4.DZ

0036513702
IVAN KLABUČAR

1.

a) $S\left[x[n]\right] = y[n]$

$x[n] = z^n$

$$y[n] = S\left[x[n]\right] = S\left[z^n\right] = \overbrace{\sum_{m=-\infty}^{+\infty} h[m] z^{n-m}}^{\text{konvolucija}} =$$

$$= z^n \underbrace{\sum_{m=-\infty}^{+\infty} h[m] z^{-m}}_{H(z)} = z^n \underbrace{H(z)}_{=\lambda \in \mathbb{C}} //$$

$x[n] = z^n$ jest svojstvena funkcija.

$x(t) = e^{st}, s \in \mathbb{C}$

b) $$y(t) = S\left[x(t)\right] = h(t) * e^{st} = \int_{-\infty}^{+\infty} h(\tau) e^{s(t-\tau)} d\tau =$$

$$= e^{st} \int_{-\infty}^{+\infty} h(\tau) e^{-s\tau} d\tau = e^{st} \underbrace{H(s)}_{\lambda \in \mathbb{C}} //$$

$x(t) = e^{st}$ jest svojstvena funkcija! //

1. 4. DZ

IVAN KLABUČAR
0036513702

c) $S[x(t)] = y(t) = \int_{-\infty}^{+\infty} x(\tau) e^{-jt\tau} d\tau = \lambda \cdot x(t)$

$$= e^{t} \cdot \underbrace{\int_{-\infty}^{+\infty} x(\tau) e^{-j\tau} d\tau}_{X(j)} = e^{t} \cdot X(j) = \lambda x(t)$$

$$\Rightarrow x(t) = \frac{e^{t} X(j)}{\lambda}$$

2. a) $S\left[x[n]\right] = \sum_{m=-\infty}^{n} x[m] = y[n]$

$$S\left[a x_1[n] + b x_2[n]\right] = \sum_{m=-\infty}^{n} \left(a x_1[m] + b x_2[m]\right) =$$

$$= \sum_{m=-\infty}^{n} a x_1[m] + \sum_{m=-\infty}^{n} b x_2[m] = a y_1[n] + b y_2[n]$$

Sustav je linearan

$$\underbrace{S\left[x[n-k]\right] = \sum_{m=-\infty}^{n-k} x[m] = y[n-k]}$$

Sustav je vremenski ne promjenjiv

Sustav je kauzalan jer $y[n]$ ne traži poznavanje $x[m]$ za $m > n$.

4. DZ

IVAN KLABUČAR
0036513702

(2) b) $S[x[n]] = \sum_{m=n}^{+\infty} x[m]$

Sustav nije kauzalan jer $y[n]$ traži poznavanje pobuda koje se još nisu dogodile
$y[n]$ ovisi o $x[m]$, $m > n$

$S[x[n-k]] = \sum_{m=n-k}^{+\infty} x[m] = y[n-k]$ → $\boxed{\text{Sustav je vremenski nepromjenjiv!}}$

$S[ax_1[n] + bx_2[n]] = \sum_{m=n}^{+\infty} ax_1[m] + \sum_{m=n}^{+\infty} bx_2[m] = ay_1[n] + by_2[n]$

→ $\boxed{\text{Sustav je linearan!}}$

c) $S[x[n]] = 2x[n] + 5x[n-2] = y[n]$

→ Sustav je kauzalan jer $y[n]$ ne zahtjeva poznavanje $x[m]$, $m > n$!

$S[x[n-k]] = 2x[n-k] + 5x[n-k-2] = y[n-k]$

→ $\boxed{\text{Sustav je vremenski nepromjenjiv!}}$

$S[ax_1[n] + bx_2[n]] = 2ax_1[n] + 2bx_2[n] + 5ax_1[n-2] + 5bx_2[n-2]$

$= a(2x_1[n] + 5x_1[n-2]) + b(2x_2[n] + 5x_2[n-2])$

$= ay_1[n] + by_2[n]$ → $\boxed{\text{Sustav je linearan!}}$

4. DZ

IVAN KLABUČAR
0036513702

2. d) $S[x[n]] = x[n] \cdot x[n+2]$ → $\boxed{\text{Sustav nije kauzalan jer } y[n] \text{ traži poznavanje } x[m],\ m>n}$

$S[x[n-k]] = x[n-k] \cdot x[n-k+2] = y[n-k]$ → $\boxed{\text{Sustav je vremenski nepromjenjiv}}$

$S[a x_1[n] + b x_2[n]] =$

$= (a x_1[n] + b x_2[n]) \cdot (a x_1[n+2] + b x_2[n+2])$

$= a^2 x_1[n] \cdot x_1[n+2] + ab\, x_1[n+2] x_2[n] + ab\, x_1[n] x_2[n+2] + b^2 x_2[n] \cdot x_2[n+2]$

$= a^2 y_1[n] + b^2 y_2[n] + \dots \neq a y_1[n] + b y_2[n]$

$\boxed{\text{Sustav nije linearan}}$

e) $S[x[n]] = \sum\limits_{m=0}^{n} x[m] \cdot 2^{n-m}$

→ $\boxed{\text{Sustav je kauzalan jer } y[n] \text{ ne traži poznavanje } x[m],\ m>n}$

$S[x[n-k]] = \sum\limits_{m=0}^{n-k} x[m] \cdot 2^{n-k-m} = y[n-k]$

Sustav je vremenski nepromjenjiv

$S[a x_1[n] + b x_2[n]] = a \sum\limits_{m=0}^{n} x_1[m] \cdot 2^{n-m} + b \sum\limits_{m=0}^{n} x_2[m] \cdot 2^{n-m}$

$= a y_1[n] + b y_2[n]$ → $\boxed{\text{Sustav je linearan}}$

4. DZ

IVAN KLABUČAR

4. $y_h[n] = C \cdot z^n, \; C, z \in \mathbb{C}$

a) 1. $6y[n] + 5y[n-1] + y[n-2] = x[n]$ $\boxed{x[n] = 0}$

$$C \cdot z^{n-2} \cdot (a_0 z^2 + a_1 \cdot z^1 + a_2) = 0$$

$$C \cdot z^{n-2} \cdot \underbrace{(6z^2 + 5z + 1)}_{} = 0$$

$\boxed{\text{korjeni} \quad z_1 = -\frac{1}{3} \quad z_2 = -\frac{1}{2}}$

2. $4y[n] + 4y[n-1] + y[n-2] = x[n]$ $x[n] = 0$

$$C \cdot z^{n-2} \cdot \underbrace{(4z^2 + 4z + 1)}_{} = 0$$

korjeni $z_1 = -\frac{1}{2}$ $z_2 = -\frac{1}{2}$ → dvostruki korjen

3. $y[n] + 4y[n-1] + 4y[n-2] = x[n]$, $x[n] = 0$

$$C \cdot z^{n-2} \cdot (z^2 + 4z + 4) = 0$$

$z_{1,2} = -2$ → dvostruki korjen

4. $2y[n] - 2y[n-1] + y[n-2] = x[n]$ $x[n] = 0$

$$C \cdot z^{n-2} \cdot (2z^2 - 2z + 1) = 0$$

$z_1 = \frac{1+j}{2}$ $z_2 = \frac{1-j}{2}$

4. DZ

0036513702

IVAN KLABUČAR

4. a) 5. $y[n] - 2y[n-1] + 2y[n-2] = x[n] \quad x[n] = 0$

$$C \cdot z^{n-2} \cdot \underbrace{\left(z^2 - 2z + 2\right)}_{} = 0$$

$$z_1 = 1 + j \quad z_2 = 1 - j$$

b) 1. Jezgra: $y_h[n] = C_1 \cdot \left(-\frac{1}{3}\right)^n + C_2 \cdot \left(-\frac{1}{2}\right)^n$

2. Jezgra: $y_h[n] = C_1 \cdot \left(-\frac{1}{2}\right)^n$

3. Jezgra: $y_h[n] = C_1 \cdot (-2)^n$

4. Jezgra: $y_h[n] = C_1 \cdot \left(\frac{1+j}{2}\right)^n + C_2 \cdot \left(\frac{1-j}{2}\right)^n$

5. Jezgra: $y_h[n] = C_1 \cdot (1+j)^n + C_2 \cdot (1-j)^n$