computational mathematics

Prof. Dr. S. Sauter
Institut für Mathematik
Universität Zürich

Numerik I – Homework 2

Deadline: 8.3.2019, 13:00

Note: Do not use implemented Matlab functions (e.g. `polyfit`, `polyval`) to solve the exercises.

Exercise 1

(a) Write a Matlab function that takes nodes $(x_i)_{i=0}^n$ and values $(f_i)_{i=0}^n$ and computes the Newton differences $(b_i)_{i=0}^n$ from formula (2.7) of the lecture notes.

Write another Matlab function that *evaluates* at x the interpolation polynomial defined by the nodes $(x_i)_{i=0}^n$ and the Newton differences $(b_i)_{i=0}^n$ (as in Satz 2.5). This function should take as input the point x and the nodes and values $(x_i)_{i=0}^n$ and $(b_i)_{i=0}^n$.

(b) Let $d > 0$ and $f : [-d, d] \to \mathbb{R}$, $f(x) := \frac{1}{1+25x^2}$. Denote by $p_{d,n} \in \mathbb{P}_n$ the polynomial of degree n satisfying

$$p_{d,n}(x_i) = f(x_i) \quad \text{for all nodes} \quad x_i = -d + \frac{2d}{n} i \quad 0 \leq i \leq n.$$

Use your codes from Exercise 1.a to compute the interpolation polynomial of f for $d = 1$ and $n = 2, 4, 6, 8, 10$. Evaluate the polynomials $p_{d,n}$ and f in $\tilde{x}_j = -d + \frac{2d}{1000} j$, $0 \leq j \leq 1000$ and show all graphs in one plot. Enter the error

$$E_{d,n} := \max_{0 \leq j \leq 1000} |(f - p_{d,n})(\tilde{x}_j)| \tag{1}$$

with respect to n into a table. What do you observe?

(c) Repeat Exercise 1.b with $d = 0.1$. Use your table to derive (approximately) the rate of convergence with respect to the number of nodes n.

(d) Find an approximate value of $d_{\max}$, such that the error $E_{d,n}$ decreases for increasing n. Plot the logarithm of the the error, $\log(E_{d,n})$, for $d = d_{max}/2$ with respect to n and determine the constants C_1, β in the estimate

$$E_{d,n} \leq C_1 e^{-\beta n}.$$

Exercise 2

(a) Let $x_0 = t$ and $x_1 = 1 - t$. Find $t \in [0, 1]$ such that $\max_{x \in [0,1]} |\omega_1(x)|$ is minimal.

(b) Show in one plot the graph of $\omega_{10}(x)$ for equidistant nodes on the interval [-1,1] and for the Chebishev nodes, defined by:

$$x_i = \cos\left(\frac{2i+1}{2(n+1)}\pi\right), \quad 0 \leq i \leq n, \text{ for } n = 10. \tag{2}$$

What do you conclude?

(c) Let f be as in Exercise 1.b, $d = 1$. Compute the error (see (1)), where $p_{d,n}$ is the interpolation polynomial of degree 10 with the nodes defined in (2). What can you say about the error compared to your results of Exercise 1.b?