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Numerik I – Homework 5

Deadline: 29.3.2019, 13:00

Restriction   The use of built-in functions for Fourier transform is not allowed.

Exercise 1 (6 P.) (Programming task)
Write a function that implements the Fourier transform given by Definition 5.2 in the lecture notes. The function should take the real-valued vector $(f_k)_{k=0}^{N-1}$ as the input argument and return as the output

(a) the Fourier coefficients $\hat{f}_k$;

(b) the coefficients $\hat{A}_m$ and $\hat{B}_m$ of the trigonometric polynomial $T(t)$ for real-valued f_k (Equation 5.5 in the lecture notes);

(c) a plot of the trigonometric polynomial $T(t)$.

Exercise 2 (6 P.) (Programming task)
On the course homepage there is a data file *f.dat* which stores the evaluations of a continuous function $f : [0, 2x_0] \to \mathbb{R}$

$$f_k = f\left(\frac{2x_0 k}{N}\right) \quad \text{where } k = 0, 1, \cdots, N-1$$

for $N = 2^{16}$. The function has the property $f(0) = f(2x_0)$, $x_0 = 101.15520937701599$.

For the refinement level $l = 2, 4, 6, 8, 10, 12, 14, 16$,

(a) Use your program in Exercise 1 to calculate the trigonometric polynomial $T(t)$ by using the following data sets respectively as the input argument

$$f\left(\frac{2x_0 k}{2^l}\right) \quad \text{where } k = 0, 1, \cdots, 2^l - 1;$$

and show the trigonometric polynomials in the plot.

(b) Calculate the approximation error for each data set

$$\max_{0 \le k \le N-1} \left| T_l\left(\frac{2x_0 k}{N}\right) - f_k \right|$$

and record them in a table.