Foam: A Python package for forward asteroseismic modelling of gravity modes

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Summary

Asteroseismology, the study of stellar pulsations, offers insights into the internal structures and evolution of stars. Analysing the variations in a star's brightness allows the determination of fundamental properties such as mass, radius, age, and chemical composition. Asteroseismology heavily relies on computational tools, but a significant number of them are closed-source, thus inaccessible to the broader astronomic community. This manuscript presents Foam, a python package designed to perform forward asteroseismic modelling of stars exhibiting gravity modes. It automates and streamlines a considerable fraction of the modelling process, comparing grids of theoretical stellar models and their oscillation frequencies to observed frequency sets in stars.

Foam offers the flexibility to employ diverse modelling approaches, allowing users to choose different methodologies for matching theoretically predicted oscillations to observations. It provides options to utilise various sets of observables for comparison with their theoretical counterparts, employ different merit functions for assessing goodness of fit, and to incorporate nested subgrids in a statistically rigorous manner. For applications of these methodologies in modelling observed gravity modes, refer to Michielsen et al. (2021) and Michielsen et al. (2023).

Introduction

Stars spend approximately 90% of their evolution on their so called *main sequence*, during which they fuse hydrogen into helium in their cores. In stars with masses above about 1.2 times the mass of the sun, the stellar core in which these fusion processes take place becomes convective. Macroscopic element transport in and near the convective cores of these stars has a large influence on their life, since it transports additional hydrogen from outside of the nuclear fusion region into this region. In this way it both prolongs the main-sequence lifetime of stars and enlarges the mass of the helium core at the end of the main sequence, which significantly influences all later stages of their evolution. However, these transport processes provide the largest uncertainties in stellar structure and evolution models for stars with convective cores, due to our poor understanding of macroscopic element transport and limited number of useful observations to test the theories. (See e.g. Anders & Pedersen, 2023 for a review on this topic.)

Through asteroseismology, we gain the means to unravel the interior structure of stars (Aerts et al., 2010; Aerts, 2021). Gravity (g-) modes in particular have a high sensitivity to the properties of the near-core region. These modes have buoyancy as their restoring force, have dominantly horizontal displacements, and oscillate with a period of several hours to a few days. Additionally, they can only propagate in the non-convective regions in the star, which makes their propagation cavity very sensitive to the size of the convective core. We can exploit the probing power of g-modes, observed in e.g. Slowly Pulsating B-type stars (Waelkens, 1991), to

41 investigate the physics in the interior of these stars, particularly the transition region between
42 the convective core and radiative envelope.

Statement of need

44 Some tools have been developed and made publicly available to model and determine stel-
45 lar parameters of solar-like oscillators, such as AIMS (Rendle et al., 2019), BASTA (Aguirre
46 Børsen-Koch et al., 2022), and pySYD (Chontos et al., 2022). However, there are several key
47 differences between the modelling of the pressure (p-) modes observed in solar-like oscillators,
48 and the modelling of the g-modes observed in more massive stars. First and foremost, the
49 well-known asteroseismic scaling relations used for solar-like oscillators cannot be extrapolated
50 to main-sequence stars with a convective core. Secondly, the effect of rotation on p-modes is
51 often included in a perturbative way, whereas the g-mode frequencies are strongly dependent
52 on rotation and require the inclusion of the Coriolis acceleration in a non-perturbative way.
53 Additionally the mass regime of stars with convective cores is subject to strong correlations
54 between several model parameters, which sometimes follow non-linear relationships. In this con-
55 text, the Mahalanobis distance (MD) (see Aerts et al., 2018 for its application to asteroseismic
56 modelling) provides a more appropriate merit function than the often used χ^2, since it tackles
57 both these non-linear correlations and includes uncertainties for the theoretical predictions.
58 The use of a different, more appropriate merit function significantly impacts modelling results.
59 This is demonstrated by Michielsen et al. (2021) in their comparison between the results
60 obtained by employing the MD versus χ^2, applied in the modelling of an observed star.

61 Foam was developed to be complimentary to the available modelling tools for solar-like oscillators.
62 It provides a framework for the forward modelling of g-modes in main-sequence stars with
63 convective cores, and tackles the differences in the modelling approach as compared to the
64 case of solar-like oscillators. Foam therefore extends the efforts to provide publicly available,
65 open-source tools for asteroseismic modelling to the g-mode domain, given that the currently
66 available tools predominantly concern the solar-like oscillators.

Software package overview

68 Foam is designed as a customisable pipeline. It will match theoretical models to observations,
69 computing the goodness of fit of each model based on the selected merit function. Afterwards
70 it will determine the best model alongside the uncertainty region of this solution based on
71 statistical criteria. On the observational side, it will take a list of frequencies as an input,
72 optionally complemented by additional information such as a set of surface properties (effective
73 temperature, surface gravity, luminosity, element surface abundances…). On the theoretical
74 side Foam will use a grid of theoretical stellar models, calculated by the user to suit their
75 specific needs. Although the current implementation is made for a grid of stellar equilibrium
76 models computed by MESA(Jermyn et al., 2023; Paxton et al., 2011, 2013, 2015, 2018, 2019),
77 whose pulsation frequencies are computed with GYRE(Townsend et al., 2018; Townsend &
78 Teitler, 2013), the majority of the code is not inherently dependent on MESA. By making
79 certain adjustments to the modelling pipeline, Foam could potentially employ grids generated
80 by different stellar evolution codes. Some suggestions how to approach this are given in
81 the description of the theoretical model grid in the online documentation. However, the
82 implementation of such functionality currently remains out of the scope of the project.

83 The script to run the pipeline can be altered in order to change the modelling approach you want
84 to take. The various configuration options, the installation procedure, and a walkthrough of how
85 to create your own modelling setup, are described in more detail in the online documentation.
86 Although it relies on grids of stellar equilibrium models computed by MESA as the source of
87 the theoretical model grid, MESA's installation itself is not required for Foam to function. The
88 installation of GYRE is however required, specifically since Foam relies on the `tar_fit.mX.kX.h5`

files included in the GYRE installation. This allows us to rescale the g-modes for various stellar rotation rates, following the traditional approximation of rotation Townsend (2020) and assuming rigid rotation. This facilitates computing the oscillation frequencies for the grid of stellar equilibrium models only once, and subsequently rescaling them to find the optimised rotation rate (see Michielsen et al., 2023). This approach avoids repeating the oscillation computations for a variety of rotation values, which would introduce extra dimensionality in the modelling problem in the form of adding the rotation rate as an additional free parameter.

Foam's modelling procedure can be broken down into following sequential steps of the pipeline:

- Extract all required parameters and quantities from the files in the theoretical MESA and GYRE grids.
- Construct the theoretical pulsation patterns for each stellar model. Thereafter select theoretical pulsation patterns matching the observational pattern whilst optimising their rotation rates. Finally combine this information with the models' surface properties.
- Calculate the likelihood of all the theoretical patterns according to the specified merit functions and observables. This list of observables consist of the pulsations, but can optionally be extended with spectroscopic or astrometric information.
- Exclude all the models that fall outside an n-sigma error box on the spectroscopic and astrometric constraints as acceptable solutions.
- Calculate the Akaike information criterion (AIC) (Claeskens & Hjort, 2008) corrected for small sample size. This statistical criterion rewards goodness of fit, but penalises model complexity in the form of additional free parameters. The AIC thus allows a statistical comparison between models of different (nested) grids where the number of free parameters is not the same.
- Calculate the 2 sigma uncertainty region of the maximum likelihood solution using Bayes' theorem.
- Make corner plots for all combinations of the different modelling choices (See Figure 1 for an example).
- Construct a table with the best model of the grid for each combination of different modelling choices.

Next to the tables with the best model parameters and their AIC values, the cornerplots provide a quick way to assess the output of the pipeline and visualise the modelling results. Figure 1 shows an example of such a cornerplot for the modelling of KIC 4930889 performed by Michielsen et al. (2023). It gives a clear indication of which models are included (coloured) or excluded (greyscale) from the uncertainty region, and indicates what the best models of the grid are (yellow, see the colour bar).
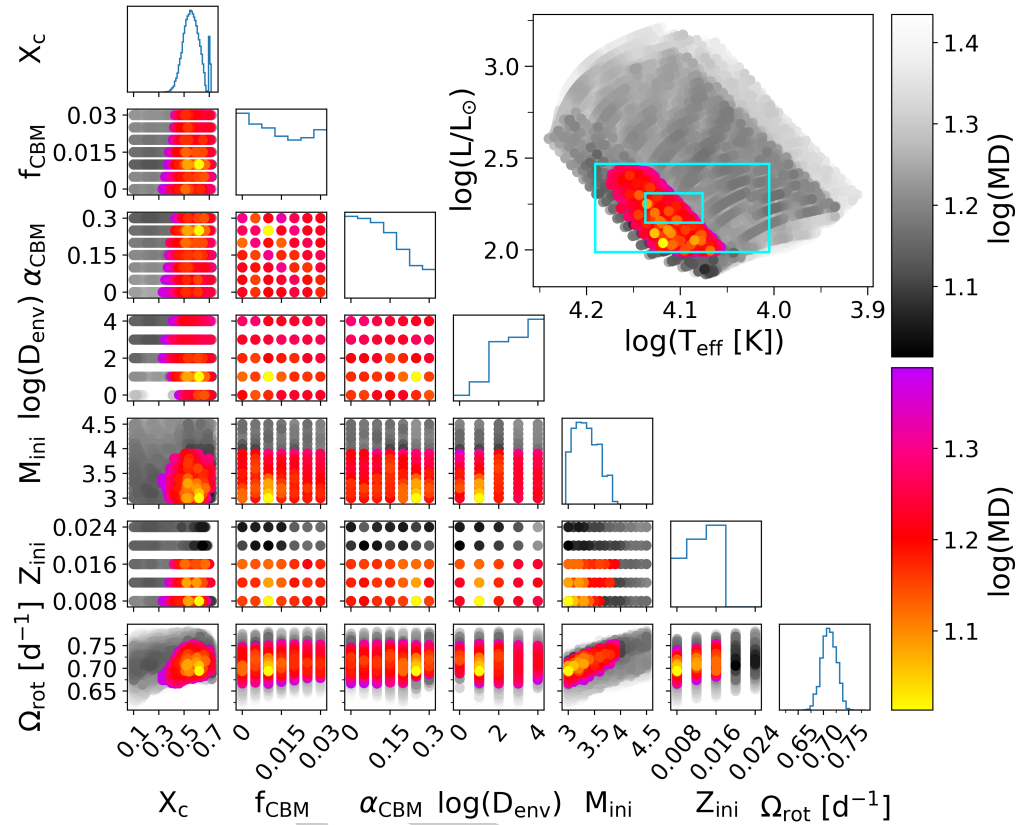
Figure 1: Cornerplot with the parameters in the grid and the rotation. The 50% best models are shown, colour-coded according to the log of their merit function value. Models in colour fall within the 2 sigma error ellipse, while those in greyscale fall outside of it. Figures on the diagonal show binned parameter distributions of the models in the error ellipse, and the panel at the top right shows an Hertzsprung-Russell (HR) diagram with 1 and 3 sigma observational error boxes. Figure taken from Michielsen et al. (2023).

Acknowledgements

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