MAT801

Numerik I – Assignment 2

Deadline: 08.03.2018, 12:00

Please note that the use of Matlab internal functions like `polyfit` or `polyval` is not permitted to solve these exercises.

Exercise 1

a) Write a Matlab function that computes the Newton differences as in formula (2.7) of the lecture notes. Input: nodes $(x_i)_{i=0}^n$ as vector, function values $(f_i)_{i=0}^n$ as vector. Output: Newton differences $(b_i)_{i=0}^n$ as vector. (3 points)

b) Write a Matlab function that evaluates the interpolation polynomial defined via the nodes $(x_i)_{i=0}^n$ and Newton differences $(b_i)_{i=0}^n$ at a given point x (Satz 2.5 in the lecture notes).
Verify the functionality of your programs by interpolating the function $f(x) = x(x-1)$ at $x_0 = 0$, $x_1 = 1$ and $x_2 = 2$. Evaluate the interpolation at $x = 0.5$. What do you expect as result? (2 points)

Please use the code that you developed above to solve the following exercises.

Exercise 2

Let the function

$$f(x) := \frac{x}{1 + \cos(x)^2},$$

and intervals $I_1 := [-2, 2]$ and $I_2 := [-4, 4]$ be given.

a) Construct the Newton polynomials of degree $n = 5, 10, 15$ on $n+1$ equispaced nodes in I_1 (the nodes should include the endpoints of the interval). Plot the polynomials and f in the same graph. (2 points)

b) Construct the Newton polynomials of degree $n = 5, 10, 15$ on $n+1$ equispaced nodes in I_2. Plot the polynomials and f in the same graph. (1 point)

c) Explain the behaviour you observed in b). Use error estimate (2.13) from the lecture notes in your reasoning. (2 points)

d) Construct the Newton polynomials P_n of degree $n = 5, 10, 15, 20, 25$ on $n+1$ Chebyschev nodes in I_2. Plot the approximations and f in the same graph. Estimate the maximum norm

$$\|f - P_n\|_{\max}, \quad n = 5, 10, 15, 20, 25$$

by taking the maximum of the absolute value of the difference of f and P_n on 1000 equispaced nodes on I_2. Display the errors in a table. What do you observe an why? (3 points)

Exercise 3

Let $g(x) := \sin(2\pi x)$.

a) Evaluate the function g at 21 equispaced nodes in $[-1, 1]$ (the nodes should contain the endpoints -1 and 1). Compute the Newton polynomial P of degree 20. Plot P and f in the same graph. (1 point)

b) Load the file 'measurements.dat' (use the Matlab command `load`). It contains 21 numbers that can be interpreted as function values of an unknown function $\bar{g}$ at the 21 equispaced nodes in $[-1, 1]$ from a). Approximate $\bar{g}$ by computing the interpolating polynomial and plot this approximation and g in the same graph. Knowing a posteriori that the set of 'mesured' data came from the function $\bar{g}(x) = \sin(2\pi x) + (-1)^{i+1}10^{-4}$, comment on the result of your plot. (2 points)