# Machine learning from scratch

## Lecture 6: Non-linear models, parameter selection

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# Course outline

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This lecture will go a bit further by introducing:

- Non linear models (polynomial kernels)
- Model evaluation
- Parameter selection

# More complex models

# Outliers and overfitting

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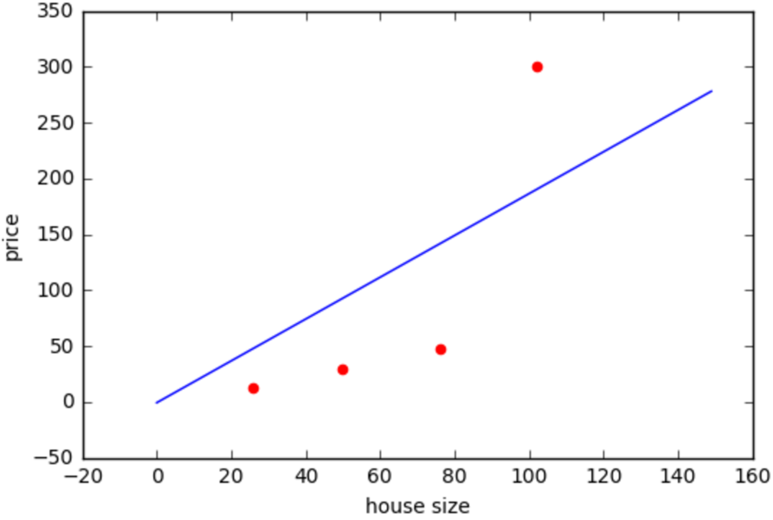

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where $\lambda$ is a **hyper-parameter** that quantifies how much we want to penalize big values of $\theta$.

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A commonly used regularization term $R$ is often the squared $\ell_2$ norm given by

$$R(\theta) = \|\theta\|_2^2 = \sum_{j=1}^{d} \theta_j^2$$

# Model evaluation
# Parameter selection

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This is often referred to as **cross-validation**.

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Another standard technique: $k$-**fold cross-validation**

- Split the data into $k$ (equally-sized) folds
- Remove 1 fold (= test fold)
- Train on the other folds
- Test on the removed fold
- Do it for all the folds

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- **Remove 1 sample** from the data set
- Train on **all the other samples**
- Test on the sample you've removed
- **Evaluate** the prediction
- Do it **for each sample of the data set**
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**Alternative**: Leave-$p$-out (LPO). LOO is LPO with $p = 1$.

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**Important note**: Hyper-parameter ranges vary a lot from an application to another. It is **data-dependent**.

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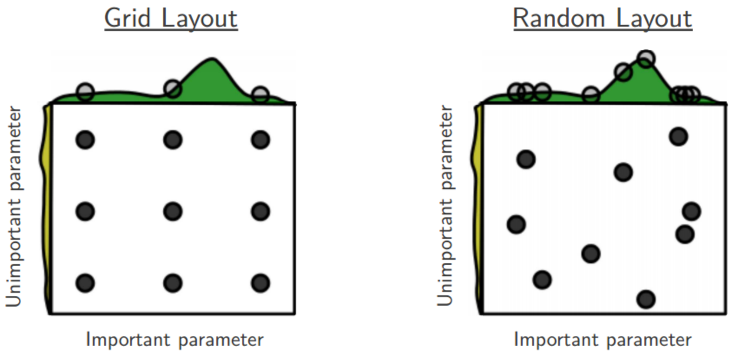

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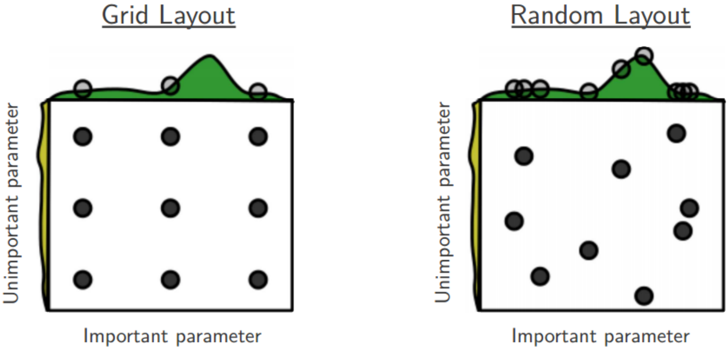


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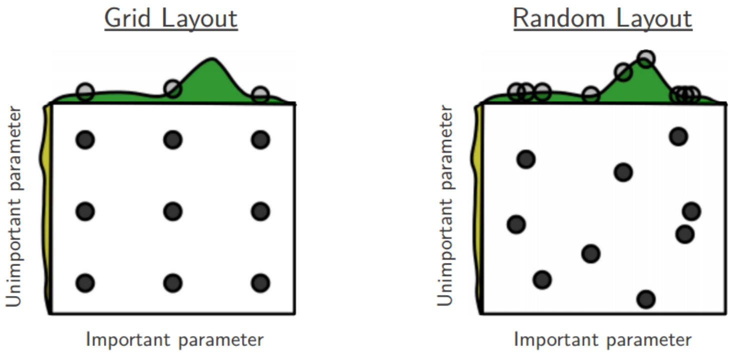


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**Practical note**: Each parameter combination can be trained/tested separately => possibility to distribute the tasks

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During the next lecture, we will work on implementing regularization to the OLS algorithm and cross-validating it and switch to classification if the time allows it.

Thank you! Questions?