computational mathematics

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Numerik I – Homework 1

Deadline: 01.03.2019, 13:00

Exercise 1 (4 P.)
Define

$$y := 333.75b^6 + a^2(11a^2b^2 - b^6 - 121b^4 - 2) + 5.5b^8 + \frac{a}{2b}.$$

(a) Compute y for $a = \sqrt{4950000001}$ and $b = 30000$ using two different tools (e.g. Matlab, Maple/Mathematica). What do you observe? Why?

(b) The parameters $a = 10^6$, $b = 10^6 + 10^{-2}$ are rounded to a^*, b^* according to

$$a^* = a(1 + \epsilon_1)$$
$$b^* = b(1 + \epsilon_2),$$

where $|\epsilon_1|$, $|\epsilon_2| \ll 1$. Use error analysis of first order[1] to find ϵ_1, ϵ_2 s.t. $z = a^2 - b^2$ and $z^* = a^{*2} - b^{*2}$ satisfy

$$z^* \doteq z(1 + \epsilon_*), \quad |\epsilon_*| \leq 10^{-7}.$$

(c) Let

$$y = \cos(x + \delta) - \cos(x), \tag{1}$$

with $\delta > 0$ small. Find an equivalent expression of (**??**) that avoids round-off errors.

Exercise 2 (5 P.)
We consider the integrals defined by

$$I_n = \int_0^1 x^n e^x \mathrm{d}x, \tag{2}$$

for $n = 0, 1, 2, 3, ...$

(a) Prove that

$$|I_n| \leq \frac{e}{n+1}. \tag{3}$$

(b) Prove, by using an integration by parts, the recurrence relation

$$I_k = e - kI_{k-1}, \quad k \geq 1. \tag{4}$$

Compute (with Matlab) I_{100} using formula (**??**) starting with $I_0 = e - 1$. Compare your result to the "exact" value $I_{100} = 0.026652359191789498$ and comment.

(c) Let us introduce $\nu > n$, $\nu \in \mathbb{N}$. Using the reversed (backward) recurrence

$$I_k = (e - I_{k+1})/(k + 1), \quad k = \nu - 1, \nu - 2, .., n. \tag{5}$$

[1] ignoring quadratic terms of ϵ

compute (with Matlab) an approximation I_{100}^* of I_{100} starting with $I_\nu^* = 0$, for $\nu = 200$. Compare the results of (b) and (c).

To explain the differences, derive the relation

$$\delta_n = \left| \frac{I_n - I_n^*}{I_n} \right| \leq \left(\prod_{l=n+1}^{\nu} \left| \frac{I_l}{e - I_l} \right| \right) \left| \frac{I_\nu - I_\nu^*}{I_\nu} \right| \tag{6}$$

and show , using (**??**), that

$$\left(\prod_{l=n+1}^{\nu} \left| \frac{I_l}{e - I_l} \right| \right) \leq \left(\frac{1}{n} \right)^{\nu - n - 1} \tag{7}$$

(d) How to choose ν to satisfy $|\delta_{100}| \leq \epsilon$, for given $\epsilon > 0$?