PLSC 504 – Fall 2020
Panel/TSCS Data:
Unit Effects + Dynamics

October 7, 2020

- "Longitudinal" $\neq$ "Time Series"
- Terminology:
  - "Unit" / "Units" / "Units of observation" / "Panels" = Things we observe repeatedly
  - "Observations" = Each (one) measurement of a unit
  - "Time points" = When each observation on a unit is made
  - $i \in \{1....N\}$ indexes units
  - $t \in \{1...T\}$ or $\{1...T_i\}$ indexes observations / time points
  - If $T_i = T \; \forall i$ then we have "balanced" panels / units
  - NT = Total number of observations (if balanced)
- Averages:
  - Y_{it} indicates a variable that varies over both units and time,
  - $\bar{Y}_i = \frac{1}{T}\sum_{t=1}^{T} Y_{it}$ = the over-time mean of Y,
  - $\bar{Y}_t = \frac{1}{N}\sum_{i=1}^{N} Y_{it}$ = the across-unit mean of Y, and
  - $\bar{Y} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} Y_{it}$ = the grand mean of Y.

More Terminology

- $N >> T \rightarrow$ "panel" data
  - NES panel studies ($N = 2000$, $T = 3$)
  - Panel Study of Income Dynamics ($N =$ large, $T \approx 12$)

- $T >> N$ or $T \approx N \rightarrow$ "time-series cross-sectional" ("TSCS") data

- $N = 1 \rightarrow$ "time series" data

Variation: A Tiny (Fake) Example

id	year	gender	pres	pid	approve
1	2014	female	obama	dem	3
1	2016	female	obama	dem	3
1	2018	female	trump	dem	5
1	2020	female	trump	dem	3
2	2014	male	obama	gop	2
2	2016	male	obama	gop	1
2	2018	male	trump	gop	4
2	2020	male	trump	gop	3
3	2014	male	obama	gop	2
3	2016	male	obama	gop	2
3	2018	male	trump	gop	4
3	2020	male	trump	dem	1

Within- and Between-Unit Variation

Define:

$$\bar{Y}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} Y_{it}$$

Then:

$$Y_{it} = \bar{Y}_i + (Y_{it} - \bar{Y}_i).$$

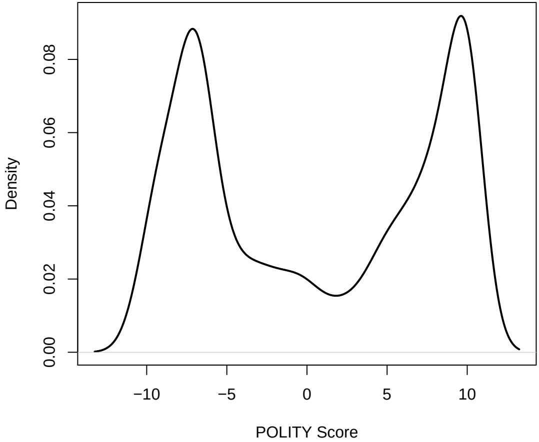
- The *total* variation in Y_{it} can be decomposed into
- The *between-unit* variation in the $\bar{Y}_i$s, and
- The *within-unit* variation around $\bar{Y}_i$ (that is, $Y_{it} - \bar{Y}_i$).

Variation:TSCS Data

```
> summary(Demos)
     ccode            Year          POLITY            GDP
 Min.   :  2   Min.   :1945   Min.   :-10.0   Min.   :   185
 1st Qu.:235   1st Qu.:1969   1st Qu.: -7.0   1st Qu.:  1579
 Median :451   Median :1985   Median :  0.0   Median :  4000
 Mean   :456   Mean   :1984   Mean   :  0.6   Mean   :  8118
 3rd Qu.:663   3rd Qu.:2000   3rd Qu.:  8.0   3rd Qu.: 10361
 Max.   :950   Max.   :2014   Max.   : 10.0   Max.   :134040
                              NA's   :111     NA's   :2349
    Monarch        lnDemons        ColdWar
 Min.   :0      Min.   :0      Min.   :0.000
 1st Qu.:0      1st Qu.:0      1st Qu.:0.000
 Median :0      Median :0      Median :1.000
 Mean   :0      Mean   :0      Mean   :0.563
 3rd Qu.:0      3rd Qu.:0      3rd Qu.:1.000
 Max.   :1      Max.   :4      Max.   :1.000
 NA's   :1198   NA's   :1149
```

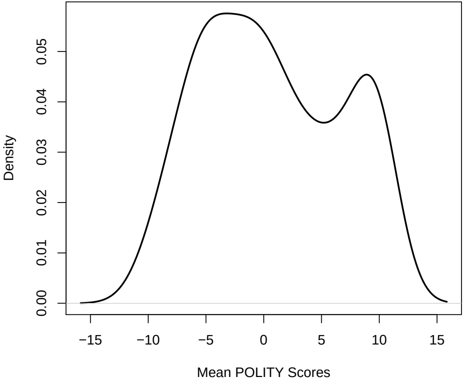
POLITY: Total Variation

```
> with(Demos, describe(POLITY)) # all variation
   vars    n mean   sd median trimmed  mad min max range skew kurtosis   se
X1    1 9260 0.63 7.47      0    0.71 10.4 -10  10    20    0    -1.68 0.08
```



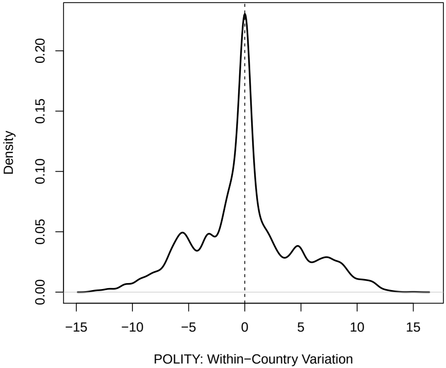
POLITY: “Between” Variation

```
> POLITYmeans <- ddply(Demos,.(ccode),summarise,
+                  POLITYmean = mean(POLITY))
>
> with(POLITYmeans, describe(POLITYmean)) # "between" variation
   vars   n mean   sd median trimmed  mad min max range skew kurtosis   se
X1    1 162 0.73 5.99   -0.1     0.7 7.36 -10  10    20 0.16    -1.21 0.47
```



POLITY: "Within" Variation

```
> Demos <- ddply(Demos, .(ccode), mutate,
+                 POLITYmean = mean(POLITY))
> Demos$POLITYwithin <- with(Demos, POLITY-POLITYmean)
>
> with(Demos, describe(POLITYwithin))
   vars    n mean   sd median trimmed  mad   min max range skew kurtosis   se
X1    1 8404    0 4.48      0   -0.11 3.16 -13.5  15  28.5 0.19     0.27 0.05
```



Model

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

assumes:

- All the usual OLS assumptions, plus
- $\beta_{0i} = \beta_0 \ \forall \ is$
- $\beta_{1i} = \beta_1 \ \forall \ is$

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

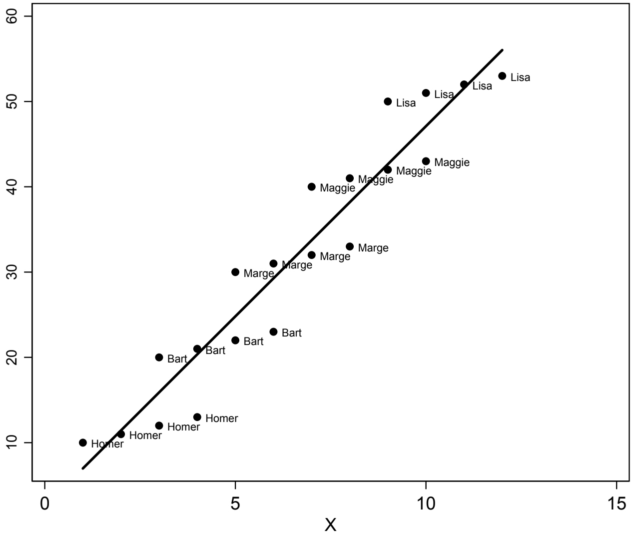
(same)

$$Y_{it} = \beta_{0i} + \beta_1 X_{it} + u_{it} \qquad \text{(unit-level)}$$

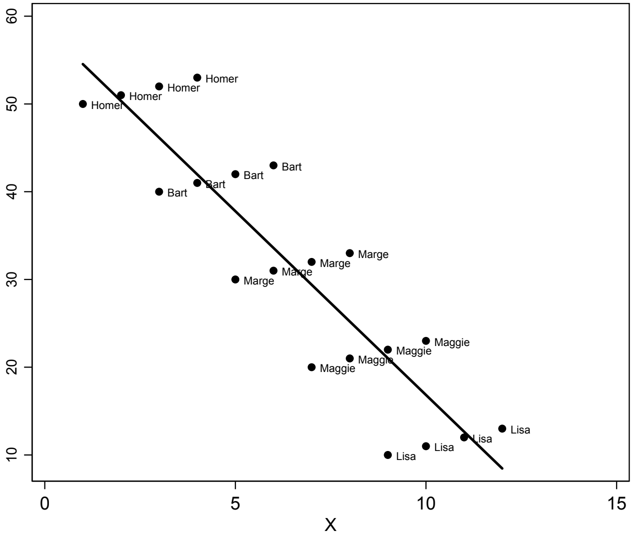
$$Y_{it} = \beta_{0t} + \beta_1 X_{it} + u_{it} \qquad \text{(time-level)}$$

$$Y_{it} = \beta_{0it} + \beta_1 X_{it} + u_{it} \qquad \text{(unit- and time-level)}$$

Varying Intercepts



Varying Intercepts



Varying Slopes (+ Intercepts)

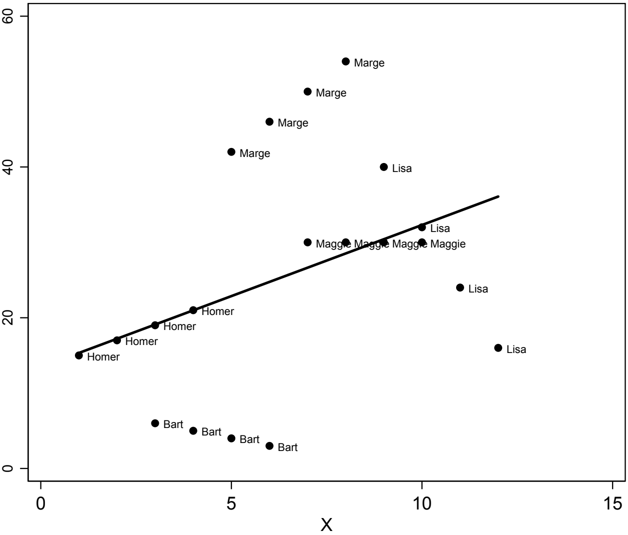
$$Y_{it} = \beta_0 + \beta_{1i} X_{it} + u_{it} \qquad \text{(unit-level slopes)}$$

$$Y_{it} = \beta_{0i} + \beta_{1i} X_{it} + u_{it} \qquad \text{(unit-level slopes and intercepts)}$$

$$Y_{it} = \beta_{0t} + \beta_{1t} X_{it} + u_{it} \qquad \text{(time-level slopes and intercepts)}$$

$$Y_{it} = \beta_{0it} + \beta_{1it} X_{it} + u_{it} \qquad \text{(unit- and time-level slopes and intercepts)}$$

Varying Slopes + Intercepts



The Error

$$u_{it} \sim \text{i.i.d.} N(0, \sigma^2) \;\forall\; i, t$$

$$\begin{aligned}
\text{Var}(u_{it}) &= \text{Var}(u_{jt}) \;\forall\; i \neq j \text{ (i.e., no cross-unit heteroscedasticity)} \\
\text{Var}(u_{it}) &= \text{Var}(u_{is}) \;\forall\; t \neq s \text{ (i.e., no temporal heteroscedasticity)} \\
\text{Cov}(u_{it}, u_{js}) &= 0 \;\forall\; i \neq j, \;\forall\; t \neq s \text{ (i.e., no auto- or spatial correlation)}
\end{aligned}$$

Pooling

- Adds data
- Enhances generalizability

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

Implies

- that the process governing the relationship between X and Y is exactly the same for each i,
- that the process governing the relationship between X and Y is the same for all t,
- that the process governing the us is the same $\forall$ i and t as well.

Two regimes:

$$Y_A = \beta'_A \mathbf{X}_A + u_A$$

$$Y_B = \beta'_B \mathbf{X}_B + u_B$$

with $\sigma^2_A = \sigma^2_B$, and $\text{Cov}(u_A, u_B) = 0$.

Estimators:

$$\hat{\beta}_{A,B} = (\mathbf{X}'_{A,B} \mathbf{X}_{A,B})^{-1} \mathbf{X}'_{A,B} Y_{A,B}$$

and

$$\widehat{\text{Var}(\beta_{A,B})} = \hat{\sigma}^2_{A,B} (\mathbf{X}'_{A,B} \mathbf{X}_{A,B})^{-1},$$

A Pooled Estimator

$$\begin{aligned}\hat{\beta}_P &= (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}(\mathbf{X}_A' Y_A + \mathbf{X}_B' Y_B) \\ &= (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}[\beta_A(\mathbf{X}_A'\mathbf{X}_A) + \beta_B(\mathbf{X}_B'\mathbf{X}_B)],\end{aligned}$$

$$\begin{aligned}\mathrm{E}(\hat{\beta}_P) &= \beta_A + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_B'\mathbf{X}_B(\beta_B - \beta_A) \\ &= \beta_B + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_A'\mathbf{X}_A(\beta_A - \beta_B)\end{aligned}$$

...and a test:

$$F = \frac{\frac{\hat{\mathbf{u}}_P'\hat{\mathbf{u}}_P - (\hat{\mathbf{u}}_A'\hat{\mathbf{u}}_A + \hat{\mathbf{u}}_B'\hat{\mathbf{u}}_B)}{K}}{\frac{(\hat{\mathbf{u}}_A'\hat{\mathbf{u}}_A + \hat{\mathbf{u}}_B'\hat{\mathbf{u}}_B)}{(N_A + N_B - 2K)}} \sim F_{[K,(N_A+N_B-2K)]}$$

$$\hat{\beta}_\lambda = (\lambda^2 \mathbf{X}_A' \mathbf{X}_A + \mathbf{X}_B' \mathbf{X}_B)^{-1} (\lambda^2 \mathbf{X}_A' Y_A + \mathbf{X}_B' Y_B)$$

with $\lambda \in [0, 1]$:

- $\lambda = 0 \ \rightarrow$ separate estimators for $\hat{\beta}_A$ and $\hat{\beta}_B$,
- $\lambda = 1 \ \rightarrow$ "fully pooled" estimator $\hat{\beta}_P$,
- $0 < \lambda < 1 \ \rightarrow$ a regression where data in regime A are given some "partial" weighting in their contribution towards an estimate of β.

Pooling, Summarized

"(R)oughly speaking, it makes sense to pool disparate observations if the underlying parameters governing those observations are sufficiently similar, but not otherwise."

One- and Two-Way Unit Effects

Two-way variation:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \gamma V_i + \delta W_t + u_{it}$$

$\longrightarrow$ two-way effects:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + \eta_t + u_{it}$$

One-way effects:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \eta_t + u_{it} \quad \text{(time)}$$

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it} \quad \text{(units)}$$

“Fixed” Effects

“Brute force” model:

$$\begin{aligned} Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it} \\ &= \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_1 I(i=1)_i + \alpha_2 I(i=2)_i + \ldots + u_{it} \end{aligned}$$

Alternatively:

$$\bar{X}_i = \frac{\sum_{N_i} X_{it}}{N_i}$$

and

$$\tilde{X}_{it} = X_{it} - \bar{X}_i.$$

Yields:

$$Y_{it} = \bar{\mathbf{X}}_i \boldsymbol{\beta}_B + (\mathbf{X}_{it} - \bar{X}_i)\boldsymbol{\beta}_W + \alpha_i + u_{it}$$

Means that:

$$\begin{aligned} Y^*_{it} &= Y_{it} - \bar{Y}_i \\ \mathbf{X}^*_{it} &= \mathbf{X}_{it} - \bar{\mathbf{X}}_i \end{aligned}$$

$$Y^*_{it} = \boldsymbol{\beta}_{FE} \mathbf{X}^*_{it} + u_{it}.$$

$\equiv$ “Within-Effects” Model.

Standard F-test for

$$H_0 : \alpha_i = \alpha_j \forall i \neq j$$

versus

$$H_A : \alpha_i \neq \alpha_j \text{ for some } i \neq j$$

is $\sim F_{N-1, NT-(N-1)}$.

An Example: Demonstrations, 1945-2014

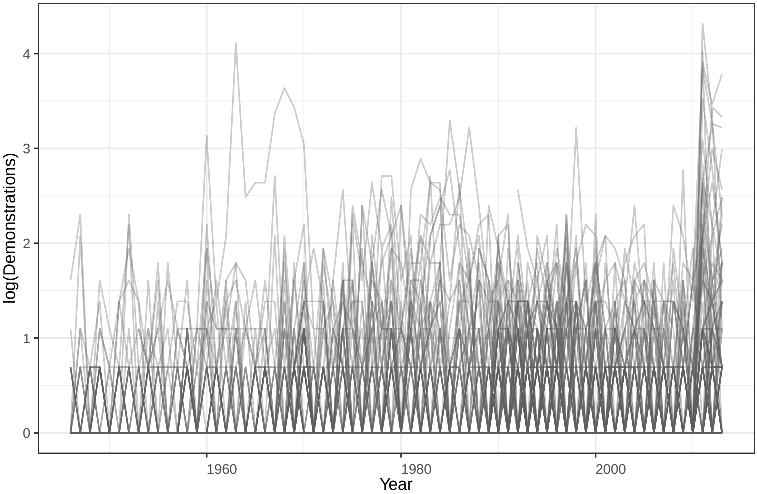
Data:

- 180 countries
- 70 years
- i indexes countries, t indexes years

Model:

$$\begin{aligned} ln(\text{Demonstrations} + 1)_{it} &= \beta_0 + \beta_1 \text{POLITY}_{it} + \beta_2 \text{POLITY}^2_{it} + \beta_3 \ln(\text{GDP})_{it} + \\ &\quad \beta_4 \text{Monarch}_{it} + \beta_5 \text{Cold War}_{it} + u_{it} \end{aligned}$$

Visualizing Panel Data: Continuous X



Visualizing Panel Data: Discrete X

(Created using `panelView`.)

```
> OLS<-lm(lnDemons~POLITY+I(POLITY^2)+lnGDP+Monarch+ColdWar,
+           data=PDF)
>
> summary(OLS)

Call:
lm(formula = lnDemons ~ POLITY + I(POLITY^2) + lnGDP + Monarch +
    ColdWar, data = PDF)

Residuals:
   Min     1Q Median     3Q    Max
-0.450 -0.293 -0.218 -0.075  4.107

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.124639   0.058208   -2.14    0.032 *
POLITY       0.006296   0.001179    5.34  9.5e-08 ***
I(POLITY^2) -0.002267   0.000255   -8.90  < 2e-16 ***
lnGDP        0.057679   0.007513    7.68  1.9e-14 ***
Monarch     -0.046393   0.028572   -1.62    0.104
ColdWar      0.027883   0.013961    2.00    0.046 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.526 on 6499 degrees of freedom
  (2863 observations deleted due to missingness)
Multiple R-squared:  0.0261,Adjusted R-squared:  0.0253
F-statistic: 34.8 on 5 and 6499 DF,  p-value: <2e-16
```

“Fixed” (Within) Effects

```
> FE<-plm(lnDemons~POLITY+I(POLITY^2)+lnGDP+Monarch+ColdWar,
+             data=PDF, effect="individual",model="within")
>
> summary(FE)
Oneway (individual) effect Within Model

Call:
plm(formula = lnDemons ~ POLITY + I(POLITY^2) + lnGDP + Monarch +
    ColdWar, data = PDF, effect = "individual", model = "within")

Unbalanced Panel: n = 145, T = 1-62, N = 6505

Residuals:
   Min. 1st Qu.  Median 3rd Qu.    Max.
-1.3556 -0.2120 -0.0768  0.0193  4.0496

Coefficients:
             Estimate Std. Error t-value Pr(>|t|)
POLITY       0.001526   0.001553    0.98  0.32604
I(POLITY^2) -0.001942   0.000296   -6.55  6.1e-11 ***
lnGDP        0.054586   0.015200    3.59  0.00033 ***
Monarch      0.047976   0.068071    0.70  0.48097
ColdWar     -0.035487   0.016235   -2.19  0.02887 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    1410
Residual Sum of Squares: 1400
R-Squared:      0.013
Adj. R-Squared: -0.0102
F-statistic: 16.7177 on 5 and 6355 DF, p-value: <2e-16
```

A Nicer Table

Table: Models of Demonstrations

	OLS	FE
POLITY	0.006***	0.002
	(0.001)	(0.002)
POLITY Squared	−0.002***	−0.002***
	(0.0003)	(0.0003)
ln(GDP)	0.058***	0.055***
	(0.008)	(0.015)
Monarch	−0.046	0.048
	(0.029)	(0.068)
Cold War	0.028**	−0.035**
	(0.014)	(0.016)
Constant	−0.125**	
	(0.058)	
Observations	6,505	6,505
R^2	0.026	0.013
Adjusted R^2	0.025	-0.010
Residual Std. Error	0.526 (df = 6499)	
F Statistic	34.820*** (df = 5; 6499)	16.720*** (df = 5; 6355)

*p<0.1; **p<0.05; ***p<0.01

Issues (?) with “Fixed” Effects

Pros:

- Specification Bias
- Intuitive
- Widely Used/Understood

Cons:

- Can’t Estimate $\boldsymbol{\beta}_B$
- Slowly-Changing **X**s
- (In)Efficiency / Inconsistency (Incidental Parameters)

“Between” Effects

From:

$$Y_{it} = \bar{\mathbf{X}}_i \boldsymbol{\beta}_B + (\mathbf{X}_{it} - \bar{\mathbf{X}}_i)\beta_W + \alpha_i + u_{it}.$$

“Between” effects:

$$\bar{Y}_i = \bar{\mathbf{X}}_i \boldsymbol{\beta}_B + u_{it}$$

- Essentially cross-sectional
- Based on N observations

"Between" Effects

```
> BE<-plm(lnDemons~POLITY+I(POLITY^2)+lnGDP+Monarch+ColdWar,
+           data=PDF, effect="individual",model="between")
>
> summary(BE)
Oneway (individual) effect Between Model

Call:
plm(formula = lnDemons ~ POLITY + I(POLITY^2) + lnGDP + Monarch +
    ColdWar, data = PDF, effect = "individual", model = "between")

Unbalanced Panel: n = 145, T = 1-62, N = 6505
Observations used in estimation: 145

Coefficients:
            Estimate Std. Error t-value Pr(>|t|)
(Intercept) -0.30601    0.20837   -1.47   0.1442
POLITY       0.00597    0.00489    1.22   0.2244
I(POLITY^2) -0.00302    0.00112   -2.69   0.0079 **
lnGDP        0.06883    0.02734    2.52   0.0130 *
Monarch     -0.04966    0.10320   -0.48   0.6312
ColdWar      0.25872    0.08482    3.05   0.0027 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    8.75
Residual Sum of Squares: 7.64
R-Squared:      0.127
Adj. R-Squared: 0.0961
F-statistic: 4.06164 on 5 and 139 DF, p-value: 0.0018
```

A Nicer Table (Again)

Table: Models of Demonstrations

	OLS	FE	BE
POLITY	0.006***	0.002	0.006
	(0.001)	(0.002)	(0.005)
POLITY Squared	−0.002***	−0.002***	−0.003***
	(0.0003)	(0.0003)	(0.001)
ln(GDP)	0.058***	0.055***	0.069**
	(0.008)	(0.015)	(0.027)
Monarch	−0.046	0.048	−0.050
	(0.029)	(0.068)	(0.103)
Cold War	0.028**	−0.035**	0.259***
	(0.014)	(0.016)	(0.085)
Constant	−0.125**		−0.306
	(0.058)		(0.208)
Observations	6,505	6,505	145
R^2	0.026	0.013	0.127
Adjusted R^2	0.025	-0.010	0.096
Residual Std. Error	0.526 (df = 6499)		
F Statistic	34.800*** (df = 5; 6499)	16.700*** (df = 5; 6355)	4.060*** (df = 5; 139)

*p<0.1; **p<0.05; ***p<0.01

Model:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}$$

with:

$$u_{it} = \alpha_i + \lambda_t + \eta_{it}$$

and

$$\begin{aligned}
E(\alpha_i) = E(\lambda_t) = E(\eta_{it}) &= 0, \\
E(\alpha_i\lambda_t) = E(\alpha_i\eta_{it}) = E(\lambda_t\eta_{it}) &= 0, \\
E(\alpha_i\alpha_j) &= \sigma_\alpha^2 \text{ if } i = j, \ 0 \text{ otherwise}, \\
E(\lambda_t\lambda_s) &= \sigma_\lambda^2 \text{ if } t = s, \ 0 \text{ otherwise}, \\
E(\eta_{it}\eta_{js}) &= \sigma_\eta^2 \text{ if } i = j, \ t = s, \ 0 \text{ otherwise}, \\
E(\alpha_i\mathbf{X}_{it}) = E(\lambda_t\mathbf{X}_{it}) = E(\eta_{it}\mathbf{X}_{it}) &= 0.
\end{aligned}$$

“Variance Components”:

$$Var(Y_{it}|\mathbf{X}_{it}) = \sigma^2_\alpha + \sigma^2_\lambda + \sigma^2_\eta$$

If we assume $\lambda_t = 0$, then we get a model like:

$$Y_{it} = \mathbf{X}_{it}\beta + \alpha_i + \eta_{it}$$

with total error variance:

$$\sigma^2_u = \sigma^2_\alpha + \sigma^2_\eta.$$

$$
\begin{aligned}
E(\mathbf{u}_i \mathbf{u}_i') \equiv \mathbf{\Sigma}_i &= \sigma_\eta^2 \mathbf{I}_T + \sigma_\alpha^2 \mathbf{i}\mathbf{i}' \\
&= \begin{pmatrix}
\sigma_\eta^2 + \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\
\sigma_\alpha^2 & \sigma_\eta^2 + \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\eta^2 + \sigma_\alpha^2
\end{pmatrix}
\end{aligned}
$$

$$
\text{Var}(\mathbf{u}) \equiv \mathbf{\Omega} = \begin{pmatrix}
\mathbf{\Sigma}_1 & 0 & \cdots & 0 \\
0 & \mathbf{\Sigma}_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \mathbf{\Sigma}_N
\end{pmatrix}
$$

Can estimate:

$$\boldsymbol{\Sigma}^{-1/2} = \frac{1}{\sigma_\eta}\left[\mathbf{I}_T - \left(\frac{\theta}{T}\mathbf{ii}'\right)\right]$$

where

$$\theta = 1 - \sqrt{\frac{\sigma_\eta^2}{T\sigma_\alpha^2 + \sigma_\eta^2}}.$$

With $\hat{\theta}$, calculate:

$$\begin{aligned} Y_{it}^* &= Y_{it} - \hat{\theta}\bar{Y}_i \\ X_{it}^* &= X_{it} - \hat{\theta}\bar{X}_i, \end{aligned}$$

estimate:

$$Y_{it}^* = (1-\hat{\theta})\alpha + X_{it}^*\beta_{RE} + [(1-\hat{\theta})\alpha_i + (\eta_{it} - \hat{\theta}\bar{\eta}_i)]$$

and iterate...

"Random" Effects: An Alternative View

```
> RE<-plm(lnDemons~POLITY+I(POLITY^2)+lnGDP+Monarch+ColdWar,
+               data=PDF, effect="individual", model="random")

> summary(RE)
Oneway (individual) effect Random Effect Model
   (Swamy-Arora's transformation)

Call:
plm(formula = lnDemons ~ POLITY + I(POLITY^2) + lnGDP + Monarch +
    ColdWar, data = PDF, effect = "individual", model = "random")

Unbalanced Panel: n = 145, T = 1-62, N = 6505

Effects:
                 var std.dev share
idiosyncratic 0.2197  0.4687   0.8
individual    0.0563  0.2373   0.2
theta:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  0.108   0.708   0.736   0.724   0.757   0.757

Coefficients:
             Estimate Std. Error z-value Pr(>|z|)
(Intercept) -0.131708   0.104987   -1.25    0.210
POLITY       0.002574   0.001450    1.78    0.076 .
I(POLITY^2) -0.001953   0.000287   -6.81  9.6e-12 ***
lnGDP        0.057117   0.012443    4.59  4.4e-06 ***
Monarch     -0.006937   0.053291   -0.13    0.896
ColdWar     -0.023580   0.015008   -1.57    0.116
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    1440
Residual Sum of Squares: 1430
R-Squared:      0.0124
Adj. R-Squared: 0.0117
Chisq: 81.0788 on 5 DF, p-value: 4.99e-16
```

Random Effects Remix (using `lmer`)

```
> library(lme4)

> AltRE<-lmer(lnDemons~POLITY+I(POLITY^2)+lnGDP+Monarch+ColdWar+
+                  (1|ccode), data=Demos)
>
> summary(AltRE)
Linear mixed model fit by REML ['lmerMod']
Formula: lnDemons ~ POLITY + I(POLITY^2) + lnGDP + Monarch + ColdWar +
    (1 | ccode)
   Data: Demos

REML criterion at convergence: 9005

Random effects:
 Groups   Name        Variance Std.Dev.
 ccode    (Intercept) 0.0536   0.232
 Residual             0.2200   0.469
Number of obs: 6507, groups:  ccode, 145

Fixed effects:
             Estimate Std. Error t value
(Intercept) -0.133634   0.104246   -1.28
POLITY       0.002623   0.001447    1.81
I(POLITY^2) -0.001972   0.000287   -6.88
lnGDP        0.057513   0.012371    4.65
Monarch     -0.015175   0.052863   -0.29
ColdWar     -0.022225   0.014986   -1.48

Correlation of Fixed Effects:
            (Intr) POLITY I(POLI lnGDP  Monrch
POLITY       0.109
I(POLITY^2)  0.134 -0.135
lnGDP       -0.968 -0.140 -0.270
Monarch      0.004  0.172 -0.163 -0.022
ColdWar     -0.391  0.387 -0.210  0.351  0.014
```

A Nicer Table (Yet Again)

Table: Models of Demonstrations

	OLS	FE	BE	RE
POLITY	0.006***	0.002	0.006	0.003*
	(0.001)	(0.002)	(0.005)	(0.001)
POLITY Squared	−0.002***	−0.002***	−0.003***	−0.002***
	(0.0003)	(0.0003)	(0.001)	(0.0003)
ln(GDP)	0.058***	0.055***	0.069**	0.057***
	(0.008)	(0.015)	(0.027)	(0.012)
Monarch	−0.046	0.048	−0.050	−0.007
	(0.029)	(0.068)	(0.103)	(0.053)
Cold War	0.028**	−0.035**	0.259***	−0.024
	(0.014)	(0.016)	(0.085)	(0.015)
Constant	−0.125**		−0.306	−0.132
	(0.058)		(0.208)	(0.105)
Observations	6,505	6,505	145	6,505
R^2	0.026	0.013	0.127	0.012
Adjusted R^2	0.025	-0.010	0.096	0.012
Residual Std. Error	0.526 (df = 6499)			
F Statistic	34.800*** (df = 5; 6499)	16.700*** (df = 5; 6355)	4.060*** (df = 5; 139)	81.100***

*p<0.1; **p<0.05; ***p<0.01

“Random” Effects: Testing

Hausman test (FE vs. RE):

$$\hat{W} = (\hat{\beta}_{\mathbf{FE}} - \hat{\beta}_{\mathbf{RE}})'(\hat{\mathbf{V}}_{\mathbf{FE}} - \hat{\mathbf{V}}_{\mathbf{RE}})^{-1}(\hat{\beta}_{\mathbf{FE}} - \hat{\beta}_{\mathbf{RE}})$$

$$W \sim \chi^2_k$$

Issues:

- Asymptotic
- No guarantee $(\hat{\mathbf{V}}_{\mathbf{FE}} - \hat{\mathbf{V}}_{\mathbf{RE}})^{-1}$ is positive definite
- A <u>general</u> specification test...

Hausman test (FE vs. RE):

```
> phtest(FE, RE)

Hausman Test

data:  lnDemons ~ POLITY + I(POLITY^2) + lnGDP + Monarch + ColdWar
chisq = 10, df = 5, p-value = 0.05
alternative hypothesis: one model is inconsistent
```

Practical “Fixed” vs. “Random” Effects

- “Panel” vs. “TSCS” Data
- Data-Generating Process
- Covariate Effects

Unit Effects Models: Software

R :

- the `lme4` package; command is `lmer`
- the `plm` package; `plm` command
- the `nlme` package; command `lme`

Stata : `xtreg`

- the `re` (the default) = random effects
- the `fe` = fixed (within) effects
- the `be` = between-effects

Dynamics

Lagged: Y?

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

If ϵ_{it} is perfect...

- $\hat{\beta}_{LDV}$ is biased (but consistent),
- O(bias) $= \frac{-1+3\boldsymbol{\beta}_{LDV}}{T}$

If ϵ_{it} is autocorrelated...

- $\hat{\beta}_{LDV}$ is biased and <u>inconsistent</u>
- IV is one (bad) option...

Lagged Ys and GLS-ARMA

Can rewrite:

$$\begin{aligned} Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + u_{it} \\ u_{it} &= \phi u_{it-1} + \eta_{it} \end{aligned}$$

as

$$\begin{aligned} Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi u_{it-1} + \eta_{it} \\ &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi(Y_{it-1} - \mathbf{X}_{it-1}\boldsymbol{\beta}_{AR}) + \eta_{it} \\ &= \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \mathbf{X}_{it-1}\psi + \eta_{it} \end{aligned}$$

where $\psi = \phi\beta_{AR}$ and $\psi = 0$ (by assumption).

Lagged Ys and World Domination

In:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it} \boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

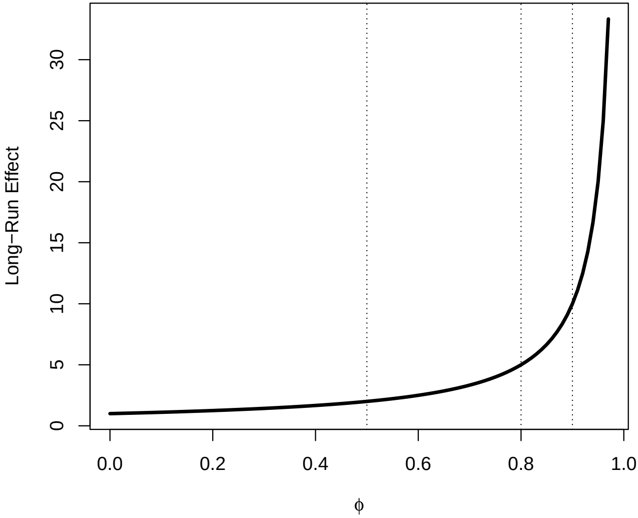
Achen: Bias "deflates" $\hat{\boldsymbol{\beta}}_{LDV}$ relative to $\hat{\phi}$, "suppress" the effects of $\mathbf{X}$...

Keele & Kelly (2006):

- Contingent on ϵs having autocorrelation
- Key: In LDV, *long-run impact of a unit change in X is*:

$$\hat{\beta}_{LR} = \frac{\hat{\beta}_{LDV}}{1 - \hat{\phi}}$$

Long-Run Impact for $\hat{\beta} = 1$



Lagged Ys and Unit Effects

Consider:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it}.$$

If we omit the unit effects, we have:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + u^*_{it}$$

with

$$u^*_{it} = \alpha_i + u_{it}$$

Lagging yields:

$$Y_{it-1} = \phi Y_{it-2} + \mathbf{X}_{it-1}\boldsymbol{\beta} + \alpha_i + u_{it-1}$$

which means

$$\text{Cov}(Y_{it-1}, u^*_{it}) \neq 0. \ \rightarrow \ \text{bias in } \hat{\phi}, \hat{\boldsymbol{\beta}}$$

"Nickell" Bias

Bias in $\hat{\phi}$ is

- toward zero when $\phi > 0$,
- increasing in ϕ.

<u>Including</u> unit effects still yields bias in $\hat{\phi}$ of $O(\frac{1}{T})$, and bias in $\hat{\boldsymbol{\beta}}$.

Solutions:

- Difference/GMM estimation
- Bias correction approaches

First Difference Estimation

$$
\begin{aligned}
Y_{it} - Y_{it-1} &= \phi(Y_{it-1} - Y_{it-2}) + (\mathbf{X}_{it} - \mathbf{X}_{it-1})\boldsymbol{\beta} + (\alpha_i - \alpha_i) + (u_{it} - u_{it-1}) \\
\Delta Y_{it} &= \phi \Delta Y_{it-1} + \Delta \mathbf{X}_{it}\boldsymbol{\beta} + \Delta u_{it}
\end{aligned}
$$

Anderson/Hsiao: If $\nexists$ autocorrelation, then use ΔY_{it-2} or Y_{it-2} as instruments for ΔY_{it-1}...

- Consistent in theory,
- in practice, the former is preferred, and both have issues if ϕ is high;
- both are inefficient.

A&H $\rightarrow$ A&B

Arellano & Bond (also Wawro): Use *all* lags of Y_{it} and $\mathbf{X}_{it}$ from $t-2$ and before.

- "Good" estimates, better as $T \rightarrow \infty$,
- Easy to handle higher-order lags of Y,
- Easy software (`plm` in R , `xtabond` in Stata).
- Model *is* fixed effects...
- $\mathbf{Z}_i$ has $T-p-1$ rows, $\sum_{i=p}^{T-2} i$ columns $\rightarrow$ difficulty of estimation declines in p, grows in T.

Another Approach: Orthogonalization

Note: We're rarely substantively interested in the fixed effects $\hat{\alpha}$...

- $\rightarrow$ reparameterize the αs so that they are *information-orthogonal* to the other parameters in the model (including the βs and ϕ)
- Key idea: Transform the αs so that (for example):

$$\mathrm{E}\left(\frac{\partial^2 L_i}{\partial\alpha\partial\beta}\right) = 0$$

- Can do this via imposition of priors, in a Bayesian framework...
- **In general**, this approach is less assumption-laden and more efficient than the IV/GMM-based approaches discussed above.

<u>References</u>:

- Lancaster, T. 2002. "Orthogonal Parameters and Panel Data." *Review of Economic Studies* 69:647-666.
- Pickup et al. (2017) [the "orthogonalized panel model" ("OPM")]

Stationarity and Unit Roots in Panel Data

- Short series + Asymptotic tests $\rightarrow$ "borrow strength"
- Requires uniform unit roots across cross-sectional units
- Various alternatives:
  - Maddala and Wu (1999)
  - Hadri (2000)
  - Levin, Lin and Chu (2002)
- What to do?
  - Difference the data...
  - Error-correction models

Demonstrations: Panel Unit Root Tests

```
> purtest(lnDemons,exo="trend",test=c("levinlin"))

Levin-Lin-Chu Unit-Root Test (ex. var.: Individual Intercepts and
Trend)

data:  lnDemons
z = -3.2, p-value = 0.0007
alternative hypothesis: stationarity

Warning message:
In selectT(l, theTs) : the time series is long


> purtest(lnDemons,exo="trend",test=c("hadri"))

Hadri Test (ex. var.: Individual Intercepts and Trend)
(Heterosked. Consistent)

data:  lnDemons
z = 671, p-value <2e-16
alternative hypothesis: at least one series has a unit root


> purtest(lnDemons,exo="trend",test=c("ips"))

Im-Pesaran-Shin Unit-Root Test (ex. var.: Individual Intercepts
and Trend)

data:  lnDemons
Wtbar = -24, p-value <2e-16
alternative hypothesis: stationarity
```

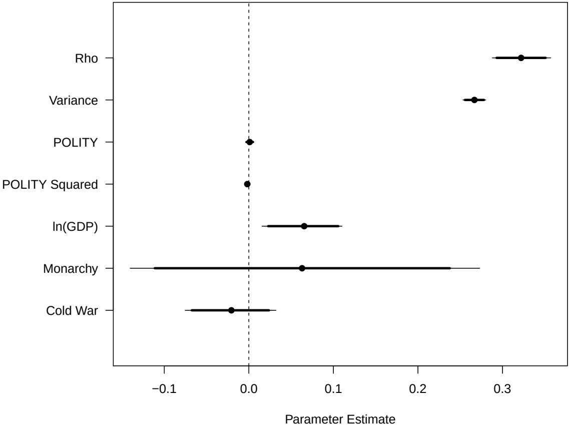
Some Dynamic Models

	LDV	First Difference	FE	LDV + FE
Intercept	-0.104	0.010		
	(0.053)	(0.007)		
Lagged ln(Demonstrations)	0.440^{*}			0.267^{*}
	(0.012)			(0.013)
POLITY	0.003^{*}	0.001	0.002	< 0.001
	(0.001)	(0.004)	(0.002)	(0.002)
POLITY Squared	-0.001^{*}	-0.003^{*}	-0.002^{*}	-0.002^{*}
	(< 0.001)	(0.001)	(< 0.001)	(< 0.001)
ln(GDP)	0.038^{*}	-0.108	0.055^{*}	0.049^{*}
	(0.007)	(0.079)	(0.015)	(0.015)
Monarch	-0.017	-0.004	0.048	0.070
	(0.026)	(0.139)	(0.068)	(0.067)
Cold War	0.011	-0.134^{*}	-0.035^{*}	-0.029
	(0.013)	(0.052)	(0.016)	(0.016)
R^2	0.200	0.004	0.013	0.077
Adj. R^2	0.199	0.003	-0.010	0.055
Num. obs.	6419	6360	6505	6419

$^{*}p < 0.05$

FE + Dynamics Using Orthogonalization

```
> library(OrthoPanels)
> set.seed(7222009)
> OPM.fit <- opm(lnDemons~POLITY+POLITYSQ+lnGDP+Monarch+ColdWar,
              data=PDF,index=c("ccode","Year"),n.samp=1000)
```



OPM Results: Short- and Long-Run Effects

For $\hat{\phi} \approx 0.32$:

Parameter	Short-Run	Long-Run
POLITY	0.0010	0.0015
POLITY Squared	-0.0018	-0.0027
ln(GDP)	0.0655	0.0956
Monarch	0.0629	0.0913
Cold War	-0.0206	-0.0310

Final Thoughts: Dynamic Panel Models

- N vs. T...
- Are dynamics nuisance or substance?
- What problem(s) do you *really* care about?