Abstract and Keywords

Researchers interested in statistically modeling network data have a well-established and quickly growing set of approaches from which to choose. Several of these methods have been regularly applied in research on political networks, while others have yet to permeate the field. This chapter reviews the most prominent methods of inferential network analysis for both cross-sectionally and longitudinally observed networks, including (temporal) exponential random graph models, latent space models, the quadratic assignment procedure, and stochastic actor oriented models. For each method, the chapter summarizes its analytic form, identifies prominent published applications in political science, and discusses computational considerations. It concludes with a set of guidelines for selecting a method for a given application.

Keywords: network analysis, political networks, network models, social networks, ERGM, latent space, quadratic assignment procedure, inferential network analysis

Introduction

NETWORKS are systems that exhibit complex, interwoven structures. These systems can be illuminating to describe and visualize, but most research agendas eventually arrive at either precise hypotheses regarding endogenous network structures (e.g., the structure of connections within the outcome network of interest) or similarly precise hypotheses regarding exogenous factors but requiring adjustment for endogenous dependencies. The conventional toolkit for statistical inference is poorly suited to evaluating such hypotheses. The core problem is that the classical machinery of hypothesis testing was designed around the assumption that the data set to be used is populated with independent replicates from a datagenerating process. Network data could not be further from this assumption. The observations, which are often organized as dyads, are individual components of a single, broader system. Even if a researcher has several replicates of the system, they are often dependent (e.g., longitudinal network data). Because the interdependencies inherent in networks violate a core assumption upon which standard regression models are built, interest in networks has sparked a wave of methodological innovations

aimed at statistical inference, modeling, and hypothesis testing with networks. Here we review the most established of these methods, briefly discuss their structure and capabilities, and highlight selected applications of these tools in political science research. Obviously a review chapter such as this cannot discuss the intricacies of each model exhaustively, but we hope to provide a general orientation to the set of techniques available and point the reader to where further details and examples may be had.

(p. 204) Cross-Sectional Models

The basic approaches to statistical inference with networks are differentiated by the objectives of the researcher vis-à-vis the network structure and dependencies. The approaches that we review here permit researchers to test precise theories of network formation that involve both the effects of covariates on tie formation and relationships among ties themselves. We also present methods that permit inference when the researcher is uncertain about the forms of dependence that underlie tie formation in the network. Here we provide a very brief overview of the three major methods of cross-sectional network analysis and provide some discussion about their use in the political science literature. For a more detailed discussion of how these methods relate to each other and an illustrative application of all three to the same data set, see Cranmer et al. (2016).

The Exponential Random Graph Model (ERGM)

We consider the exponential random graph model (ERGM) (Holland and Leinhardt, 1981; Wasserman and Pattison, 1996) to be the canonical model for network data. The ERGM provides a modeling framework that, just like conventional regression, can accommodate the effects of covariates on the status of relationships, but unlike regression, it can model the prominence and significance of structural dependencies such as reciprocity and transitivity. Furthermore, the ERGM has been extended to accommodate longitudinal relational data (i.e., time series of networks) (Robins and Pattison, 2001; Hanneke, Fu, and Xing, 2010a; Desmarais and Cranmer, 2012b) and networks with edge weights (Wyatt, Choudhury, and Bilmes, 2010; Desmarais and Cranmer, 2011; Krivitsky, 2012).

The mathematical form of the ERGM makes clear its potential as a general framework for statistical inference with networks. The ERGM is a probability model defined on the support of possible adjacency matrices. Let there be n actors/nodes/vertices, among which we represent dichotomous ties as Y; an $n \times n$ matrix with $Y_{ij} = 1$ if there is a tie from i to j and 0 otherwise. The probability of observing Y according to an ERGM is

$$P(Y) = \frac{\exp\left(\sum_{j=1}^{k} \theta_j h_j(Y)\right)}{\sum_{Y* \in \mathcal{Y}} \exp\left(\sum_{j=1}^{k} \theta_j h_j(Y*)\right)}.$$

(1)

The flexibility of the ERGM for capturing covariate effects and interdependencies in networks arises from the general nature of the h_j. These are statistics—functions applied to the adjacency matrix and optionally covariates—that are theorized to influence the likelihood of observing a configuration of the adjacency matrix (e.g., a network with thirty (p. 205) triangles, one hundred edges, and twenty-five reciprocal dyads). The θ are parameters to be estimated that, similar to regression coefficients, give the effects of the network statistics on the likelihood of observing a particular instance of the adjacency matrix. The larger θ_j, the higher the likelihood of observing subgraph configurations that contribute positively to h_j. The flexibility and grounding in theory come at a cost. An identifying assumption, one also common to the regression framework, is that the model is correctly specified. That is, it is assumed that every feature of a network configuration that is relevant to determining whether we will observe that network configuration is represented by an element of h. Misspecification can result in biased estimates of parameters as well as subpar performance of hypothesis tests. A detailed review of the ERGM is provided by a number of sources, including Goodreau et al. (2008); Hunter et al. (2008); Cranmer and Desmarais (2011); and Lusher, Koskinen, and Robins (2013).

For applications of ERGM in political networks research, see Thurner and Binder (2009); Lazer et al. (2010); Cranmer and Desmarais (2011); Gerber, Henry, and Lubell (2013); Kirkland and Gross (2014); Box-Steffensmeier and Christenson (2014); and Song (2015).

Let us consider the example of Box-Steffensmeier and Christenson (2014) in detail. Recently, scholars have begun to construct networks of interest groups based on cosigning the same amicus curiae brief. Amicus curiae or "friend of the court" legal briefs are written by actors who are not party to a case but believe they can offer information relevant to it. One contribution of this work is that the network measure constructed from amicus curiae briefs is both coordinated and purposive; the process requires negotiation between cosignatories and agreement on the direction, argument, and details of the brief. They find evidence of an increasingly transitive network resembling a host of tightly grouped factions and leadership hub organizations employing mixed coalition strategies. Egocentric networks of organizations show that three major theoretically posited coalition strategies are present in the data: *lone wolves*, who work alone; *teammates*, who work with cliques of varying sizes; and *leaders,* who pull together otherwise disparate groups. They further utilize business directories to gather attribute data on signers from the first decade of the millennium. In doing so, they find a broad range of industries, with the greatest number of signers in the services division of the Standard Industrial Classification (SIC). They also use the attribute data to test homophily hypotheses for various business characteristics in the ERGM. The paper finds that driving the network formation is assortative mixing based on industry area, budget, sales, and membership characteristics.

Substantively, Box-Steffensmeier and Christenson (2014) contribute to the discipline by shedding light on how to measure the networks of the vast expanse of special interests, how the networks have changed over time, and which characteristics are sought out as complements and which are considered threatening to cooperation. In addition, they pro-

vide insights into more micro-level considerations about which groups are the most attractive partners and in what capacity.

(p. 206) The Latent Space Model (LSM)

The latent space model, introduced by Hoff, Raftery, and Handcock (2002), is a latent variable model for tie formation in networks that exploits the near ubiquity of homophily in networks: the tendency for nodes to tie to others that are similar on one or more attribute. Nodes are positioned in a k-dimensional latent space, and the probability of a tie between any two nodes is inversely related to the distance between their positions in this latent space. Each dimension in the latent space can be thought of as an unmeasured attribute of nodes. For example, such attributes might be income or age if nodes are people, or level of authoritarianism and unemployment rate if nodes are countries. Under the latent space model, the log odds of a tie between nodes i and j is:

$$\eta_{i,j} = \log \text{odds}(y_{i,j} = 1 | z_i, z_j, x_{i,j}, \alpha, \beta)$$

(2)

$$= \alpha + \beta' x_{i,j} - |z_i - z_j|$$

(3)

where α is an intercept term controlling the overall density of the network, β is a vector of regression coefficients, $x_{i,j}$ is the vector of measured covariates, and $|z_i - z_j|$ is the Euclidean distance between nodes in the latent space. Interpretation of the LSM is similar to interpreting logistic regression. The β is interpreted as the change in the log odds of a tie given a one-unit increase in the value of a covariate.

For applications of LSM in political networks research, see Ward, Siverson and Cao (2007); Kirkland (2012); Krafft et al. (2012); Ward, Ahlquist, and Rozenas (2013); Kirkland and Gross (2014); and Cao and Ward (2014).

To consider an example of such work in detail, let us examine Kirkland (2012). Kirkland was primarily interested in the effect of an exogenous variable, the sharing of a constituency, on the network of cosponsorships in the US Congress. In terms of design, he used a natural experiment. During the 2000–2002 redistricting cycle, North Carolina eliminated its small set of multi-member districts as a result of a court intervention, allowing Kirkland to compare the behavior of legislators in multi-member districts to those in single-member districts (since not all North Carolina districts were multi-member), and to compare legislators from multi-member districts to themselves after the switch. His theory, being about exogenous predictors, did not provide specific hypotheses about the structure of the endogenous network dependencies, making the proper specification of an ERGM difficult. Treating the network dependencies as a nuisance, rather than as an object of interest, he estimated an LSM on each year in the sample. This setup allowed Kirkland to test the effects of the exogenous variable of interest on the network while control-

ling for the interdependence known to be present. To select the dimensionality of the LSM, Kirkland increased the number of dimensions in the model until the new dimensions stopped producing significant gains in model fit; two dimensions, as should happen.

(p. 207) Substantively, Kirkland (2012) demonstrates that sharing a constituency produces sizable increases in collaboration via cosponsorship. There is some research both in American politics and in the comparative literature that suggests that multi-member districts produce representatives who are likely to compete with one another to build a distinct "brand" from their district partner, and that these districts produce representatives who lack strong geographic ties to their constituents, allowing them to adhere to party lines more frequently. Alternatively, Kirkland hypothesized that the advantages of working together to gain on policy outweighed the potential gains from adhering to party lines and building distinct individual brands and found that multi-member districts provide legislators with natural coalition partners, who stand to gain on policy through collaboration.

The Quadratic Assignment Procedure (QAP)

The quadratic assignment procedure (QAP) (Krackardt, 1987) is a method of hypothesis testing that builds on Mantel's (1967) permutation procedure and computes the statistical significance of parameter estimates when the dependent variable is itself a relational matrix (e.g., distance, correlation, network adjacency matrices). The QAP is a form of non-parametric permutation testing in which row and column shuffles of the dependent and independent variables are used to simulate the null condition, which results in a simulated null distribution of regression coefficients.

To illustrate the basic methodology of QAP, we offer an example of bivariate permutation in figure 8.1. Randomly shuffling rows and columns of the adjacency matrix (Y) maintains the structure of the network right down to the edges, but by shuffling the nodes, the relationship between Y and a hypothetical dyadic covariate X is broken. In the QAP, some measure of relationship (e.g., correlation coefficient, regression) between Y and X is calculated on the unpermuted data, then the same measure is calculated on many permutation-based replications of the data. A two-tailed p-value to assess the null hypothesis that there is no relationship between Y and X can be calculated as the proportion of relationships calculated under permutations that are at least as large, in magnitude, as the value calculated on the observed data.

Using QAP with three or more values is more complicated than using either the Y or the X permutations, which are equivalent in the bivariate case. Suppose there are three relational variables—the dependent network Y, the independent variable of interest X, and a potential confounder Z, for which we would like to control in a regression calculating the effect of X on Y. Figure 8.2 depicts the causal diagram of Z inducing a spurious correlation between X and Y. Consider Y permutation, in which the rows of Y are permuted to reflect the null hypothesis of no relationship between Y and X. Doing this also breaks the relationship between Z and Y, which amounts to assuming away one leg in the confounder di-

agram. Now consider X permutation, in which just X is shuffled. This would break the relationship between Z and X, also assuming away a leg in the confounding diagram. (p. 208)

Y	A	B	C
A	0	1	1
B	1	0	0
C	0	1	0

X	A	B	C
A	0	3	-2
B	1.2	0	-4
C	0	6	0

⋮

	B	C	A
B	0	0	1
C	1	0	0
A	1	1	0

	A	B	C
A	0	3	-2
B	1.2	0	-4
C	0	6	0

Figure 8.1 Illustration of bivariate QAP. Example of dependent network (i.e., *Y*) permutation.
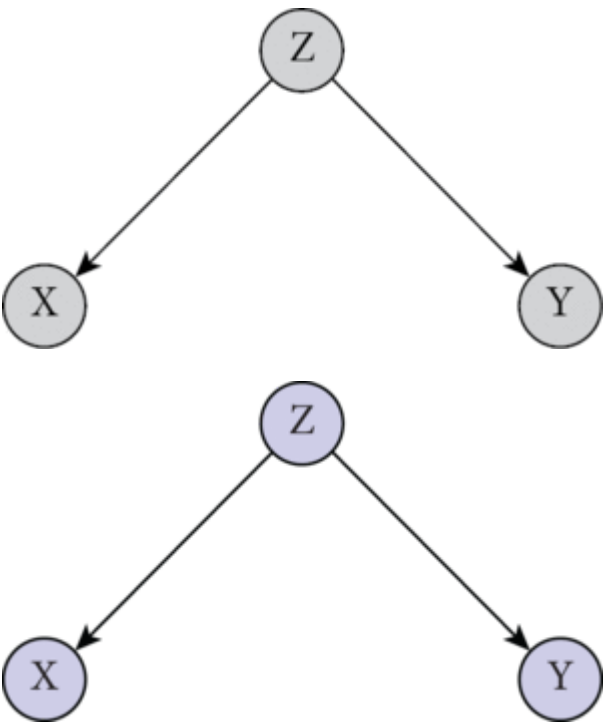

Figure 8.2 Causal diagram illustrating the role of a confounder (Z) in creating a spurious correlation between X and Y.

Dekker, Krackhardt, and Snijders (2007) propose an algorithm, termed double semi-partialing, that is intended to balance between X and Y permutations, and they show that it performs better than X or Y permutations. Let $\widehat{\delta Z}$ be the estimate from $Y = \delta Z + E$. Define $\widehat{\in}_{XZ} = X - \widehat{\delta Z}$. Define $\pi(\widehat{\in}_{XZ})$ to be a matrix permutation of $\widehat{\in}_{XZ}$. The nonparametric

null distribution of β is derived by estimating $Y = \pi(\overset{\wedge}{\in}_{XZ})\beta + \delta Z + E$. QAP is a very useful methodology when the researcher is uncertain which dependencies to include in ERGM, or the LSM does not provide a good fit.

For applications of QAP in political networks research, see Mizruchi (1990); Weible and Sabatier (2005); Bochsler (2009); Shrestha and Feiock (2009); Henry (2011); Lee, Feiock, and Lee (2012); Heaney et al. (2012); Desmarais et al. (2015); Andrew et al. (2015); and Chen, Suo, and Ma (2015).

Considering the work of Henry (2011) as an example, the authors were interested in understanding the conditions under which party activists are members of the same associations and particularly the extent to which copartisanship affected this likelihood. Drawing on surveys they had conducted at the 2008 Democratic and Republican National Conventions, Henry (2011) estimated models predicting organizational comembership as a function of copartisanship. They also controlled for ideology, sex/gender, race/ethnicity, age, educational attainment, income, and religious participation. The empirical analysis is particularly interesting given our discussion above, but the authors specified three types of models: logistic regression, the QAP, and an ERGM. Results from all three models showed that being in the same party is a strong positive (p. 209) predictor of being in the same organization, even after controlling for alternative explanations for comembership. In fact, only 1.78 percent of comemberships crossed party lines; the association network is strongly polarized by party. The results illustrate one of the many barriers to partisan cooperation: party activists rarely come into contact with activists from the other party in their civic and political associations.

Dynamic Models

The statistical analysis of longitudinal networks has a smaller literature than that for cross-sectional networks. Two models that build on the ERGM, however, have been applied with some frequency in political science. Here we discuss each model and its applications to political science briefly. For a direct contrast and comparison of the two models, see Desmarais and Cranmer (2012a) and Leifeld and Cranmer (2014).

The Temporal ERGM (TERGM)

The temporal exponential random graph model (TERGM) is a straightforward longitudinal extension of the ERGM. The defining feature of the TERGM vis-à-vis the ERGM is that the statistics in the TERGM model the ways in which past realizations of the network influence the current network.

To modify the ERGM for modeling how the network N at time t, denoted N^t, depends on previous networks $K \in \{1, \ldots, T-1\}$, we simply introduce the lagged networks into the **h**.

$$P(Y^t|Y^{t-K}, \ldots, Y^{t-1}, \boldsymbol{\theta}) = \frac{\exp(\boldsymbol{\theta}^\top \mathbf{h}(Y^t, Y^{t-1}, \ldots, Y^{t-K}))}{c(\boldsymbol{\theta}, Y^{t-K}, \ldots, Y^{t-1})}.$$

(4)

The specification of K is important because Y^t networks in the time series preceding time point $K + 1$ cannot be modeled with the correct model.

Equation 4 specifies a TERGM for a single time point, Y^t. The joint likelihood of the time series of networks spanning times $K + 1$ and T is derived as the product of the conditional probabilities, conditioning on past networks:

$$P(Y^{K+1}, \ldots, Y^T | Y^1, \ldots, Y^K,) = \prod_{t=K+1}^{T} P(Y^t|Y^{t-K}, \ldots, Y^{t-1},).$$

(5)

A TERGM can be applied to just a single network, which is equivalent to using ERGM with statistics involving one or more lagged networks, or a time series of networks with dozens or even hundreds of networks. For more detailed methodological treatment of the TERGM, see Hanneke, Fu, and Xing (2010b); Cranmer and Desmarais (2011); and (p. 210) Desmarais and Cranmer (2012c). The bootstrap pseudo likelihood methods introduced in Desmarais and Cranmer (2012c) permit computationally efficient application to long time series.

Dependence on lagged networks can be incorporated in a fashion similar to including a covariate in a cross-sectional ERGM. For example, reciprocation is a nearly ubiquitous process in social networks. However, in time-stamped data, it is not clear whether the reciprocation of a tie will occur within one time period—especially if ties take a long time to form (e.g., links in an annually measured scientific paper citation network) or if time periods are notably short (e.g., an email network with hourly time stamps). To observe reciprocity, we may need to measure it with a lag. Single-period delayed reciprocity is modeled as

$$h_r(Y^t, Y^{t-1}) = \sum_{ji} Y_{ij}^t Y_{ji}^{t-1}.$$

(6)

The lagged effects in a TERGM specification—the components of $\mathbf{h}$—can vary in terms of their order, from 1 to K. For example, if $K = 4$, we could model star-formation processes that aggregate over four periods, but also limit reciprocity effects to a single period. See Morris, Handcock, and Hunter (2008) for an extensive review of endogenous effect specification.

For applications of the TERGM in political networks research, in which myriad examples of lagged endogenous effects can be found, see, for example, Cranmer, Desmarais, and Menninga (2012); Cranmer, Desmarais, and Kirkland (2012); Almquist and Butts (2013); Clark and Caro (2013); Corbetta (2013); Cranmer, Heinrich, and Desmarais (2014); Masket and Shor (2014); and Ingold and Leifeld (2016).

Consider the work of Ingold and Leifeld (2016) as an example. Dozens of policy network case studies in continental Europe have used almost identical survey questionnaires to collect data over the last three decades. In addition to data on collaboration, information exchange, and a variety of actor attributes, respondents were usually asked whom they would judge as particularly important in the rest of the network. While this is obviously a network relation, earlier studies aggregated it into a node attribute in order to measure an actor's overall importance in the policy network. Ingold and Leifeld (2016) exploited the fact that this is a network relation and modeled it using ERGMs for the cross-sectional data sets and TERGMs for the longitudinal data set. Specifically, they used four different data sets from different countries (institutional settings), levels (local to national), and policy stages (implementation and policy formulation decision making) to assure validity of results across context. In terms of specification, they estimated an ERGM of their second temporal observation conditional on the first, thus modeling the changes between periods one and two. The TERGM examined whether changes in the betweenness centrality of actors in the collaboration network could explain their perception as being important at the second time point. In other words, it is not simply collaboration centrality that makes one important, it is not simply collaboration ties with alters that make one regard that other organization as important, and it's not simply one's institutional role that makes one important (p. 211) (e.g., being a decision maker)—it's also that becoming more central over time makes alters perceive an ego as being an important player in the policy process. As the predictor of interest was betweenness centrality, this reflects the ability to tie different communities in the network together.

Stochastic Actor Oriented Models (SAOMs)

The stochastic actor oriented model (SAOM) (Snijders, van de Bunt, and Steglich, 2010) is a statistical model for longitudinal network data that has had considerable application in political networks research and can be considered an alternative to TERGM. The SAOM is a structurally detailed model that is focused on interpretation at the node level. The SAOM and TERGM are similar in many underlying facets. Indeed, the equilibrium distribution of the SAOM is an exponential family random graph (Snijders, 2001). The "actor orientation" in the SAOM begins with an assumption about how the network changes. First, ties change one at a time. An actor is selected at random according to a Poisson process. The actor can then choose to leave its tie profile unchanged or change a single outgoing tie. The rates that parameterize the Poisson processes can be actor specific, modeling how tie volatility varies based on actor-level covariates. Let $Y^{(ij)}$ represent the network in which element ij of the adjacency matrix is toggled (changed from 0 to 1 or from 1 to 0). If selected to consider a tie change, the probability that actor i changes its

tie status with j is proportional to $\exp\left(\sum_{h=1}^{k} \theta_h \Gamma_{ih}(Y^{(ij)})\right)$, where $\sum_{h=1}^{k} \theta_h \Gamma_{ih}(Y^{(ij)})$ is the actor "objective function" (Snijders, van de Bunt, and Steglich, 2010). Like the ERGM, the building blocks are regression coefficient style parameters θ and network statistics $\Gamma()$. The SAOM objective function is a dot product of network statistics that change with the ties sent by actor i and real-valued parameters.

An added feature of the SAOM relative to TERGM is that it offers the option of specifying a dynamic model for time-dependent actor attributes termed "behavior." The behavior equation includes both other node covariates and functions of the network designed to evaluate how the network influences behavior. To develop the idea of the behavior model, first consider a regression for a node attribute at time t, which includes as covariates other node attributes as well as node-level statistics calculated on the network at time $t - 1$ (e.g., node centrality, the average attribute value of a node's partners in the network). Instead of directly measuring the network at $t - 1$ in order to model node attributes at time t, intermediate changes in both the network and behavior are simulated between times t and $t - 1$. The model for the change in behavior between times $t - 1$ and t includes an iterative process of change between the network and behavior—changes that are simulated/inferred, as they are not observed (Steglich, Snijders, and Pearson, 2010). This simulation of intermediate changes is the modeling feature that differentiates TERGM and SAOM. The canonical form of data for which the SAOM is designed is network snapshot data, in which the ties in a network are sampled at different points in time, and the researcher does not know how the network changed between (p. 212) snapshots. The changes are simulated according to the detailed structural assumptions in the SAOM. Finally, one should also note the widespread misconception that this coevolutionary model can identify separate influences for homophily and influence, two forces that are usually confounded in network research. As Snijders, van de Bunt, and Steglich (2010) point out, the SAOM can only make this differentiation in extremely limited cases, and the model is not generally capable of this differentiation in applied research contexts.

Political science applications of SAOM include Berardo and Scholz (2010), Fischer et al. (2012), Fischer and Sciarini (2013), Berardo (2013), Kinne (2014), and Manger and Pickup (2014).

As a last example from the literature, consider Berardo (2013), who (2013) studied perceptions of procedural fairness in self-organized networks of stakeholders in five US estuaries where environmental problems are common. How members of a group perceive fairness in decision-making processes is a key variable to study for researchers interested in collaborative processes, given that those processes in which procedural fairness is low usually are less likely to be sustained in time. Berardo finds that participation in networks does not alter perceptions of procedural fairness. Instead, perceptions of procedural fairness impact participation in networks, with homophily seeming to drive the process. In other words, similar perceptions of fairness are a strong predictor of the establishment of ties among actors. Berardo claims that this is an important result, because it hints at the fact that in the absence of high levels of procedural fairness, the network is likely to be

fragmented into groups of "winners" and "losers" in the decision-making process, which may make long-term collaboration at a systemic level difficult to sustain.

Selecting an Appropriate Approach

The methods described above differ along a couple of important dimensions, including (1) the role of structural network theory in developing a model specification and (2) the degree to which interdependence is accounted for through the model parameters versus through the hypothesis testing framework. Considering how a given application aligns along these dimensions, the researcher can determine which of these methods is most appropriate.

The first major consideration in selecting a model for a network regards whether one of the objectives of the research is to conduct inference regarding precise patterns of interdependence among the ties in the network. For example, Song (2015) states several hypotheses regarding interdependence between ties in networks of political discussion. These include popularity (the tendency to send ties to nodes with high in-degree), activity (the tendency for nodes with high out-degree to send new ties at a higher rate), and transitivity. When the researcher has formed precise hypotheses regarding dependence among the ties, the only models that we have discussed that can be used are those based on the ERGM—ERGM, TERGM, and SAOM—the latter two in the case of longitudinal (p. 213) networks. The LSM, though useful for the purpose of exploring the structure of the network that is not explained through the use of exogenous node and dyad covariates, cannot be used to test hypotheses regarding the interdependence among ties. The QAP represents a method of hypothesis testing that is robust to the presence of interdependence among the ties in the network, but there is no way to use QAP results in diagnosing the forms of interdependence that exist among the ties.

If the researcher is only interested in testing the effects of covariates, but would like to account for network dependence, the second important consideration is the choice between parametric and nonparametric methods. Both the ERGM and LSM use precise parameterizations to model away the interdependence for which the covariates do not account. In the ERGM, the researcher must specify the exact network statistics that correspond to the forms of interdependence, a process that may sometimes be burdensome. In the LSM, the interdependence is absorbed by a large latent parameter space of unmeasured node attributes, along which nodes are assumed to be homophilous.

With both the LSM and ERGM, it is possible that the researcher will fail to specify a model that results in the good fit required to draw inferences with a parametric model. As noted previously, specification of an ERGM requires precise theory regarding the forms of interdependence in a network. The LSM demands less in terms of precise assumptions regarding structural dependencies, but it does impose some limitations. The dependence structures in the LSM must conform closely to a metric space in which the probability of a tie is inversely proportional to distance in the space. This creates two limitations on the LSM in accounting for dependence. These limitations derive from the properties of metric

spaces (Bryant, 1985). The first property is *symmetry*, which means that the distance from *i* to *j* is equivalent to the distance from *j* to *i*. If the probability of a tie is linked to these symmetrical distances in the latent space, then the LSM implies that ties exhibit a tendency toward reciprocity. Symmetric dependence would be inappropriate for a hierarchical network, for example, in which ties flow along paths that do not loop back (e.g., an organizational chart) (Krackhardt, Carley, and Prietula, 1994). The other relevant property of a metric space is the *triangle inequality*, which indicates that the distance between *i* and *j* can be at most the distance between *i* and *k* and *j* and *k*, where *k* is any other node in the network. This means that if, according to the latent space, *i* is very likely to tie with *k* and *j* is very likely to tie with *k*, then *i* and *j* will also be likely to tie. The triangle inequality embeds transitivity into the structural assumptions underlying the LSM. Such an assumption would be inappropriate for a conflictual network, in which the enemy of your enemy is likely your friend (Cranmer and Desmarais, 2011). If the researcher's theory of network dependence is not precise enough to specify an ERGM and the structural assumptions associated with the LSM are not suitable in a given application, then QAP is the best option.

Model dependence is the term used to describe the conditions under which the conclusions drawn from a statistical modeling exercise vary across reasonable alternative specifications of the model (Ho et al., 2007). For example, if a test of the hypothesis that nodes in a network exhibit homophily with respect to a given attribute exhibited different results when using LSM and QAP, the test would be model dependent. If the (p. 214) researcher's primary interest lies in testing hypotheses regarding the effects of exogenous covariates, and the researcher has identified plausible specifications of ERGM and/or LSM, it is prudent to also evaluate hypotheses using QAP. If results are consistent across ERGM, LSM, and QAP, that is a strong sign of robustness. If they vary, the researcher should endeavor to understand why and assure that the structural assumptions represented by ERGM and/or LSM are indeed appropriate.

Turning to dynamic models, the literature on political networks presents two modeling options: SAOM and TERGM.[1] Both Desmarais and Cranmer (2012a) and Leifeld and Cranmer (2014) present thorough discussions of the ways in which researchers can draw conceptual and empirical comparisons of SAOM and TERGM. First, in terms of empirical comparison, if the researcher does not have theoretical reasons to prefer either SAOM or TERGM, it is possible to use either in- or out-of-sample comparisons to evaluate which method provides a better fit. Desmarais and Cranmer (2012a) and Leifeld and Cranmer (2014) provide extensive discussion and examples regarding the empirical comparison of SAOM and TERGM.

There are a few considerations that may allow the researcher to select between SAOM and TERGM based on theory. First, is it appropriate to assume that the network changes according to a process in which an actor considers changing one outgoing tie at a time to stochastically optimize its objective function? If the answer to this question is yes, then the SAOM is a favorable option. Through making this assumption about the form of network change, it is possible to identify other features of the network dynamics, such as dif-

ferentiating between homophily and influence. If the answer to this question is no, then TERGM is favorable, because there are no such assumptions regarding the form of network change. Second, do the ties in the network and behavior coevolve simultaneously? As noted previously, the SAOM consists of two equations: an objective function for tie formation and another for behavior (i.e., node attribute) evolution. The main advantage of using SAOM over estimating these functions separately is that through the structural assumptions regarding change dynamics, it is possible to identify coevolution dynamics that occur at a finer time interval than that at which the data were collected. Third, is there a reason to suspect nonstationarity in the time series of networks? Underlying the TERGM is an assumption that the series of networks under analysis has reached a stationary distribution—a stable time-point-to-time-point process of network generation. In the SAOM it is assumed that change over time follows a stable/stationary process, but it is not assumed that the time series of networks has reached stationarity. Thus, if the network under study has recently come into formation—suggesting that more time may have to pass to reach stationarity, or there is some inexplicable shift in the network's structure over time, use of the SAOM would be favored.

Frontiers in Network Modeling

Scholars of government and politics are in a nearly unique position as far as access to data on ties between the actors they study. In every subfield of political science—from (p. 215) political behavior research on campaign contribution networks of individuals to intra-institutional studies of interaction within legislatures or administrative agencies, to international studies of trade flows, alliances, and migration—data are regularly available and often open by public records mandate. Furthermore, this rich archival data often represent a census of the relevant population that covers several decades or even centuries. Since such varied and rich data are available, it is safe to assume that most established methods for modeling network data could be fruitfully applied to problems in political networks research. Furthermore, as the field of automated text analysis matures, the volume and scope of network data available to political scientists are likely to explode. On the frontier of research we see great promise in the application of network methods to diverse data sets and in the generation of new political networks data.

The frontier of statistical network analysis is also developing quickly. One area in which the field is rapidly evolving is techniques for analyzing weighted (e.g., valued-edge) networks. Wyatt, Choudhury, and Bilmes (2010; Desmarais and Cranmer (2011; and Krivitsky (2012) have proposed methods for approaching this problem, and software is now available to implement these techniques. Other recent developments that show promise and deserve further consideration in political networks research include the mixed membership stochastic blockmodel (Airoldi et al., 2009), exponential family random network models (Fellows and Handcock, 2012), and relational event models for longitudinal data (Butts, 2008), but these methods have yet, as of this writing, to permeate political science.

In sum, political network analysis is developing rapidly in terms of the range of substantive problems to which network analysis is applied, the quality and volume of network data available to political scientists, and the statistical methods themselves. We think it likely that the landscape of political networks will look substantially different ten years after this writing than it does today.

Acknowledgments

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Notes:

(1.) Ward, Ahlquist, and Rozenas (2013) propose a dynamic extension of the LSM. However, within-dyad autocorrelation is the only form of network dependence for which it accounts. Since it does not account for higher order forms of interdependence, we do not consider it to be a dynamic network model along the same lines as SAOM and TERGM, which both permit the modeling of arbitrary forms of intertemporal dependence.

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