



computational mathematics

Prof. Dr. S. Sauter
Institut für Mathematik
Universität Zürich

Numerik I – Homework 11

Deadline: 17.5.2019, 13:00

Exercise 1 (5 Points, Theoretical task)

Prove Lemma 8.2 from the lecture notes.

Exercise 2 (8 Points, Computational task)

With reference to the lecture notes, consider the Poisson problem (8.2a).

- (a) Implement a function which takes as input an integer n , and two functions f, g , and constructs the $n^2 \times n^2$ matrix M and the vector r of (8.6) as explained in the lecture notes (f, g have the same meaning as in the lecture notes).
- (b) Fix $\Omega := [0, 1]^2$, $f(x, y) = 2\pi^2 \sin(\pi x) \cos(\pi y)$ for $(x, y) \in \Omega$, $g(x, y) = \sin(\pi x) \cos(\pi y)$ for $(x, y) \in \partial\Omega$. Verify that the exact solution is given by g on the whole domain Ω .
- (c) For $8 \leq n \leq 20$ solve the resulting linear system (you can use the built-in functions of matlab and python for this task), and compute the error with respect to the exact solution; plot the error versus the number of grid points n in a bilogarithmic scale.
- (d) Is the error converging polynomially or exponentially? Which is the convergence rate?
- (e) Produce a three-dimensional plot of the approximate solution.

Exercise 3 (3 Points, Theoretical task)

Prove Lemma 8.10 from the lecture notes.