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**Algorithm 1:** CTC Loss alpha computation

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**Data:**  $out_{m \times n}$  (result of softmax), where  $m = \bar{W}/4$ ,  $n = |\hat{A}|$ ,

$l$  (label encoded by alphabet),

$bl=0$  (blank index)

**begin**

$Loss = 0$

$L = 2 \times len(l) + 1$

$T = m$

$a = zeros(T, L)$

$a_0^0 = out_0^{bl}$

$a_0^1 = out_0^{l_0}$

$c = \sum_{i=0}^1 a_0^i$

**for**  $i := 0$  **to** 1 **do**

$a_0^i = \frac{a_0^i}{c}$

$Loss = Loss + c$

**for**  $t := 1$  **to**  $T$  **do**

$s = \max(0, L - 2 \times (T - t))$

$e = \min(2 \times t + 2, L)$

**for**  $s := 1$  **to**  $L$  **do**

$i = \frac{s-1}{2}$

$red = a_{t-1}^s$

$blue = 0$

**if**  $s > 0$  **then**

$blue = a_{t-1}^{s-1}$

**if**  $s \bmod 2 = 0$  **then**

$a_t^s = (red + blue) \times out_t^{bl}$

**else if**  $s = 1$  **or**  $l_i = l_{i-1}$  **then**

$a_t^s = (red + blue) \times out_t^{l_i}$

**else**

$orange = a_{t-1}^{s-2}$

$a_t^s = (red + blue + orange) \times out_t^{l_i}$

$c = \sum_{i=s}^e a_t^i$

**for**  $i := s$  **to**  $e$  **do**

$a_t^i = \frac{a_t^i}{c}$

$Loss = Loss + c$

**return**  $Loss$

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**Algorithm 2:** CTC Loss gradient computation

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**Data:**  $out_{m \times n}$  (result of softmax), where  $m=\bar{W}/4$ ,  $n=|\hat{A}|$ ,  
 $l$  (label encoded by alphabet),  
 $bl=0$  (blank index),  
 $a, b$  (alpha and beta from paper)

**begin**

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 $L = 2 \times len(l) + 1$   
 $T = m$   
 $grad = zeros(T, n)$   
 $ab = a * b$   
for  $s := 1$  to  $L$  do  
  if  $s \bmod 2 = 0$  then  
    for  $t := 1$  to  $T$  do  
       $grad_t^{bl} += ab_t^s$   
       $ab_t^s = \frac{ab_t^s}{out_t^{bl}}$   
  else  
    for  $t := 1$  to  $T$  do  
       $i = \frac{s-1}{2}$   
       $grad_t^i += ab_t^s$   
       $ab_t^s = \frac{ab_t^s}{out_t^i}$   
 $absum = zeros(T)$   
for  $t := 1$  to  $T$  do  
   $absum_t = \sum_{s=1}^L ab_t^s$   
for  $t := 1$  to  $T$  do  
  for  $i := 1$  to  $n$  do  
     $grad_t^i = out_t^i - \frac{grad_t^i}{out_t^i * absum_t}$   
return  $grad$ 
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