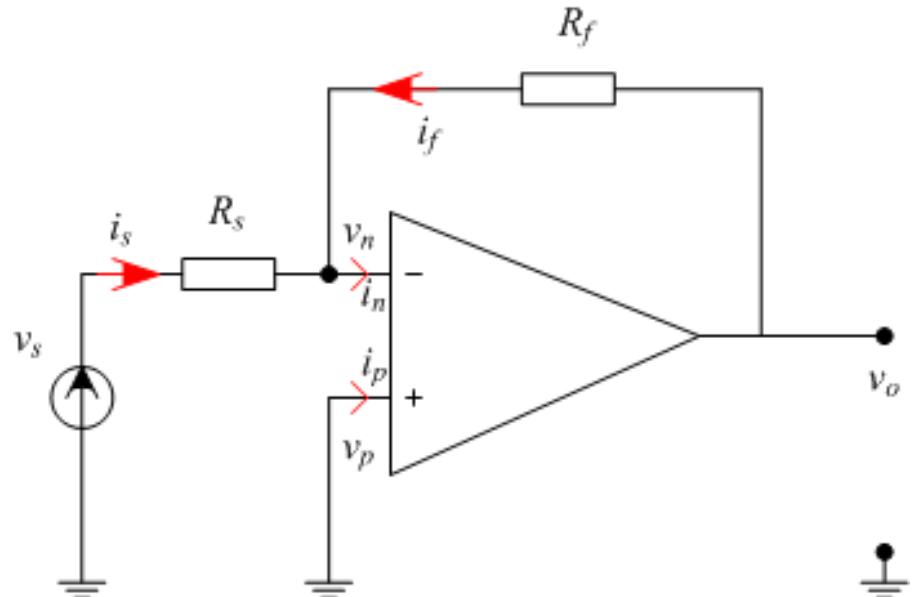
$$\begin{cases} i_s + i_f = i_n \\ v_p = 0 \Rightarrow v_n = 0 \end{cases}$$



$$\begin{cases} i_s = \frac{v_s}{R_s} \\ i_f = \frac{v_o}{R_f} \\ i_n = 0 \end{cases}$$

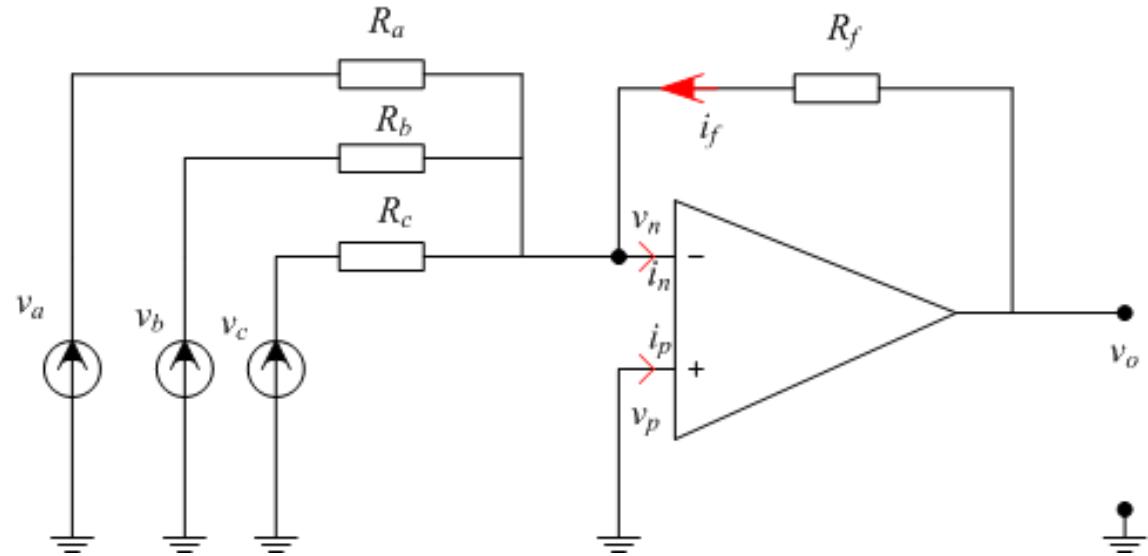


$$v_0 = -\frac{R_f}{R_s} v_s$$



Amplificatorul operational (AO)

Sumator



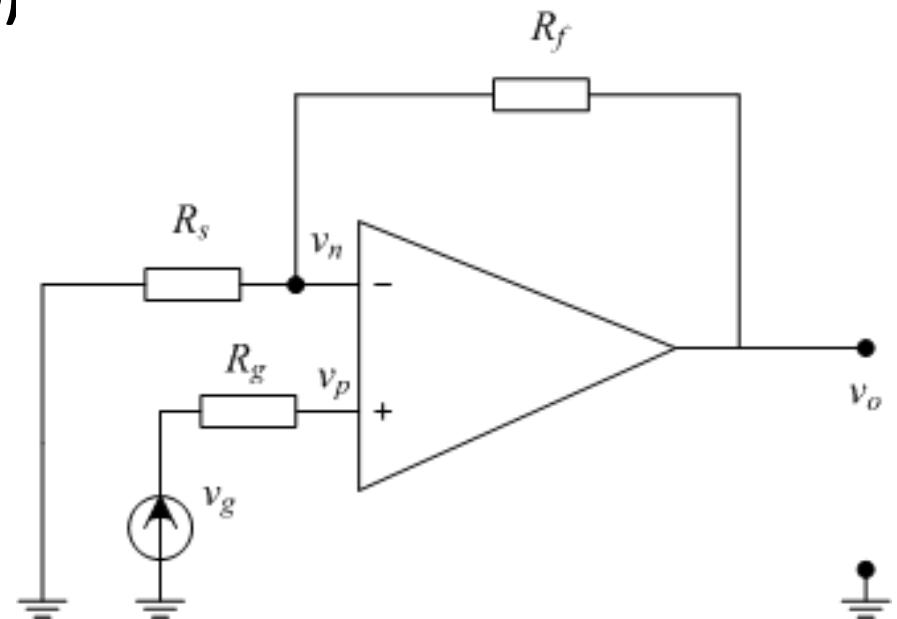
$$\frac{v_a - v_n}{R_a} + \frac{v_b - v_n}{R_b} + \frac{v_c - v_n}{R_c} + \frac{v_o - v_n}{R_f} = 0$$



$$v_o = -\left(\frac{R_f}{R_a} v_a + \frac{R_f}{R_b} v_b + \frac{R_f}{R_c} v_c \right)$$

Amplificatorul operational (AO)

Neinvensor



$$\begin{cases} i_p = 0 \Rightarrow v_p = v_g \\ v_n = v_o \frac{R_s}{R_s + R_f} \\ v_n = v_p \end{cases}$$



$$v_o = \frac{R_s + R_f}{R_s} v_g$$

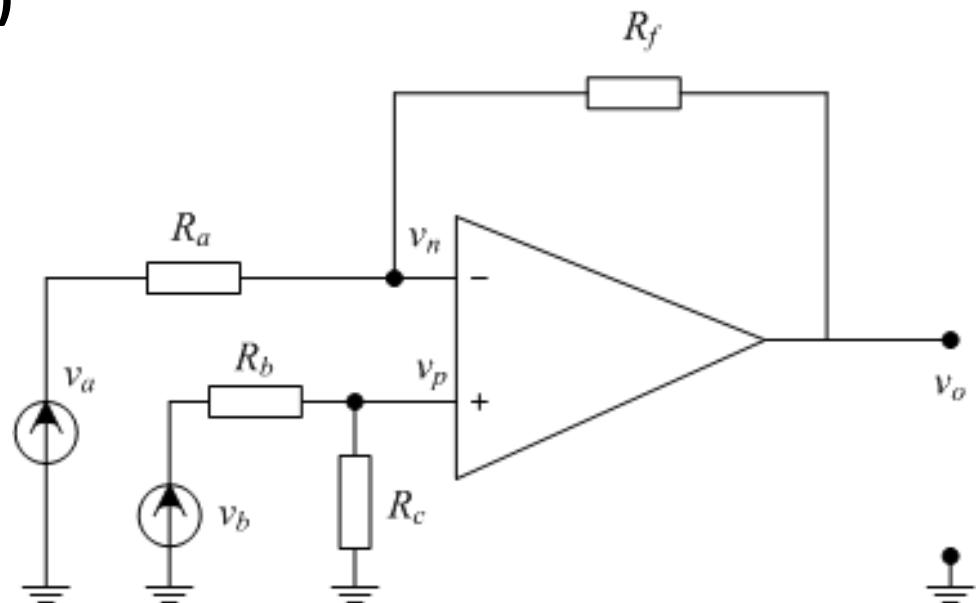
Amplificatorul operational (AO)

Diferentiator

$$\begin{cases} i_n = 0 \Rightarrow \frac{v_a - v_n}{R_a} + \frac{v_o - v_n}{R_f} = 0 \\ i_p = 0 \Rightarrow v_p = v_b \frac{R_c}{R_b + R_c} \\ v_n = v_p \end{cases}$$



$$v_o = \frac{R_c}{(R_b + R_c)} \cdot \frac{(R_a + R_f)}{R_a} v_b - \frac{R_f}{R_a} v_a$$

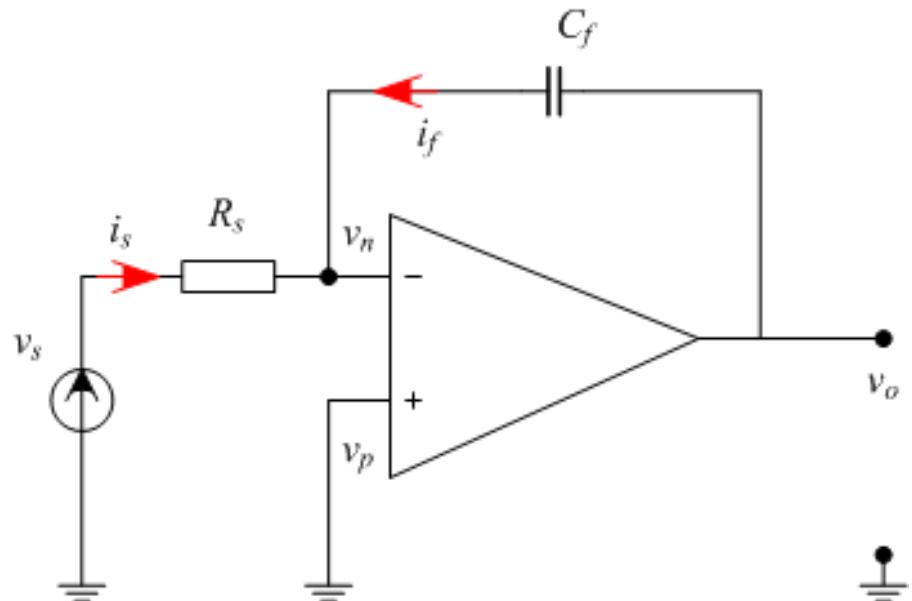


$$\frac{R_a}{R_f} = \frac{R_b}{R_c}$$

$$v_o = \frac{R_f}{R_a} (v_b - v_a)$$

Amplificatorul operational (AO)

Integrator



$$\begin{cases} i_f + i_s = 0 \\ v_n = v_p = 0 \\ i_s = \frac{v_s}{R_s} \\ i_f = C_f \frac{dv_o}{dt} \end{cases}$$

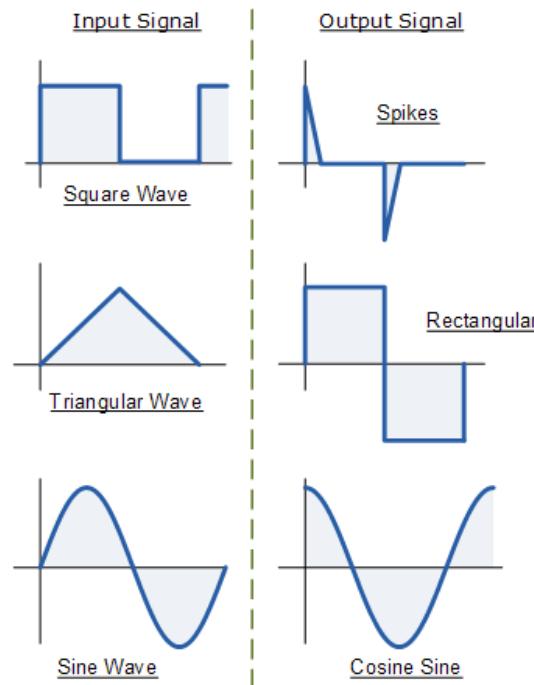
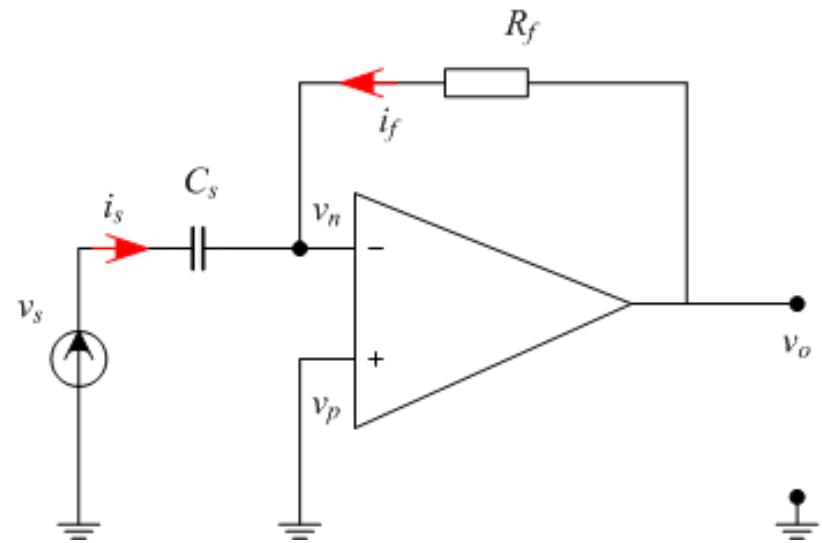


$$\frac{dv_o}{dt} = -\frac{1}{R_s C} v_s \Rightarrow v_o(t) = -\frac{1}{R_s C} \int_{t_0}^t v_s d\tau + v_0(t_0)$$

Amplificatorul operational (AO)

Derivator

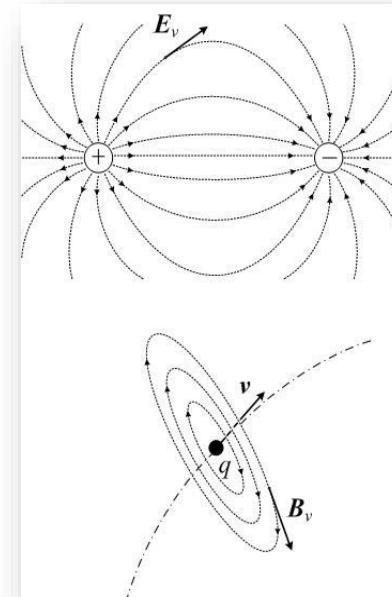
$$\left\{ \begin{array}{l} i_f + i_s = 0 \\ i_f = \frac{v_0}{R_f} \\ i_s = C_s \frac{dv_s}{dt} \end{array} \right. \quad \Rightarrow \quad v_0 = -R_f C_s \frac{dv_s}{dt}$$



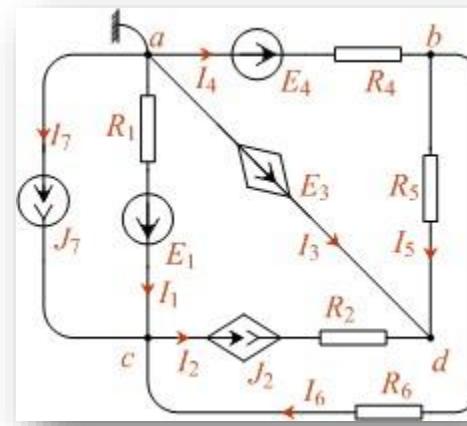
BAZELE ELECTROTEHNICII

- note de curs -

- Teoria câmpului electromagnetic

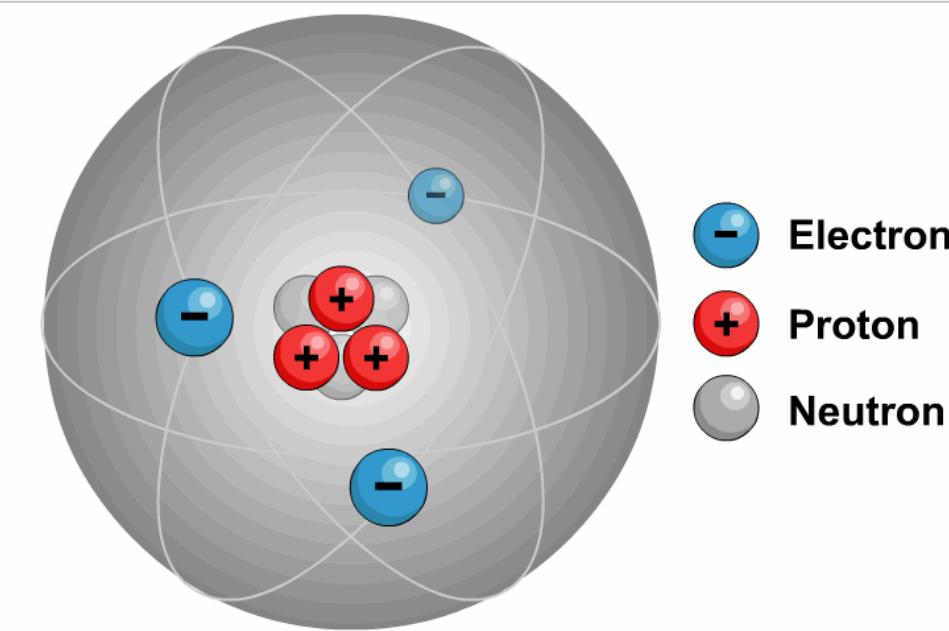


- Teoria circuitelor electrice



I. Introducere

- Sarcina electrică



Electrotehnica – este o disciplină a științelor tehnice care studiază **fenomenele electrice și magnetice**, obiectul ei fiind **analiza și modelarea matematică** a acestor fenomene, în vederea aplicării lor în **tehnică**.

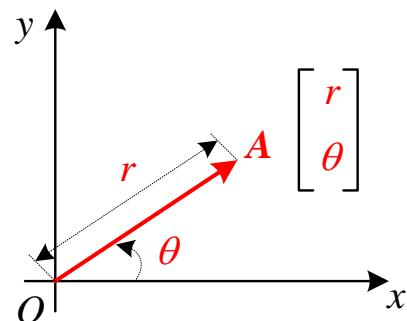
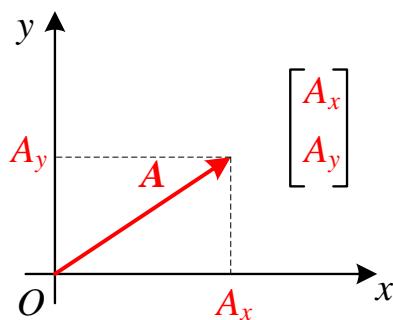


Se numește **mărime** o proprietate comună pentru o clasă de obiecte, stări, procese etc. Mărimea ce se poate evalua experimental, prin **procedeul de măsurare**, astfel încât să i se poată asocia (ataşa) o descriere numerică – o valoare numerică – se numește **mărime fizică**.



După tipul simbolului matematic folosit pentru descrierea numerică a mărimii fizice, se pot identifica:

- *mărimi scalare*
- *mărimi vectoriale*
- *mărimi tensoriale*



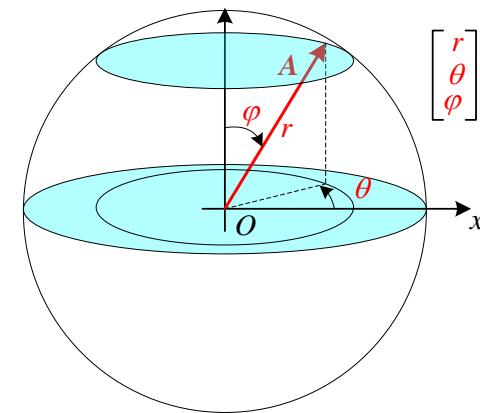
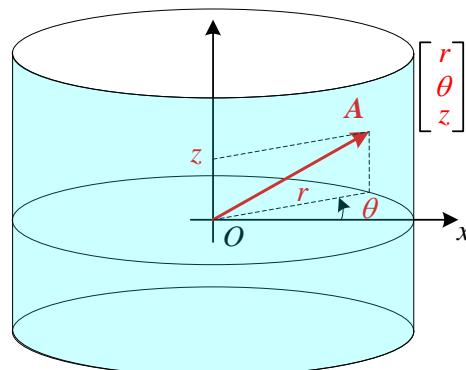
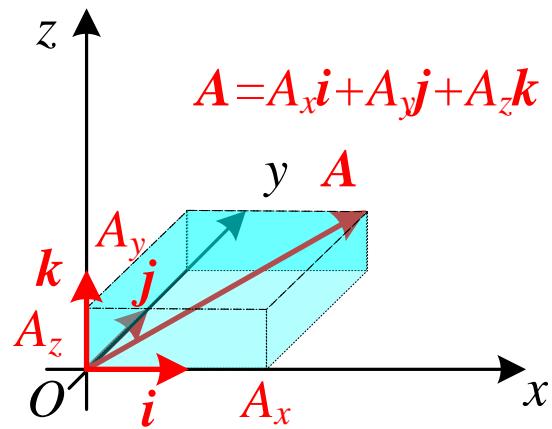
- ***mărime scalară*** – complet determinată de valoarea sa numerică și unitatea de măsură.



- ***mărime vectorială*** – complet determinată de valoarea numerică, direcție și sens



• **mărime vectorială** – complet determinată de valoarea numerică, direcție și sens



λ – numar real

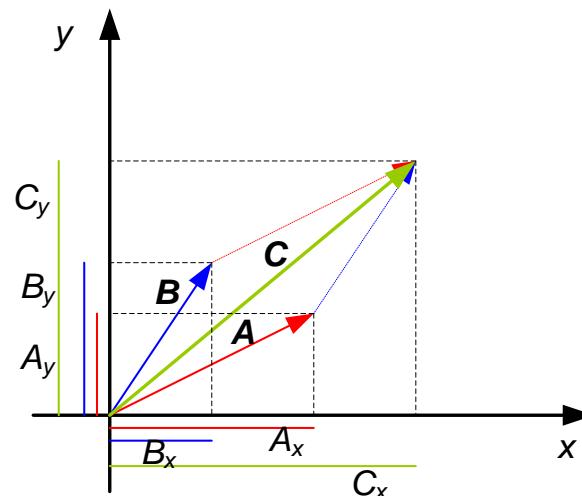
$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

$$\lambda \mathbf{A} = \lambda A_x \mathbf{i} + \lambda A_y \mathbf{j} + \lambda A_z \mathbf{k}$$

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B} = (A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} + (A_z + B_z) \mathbf{k}$$



• **mărime vectorială** – complet determinată de valoarea numerică, direcție și sens

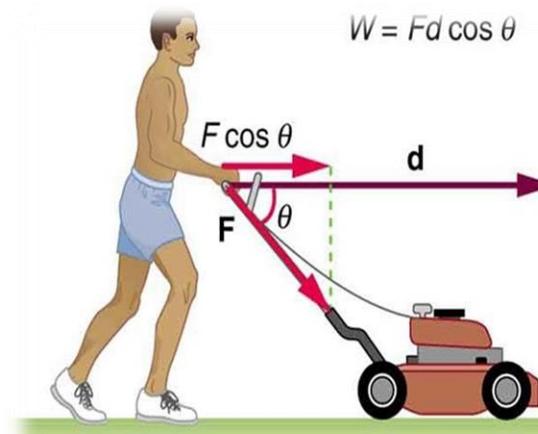
$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

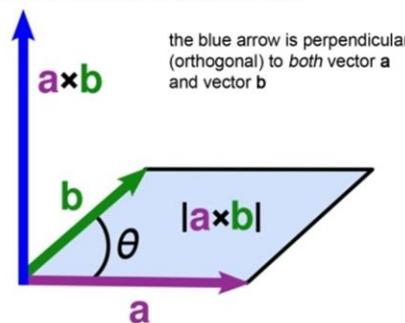
$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z \rightarrow \text{un scalar}$$

sau

$$\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}| \cdot |\mathbf{B}| \cdot \cos \alpha$$



blue arrow is the resultant vector, with scalar value a times b times sine(θ), which is the area of the parallelogram in the plane of \mathbf{a} and \mathbf{b}



Cross-product of two vectors \mathbf{a} and \mathbf{b} separated by angle θ . Vertical vector is the cross-product value & direction

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \rightarrow \text{un vector}$$

sau

$$|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}| \cdot |\mathbf{B}| \cdot \sin \theta$$

- **mărime tensorială** – descrisă printr-o valoare de tip matrice de numere.

$$\bar{A} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$$

După **modul de definire** a mărimii fizice, se pot identifica:

- mărimi **primitive** – definite prin descrierea procedeului de măsurare;
- mărimi **derivate** – definite cu ajutorul expresiilor, în funcție de alte mărimi.

După **momentul de timp**, se pot identifica:

- mărimi **instantanee** – sunt asociate unui moment de timp $u(t)$;
- mărimi **de proces** – sunt asociate unui interval de timp (*energ*).

După **spațiul unde sunt definite**, se pot identifica:

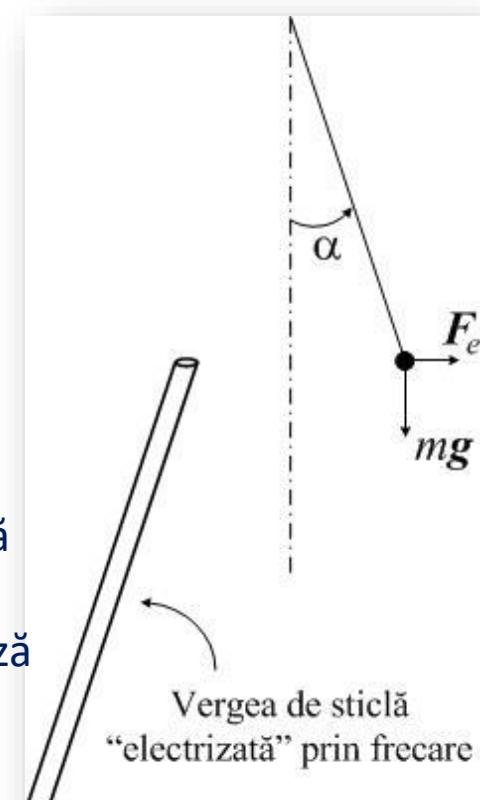
- mărimi **locale** – asociate unui singur punct din spațiu (temperatura);
- mărimi **globale** – asociate unui număr infinit de puncte (definite pe curbe, suprafețe).

STARILE ELECTROMAGNETICE ALE CORPURILOR. CAMPUL ELECTROMAGNETIC

1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

$$\mathbf{F}_e = q\mathbf{E}_v$$

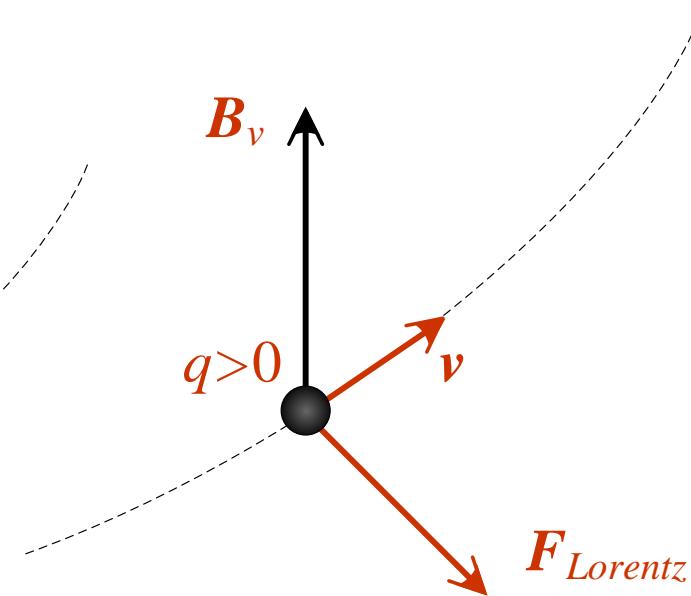
În această relație empirică mărimea q este sarcina electrică și caracterizează starea de încărcare electrică a corpului de probă [C], iar \mathbf{E}_v , vectorul câmp electric în vid, caracterizează starea locală a câmpului electric



STARILE ELECTROMAGNETICE ALE CORPURILOR. CAMPUL ELECTROMAGNETIC

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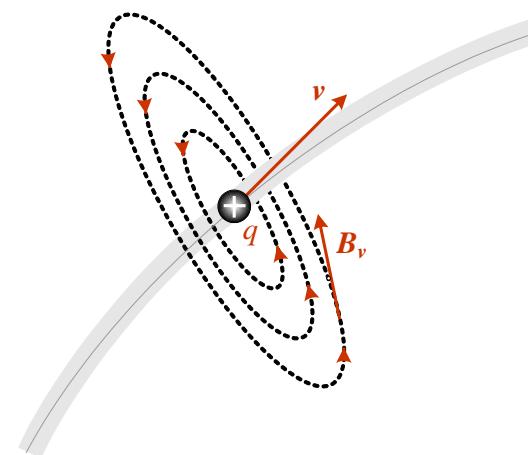
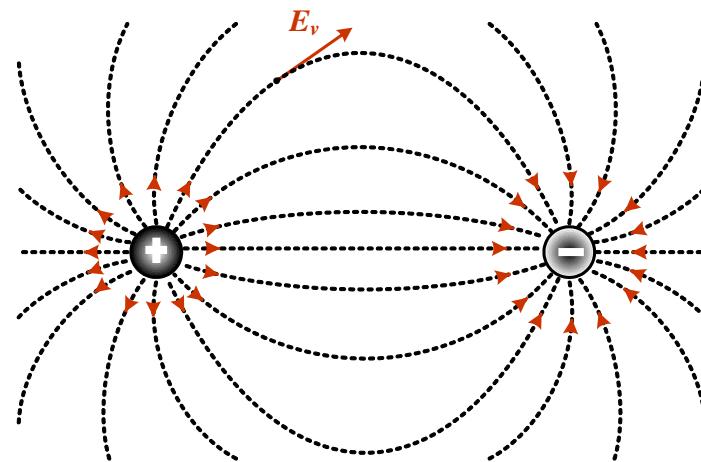
$$\mathbf{F}_m = q\mathbf{v} \times \mathbf{B}_v$$



Mărimea vectorială \mathbf{B}_v (mai precis, $\mathbf{B}_{vid}(r, t)$) este o mărime de stare locală care caracterizează câmpul magnetic în vid [T].

1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

Linii de camp



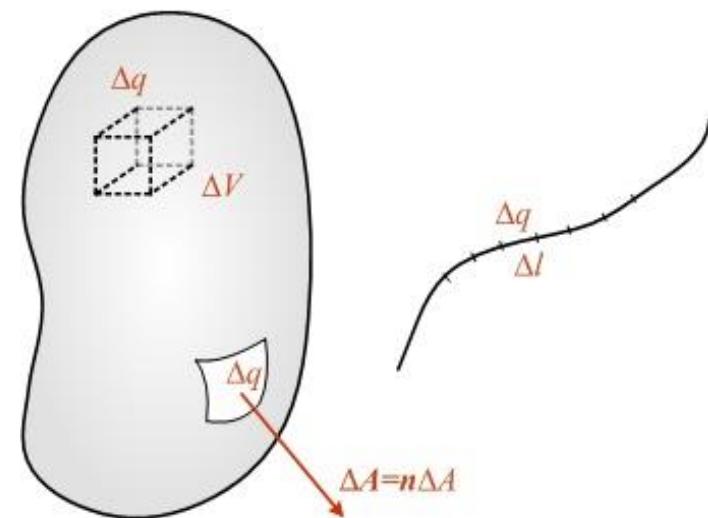
1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

Densitati de sarcina electrica

$$\rho_v = \lim_{\Delta V \rightarrow 0} \frac{\Delta q}{\Delta V} = \frac{d q}{d V} \quad [C/m^3]$$

$$\rho_s = \lim_{\Delta A \rightarrow 0} \frac{\Delta q}{\Delta A} = \frac{d q}{d A} \quad [C/m^2]$$

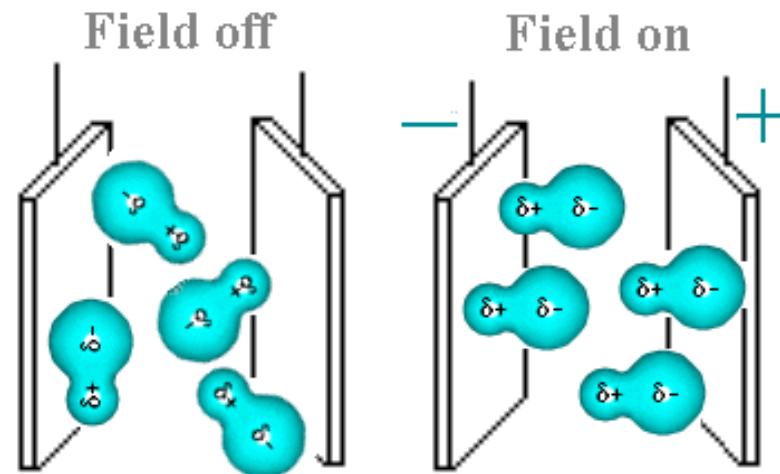
$$\rho_l = \lim_{\Delta l \rightarrow 0} \frac{\Delta q}{\Delta l} = \frac{d q}{d l} \quad [C/m]$$



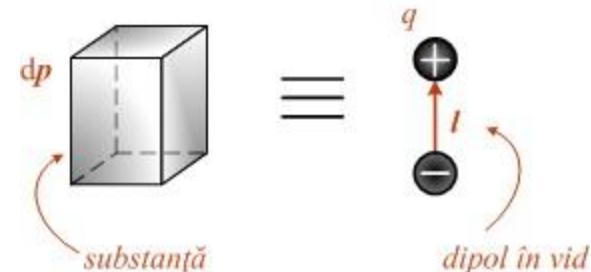
$$q = \iiint_V \rho_v dV + \iint_S \rho_s dS + \int_L \rho_l dl + \sum_{i=1}^n q_i$$

1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

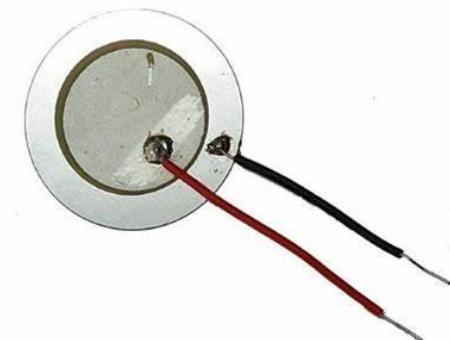
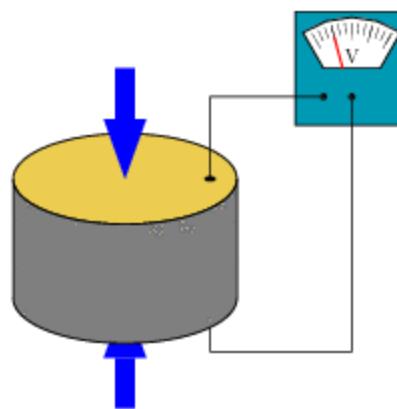
$$C_e = p \times E_v$$



$$p = p_t + p_p \quad [\text{Cm}]$$

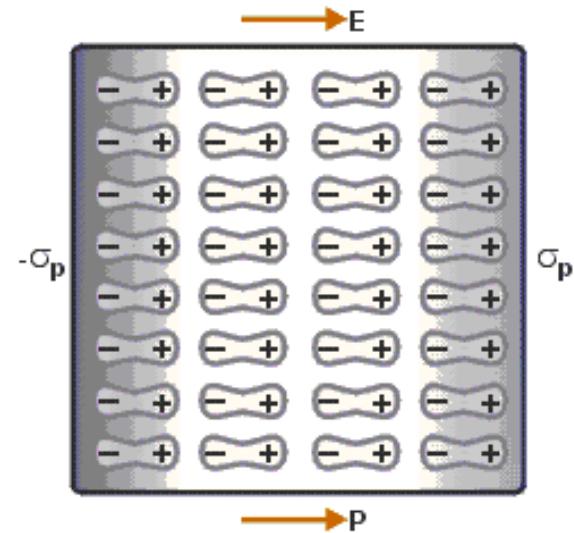


1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

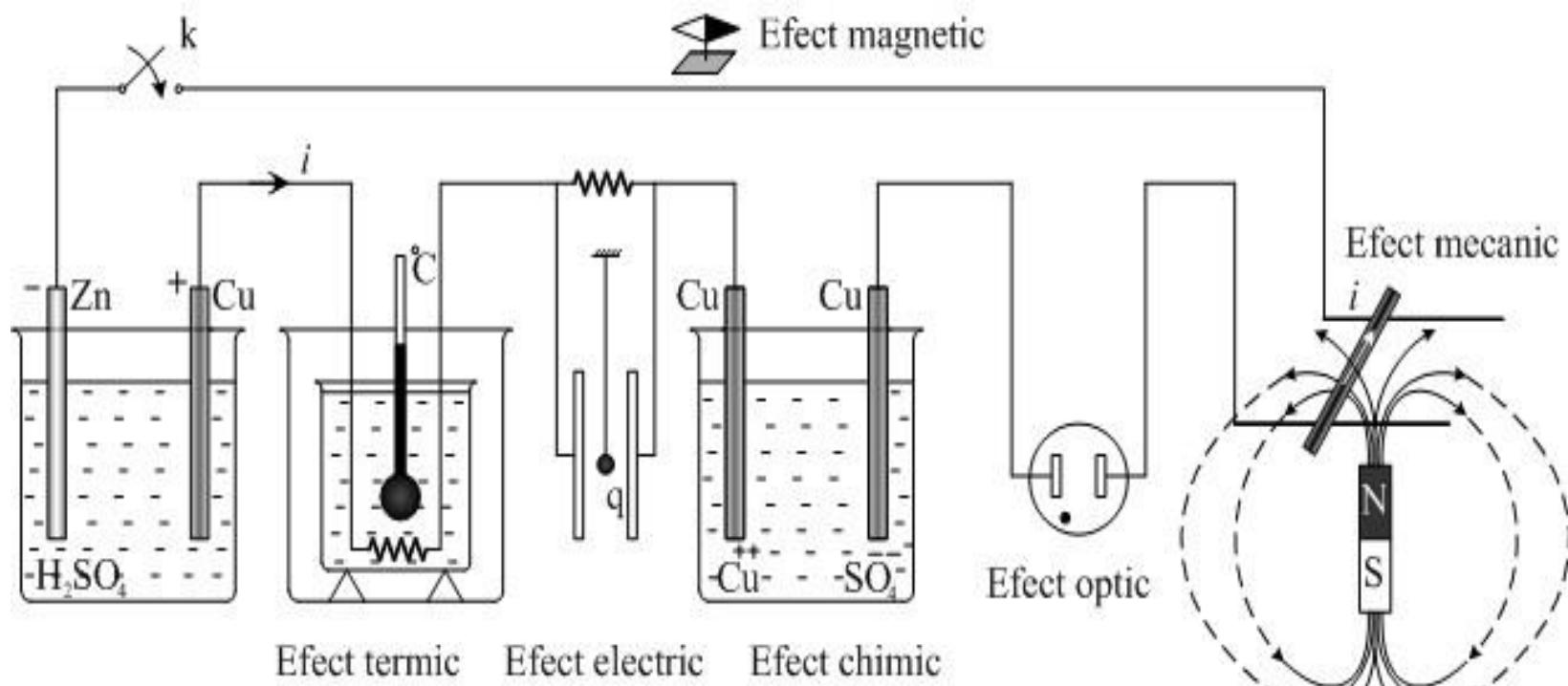


$$P = \lim_{\Delta V \rightarrow 0} \frac{\Delta p}{\Delta V} = \frac{dp}{dV}$$

$$P = P_t + P_p \quad [\text{C/m}^2]$$



1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. **Starea electrocinetica**
4. Starea de magnetizare

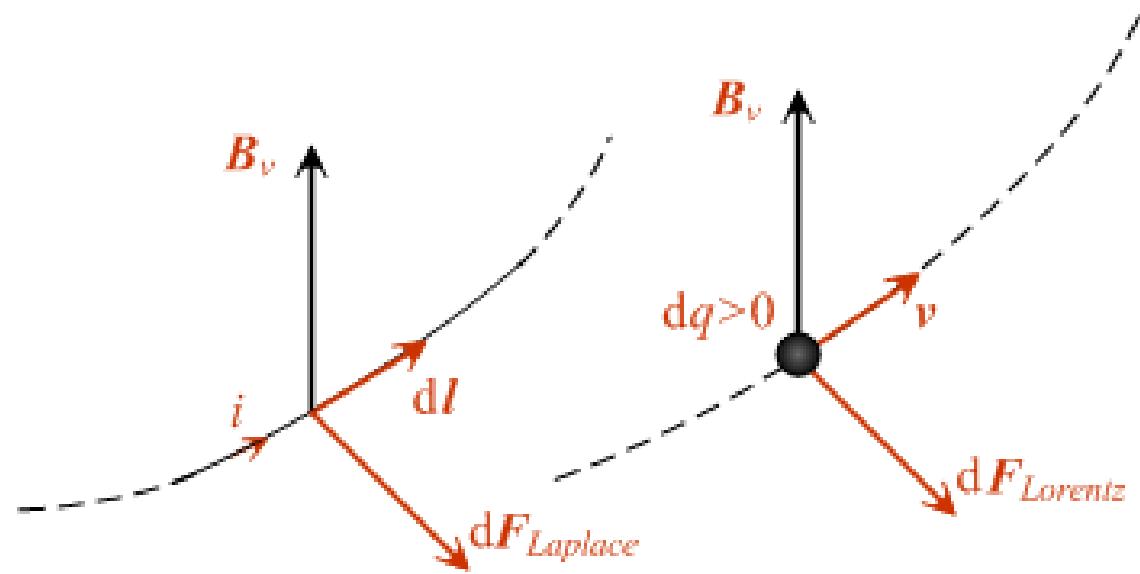


Lanțul închis de conductoare (realizat prin închiderea comutatorului) se află într-o stare nouă numită **stare electrocinetică**.

1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. **Starea electrocinetica**
4. Starea de magnetizare

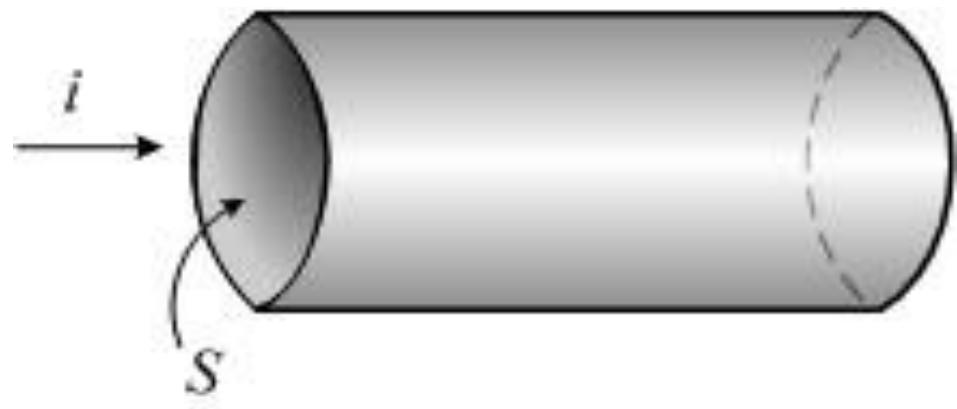
Forța Laplace

$$d\mathbf{F} = i \cdot d\mathbf{l} \times \mathbf{B}_v$$



1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. **Starea electrocinetica**
4. Starea de magnetizare

$$J = \frac{i}{S} \quad [\text{A/m}^2]$$

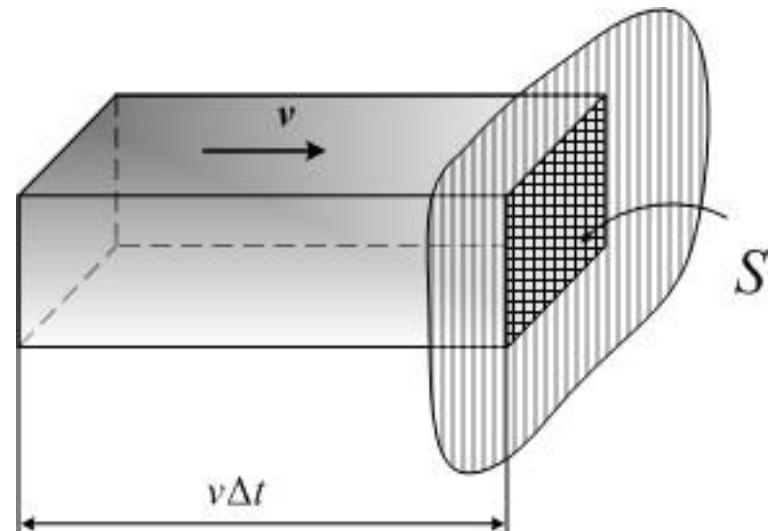


Starea electrocinetică se caracterizează local (într-un punct oarecare din conductor) printr-o mărime numită **densitatea curentului de conducție**, *J*, mărime vectorială de stare al cărui **flux** printr-o suprafață *S* este egal cu intensitatea curentului prin acea suprafață:

$$i = \int_S J \cdot dS$$

1. Starea de incarcare electrica
2. Starea de polarizare electrica
- 3. Starea electrocinetica**
4. Starea de magnetizare

$$i = \lim_{\Delta t \rightarrow 0} \frac{\Delta q_{micro}}{\Delta t} = \frac{d q_{micro}}{d t}$$



$$i = \frac{\Delta q}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{\rho(v\Delta t \cdot \Delta S)}{\Delta t} = \rho v \cdot \Delta S.$$

$$i_v = \int_{S_\Gamma} J_v dS$$

$$J_v = \rho v$$

1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

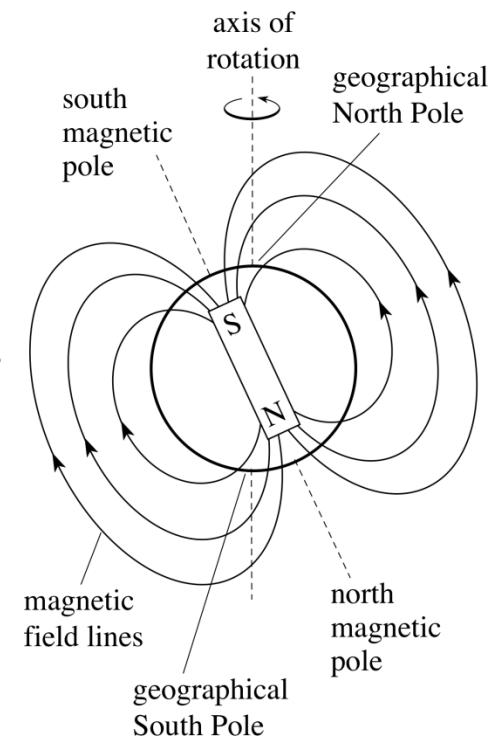
Starea de magnetizare a unui corp e caracterizată **global** printr-o mărime vectorială numită **moment magnetic**, notată cu \mathbf{m} [Am²].

$$\mathbf{C} = \mathbf{m} \times \mathbf{B}_v$$

$$\mathbf{M} = \lim_{\Delta V \rightarrow 0} \frac{\Delta \mathbf{M}}{\Delta V} = \frac{d \mathbf{m}}{d V} \quad [\text{A/m}]$$

$$\mathbf{m} = \mathbf{m}_t + \mathbf{m}_p$$

$$\mathbf{M} = \mathbf{M}_t + \mathbf{M}_p$$



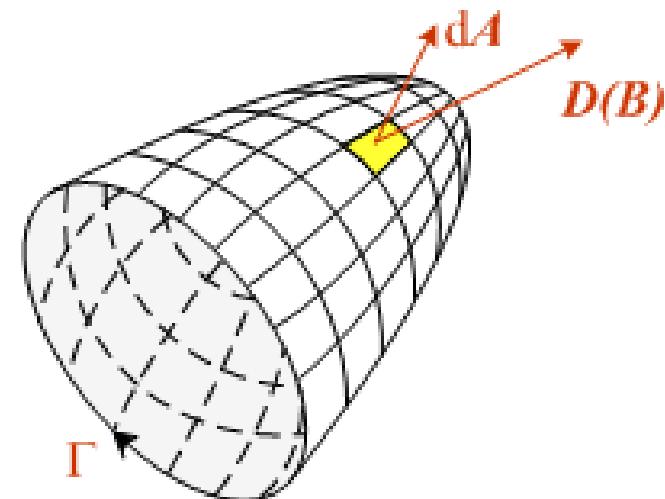
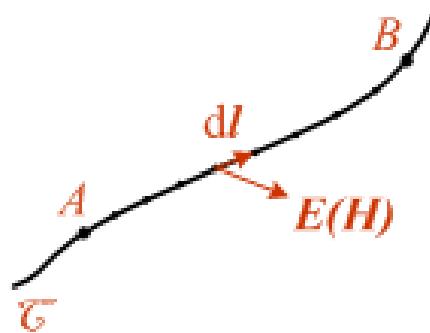
1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

Campul electric si magnetic in corpuri

În corpuri, starea câmpului electromagnetic este caracterizată local prin următoarele mărimi:

- Intensitatea câmpului electric \mathbf{E} care în vid coincide cu \mathbf{E}_v
- Inducția electrică \mathbf{D} , care în vid satisfacă relația $\mathbf{D}_v = \epsilon_0 \mathbf{E}_v$ [C/m²] $\epsilon_0 = 1/(4\pi \cdot 10^9)$ F/m permitivitatea vidului
- Intensitatea campului magnetic \mathbf{H} , care în vid satisfacă relația $\mathbf{B}_v = \mu_0 \mathbf{H}_v$ [A/m] $\mu_0 = 4\pi \cdot 10^{-7}$ H/m permeabilitatea mediului

Inductia magnetica \mathbf{B} care în vid coincide cu \mathbf{B}_v .



BAZELE ELECTROTEHNICII

- note de curs -

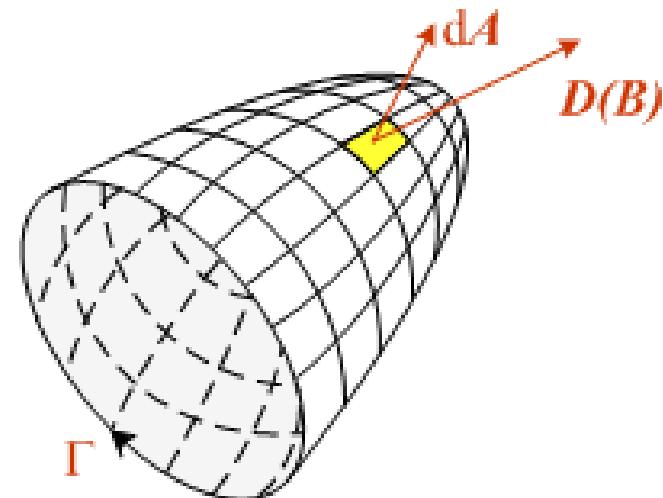
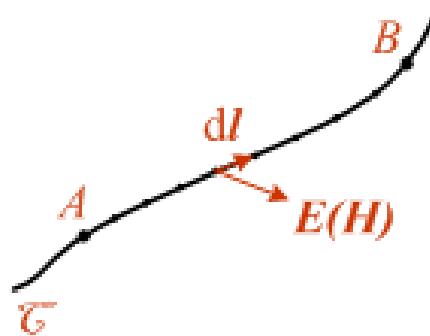
1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

Campul electric si magnetic in corperi

În corperi, starea câmpului electromagnetic este caracterizată local prin următoarele mărimi:

- Intensitatea câmpului electric E [V/m] care în vid coincide cu E_v
- Inducția electrică D [C], care în vid satisfacă relația $D_v = \epsilon_0 E_v$ [C/m²] $\epsilon_0 = 1/(4\pi \cdot 10^9)$ F/m permitivitatea vidului
- Intensitatea campului magnetic H [A/m], care în vid satisfacă relația $B_v = \mu_0 H_v$ [A/m] $\mu_0 = 4\pi \cdot 10^{-7}$ H/m permeabilitatea mediului

Inductia magnetica B [T] care în vid coincide cu B_v .

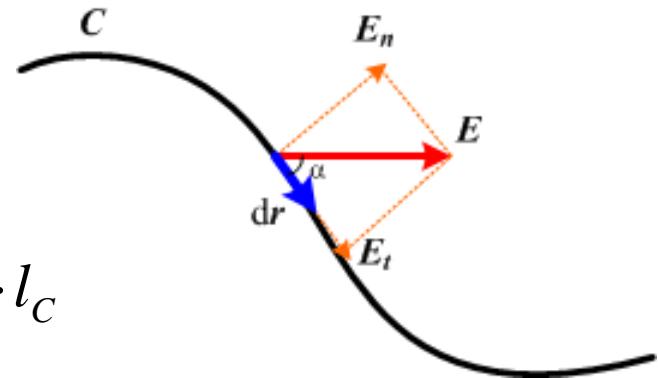


1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

Tensiunea electrică

$$u(t) = \int_C \vec{E}(r, t) d\vec{r} \quad [\text{V}]$$

$$u(t) = \int_C E \cdot dr = \int_C E dr \cos \alpha = \int_C E_t dr = E_{t_med} \cdot l_C$$



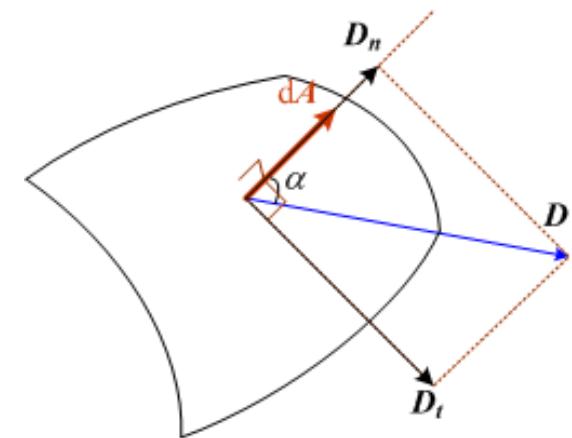
C curba inchisa $\rightarrow u(t)$ devine $e(t)$ tensiune electromotoare

$$e(t) = \oint_C \vec{E}(r, t) d\vec{r} \quad [\text{V}]$$

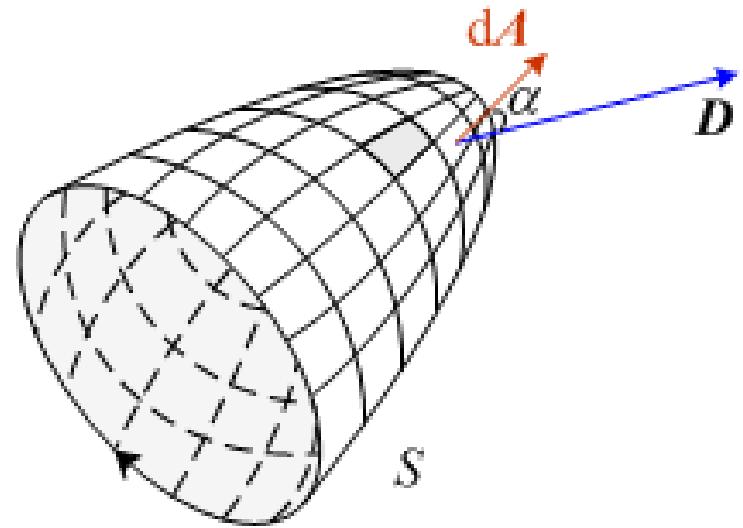
1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

Flux electric

$$\Psi_S(t) = \iint_S \vec{D}(r, t) d\vec{A} \quad [\text{C}]$$



$$\Psi_S(t) = \int_S D \cdot dA = \iint_S D dA \cos \alpha = \iint_S D_n dA = D_{n_med} \cdot A_S$$



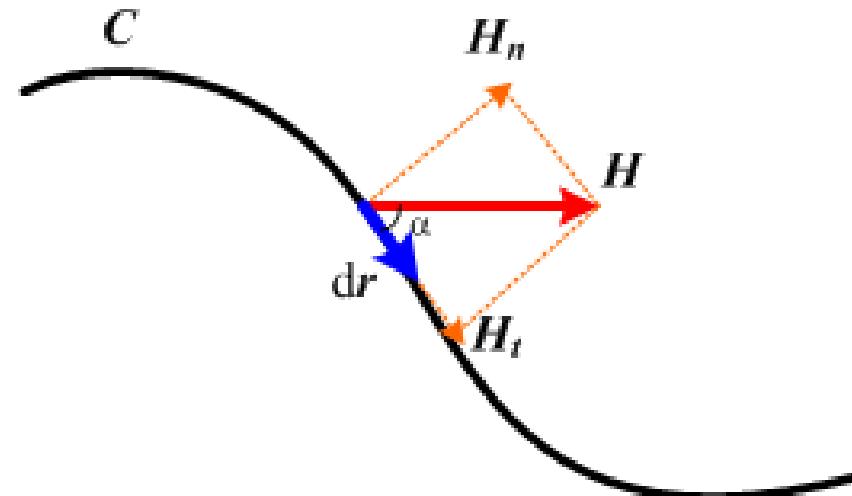
S suprafață închisă

$$\Psi_\Sigma(t) = \iint_\Sigma \vec{D}(r, t) d\vec{A} \quad [\text{C}]$$

1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

Tensiunea magnetica $u_m(t) = \int_C \vec{H}(r, t) d\vec{r} \quad [\text{A}]$

$$u_m(t) = \int_C \vec{H} \cdot d\vec{r} = \int_C H dr \cos \alpha = \int_C H_t dr = H_{t_med} \cdot l_C$$



1. Starea de incarcare electrica
2. Starea de polarizare electrica
3. Starea electrocinetica
4. Starea de magnetizare

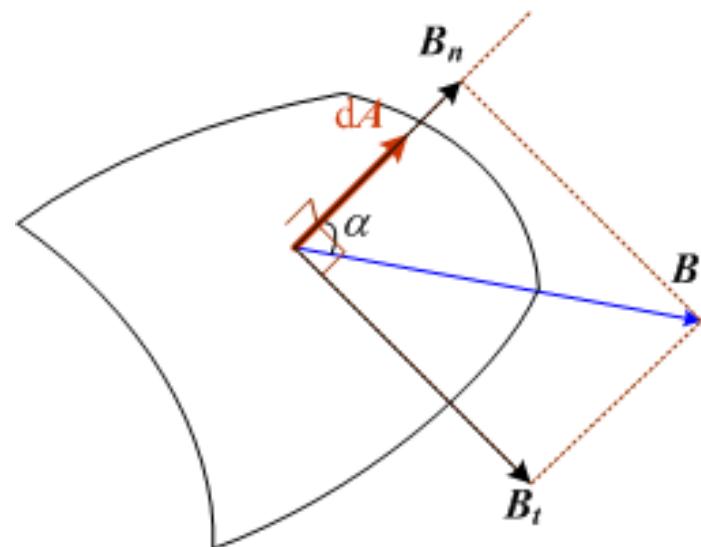
Flux magnetic

$$\Phi_S(t) = \iint_S \vec{B}(r, t) d\vec{A} \quad [\text{Wb}]$$

$$\Phi_S(t) = \int_S \vec{B} \cdot d\vec{A} = \iint_S B dA \cos \alpha = \iint_S B_n dA = B_{n_med} \cdot A_S$$

S suprafață închisă

$$\Phi_\Sigma(t) = \iint_\Sigma \vec{B}(r, t) d\vec{A} \quad [\text{Wb}]$$



LEGILE CAMPULUI ELECTROMAGNETIC**- Legea fluxului electric**

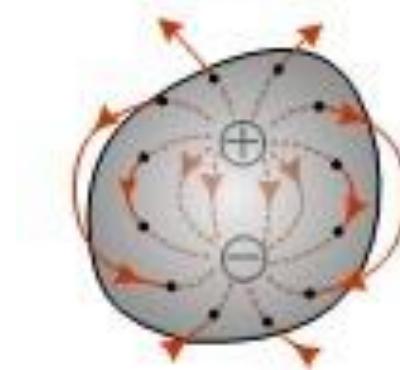
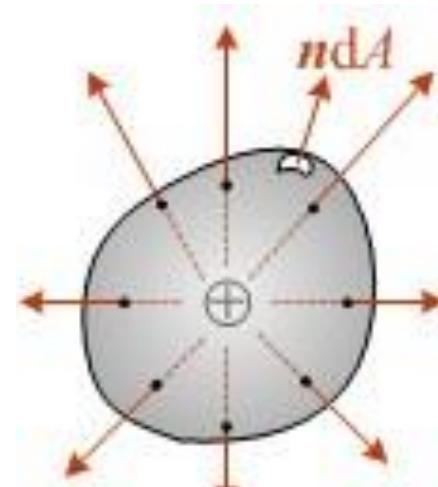
În orice moment de timp și indiferent de modul de variație al mărimilor, fluxul electric pe orice suprafață închisă Σ este egal cu sarcina electrică din domeniul delimitat de suprafața Σ .

$$\Psi_{\Sigma} = q_{D_{\Sigma}}$$

Forma globală
a legii

$$\oint_{\Sigma} \mathbf{D} \cdot \mathbf{n} dA = \int_{V_{\Sigma}} \rho_V dV$$

Forma integrală
a legii



LEGILE CAMPULUI ELECTROMAGNETIC**- Legea fluxului electric**

$$\oint_{\Sigma} \mathbf{D} \cdot \mathbf{n} dA = \int_{V_{\Sigma}} \operatorname{div} \mathbf{D} dV = \int_{V_{\Sigma}} \rho_V dV$$



$$\operatorname{div} \mathbf{D} = \rho_V$$

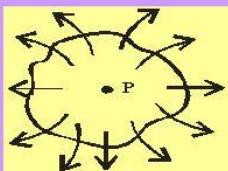
Forma locală a legii

(i) Divergence

- The divergence of a vector \mathbf{V} written as $\operatorname{div} \mathbf{V}$ represents the

$$\operatorname{div} \mathbf{V} = \nabla \bullet \mathbf{V} =$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$



(a) positive divergence



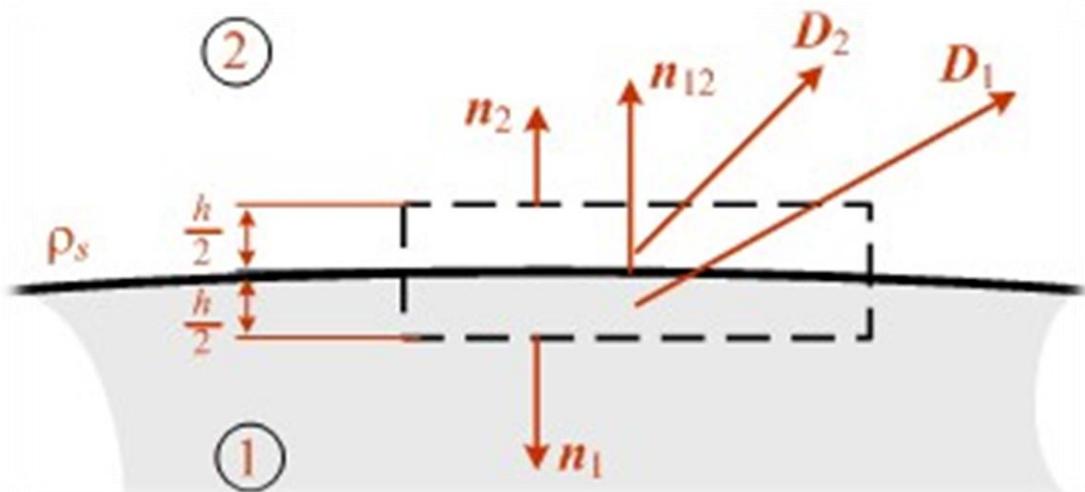
(b) negative divergence



(c) zero divergence

LEGILE CAMPULUI ELECTROMAGNETIC*- Legea fluxului electric*

$$\mathbf{n}_{12} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \rho_s$$



$$\rho_s = 0 \Rightarrow D_{2n} = D_{1n}$$

LEGILE CAMPULUI ELECTROMAGNETIC**- Legea fluxului electric**

Aplicatia 1. Teorema lui Coulomb

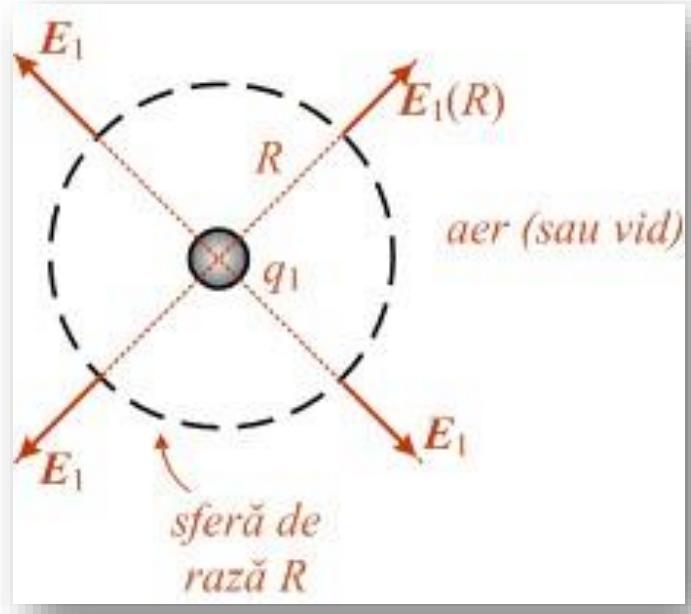
$$\oint_{\Sigma} \mathbf{D}_1 \, dS = \oint_{\Sigma} \epsilon_0 \mathbf{E}_1 \, dS = \epsilon_0 \mathbf{E}_1 \oint dS = \\ = \epsilon_0 E_1 4\pi R^2 = q_1$$



$$\mathbf{E}_1 = \frac{q_1}{4\pi\epsilon_0} \frac{\mathbf{R}}{R^3}$$



$$\mathbf{F}_2 = q_2 \mathbf{E}_1 = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\mathbf{R}}{R^3}$$



LEGILE CAMPULUI ELECTROMAGNETIC**- Legea fluxului electric**

Aplicatia 2. Distributia campului electric produs de o sfera de raza a incarcata cu ρv

Pentru $r < a$

$$\psi_{\Sigma} = \oint_{\Sigma} \mathbf{D} d\mathbf{A} = \oint_{\Sigma} D dA = D \oint_{\Sigma} dA = D 4\pi r^2$$

$$q_{D\Sigma} = \int_{D\Sigma} \rho dV = \rho \int_{D\Sigma} dV = \rho \frac{4\pi r^3}{3}$$

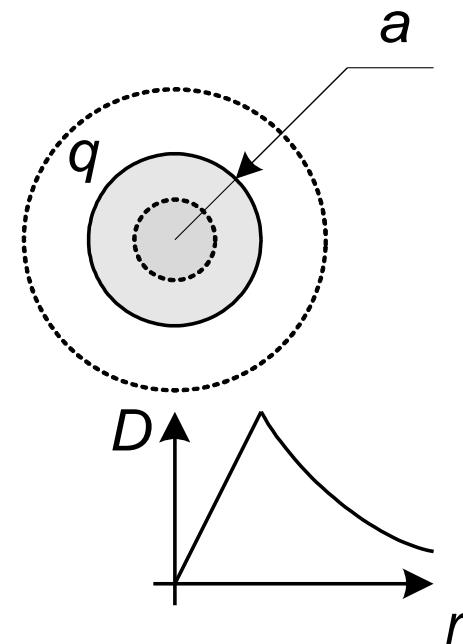
$$\psi_{\Sigma} = q_{D\Sigma} \Rightarrow D_{\text{int}} 4\pi r^2 = \rho \frac{4\pi r^3}{3} \Rightarrow D_{\text{int}} = \rho \frac{r}{3}$$

Pentru $r > a$

$$q_{D\Sigma} = \int_{D\Sigma} \rho dV = \rho \int_{D\Sigma} dV = \rho \frac{4\pi r^3}{3} = q$$

$$\psi_{\Sigma} = \oint_{\Sigma} \mathbf{D} d\mathbf{A} = \oint_{\Sigma} D dA = D \oint_{\Sigma} dA = D 4\pi r^2$$

$$\psi_{\Sigma} = q_{D\Sigma} \Rightarrow D_{\text{ext}} 4\pi r^2 = \rho \frac{4\pi a^3}{3} \Rightarrow D_{\text{ext}} = \rho \frac{a^3}{3r^2} = \frac{q}{4\pi r^2} \Rightarrow \mathbf{D} = \frac{q}{4\pi r^2} \hat{r}$$

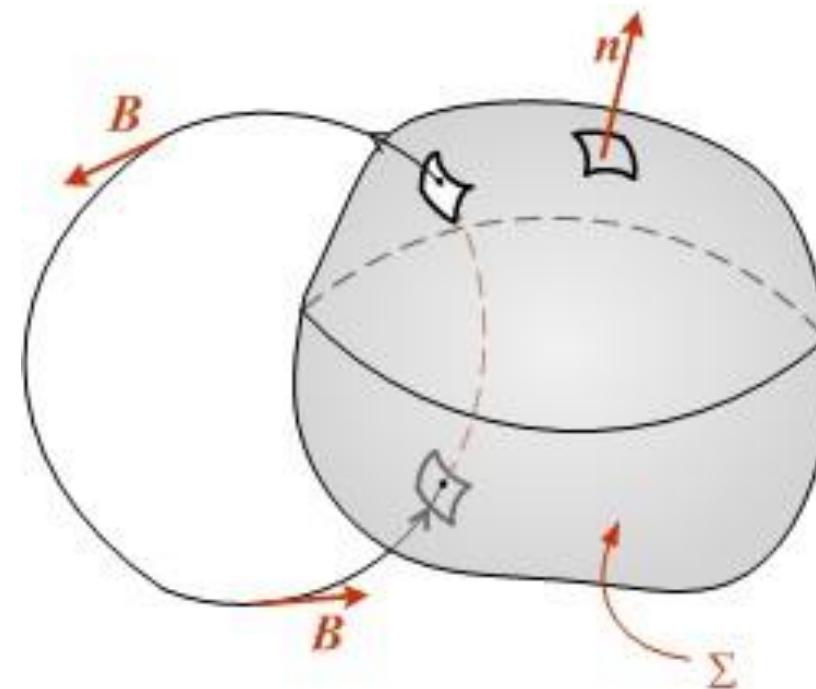


LEGILE CAMPULUI ELECTROMAGNETIC*-Legea fluxului electric**-Legea fluxului magnetic*

Fluxul magnetic prin orice suprafață închisă este nul

$$\Phi_{\Sigma} = 0 \quad \text{Forma generală}$$

$$\oint_{\Sigma} \mathbf{B} d\mathbf{S} = 0 \quad \text{Forma integrală}$$



LEGILE CAMPULUI ELECTROMAGNETIC*-Legea fluxului electric**-Legea fluxului magnetic***În domenii de continuitate**

$$\oint_{\Sigma} \mathbf{B} dS = \int_{V_{\Sigma}} \operatorname{div} \mathbf{B} dV = 0 \quad \rightarrow \quad \operatorname{div} \mathbf{B} = 0$$

Forma locală

**În domenii de discontinuitate
(în puncte situate pe o
suprafață de discontinuitate)**

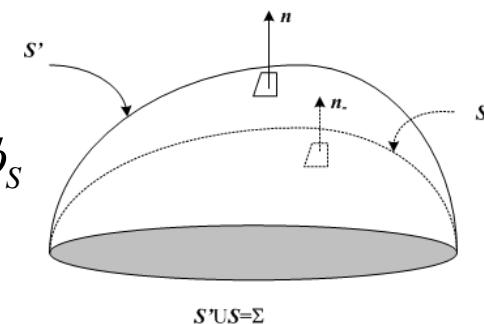
$$\mathbf{n}_{12} \times (\mathbf{B}_2 - \mathbf{B}_1) = 0$$

$$B_{2n} = B_{1n}$$

Forma locală

Aplicație: Toate suprafețele care au aceeași bordură au și același flux magnetic.

$$\oint_{\Sigma} \mathbf{B} \mathbf{n} dA = 0 = \oint_{S'} \mathbf{B} \mathbf{n}' dA + \oint_S \mathbf{B} \mathbf{n}_- dA = \oint_{S'} \mathbf{B} \mathbf{n}' dA - \oint_S \mathbf{B} \mathbf{n} dA = \phi_{S'} - \phi_S$$



LEGILE CAMPULUI ELECTROMAGNETIC

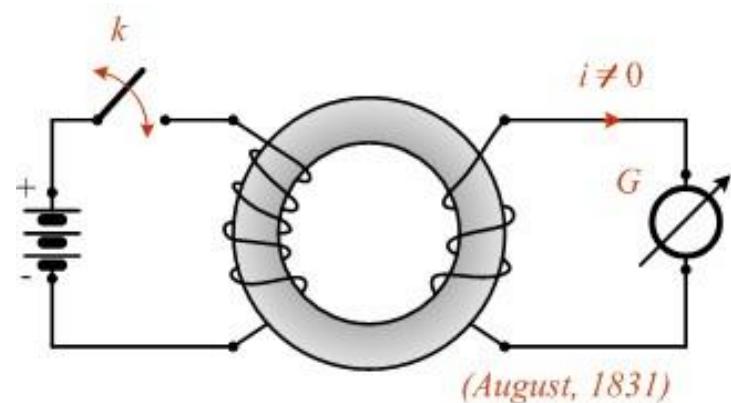
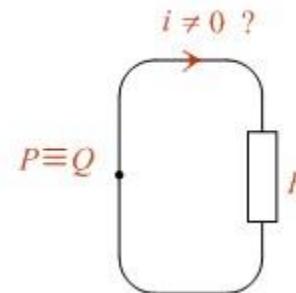
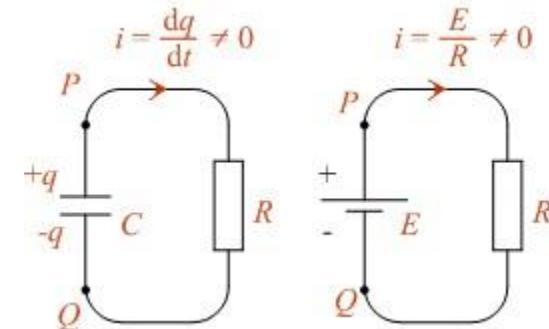
-Legea fluxului electric

-Legea fluxului magnetic

-Legea inductiei electromagnetice

Inducția electromagnetică este fenomenul prin care apare o tensiune electromotoare în lungul unei curbe închise când suprafața mărginită de curbă este străbătută de un flux magnetic variabil în timp. T.e.m. indușă are un astfel de sens încât, prin efectele sale să se opună cauzelor care au produs-o.

Fenomenul a fost descoperit de Mihail Faraday în 1831.

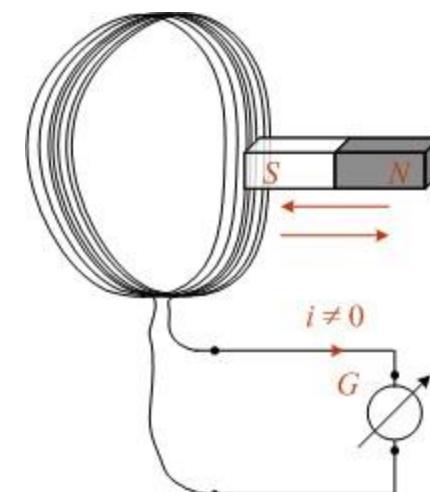
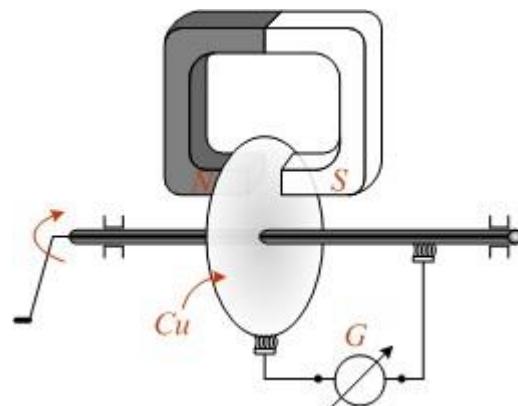
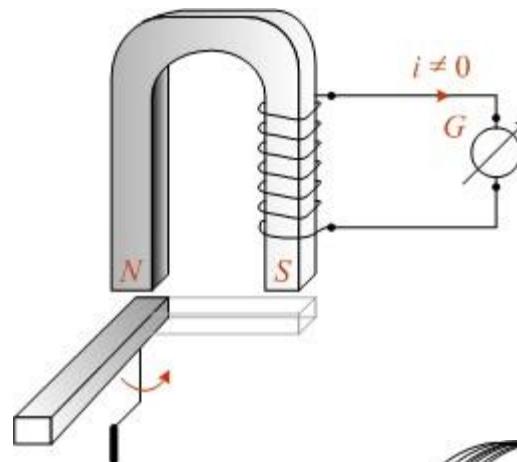
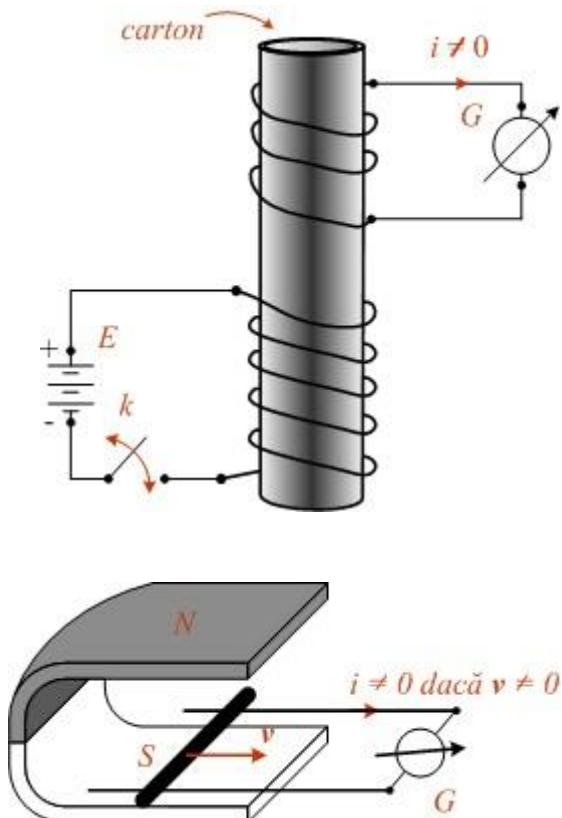


LEGILE CAMPULUI ELECTROMAGNETIC

-Legea fluxului electric

-Legea fluxului magnetic

-Legea inductiei electromagnetice

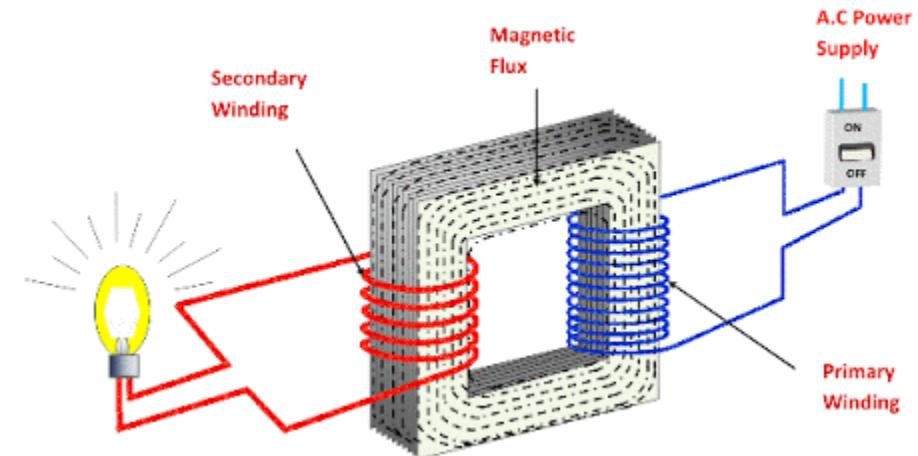
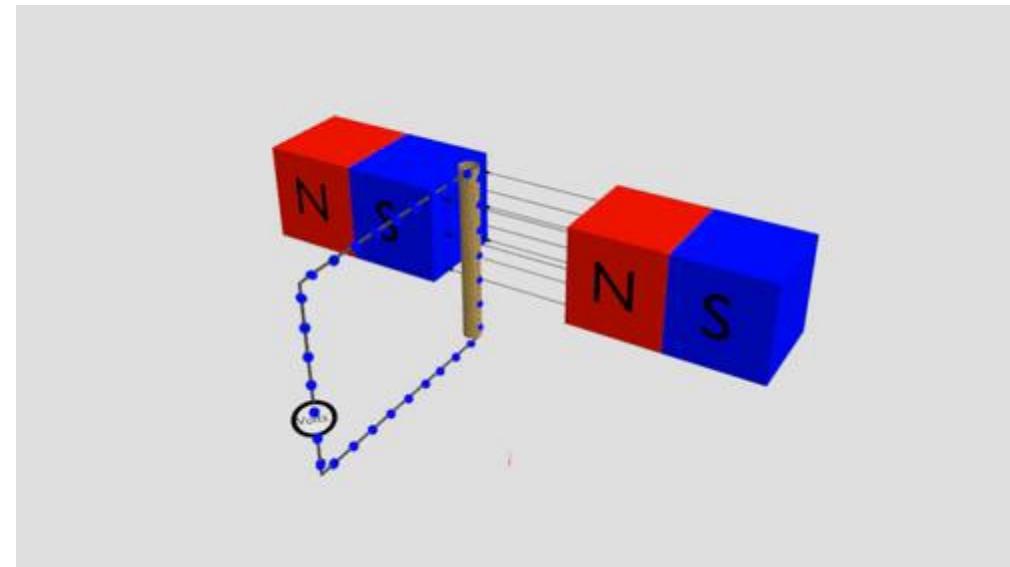


LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice



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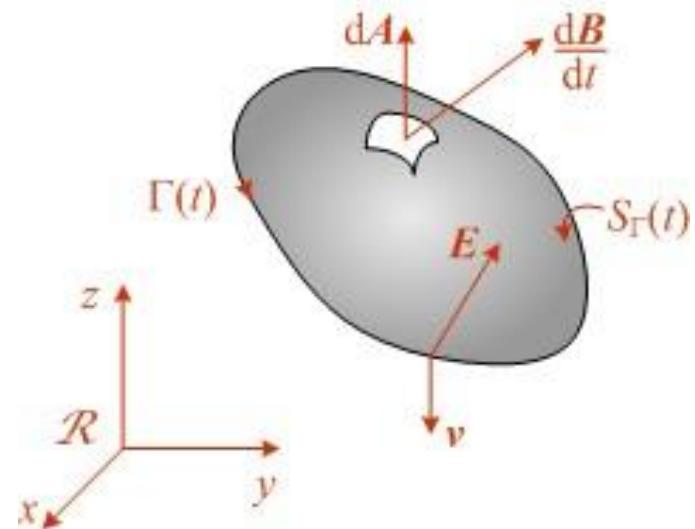
LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice

În orice moment de timp și pentru oricare curbă închisă Γ , tensiunea electromotoare atașată curbei Γ este egală cu viteza de scădere a fluxului magnetic printr-o suprafață (atașată) mărginită de curba Γ

$$e_{\Gamma} = - \frac{d\Phi_{S_{\Gamma}}}{dt}$$

Forma generală



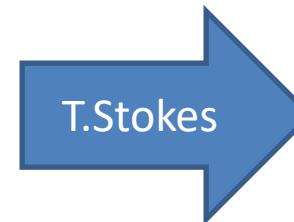
$$\oint_{\Gamma} \mathbf{E} d\mathbf{l} = - \frac{d}{dt} \int_{S\Gamma} \mathbf{B} \mathbf{n} dA$$

Forma integrală

LEGILE CAMPULUI ELECTROMAGNETIC*-Legea fluxului electric**-Legea fluxului magnetic**-Legea inductiei electromagnetice*

Pentru medii imobile:

$$\oint_{\Gamma} \mathbf{E} d\mathbf{l} = - \int_{S\Gamma} \frac{\partial \mathbf{B}}{\partial t} \mathbf{n} dA$$



$$\text{rot} \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

Pentru medii in miscare:

$$\frac{d_f B}{dt} = \frac{\partial B}{\partial t} + v \text{div} \mathbf{B} + \text{rot}(\mathbf{B} \times \mathbf{v})$$

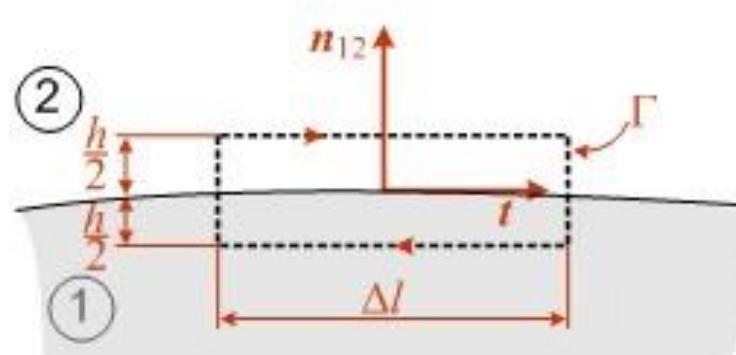
$$\text{rot} \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} - \text{rot}(\mathbf{B} \times \mathbf{v})$$

$$\oint_{\Gamma} \mathbf{E} d\mathbf{l} = \iint_{S_{\Gamma}} \text{rot} \mathbf{E} dA = - \left(\iint_{S_{\Gamma}} \frac{\partial \mathbf{B}}{\partial t} \mathbf{n} dA + \iint_{S_{\Gamma}} \text{rot}(\mathbf{B} \times \mathbf{v}) dA \right)$$

$$e_{\Gamma}^0 = e_{\Gamma_{\text{transformare}}} + e_{\Gamma_{\text{miscare}}}$$

LEGILE CAMPULUI ELECTROMAGNETIC*-Legea fluxului electric**-Legea fluxului magnetic**-Legea inductiei electromagnetice*

Pentru domenii de discontinuitate:



$$\mathbf{E}_2 \cdot \Delta \mathbf{l}_2 + \mathbf{E}_1 \cdot \Delta \mathbf{l}_1 = 0,$$

$$\Delta \mathbf{l}_2 = \mathbf{t} \Delta l; \quad \Delta \mathbf{l}_1 = (-\mathbf{t}) \Delta l,$$

$$\mathbf{t} (\mathbf{E}_2 - \mathbf{E}_1) = 0.$$

LEGILE CAMPULUI ELECTROMAGNETIC

-Legea fluxului electric

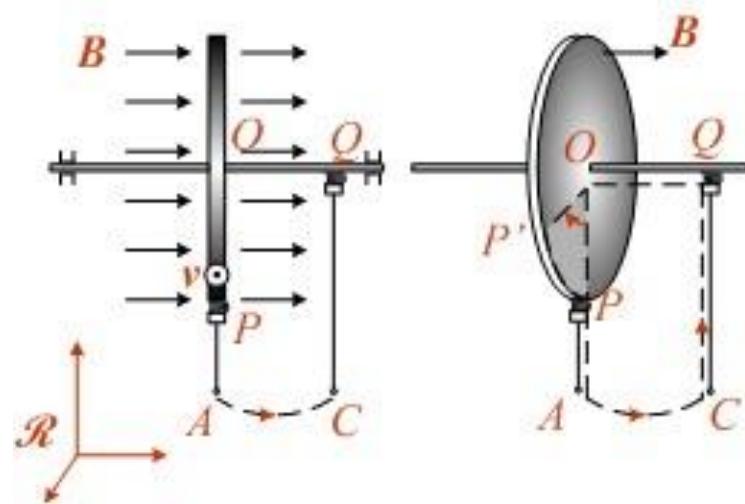
-Legea fluxului magnetic

-Legea inductiei electromagnetice

Aplicatia 1. Discul lui Faraday

$$\phi(t) = Ba^2 \frac{\Delta\alpha}{2}$$

$$e = -\frac{d\phi}{dt} = -Ba^2 \omega \frac{1}{2}$$

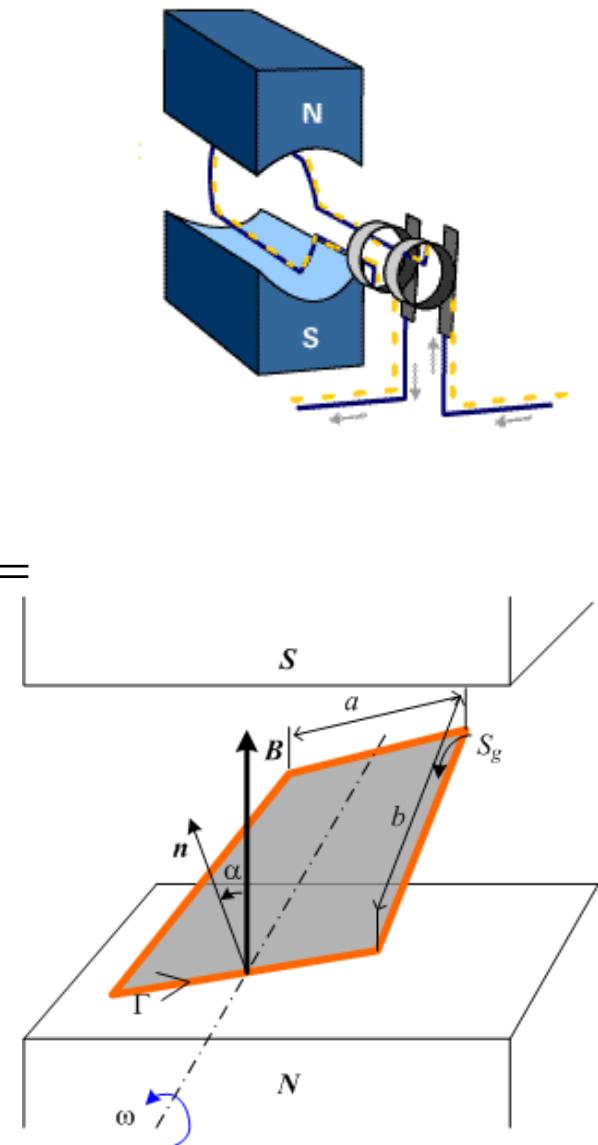


LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice

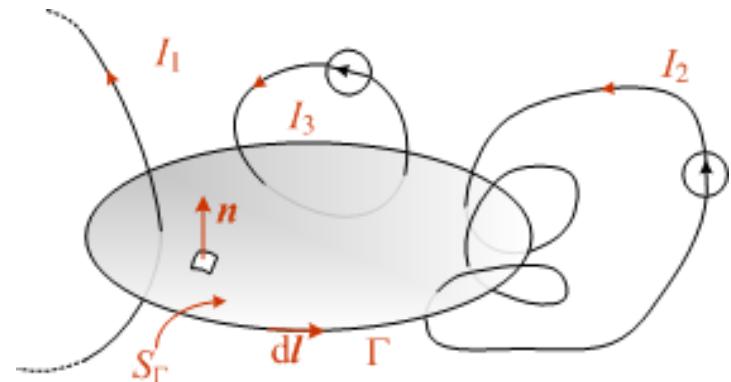
Aplicatia 2. Principiul generatorului de tensiune alternativa

$$\begin{aligned}
 e &= -\frac{d}{dt} \int_{Sg} \mathbf{B} \cdot d\mathbf{A} = -\frac{d}{dt} \int_{Sg} B \cos \alpha dA = -\frac{d}{dt} B \cos \alpha \int_{Sg} dA = \\
 &= -\frac{d}{dt} B \cos \alpha ab = abB \sin \alpha \frac{d\alpha}{dt} = abB\omega \sin \alpha
 \end{aligned}$$



LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic



Tensiunea magnetomotoare de-a lungul unei curbe închise este egală cu suma dintre curentul electric printr-o suprafață arbitrară care se sprijină pe acea curbă și viteza de variație a fluxului electric de pe acea suprafață.

$$u_{m\Gamma} = i_{S_\Gamma} + \frac{d\Psi_{S_\Gamma}}{dt}$$

Forma generală

$$\oint_{\Gamma} \mathbf{H} dl = \int_{S_\Gamma} \mathbf{J} dA + \frac{d}{dt} \int_{S_\Gamma} \mathbf{D} dA$$

Forma integrală

LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic

Pentru medii imobile:

$$\int_{\Gamma} \vec{H} d\vec{l} = \iint_{S_{\Gamma}} \text{rot} \vec{H} d\vec{A} = \iint_{S_{\Gamma}} \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) d\vec{A} \quad \xrightarrow{\text{Forma locala}} \quad \text{rot} \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Pentru medii in miscare:

$$\text{rot} \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} + \rho \vec{v} + \text{rot}(\vec{D} \times \vec{v})$$

Pentru domenii de discontinuitate:

$$(\vec{H}_2 - \vec{H}_1)_{\text{tangential}} = \vec{J}_s \times \vec{n}_{12}$$

Forma locala

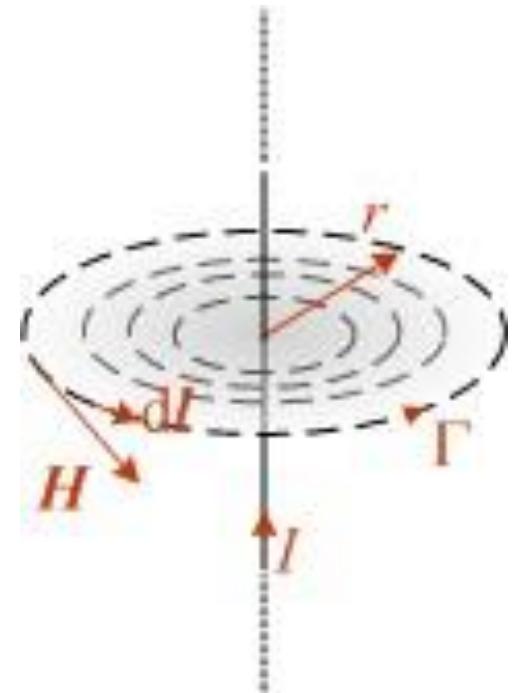
LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic

Aplicatie - Calculul câmpului magnetic al unui fir conductor, rectiliniu, infinit lung, parcurs de curent continuu.

$$\oint_{\Gamma} \mathbf{H} \cdot d\mathbf{l} = \oint_{\Gamma} H \, dl = H \oint_{\Gamma} dl = H \cdot 2\pi r$$

$$\mathbf{H} = \frac{I}{2\pi r} \mathbf{u}_{\varphi}$$

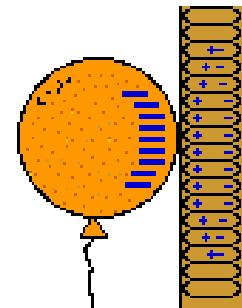


BAZELE ELECTROTEHNICII

- note de curs -

LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic
- Legea polarizatiei electrice temporare



The charged balloon causes the wood molecules to become polarized. Attraction is then possible.

În orice regim și în orice moment, polarizația electrică temporară dintr-un punct este o funcție de intensitatea câmpului electric din punctul respectiv

$$\mathbf{P}_t(t, \mathbf{r}) = f(\mathbf{E}(t, \mathbf{r}))$$

Susceptivitate electrica
(adimensional)

Pentru medii liniare și izotrope

$$\mathbf{P}_t(t, \mathbf{r}) = \epsilon_0 \chi_e \mathbf{E}(t, \mathbf{r})$$

Pentru medii neliniare

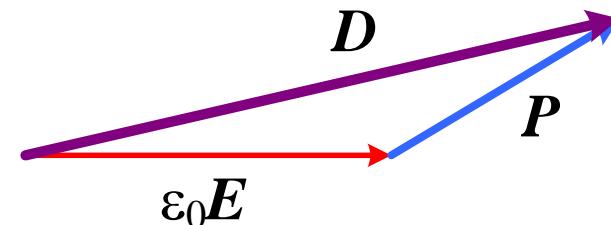
$$\mathbf{P}_t(t, \mathbf{r}) = \epsilon_0 \cdot \chi_e(\mathbf{E}) \cdot \mathbf{E}(t, \mathbf{r})$$

Pentru medii anizotrope

$$\mathbf{P}_t(t, \mathbf{r}) = \epsilon_0 \cdot \overline{\chi_e} \cdot \mathbf{E}(t, \mathbf{r})$$

LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic
- Legea polarizatiei electrice temporare
- Legea legaturii in camp electric**



$$\mathbf{D}(t, \mathbf{r}) = \epsilon_0 \cdot \mathbf{E}(t, \mathbf{r}) + \mathbf{P}(t, \mathbf{r})$$

În orice moment și în orice punct, inducția electrică este egală cu suma vectorială dintre polarizația și intensitatea câmpului electric multiplicată cu permitivitatea vidului, din acel punct

Permitivitatea
absolută a vidului

$$\epsilon_0 = \frac{1}{4\pi \cdot 9 \cdot 10^9} [F/m]$$

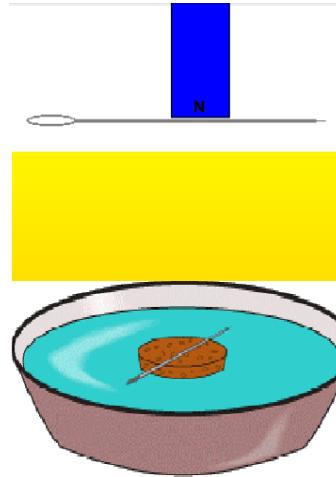
În vid $\mathbf{D}(t, \mathbf{r}) = \epsilon_0 \cdot \mathbf{E}(t, \mathbf{r})$

Permitivitatea
mediului

Pentru medii liniare fără polarizație permanentă

$$\mathbf{D}(t, \mathbf{r}) = \epsilon_0 \cdot \mathbf{E}(t, \mathbf{r}) + \mathbf{P}_t(t, \mathbf{r}) = \epsilon_0(1 + \chi_e) \mathbf{E}(t, \mathbf{r}) = \epsilon_0 \epsilon_r \mathbf{E}(t, \mathbf{r}) = \epsilon \mathbf{E}(t, \mathbf{r})$$

Pentru medii liniare cu polarizație permanentă $\mathbf{D} = \epsilon \mathbf{E} + \mathbf{P}_p$



LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic
- Legea polarizatiei electrice temporare
- Legea legaturii in camp electric
- Legea magnetizatiei temporare**

În orice regim și în orice moment, magnetizatia temporară dintr-un punct este o funcție de intensitatea câmpului magnetic din punctul respectiv

Pentru medii liniare și izotrope

Pentru medii neliniare

Pentru medii anizotrope

$$\mathbf{M}_t(t, \mathbf{r}) = f(\mathbf{H}(t, \mathbf{r}))$$

Susceptivitatea magnetica
(adimensional)

$$\mathbf{M}_t(t, \mathbf{r}) = \chi_m \mathbf{H}(t, \mathbf{r})$$

$$\mathbf{M}_t(t, \mathbf{r}) = \chi_m(\mathbf{H}) \mathbf{H}(t, \mathbf{r})$$

$$\mathbf{M}_t(t, \mathbf{r}) = \overline{\overline{\chi_m}} \mathbf{H}(t, \mathbf{r})$$

LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic
- Legea polarizatiei electrice temporare
- Legea legaturii in camp electric
- Legea magnetizatiei temporare
- Legea legaturii in camp magnetic**

În orice moment și în orice punct, inducția magnetică este egală cu suma vectorială dintre intensitatea câmpului magnetic și magnetizatie, multiplicată cu permeabilitatea vidului, din acel punct.

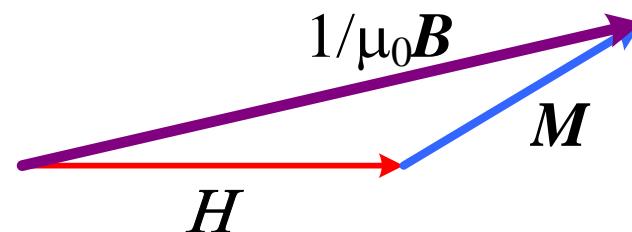
În vid $B(t, r) = \mu_0 H(t, r)$

Pentru medii liniare fără magnetizatie permanentă

$$B = \mu_0 (H + M_t) = \mu_0 (H + \chi_m H) = \mu_0 (1 + \chi_m) H = \mu H$$

Pentru medii liniare cu magnetizatie permanentă

$$B = \mu_0 (1 + \chi_m) H + \mu_0 M_p = \mu H + \mu_0 M_p = \mu H + I_p$$



$$B(t, r) = \mu_0 (H(t, r) + M(t, r))$$

Permeabilitatea
absolută a vidului

Permeabilitatea
mediului

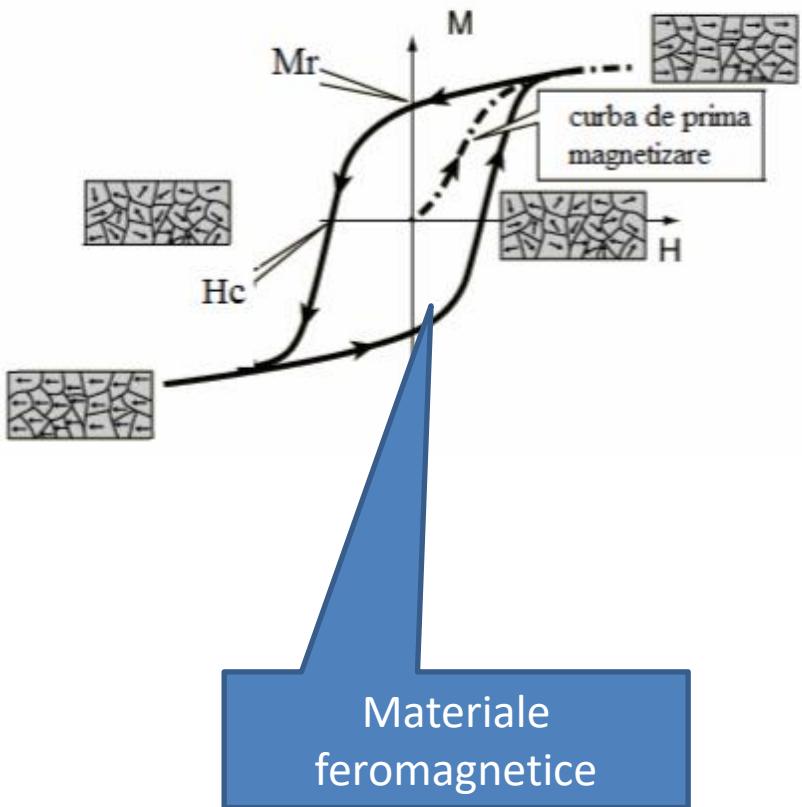
$$\mu_0 = 4\pi \cdot 10^{-7} [H / m]$$

LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic
- Legea polarizatiei electrice temporare
- Legea legaturii in camp electric
- Legea magnetizatiei temporare
- Legea legaturii in camp magnetic

- Materiale diamagnetice $(\overline{M}_t \uparrow\downarrow \overline{H})$
- Materiale paramagnetice $(\overline{M}_t \uparrow\uparrow \overline{H})$

Materiale
neferomagnetice



Materiale
feromagnetice

LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic
- Legea polarizatiei electrice temporare
- Legea legaturii in camp electric
- Legea magnetizatiei temporare
- Legea legaturii in camp magnetic
- Legea conductiei electrice**

$$\rho = \rho_0 [1 + \alpha_0 (T - T_0)]$$

$$\sigma = \frac{1}{\rho}$$

Conductivitate



Suma vectorială dintre intensitatea câmpului electric E și intensitatea câmpului electric imprimat E_i din interiorul unui conductor este proporțională în fiecare punct cu densitatea curentului de conducție din acel punct.

$$E + E_i = \rho J$$

Rezistivitatea materialului

$$J = \sigma(E + E_i)$$

În cunectoare omogene $E_i=0$

LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic
- Legea polarizatiei electrice temporare
- Legea legaturii in camp electric
- Legea magnetizatiei temporare
- Legea legaturii in camp magnetic
- Legea conductiei electrice
- Legea transferului de energie**

Puterea cedată pe unitatea de volum a conductorului de către câmpul electromagnetic, în procesul de conducție electrică, este egală cu produsul scalar dintre intensitatea câmpului electric și densitatea curentului electric de conducție.

$$p_J = \mathbf{E} \cdot \mathbf{J} \quad \left[\frac{\text{W}}{\text{m}^3} \right]$$

Conductori omogeni: $p_J = \mathbf{E} \cdot \mathbf{J} = E \cdot J = \rho J^2 > 0$

Conductori neomogeni: $\mathbf{E} = \rho \mathbf{J} - \mathbf{E}_i \Rightarrow p_J = \rho J^2 - \mathbf{E}_i \cdot \mathbf{J}$

Pentru forma integrala consideram o portiune de conductor:

$$P_J = \iiint_V p_J dV = \iiint_V E J dV = \int_1^2 E J A dl = \int_1^2 E i dl = i \int_1^2 E dl = u \cdot i$$

LEGILE CAMPULUI ELECTROMAGNETIC

- Legea fluxului electric
- Legea fluxului magnetic
- Legea inductiei electromagnetice
- Legea circuitului magnetic
- Legea polarizatiei electrice temporare
- Legea legaturii in camp electric
- Legea magnetizatiei temporare
- Legea legaturii in camp magnetic
- Legea conductiei electrici
- Legea transferului de energie
- Legea transferului de masa**

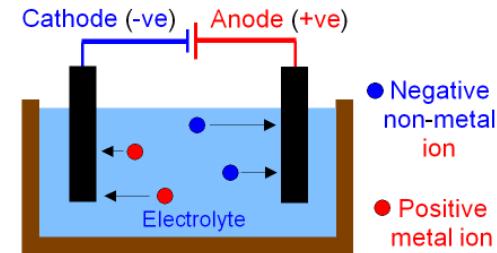
A – masa atomica a elementului

n_v – valența substantei

F – const. Faraday 96400 C

Forma locală:

$$\delta_m = k \cdot J \left[\frac{kg}{m^2 s} \right]$$



Masa de substanță depusă în unitatea de timp la unul din electrozii unei băi electrolitice este proporțională cu intensitatea curentului electric de conducție prin electrolit.

$$\frac{dm}{dt} = k \cdot i$$

$$k = \frac{A}{n_v F}$$

$$m = k \int_0^t i dt = kq$$

Echivalent electrochimic

Regimurile de desfășurare a fenomenelor electromagnetice

Regim static

- mărimele de stare nu variază în timp și nu se produc transformări energetice

Electrostatică

Magnetostatică

Regim staționar

- mărimele de stare nu variază în timp și au loc transformări energetice

Regim electric staționar

Regim magnetic staționar

Regim cvasistaționar

- mărimele de stare variază în timp dar suficient de lent pentru a se neglija undele electromagnetice

Regim anelectric

Regim amagnetic

Regim nestaționar (variabil)

- Nu este impusă nicio restricție asupra ritmului de variație în timp al mărimarilor

BAZELE ELECTROTEHNICII

- note de curs -

Regimurile de desfășurare a fenomenelor electromagnetice

Regim static

- mărimele de stare nu variază în timp și nu se produc transformări energetice

Electrostatică

Magnetostatică

Regim staționar

- mărimele de stare nu variază în timp și au loc transformări energetice

Regim electric staționar

Regim magnetic staționar

Regim cvasistaționar

- mărimele de stare variază în timp dar suficient de lent pentru a se neglija undele electromagnetice

Regim anelectric

Regim amagnetic

Regim nestaționar (variabil)

- Nu este impusă nicio restricție asupra ritmului de variație în timp al mărimarilor

ELECTROSTATICĂ

Regim static

- mărurile de stare nu variază în timp și nu se produc transformări energetice

$$\left(\frac{\partial}{\partial t} = 0; \frac{d}{dt} = 0 \right) \quad (\mathbf{v} = 0; \mathbf{J} = 0, i=0)$$



Relațiile fundamentale ale electrostaticii

Legea fluxului electric (regim static)

$$\Psi_{\Sigma} = q_{D_{\Sigma}} \quad \operatorname{div} \mathbf{D} = \rho_v$$

Legea ind. electromagn (regim static)

$$e_{\Gamma} = \oint_{\Gamma} \mathbf{E} d\mathbf{l} = 0 \quad \operatorname{rot} \mathbf{E} = 0$$

Legea cond.electrice (regim static)

$$\mathbf{E} + \mathbf{E}_i = 0 \quad \mathbf{J} = 0$$

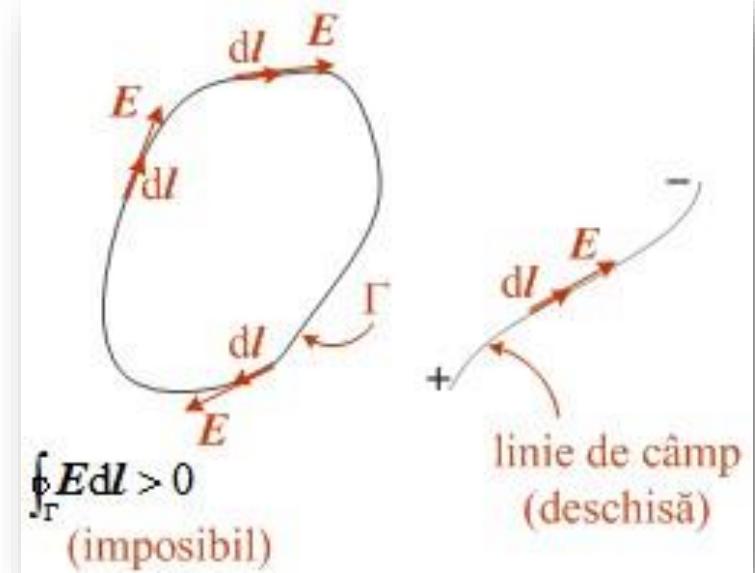
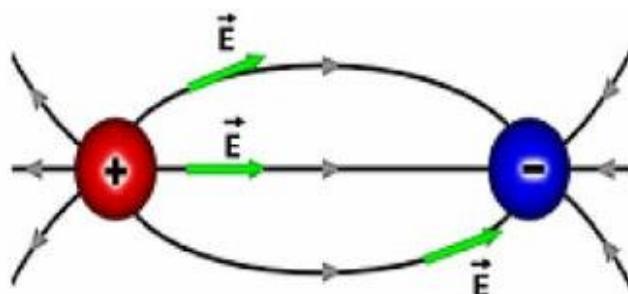
Legea legăt. câmp el. (regim static)

$$\mathbf{D} = \epsilon \mathbf{E}$$

ELECTROSTATICĂ

Consecințe:

C1. În câmp electrostatic nu există linii de câmp închise.



ELECTROSTATICĂ

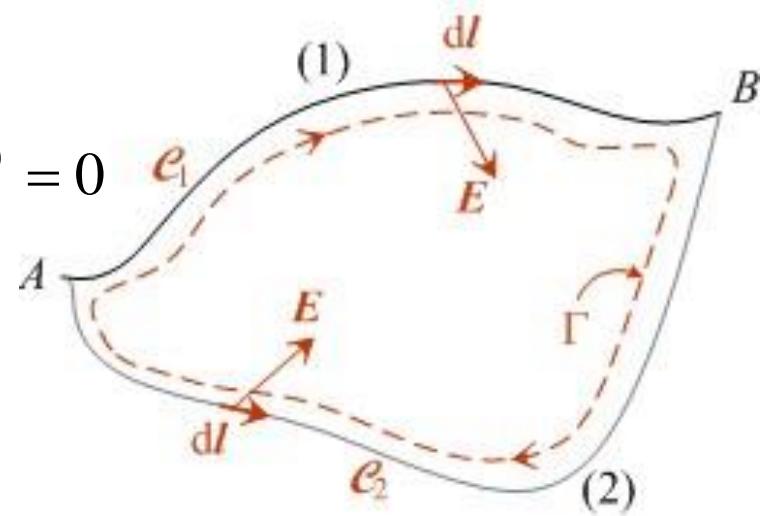
Consecințe:

C2. Tensiunea electrică nu depinde de drumul ales între două puncte date.

$$\oint_{\Gamma} \mathbf{E} d\mathbf{l} = \int_{C_1 AB} \mathbf{E} d\mathbf{l} + \int_{C_2 BA} \mathbf{E} d\mathbf{l} = u_{AB}^{(1)} - u_{AB}^{(2)} = 0$$



$$u_{AB}^{(1)} = u_{AB}^{(2)}$$

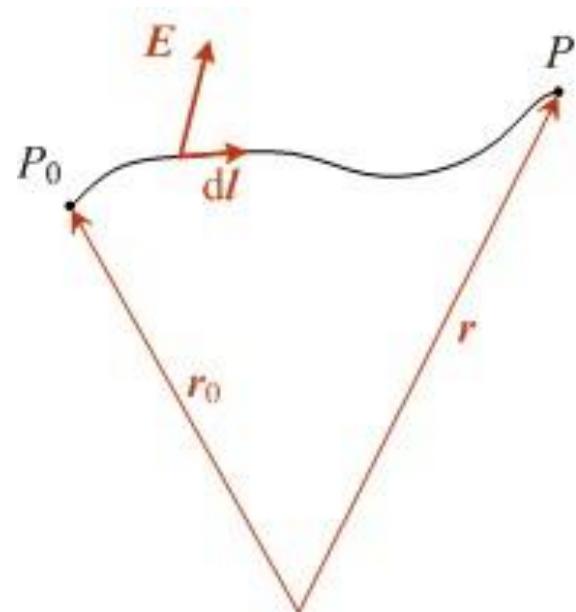
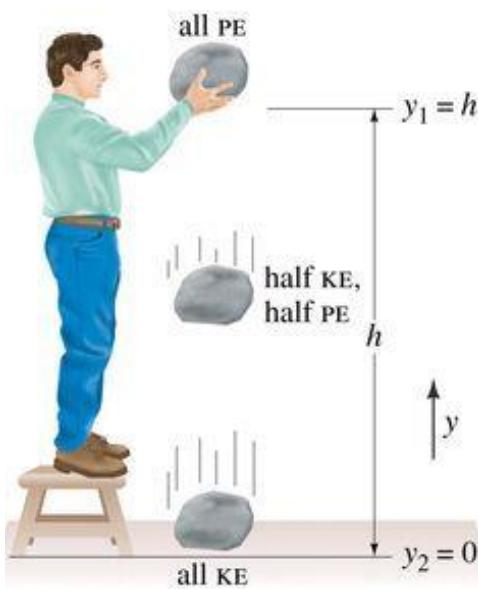


ELECTROSTATICĂ

Consecințe:

C3. Definiția potențialului electric.

$$V(\mathbf{r}) = V(\mathbf{r}_0) - \int_{P_0}^{\mathbf{r}} \mathbf{E} d\mathbf{l}$$



$$\mathbf{E} = -\nabla V$$

Semnificația fizică a potențialului electric scalar. Dacă un câmp electric acționează asupra unei particule încărcate electric cu sarcina q și o deplasează din punctul P_0 în punctul P atunci el va efectua lucru mecanic. Potențialul electric într-un punct al câmpului este egal cu raportul dintre lucrul efectual de câmp pentru a deplasa o particulă cu sarcina q din punctul considerat până în punctul în care potențialul se consideră nul și sarcina electrică.

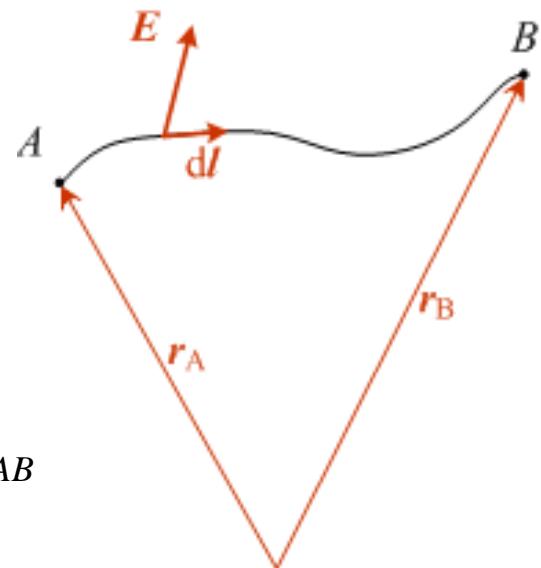
ELECTROSTATICĂ

Consecințe:

C4. Tensiunea electrică dintre două puncte situate în câmp electrostatic este egală cu diferența potențialelor din acele puncte.

$$\left. \begin{aligned} V_A &= V_{P_0} - \int_{P_0}^A \mathbf{E} d\mathbf{l} \\ V_B &= V_{P_0} - \int_{P_0}^B \mathbf{E} d\mathbf{l} \end{aligned} \right\}$$

$$V_A - V_B = \int_A^{P_0} \mathbf{E} d\mathbf{l} + \int_{P_0}^B \mathbf{E} d\mathbf{l} = \int_A^B \mathbf{E} d\mathbf{l} = u_{AB}$$



ELECTROSTATICA

Consecințe:

C5. Toate punctele unui corp conductor au același potențial electric (indiferent de dimensiuni și de forma corpului).

$$\mathbf{E} = 0, \text{ deci } U_{AB} = V(A) - V(B) = 0;$$

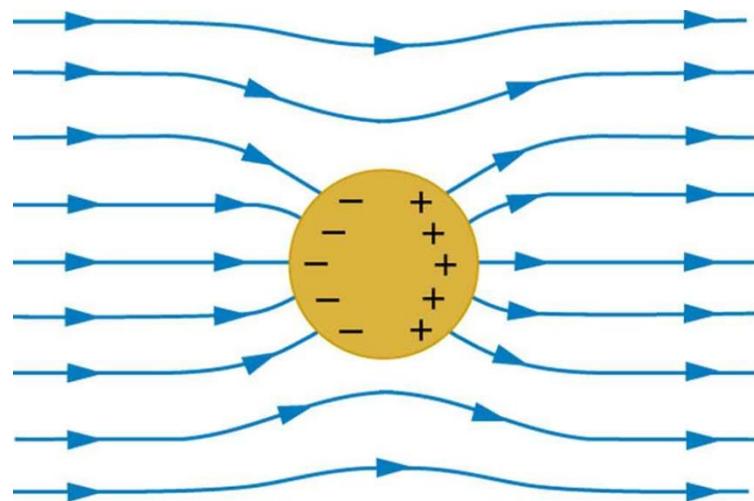


ELECTROSTATICA

Consecințe:

C 6. Liniile de câmp electric din exteriorul corpurilor conductoare sunt perpendiculare pe suprafața conductoarelor.

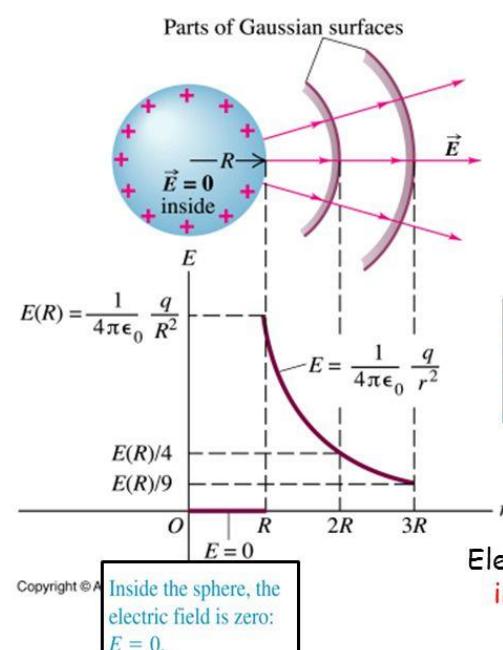
$$\left. \begin{array}{l} E_{t2} = E_{t1} \\ E_{\text{int}} = 0 \end{array} \right\} \Rightarrow E_{t2} = 0$$



ELECTROSTATICA

Consecințe:

C 7. Un corp conductor omogen încărcat electric are sarcina repartizată numai pe suprafața corpului. În interiorul conductorului $\rho = 0$.



Under electrostatic conditions the electric field inside a solid conducting sphere is zero. Outside the sphere the electric field drops off as $1 / r^2$, as though all the excess charge on the sphere were concentrated at its center.

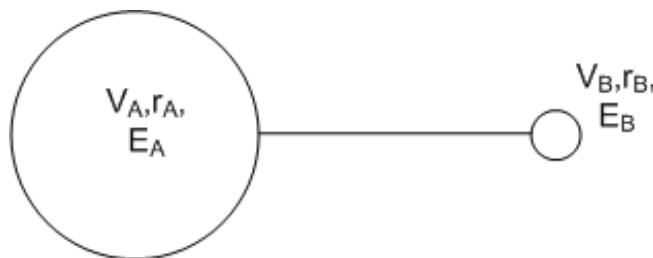
Outside the sphere, the magnitude of the electric field decreases with the square of the radial distance from the center of the sphere:

Electric field = zero (electrostatic)
inside a solid conducting sphere

ELECTROSTATICA

Consecințe:

C 8. Efectul de vârf.

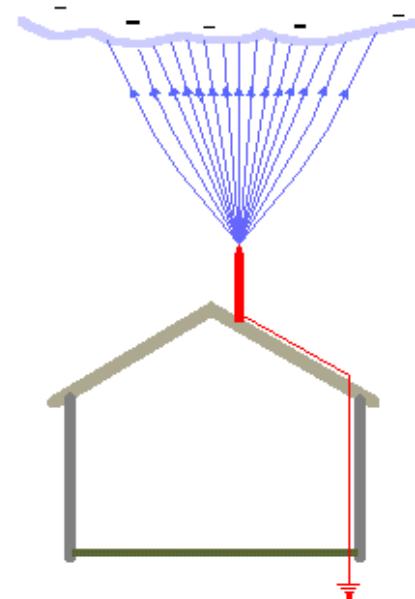


$$V_A = V_B$$

$$\frac{q_A}{4\pi\epsilon_0 r_A} = \frac{q_B}{4\pi\epsilon_0 r_B} \Rightarrow \frac{q_A}{q_B} = \frac{r_A}{r_B}$$

$$E_A = \frac{q_A}{4\pi\epsilon_0 r_A^2}, E_B = \frac{q_B}{4\pi\epsilon_0 r_B^2} \Rightarrow \frac{E_A}{E_B} = \frac{\frac{q_A}{r_A^2}}{\frac{q_B}{r_B^2}} = \frac{r_B}{r_A}$$

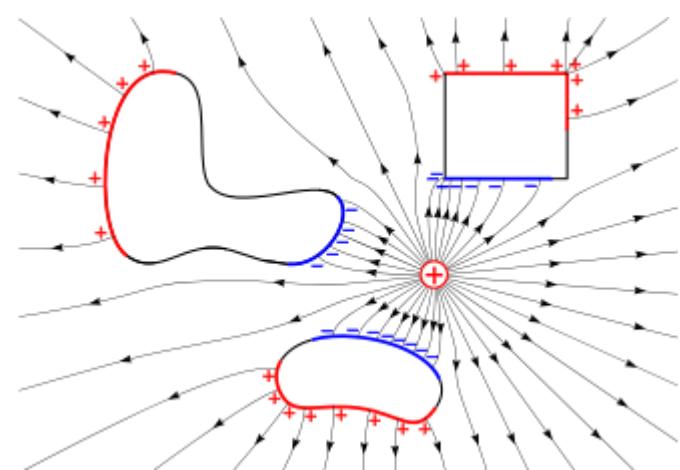
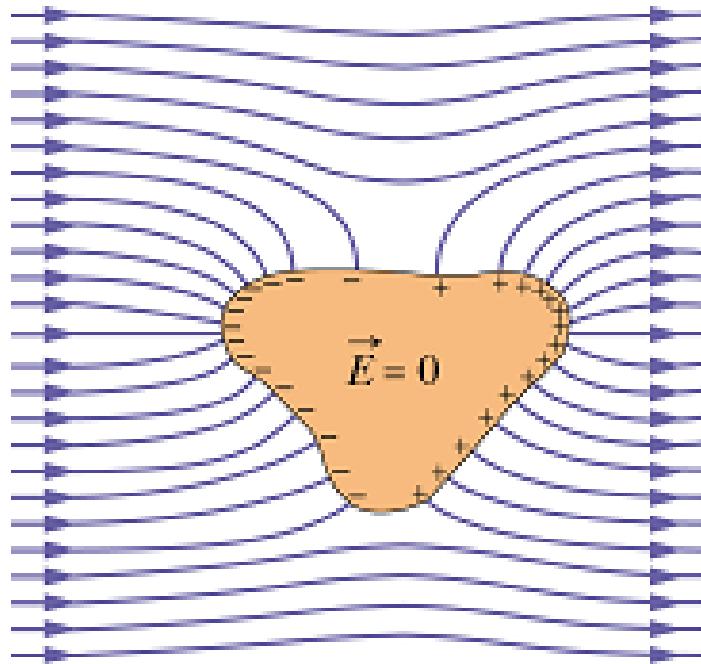
Zonele ascuțite ale corpurilor conductoare sunt regiuni în care câmpul electric este mai intens decât în zonele în care razele de curbură sunt mari.



ELECTROSTATICA

Consecințe:

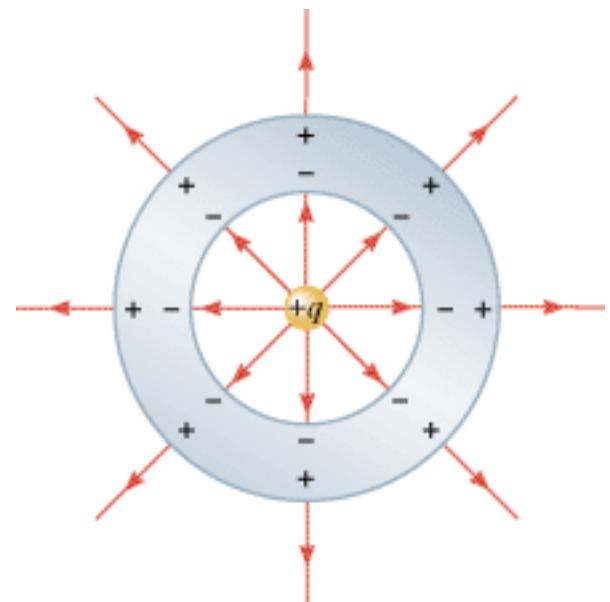
C 9. Fenomenul de influență electrostatică.



ELECTROSTATICA

Consecințe:

C 10. Ecranul electrostatic.

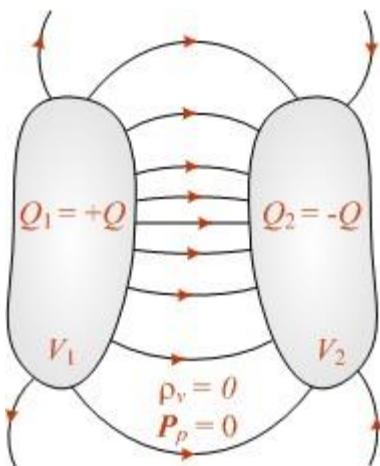


ELECTROSTATICĂ

Condensatoare electrice

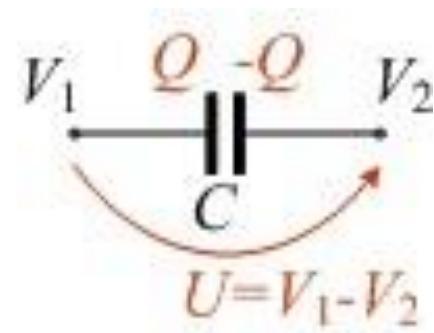
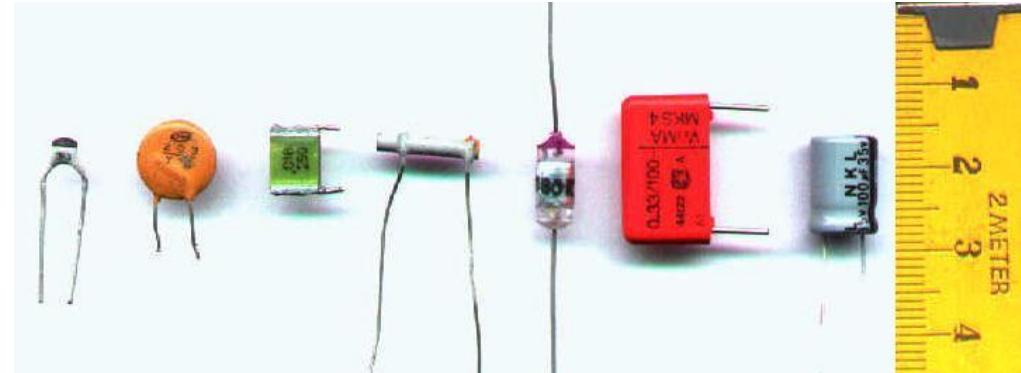
Condensatorul este un dispozitiv electric realizat din două corpuri conductoare omogene (metalice) încărcate cu sarcini electrice egale și de semn contrar între care există un mediu dielectric neîncărcat electric. Corpurile conductoare se numesc armături; toate liniile care pornesc de la armătura pozitivă ajung la armătura negativă.

Dielectricul poate să fie omogen sau neomogen și polarizația lui permanentă trebuie să fie nulă.



$$C = \frac{Q_1}{V_1 - V_2} = \frac{Q_2}{V_2 - V_1},$$

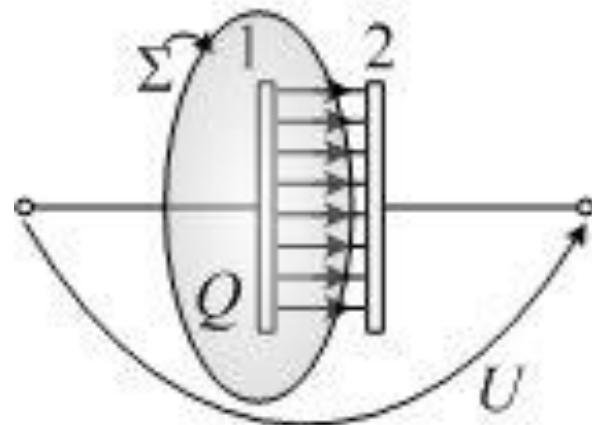
$$\left(C = \frac{Q}{U} \right)$$



ELECTROSTATICA

Condensatoare electrice

Calculul capacității condensatorului plan



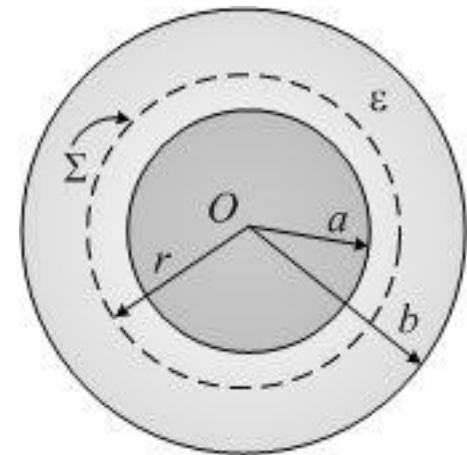
$$\psi_{\Sigma} = \int_{\Sigma} D dA = D \cdot A = Q \quad \rightarrow \quad D = \frac{Q}{A} \quad \text{si} \quad E = \frac{D}{\epsilon}$$

$$U_{1-2} = \int_1^2 Edl = Ed = \frac{Qd}{\epsilon A} \quad \rightarrow \quad C = \frac{Q}{U_{1-2}} = \frac{\epsilon A}{d}$$

ELECTROSTATICA

Condensatoare electrice

Calculul capacității condensatorului cilindric



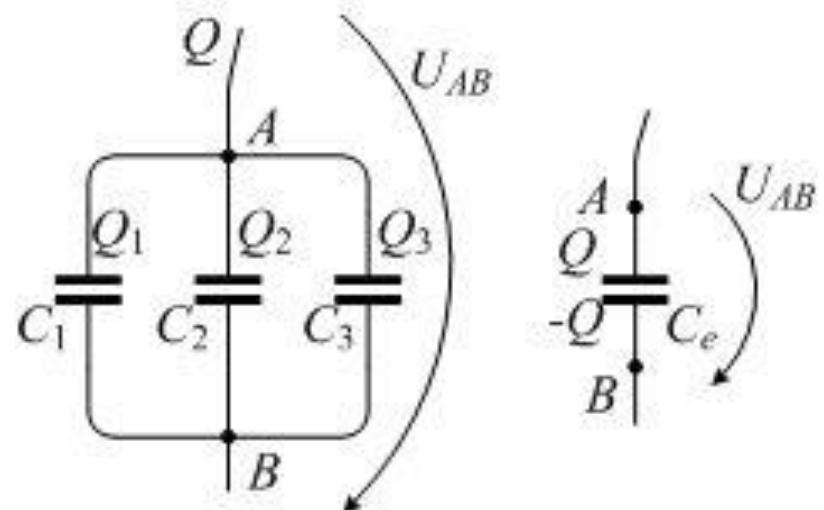
$$\psi_{\Sigma} = \int_{\Sigma} D dA = D \cdot 2\pi r l = Q \quad \rightarrow \quad D = \frac{Q}{2\pi r l} \quad \text{si} \quad E = \frac{D}{\epsilon}$$

$$U_{12} = \int_{r=a}^{r=b} E dr = \frac{Q}{2\pi\epsilon l} \int_a^b \frac{dr}{r} = \frac{Q}{2\pi\epsilon l} \ln \frac{b}{a} \quad \rightarrow \quad C = \frac{Q}{U_{12}} = \frac{2\pi\epsilon l}{\ln \frac{b}{a}}$$

ELECTROSTATICA

Condensatoare electrice

Condensatoare conectate in paralel



$$Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)U_{AB},$$

$$Q = C_e U_{AB},$$

$$C_e = \sum_{k=1}^n C_k$$

ELECTROSTATICA

Condensatoare electrice

Condensatoare conectate in serie

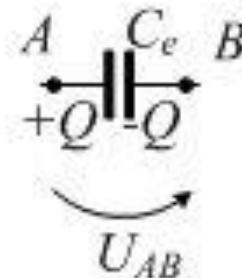
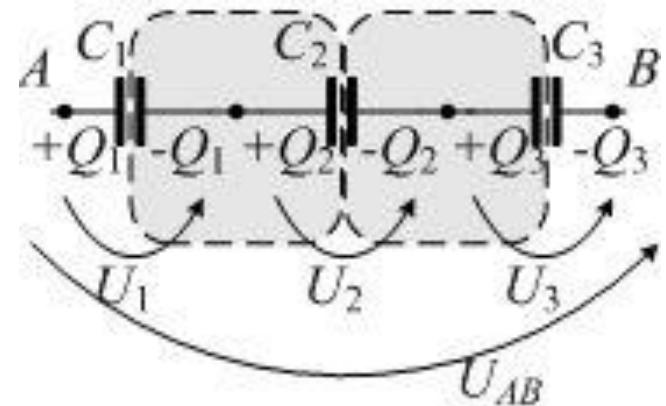
$$Q_1 = Q_2 = Q_3 = Q.$$

$$U_{AB} = U_1 + U_2 + U_3$$

$$\frac{Q}{C_e} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3},$$

sau

$$\frac{1}{C_e} = \sum_{k=1}^n \frac{1}{C_k}.$$



ELECTROSTATICĂ

Condensatoare electrice

Aplicații

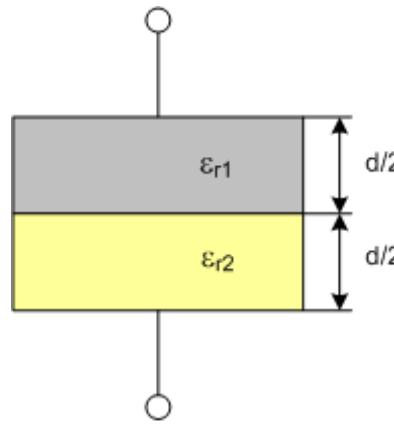
Determinați capacitatea fiecărui condensator din fig.

Cazul (a)

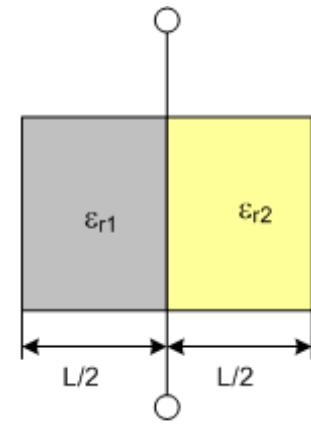
$$C_1 = \frac{\epsilon_0 \epsilon_{r1} S}{d/2}, \quad C_2 = \frac{\epsilon_0 \epsilon_{r2} S}{d/2}$$

Cele două condensatoare sunt în serie

$$C = \frac{C_1 C_2}{C_1 + C_2} = 25,46 \text{ pF}$$



(a)



(b)

$$\epsilon_{r1} = 4 \quad \epsilon_{r2} = 6$$

$$d = 5 \text{ mm} \quad S = 30 \text{ cm}^2$$

Cazul (b)

$$C_1 = \frac{\epsilon_0 \epsilon_{r1} S / 2}{d}, \quad C_2 = \frac{\epsilon_0 \epsilon_{r2} S / 2}{d}$$

Cele două condensatoare sunt în paralel

$$C = C_1 + C_2 = 26,53 \text{ pF}$$

ELECTROSTATICA

Condensatoare electrice

Energia câmpului electric din dielectricul unui condensator

$$i(t) = \frac{dq}{dt} \quad p_{abs} = ui = u \frac{dq}{dt} \quad p_{abs} = \frac{dW_e}{dt} \quad u \cdot dq = dW_e$$

$$W_e = \int_{St.\text{initiala}}^{St.\text{finala}} u \cdot dq = \int_0^Q u \cdot dq$$

Energia câmpului electric din dielectricul condensatorului plan

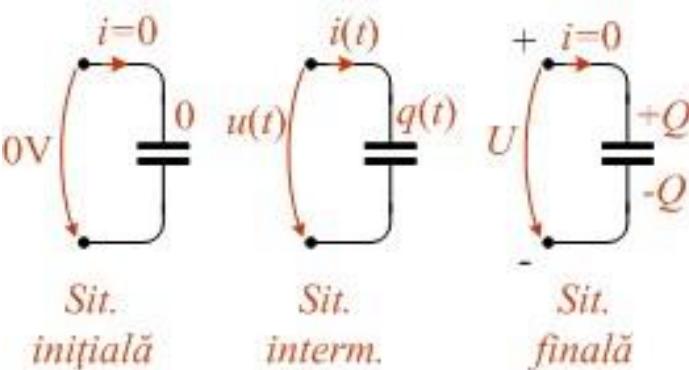
$$q = S \cdot D$$

$$dq = S \cdot dD$$

$$U = E \cdot d$$

$$We = \int_{St.\text{initiala}}^{St.\text{finala}} Ed \cdot S dD = \int_0^D E \cdot dD \cdot Vol.$$

$$W_e = \int_0^Q u dq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{QU}{2} = \frac{CU^2}{2} [J]$$

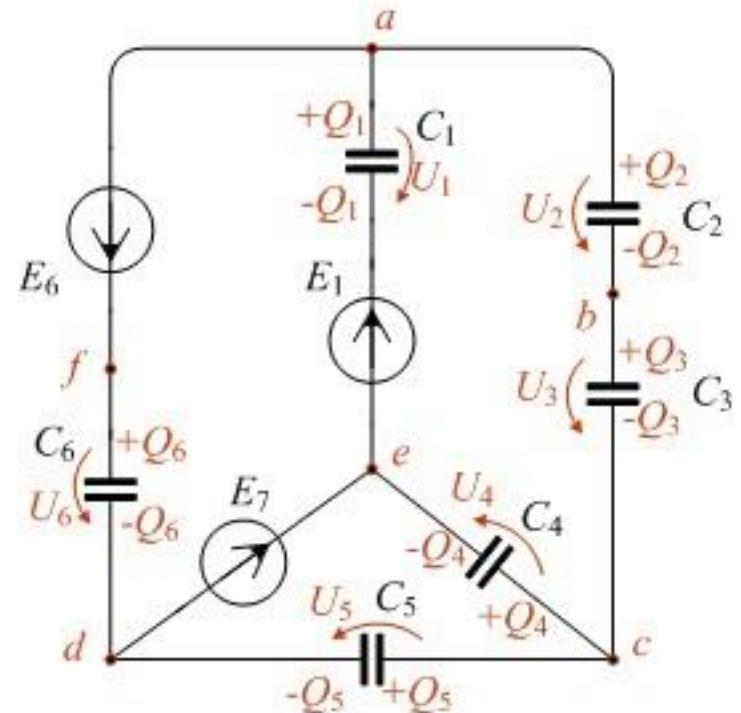


ELECTROSTATICĂ

Condensatoare electrice

Rețele cu condensatoare (Circuite statice)

Înainte de conectarea surselor de tensiune în circuit, condensatoarele pot avea o stare de încărcare inițială. Într-un circuit cu condensatoare se cunosc capacitățile condensatoarelor, sarcinile lor inițiale, tensiunile surselor și se cer sarcinile finale ale condensatoarelor sau tensiuni electrice între armăturile condensatoarelor. Pentru rezolvarea acestei probleme se folosesc „teoremele lui Kirchhoff” pentru circuitele cu condensatoare.



ELECTROSTATICĂ***Condensatoare electrice*****Rețele cu condensatoare (Circuite statice)****Teorema I Kirchhoff pt circuite statice**

$$\sum_{k \in a} Q_k^0 = \sum_{k \in a} Q_k ,$$

Teorema II Kirchhoff pt circuite statice

$$\oint_{\Gamma} E \cdot d\Gamma = 0, \text{ sau } \sum_{k \in p} U_k = 0.$$

suma sarcinilor inițiale de pe armăturile de condensator aflate în suprafața Σ_a este egală cu suma sarcinilor finale aflate pe armăturile acelorași condensatoare, armături cuprinse în Σ_a

Suma tensiunilor electrice pe un contur închis, (un ochi de rețea), este mereu nulă

ELECTROSTATICA

Condensatoare electrice

Rețele cu condensatoare (Circuite statice) - exemplu

$$Q_6^0 + Q_1^0 + Q_2^0 = Q_6 + Q_1 + Q_2$$

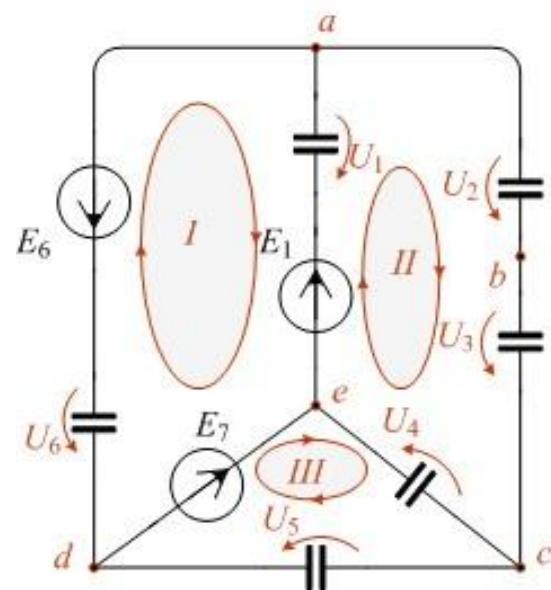
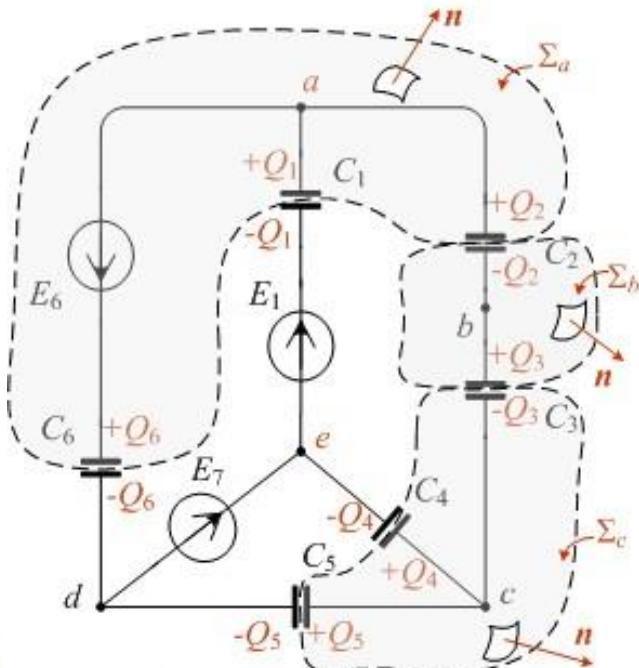
$$-Q_2^0 + Q_3^0 = -Q_2 + Q_3$$

$$-Q_3^0 + Q_4^0 + Q_5^0 = -Q_3 + Q_4 + Q_5$$

$$U_1 + E_1 + E_7 - U_6 + E_6 = 0$$

$$U_2 + U_3 + U_4 - E_1 - U_1 = 0$$

$$-U_4 + U_5 - E_7 = 0$$



BAZELE ELECTROTEHNICII

- note de curs -

Regimurile de desfășurare a fenomenelor electromagnetice

Regim static

- mărimele de stare nu variază în timp și nu se produc transformări energetice

Electrostatică

Magnetostatică

Regim staționar

- mărimele de stare nu variază în timp și au loc transformări energetice

Regim electric staționar

Regim magnetic staționar

Regim cvasistaționar

- mărimele de stare variază în timp dar suficient de lent pentru a se neglija undele electromagnetice

Regim anelectric

Regim amagnetic

Regim nestaționar (variabil)

- Nu este impusă nicio restricție asupra ritmului de variație în timp al mărimarilor

Regimul electric stationar - Electrocinetica

$$\left(\frac{\partial}{\partial t} = 0; \frac{d}{dt} = 0 \right) \quad (\mathbf{J} \neq 0)$$

Relațiile fundamentale ale electrocineticii

1. legea inducției electromagnetice – legea lui Faraday – scrisă pentru regimul electrocinetic staționar:

$$\oint_{\Gamma} \mathbf{E} d\mathbf{l} = 0, \quad \text{rot } \mathbf{E} = 0, \quad \text{rot}_S \mathbf{E} = 0$$

Ceea ce permite exprimarea intensității câmpului electric în funcție de un potențial electrocinetic staționar definit de:
 $\mathbf{E} = - \text{grad}V$.

2. legea circuitului magnetic.

$$\oint_{\Gamma} \mathbf{H} d\mathbf{l} = \int_{S\Gamma} \mathbf{J} dA + \frac{d}{dt} \int_{S\Gamma} \mathbf{D} dA$$



$$\Gamma \rightarrow P, S_{\Gamma} \rightarrow \Sigma, \Rightarrow \int_{\Gamma} \mathbf{H} d\mathbf{l} \rightarrow 0$$

$$\dot{i}_{\Sigma} = 0$$

Regimul electric stationar - Electrocinetica

$$\left(\frac{\partial}{\partial t} = 0; \frac{d}{dt} = 0 \right) \quad (\mathbf{J} \neq 0)$$

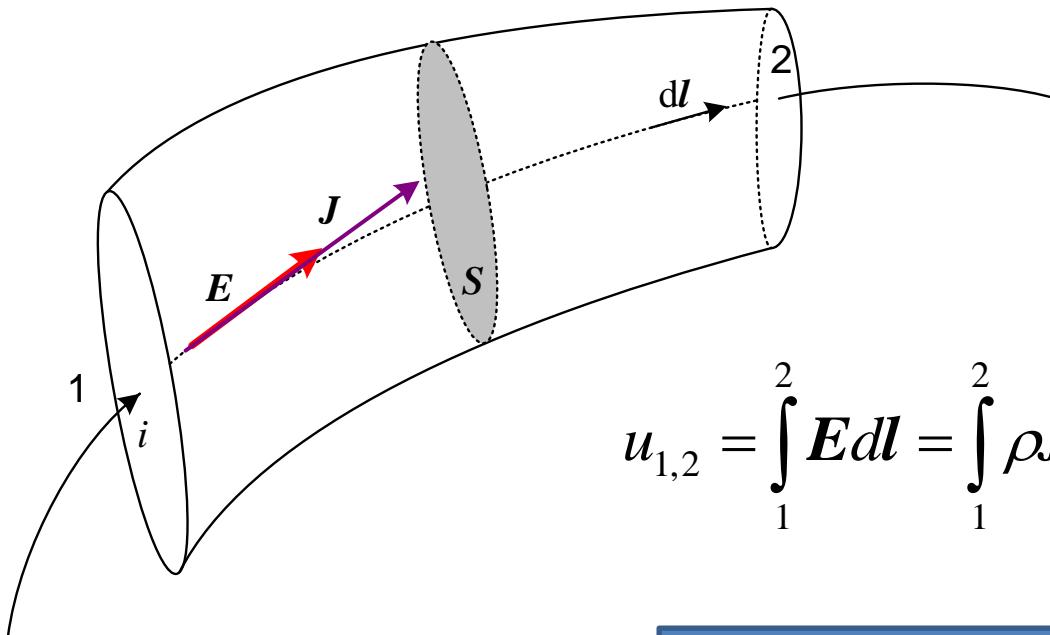
Relațiile fundamentale ale electrocineticii

3. Legea conducției electrice: $\mathbf{J} = \sigma(E + E_i)$

4. Legea transferului de energie: $p = EJ$

Electrocinetica – Regim de CURENT CONTINUU

Regimul electric stationar - Electrocinetica



$$u_{1,2} = \int_1^2 E dl = \int_1^2 \rho J dl = \int_1^2 \rho \frac{i}{S} dl = i \int_1^2 \frac{\rho}{S} dl = iR$$

Daca portiunea de conductor este omogena (ρ const) si are sectiunea transversala constata, rezistenta electrica devine:

$$R = \frac{\rho L}{S}$$

Electrocinetica – Regim de CURENT CONTINUU

Regimurile de desfășurare a fenomenelor electromagnetice

Regim static

- mărimele de stare nu variază în timp și nu se produc transformări energetice

Electrostatică

Magnetostatică

Regim staționar

- mărimele de stare nu variază în timp și au loc transformări energetice

Regim electric staționar

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- Nu este impusă nicio restricție asupra ritmului de variație în timp al mărimarilor

MAGNETOSTATICA

$$\left(\frac{\partial}{\partial t} = 0; \frac{d}{dt} = 0; J = 0 \right)$$

Relațiile fundamentale ale magnetostaticii

1. Legea circuitului magnetic – scrisă pentru regim static:

$$\oint_{\Gamma} H dl = 0, \text{rot} H = 0$$

Se poate defini o funcție scalară numită potențial magnetostatic scalar:

$$V_m(P) = V_m(P_0) - \int_{P_0}^P H dl$$

2. Legea fluxului magnetic $\phi_{\Sigma} = 0, \text{div} B = 0$

3. Legea legăturii dintre BH $B = \mu_0(H + M)$

CAMPUL MAGNETIC IN REGIM STATIONAR

$$\left(\frac{\partial}{\partial t} = 0; \frac{d}{dt} = 0; J \neq 0 \right)$$

Relațiile fundamentale

1. **Legea circuitului magnetic** – scrisă pentru regim stationar:

$$\oint_{\Gamma} H dl = \int_{S\Gamma} J dA, \text{rot } H = J$$

2. **Legea fluxului magnetic** $\phi_{\Sigma} = 0, \text{div } B = 0$

3. **Legea legăturii dintre BH** $B = \mu_0(H + M)$

CAMPUL MAGNETIC IN REGIM STATIONAR

CIRCUITE MAGNETICE

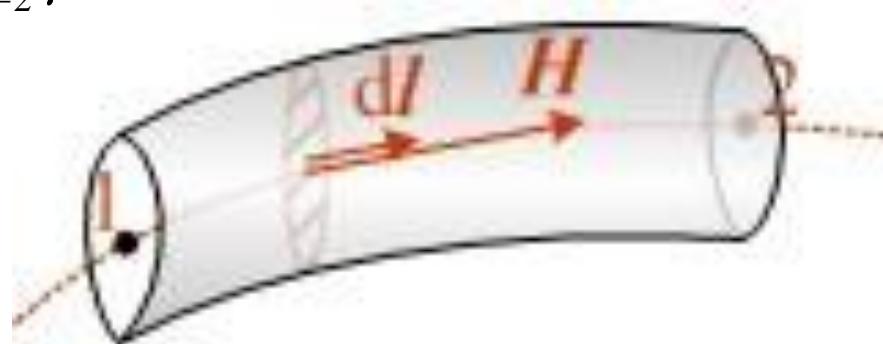
$$u_{m_{1-2}} = \int_{1-2} \mathbf{H} \cdot d\mathbf{l}$$

Se consideră un tub de flux magnetic – definit ca totalitatea liniilor de câmp magnetic care trec prin punctele unei curbe închise. Fluxul magnetic este constant în orice secțiune a tubului de flux.

$$u_{m_{1-2}} = \int_{1-2} \mathbf{H} \cdot d\mathbf{l} \cdot \frac{S}{S} \cdot \frac{\mu}{\mu} = \Phi_f \cdot \int_{1-2} \frac{d\mathbf{l}}{\mu S} = \Phi_f \cdot R_m$$

$$R_m = \frac{u_m}{\Phi_f}.$$

Reluctanța sau rezistența magnetica



CAMPUL MAGNETIC IN REGIM STATIONAR

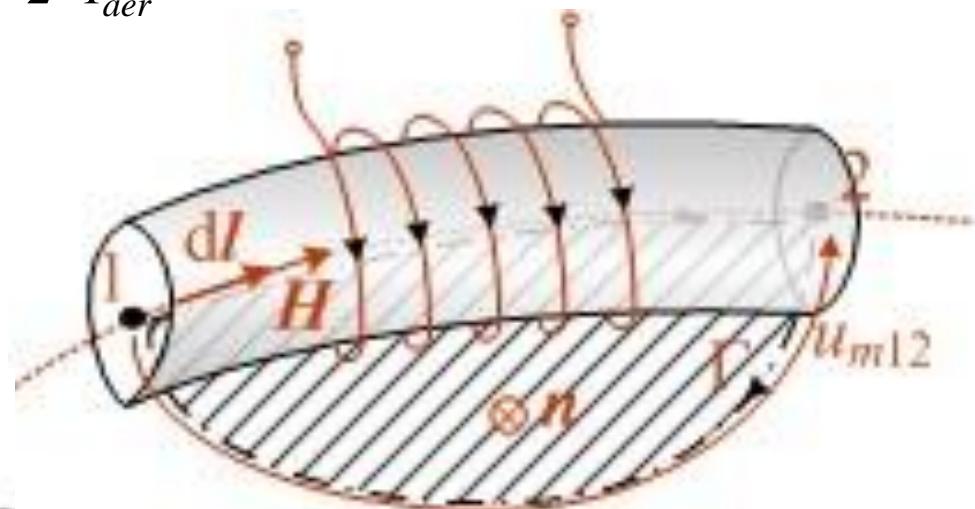
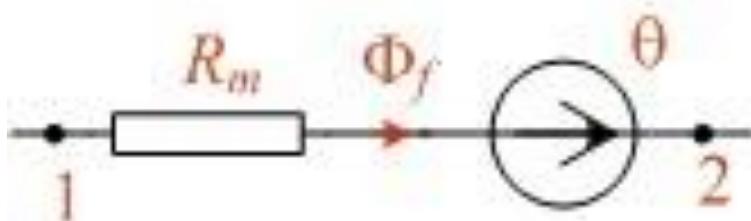
CIRCUITE MAGNETICE

$$\oint_{\Gamma} \mathbf{H} \cdot d\mathbf{l} = Ni$$

$$\oint_{\Gamma} \mathbf{H} \cdot d\mathbf{l} = \int_{1-2_{Fe}} \mathbf{H} \cdot d\mathbf{l} + \int_{2-1_{aer}} \mathbf{H} \cdot d\mathbf{l} = R_m \Phi_f - u_{m_{1-2}}$$



$$u_{m_{1-2}} = R_m \Phi_f - \vartheta$$

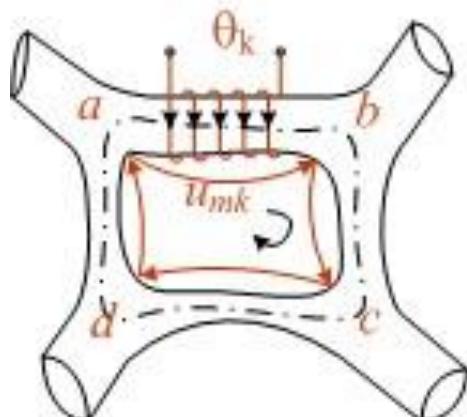
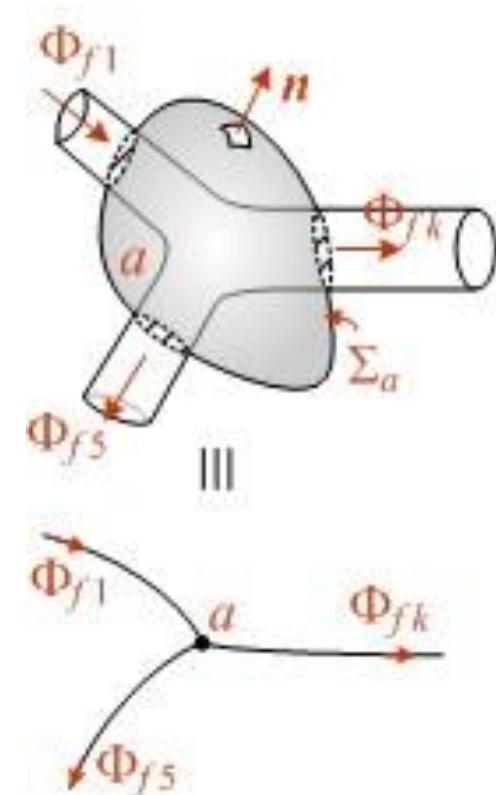


CAMPUL MAGNETIC IN REGIM STATIONAR

Teoremele lui Kirchhoff pentru circuite magnetice

$$\iint_{\Sigma} \mathbf{B} \cdot d\mathbf{S} = \sum_{k \in a} \iint_{S_k} \mathbf{B} \cdot d\mathbf{S} = \sum_{k \in a} \Phi_{f_k} = 0$$

suma algebrică a fluxurilor magnetice fasciculare care străbat laturile unui circuit magnetic, care se strâng în nodul a este nulă.

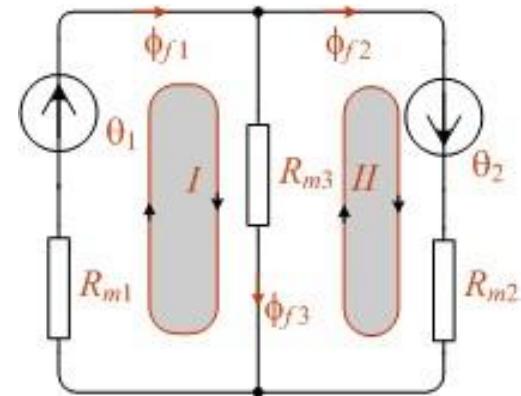
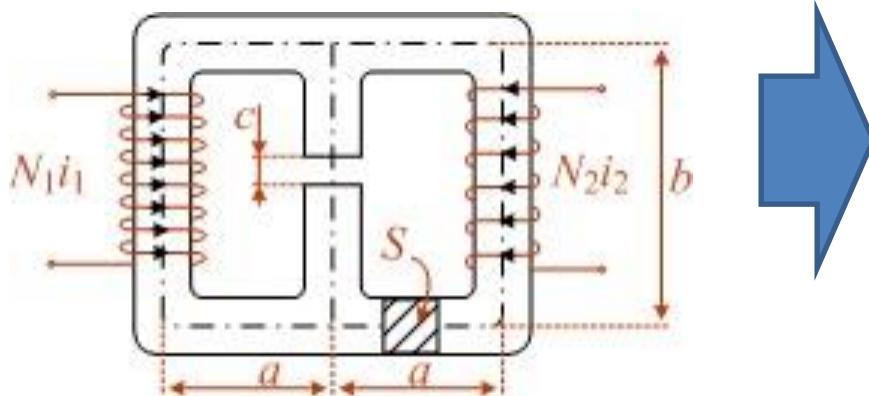


$$\oint_{\Gamma} \mathbf{H} \cdot d\mathbf{l} = \sum_{k \in (p)} u_{m_k} = 0$$

Suma algebrică a tensiunilor magnetice măsurate la bornele laturilor k, laturi care alcătuiesc ochiul (p), este zero

CAMPUL MAGNETIC IN REGIM STATIONAR

Teoremele lui Kirchhoff pentru circuite magnetice



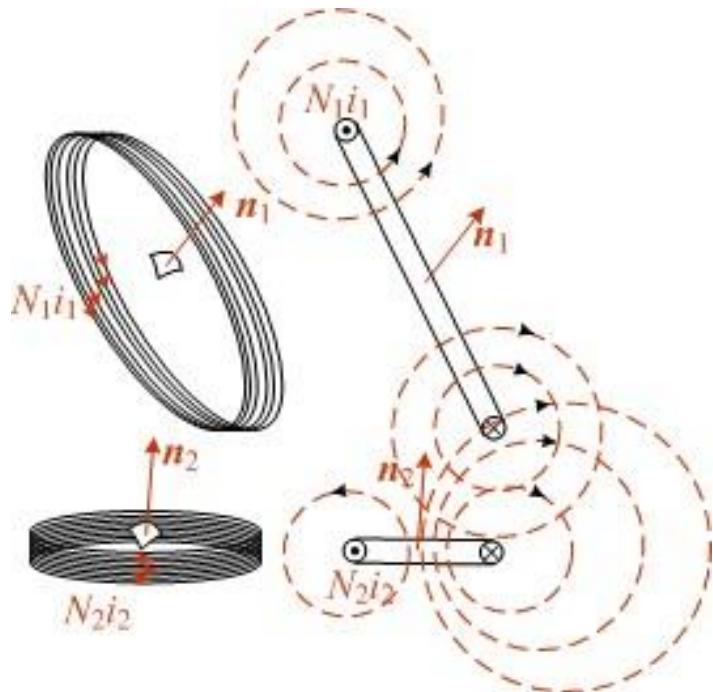
$$\begin{cases} \Phi_{f_1} = \Phi_{f_2} + \Phi_{f_3} \\ \theta_1 = R_{m_1} \Phi_{f_1} + R_{m_3} \Phi_{f_3} \\ \theta_2 = R_{m_2} \Phi_{f_2} - R_{m_3} \Phi_{f_3} \end{cases}$$

$$R_{m_1} = R_{m_2} = \frac{2a + b}{\mu S},$$

$$R_{m_3} = \frac{b - c}{\mu S} + \frac{c}{\mu_0 S}.$$

CAMPUL MAGNETIC IN REGIM STATIONAR

Bobine electrice. Inductivitati



Inductivitatea proprie a primei bobine este o mărime definită prin raportul dintre fluxul magnetic total ce străbate bobina 1 și curentul i_1 care „a produs“ fluxul magnetic:

$$L_{11} = \frac{\Phi_{t_{11}}}{i_1} = \frac{N_1 \Phi_{f_{11}}}{i_1}$$

Se numește inductivitate mutuală între bobinele 2 și 1 mărimea:

$$L_{21} = \frac{\Phi_{t_{21}}}{i_1} = \frac{N_2 \Phi_{f_{21}}}{i_1}$$

$$L_{12} = L_{21} \quad |L_{12}| = M$$

BAZELE ELECTROTEHNICII

- note de curs -

Teoremele fundamentale ale electromagnetismului

Teorema conservării sarcinii electrice

$$i_{\Sigma} = - \frac{d}{dt} q_{D\Sigma}$$

$$\int_{S\Sigma} J dA = - \frac{d}{dt} \int_{D\Sigma} \rho_v dv$$

Intensitatea curentului electric ce părăsește orice suprafață închisă este egală cu viteza de scădere a sarcinii electrice din domeniul $D_{\text{--inchis}}$ de acea suprafață:

Demonstratie:

Se consideră o suprafață deschisă $S\Gamma$ care se sprijină pe curba închisă gama. Legea circuitului magnetic pe această curbă este:

$$u_{m\Gamma} = \int_{\Gamma} H dl = H_{tmed} l_{\Gamma} \rightarrow 0$$

$$i_{\Sigma} + \frac{d}{dt} \psi_{\Sigma} = 0$$



$$\int_{\Gamma} H dl = \int_{S\Gamma} J dA + \frac{d}{dt} \int_{S\Gamma} D dA$$



Dacă diametrul curbei gama tinde către zero, aceasta se reduce la un punct, iar suprafața deschisă $S\Gamma$ tinde către o suprafață închisă Σ .

Teoremele fundamentale ale electromagnetismului

Teorema energiei electromagnetice

$$P_{\Sigma} = P_{D\Sigma} + \frac{\partial W_{em}}{\partial t}$$

Demonstratie:

$$\text{rot } E = -\frac{\partial B}{\partial t}$$



$$E \text{rot } H - H \text{rot } E = JE + E \frac{\partial D}{\partial t} + H \frac{\partial B}{\partial t}$$

$$\text{div}(E \times H) = H \text{rot } E - E \text{rot } H$$

$$E \frac{\partial D}{\partial t} = E \epsilon \frac{\partial E}{\partial t} = \frac{\epsilon}{2} \frac{\partial E^2}{\partial t} = \frac{\partial}{\partial t} \left(\frac{DE}{2} \right),$$

$$H \frac{\partial B}{\partial t} = \frac{\partial}{\partial t} \frac{BH}{2}$$

Puterea transferată de câmpul electromagnetic unui domeniu imobil prin frontieră acestuia D_{σ} este egală cu puterea transferată corporilor din domeniul P_{σ} plus viteza de creștere a energiei câmpului electromagnetic din domeniu:

$$-\text{div}(E \times H) = JE + \frac{\partial}{\partial t} \left(\frac{DE}{2} + \frac{BH}{2} \right)$$

$$-\text{div}S = p + \frac{\partial w_{em}}{\partial t}$$

Vector Poynting

Teoremele fundamentale ale electromagnetismului

Teorema forțelor generalizate

Forța generalizată X_k cu care câmpul electric acționează asupra sistemelor de corpu este:

$$X_k = -\frac{\partial W_e}{\partial x_k} \Big|_{\psi=ct} = \frac{\partial W_e}{\partial x_k} \Big|_{U=ct} \quad W_e = \int_D \frac{DE}{2} dv$$

Forța generalizată X_k cu care câmpul magnetic acționează asupra sistemelor de corpu este:

$$X_k = -\frac{\partial W_m}{\partial x_k} \Big|_{\phi=ct} = \frac{\partial W_m}{\partial x_k} \Big|_{i=ct} \quad W_m = \int_D \frac{BH}{2} dv$$

Teoremele fundamentale ale electromagnetismului

Teorema forțelor generalizate

Exemplu:

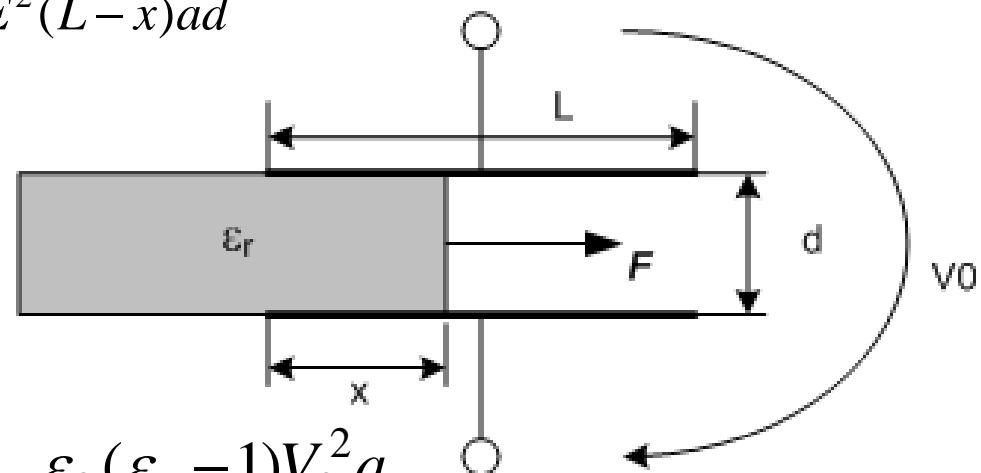
$$F = \frac{\partial W_e}{\partial x}$$

$$W_e = \int \frac{\epsilon E^2}{2} dV = \frac{1}{2} \epsilon_0 \epsilon_r E^2 x ad + \frac{1}{2} \epsilon_0 E^2 (L-x)ad$$

$$E = \frac{V_0}{d}$$



Un condensator plan cu dimensiunile L și a și cu distanța dintre armături d este menținut la o tensiune constantă V₀. Dacă o bucată de dielectric cu ϵ_r este scoasă dintre armături (rămânând x) calculați forța ce trebuie să aducă dielectricul în poziția originală.



$$F = \frac{\partial W_e}{\partial x} = \frac{1}{2} \epsilon_0 (\epsilon_r - 1) \frac{V_0^2}{d^2} da = \frac{\epsilon_0 (\epsilon_r - 1) V_0^2 a}{2d}$$

Teoremele fundamentale ale electromagnetismului

Teorema forțelor generalizate

Exemplu:

$$W_e = \frac{CV_0^2}{2}$$

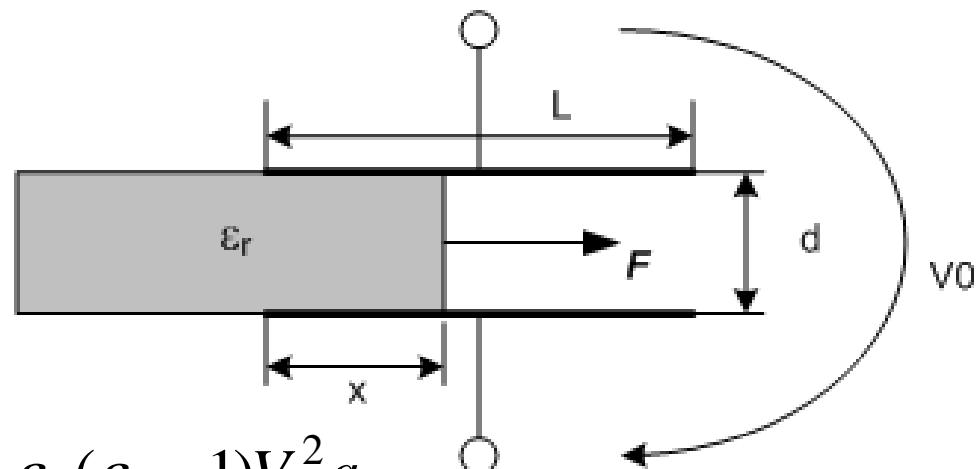
$$C = C_1 + C_2 =$$

$$= \frac{\epsilon_0 \epsilon_r a x}{d} + \frac{\epsilon_0 \epsilon_r a (L - x)}{d}$$



$$F = \frac{\partial W_e}{\partial x} = \frac{1}{2} \epsilon_0 (\epsilon_r - 1) \frac{V_0^2}{d^2} da = \frac{\epsilon_0 (\epsilon_r - 1) V_0^2 a}{2d}$$

Un condensator plan cu dimensiunile L și a și cu distanța dintre armături d este menținut la o tensiune constantă V_0 . Dacă o bucată de dielectric cu ϵ_r este scoasă dintre armături (rămânând x) calculați forța ce tinde să aducă dielectricul în poziția originală.



Regimurile de desfășurare a fenomenelor electromagnetice

Regim static

- mărimele de stare nu variază în timp și nu se produc transformări energetice

Electrostatică

Magnetostatică

Regim staționar

- mărimele de stare nu variază în timp și au loc transformări energetice

Regim electric staționar

Regim magnetic staționar

Regim cvasistaționar

- mărimele de stare variază în timp dar suficient de lent pentru a se neglija undele electromagnetice

Regim anelectric

Regim amagnetic

Regim nestaționar (variabil)

- Nu este impusă nicio restricție asupra ritmului de variație în timp al mărimarilor

$$\text{rot } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\text{rot } \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

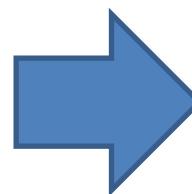
$$\text{div } \mathbf{D} = \rho_v$$

$$\text{div } \mathbf{B} = 0$$

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_i)$$



$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu \frac{\partial H_z}{\partial t}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x + \epsilon \frac{\partial E_x}{\partial t}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y + \epsilon \frac{\partial E_y}{\partial t}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z + \epsilon \frac{\partial E_z}{\partial t}$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{1}{\epsilon} \rho_v$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial z}{\partial x} - \frac{\partial E_y}{\partial y} = -\mu \frac{\partial H_z}{\partial t}$$

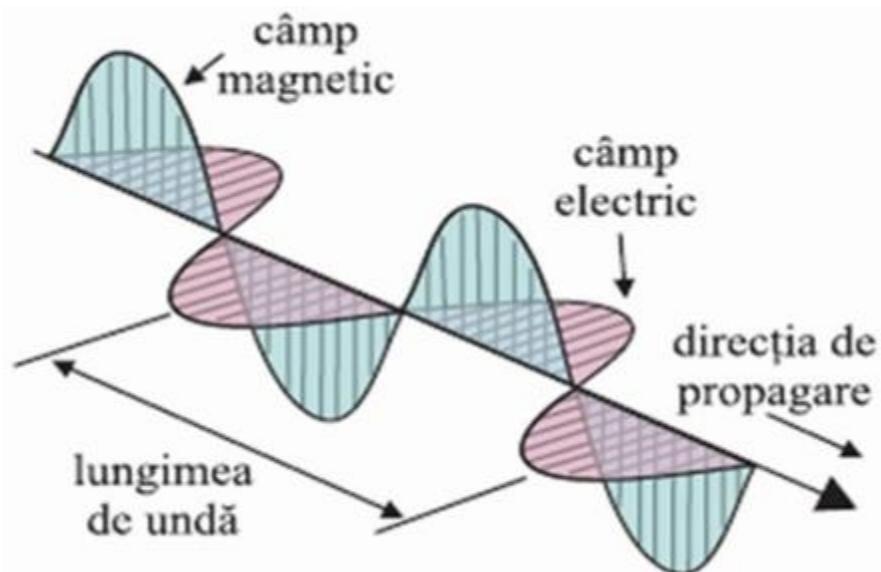
$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x + \epsilon \frac{\partial E_x}{\partial t}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y + \epsilon \frac{\partial E_y}{\partial t}$$

$$\frac{\partial z}{\partial x} - \frac{\partial H_y}{\partial y} = J_z + \epsilon \frac{\partial E_z}{\partial t}$$

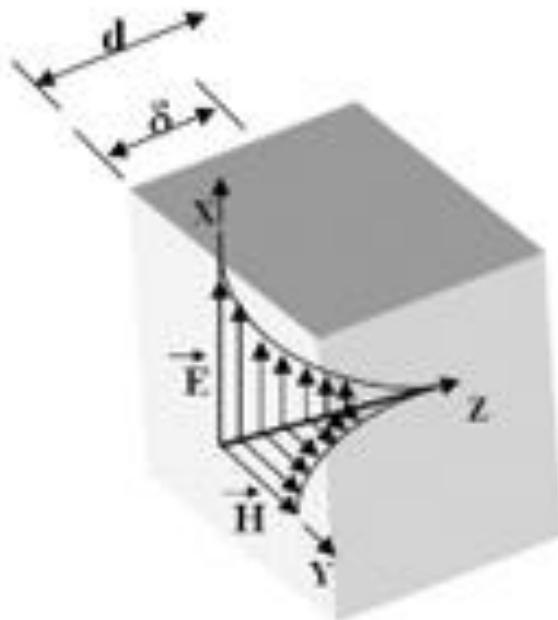
$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{1}{\epsilon} \rho_v$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$$



$$\frac{\partial^2 H_z}{\partial x^2} - \epsilon \mu \frac{\partial^2 H_z}{\partial t^2} = 0$$

$$\frac{\partial^2 E_y}{\partial x^2} - \epsilon \mu \frac{\partial^2 E_y}{\partial t^2} = 0$$



δ = Penetration depth of the electromagnetic wave

d = Material thickness

\vec{E} = Electric field

\vec{H} = Magnetic field

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$$