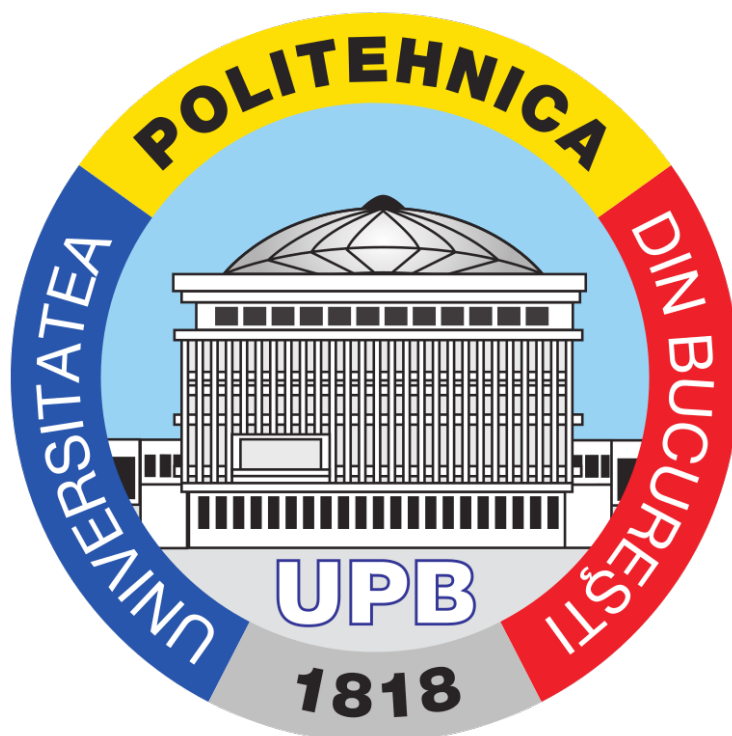


Problema curs 6
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1.

$$H(s) = \frac{1+s^2}{s^2+2s+1}$$

a) $1+s^2=0 \Rightarrow s^2=-1 \Rightarrow s = \pm i$ ← zerouri

$$s^2+2s+1=0 \Rightarrow (1+s)^2=0 \Rightarrow s_{1,2} = -1 \text{ ← pol de grad 2}$$

b) Sistemul este strict extern stabil d.p.d.v. dinamic dacă polii lui $H(s) \in C^{-\infty}$
Cum $s_{1,2} < 0$ (polii sunt în semiplanul stâng) \Rightarrow se respectă condiția.

c) Caracteristica de frecvență

$$s \rightarrow j\omega$$

$$H(j\omega) = \frac{1+(j\omega)^2}{(j\omega)^2+2j\omega+1} = \frac{1-\omega^2}{-\omega^2+2j\omega+1}$$

d) Caracteristica de amplitudine

$$H(\omega) = |H(j\omega)| = \sqrt{\frac{(1-\omega^2)^2}{(1-\omega^2)^2+4\omega^2} + \left(\frac{4\omega(1-\omega^2)}{(1-\omega^2)^2+4\omega^2}\right)^2}$$

e) Caracteristica de fază

$$f(\omega) = -\arctan \omega$$

$$f(\omega) = \pi + \arctg\left(\frac{1}{\omega}\right) - \arctg \frac{4\omega}{1-\omega^2}$$

f) Banda de frecvență

$$\frac{H(0)}{\sqrt{2}} = \frac{1}{\omega^2+1}$$

$$H(0) = 1 \Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\omega^2+1}$$

$$\frac{H(0)}{\sqrt{2}} = \sqrt{\frac{(1-\omega^2)^2}{(1-\omega^2)^2+4\omega^2}} \Rightarrow (1-\omega^2)^2 = 4\omega^2 \Rightarrow (1-\omega^2) = \pm 2\omega$$

Alegem doar componenta pozitivă.

$$\omega^2+2\omega-1=0, \omega > 0 \Rightarrow \omega = \sqrt{2}-1$$

$$f = \frac{\omega}{2\pi} \Rightarrow B \in \left[0, \frac{\sqrt{2}-1}{2\pi}\right]$$