computational mathematics

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Numerik I – Homework 9

Deadline: 3.5.2019 13:00

Exercise 1 (Theoretical task)

(a) (2 P.) Let $f \in C^0(\mathbb{R})$ be a continuous function. Suppose there exists some $X \in \mathbb{R}$ and $\varepsilon > 0$, such that $[X-\varepsilon, X+\varepsilon]$ contains exactly one simple zero of f. Write an (efficient) algorithm to find real numbers a and b with $f(a)f(b) < 0$.

(b) (2 P.) Examine how the runtime of the algorithm depend on the unknowns X and ε.

Exercise 2 (Computational task)

(a) (1 P.) Implement the bisection method for finding zeros of a continuous function.

(b) (4 P.) Implement a method that finds the k-th zero (up to an accuracy of 10^{-7}) of the Legendre polynomial P_n defined in Homework 7 Exercise 3. Test your program to find the 8-th zero of P_{10}.

Exercise 3 (Theoretical task, 3 P.)

Prove by induction the statement (7.6) (Separationseigenschaft) from the lecture notes.
Hint: Use the 3-term recursion formula and evaluate the polynomial π_{n+1} in the zeros of π_n.

Exercise 4 (Theoretical task)

We want to find the zeros of the function

$$f_a(x) = x^2 - a$$

using Newton's method.

(a) (1 P.) How does the Newton iteration look like for this problem?

(b) (3 P.) Prove that Newton's method converges to $+\sqrt{a}$ for any positive starting value x_0.