Algorithm 1: *ComputeAlpha*

Data: $out_{m \times n}$ (result of softmax), where $m = \bar{W}_{\mathbf{unpadded}}/4$, $n = |\hat{A}|$,
$label$ (encoded by alphabet),
T, L, $bl = 0$ (blank index)
begin
  $a = zeros(T, L)$
  $a[0][0] = out_{unpadded}[0][bl]$
  $a[0][1] = out_{unpadded}[0][label[0]]$
  $c = a[0][0] + a[0][1]$
  if $c > 0$ **then**
    $a[0][0] = \frac{a[0][0]}{c}$
    $a[0][1] = \frac{a[0][1]}{c}$
  for $t := 1$ **to** T **do**
    $start = \max(0, L - 2 \times (T - t))$
    $end = \min(2 \times t + 2, L)$
    for $s := start$ **to** L **do**
      $i = floor(\frac{s-1}{2})$
      $a[t][s] = a[t-1][s]$
      if $s > 0$ **then**
        $a[t][s] = a[t][s] + a[t-1][s-1]$
      if s ***mod*** $2 = 0$ **then**
        $a[t][s] = a[t][s] \times out_{unpadded}[t][bl]$
      else if $s = 1$ ***or*** $label[i] = label[i-1]$ **then**
        $a[t][s] = a[t][s] \times out_{unpadded}[t][label[i]]$
      else
        $a[t][s] = (a[t][s] + a[t-1][s-2]) \times out_{unpadded}[t][label[i]]$
    $c = \sum_{i=start}^{end} a[t][i]$
    if $c > 0$ **then**
      for $s := start$ **to** end **do**
        $a[t][s] = \frac{a[t][s]}{c}$
  return a

Algorithm 2: CTC Loss and **softmax** gradient computation

Data: $out_{M \times N}$ (result of softmax), where $M = \bar{W}_{\mathbf{padded}}/4$, $N = |\hat{A}|$,
$label$ (encoded by alphabet),
$w = \bar{W}_{\mathbf{unpadded}}/4$,
$bl = 0$ (blank index)
begin
  $L = 2 \times len(label) + 1$
  $T = w$
  $out_{unpadded} = zeros(T, N)$
  for $t := 0$ **to** T **do**
    for $n := 0$ **to** N **do**
      $out_{unpadded}[t][n] = out_{padded}[t][n]$
  $a = ComputeAlpha(out_{unpadded}, label, T, L, bl)$
  $out^{flipped}_{unpadded} = fliplr(out_{unpadded})$
  $label_{reversed} = reverse(label)$
  $b = ComputeAlpha(out^{flipped}_{unpadded}, label_{reversed}, T, L, bl)$
  $b = flipud(fliplr(b))$
  $ab = a * b$
  $lab = zeros(T, N)$
  for $s := 0$ **to** L **do**
    if s ***mod*** $2 = 0$ **then**
      for $t := 0$ **to** T **do**
        $lab[t][bl] = lab[t][bl] + ab[t][s]$
        $ab[t][s] = \frac{ab[t][s]}{out_{unpadded}[t][bl]}$
    else
      for $t := 0$ **to** T **do**
        $i = floor(\frac{s-1}{2})$
        $lab[t][i] = lab[t][i] + ab[t][s]$
        $ab[t][s] = \frac{ab[t][s]}{out_{unpadded}[t][label[i]]}$
  $lh = zeros(T)$
  for $t := 0$ **to** T **do**
    $lh[t] = \sum_{s=1}^{L} ab[t][s]$
  $loss = -\sum_{t=1}^{T} \ln lh[t]$
  $softmaxGrad = zeros(M, N)$
  for $t := 0$ **to** T **do**
    for $n := 0$ **to** N **do**
      $softmaxGrad[t][n] = out_{unpadded}[t][n] - \frac{lab[t][n]}{out_{unpadded}[t][n] * lh[t]}$
  return $loss$, $softmaxGrad$