计算物理·数值方法

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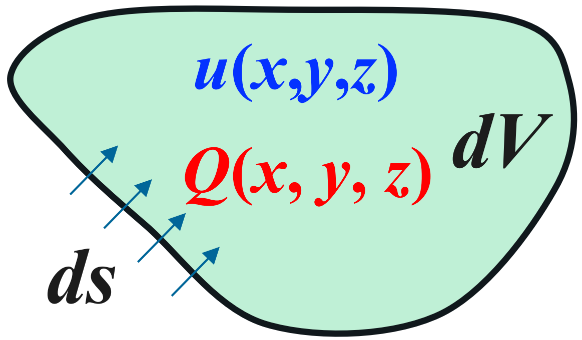
2022-04-06

热传导方程

----偏微分方程的数值解法(抛物型)

对内部有热源Q(t,x,y,z)物体：



$$\int dt \oiint_S K(t,x,y,z)\frac{\partial u}{\partial n}ds + \int dt \iiint_V Q(t,x,y,z)dV = \int dt \iiint_V c\rho\frac{\partial u}{\partial t}dV$$

傅立叶定律

K:热传导系数

Green公式

$$\int dt \iiint_V \nabla\cdot(K\nabla u)dV + \int dt \iiint_V Q(t,x,y,z)dV = \int dt \iiint_V c\rho\frac{\partial u}{\partial t}dV$$

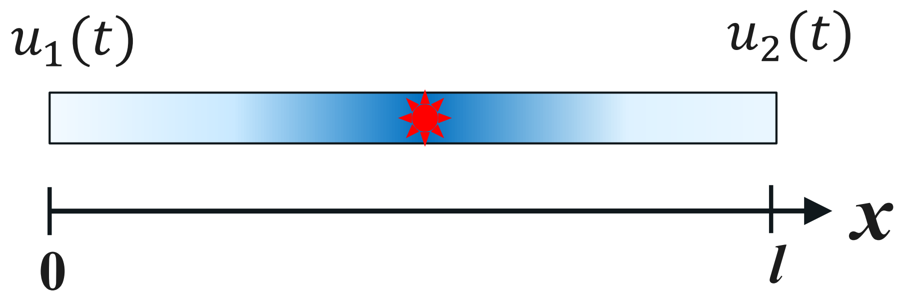
$$c\rho\frac{\partial u}{\partial t} = K\Delta u + Q(t,x,y,z)$$

$$\lambda = \frac{K}{c\rho}; \quad h(t,x,y,z) = \frac{Q(t,x,y,z)}{c\rho}$$

$$\frac{\partial u}{\partial t} = \lambda\left(\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} + \frac{\partial^2 u}{\partial^2 z}\right) + q(t,x,y,z)$$

热传导方程

一维热传导方程



$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = \lambda \dfrac{\partial^2 u}{\partial^2 x} + q(t,x) \\ u(0,x) = \varphi\,(x) \quad \text{初始条件} \\ u(t,0) = g_1(t) \quad \text{边界条件} \\ u(t,l) = g_2(t) \end{cases}$$

思路：用差分代替微分

$$\frac{\partial^2 u}{\partial x^2}\Big|_{i,k} = \frac{u_{i-1,k} - 2u_{i,k} + u_{i+1,k}}{h^2}$$

$$\frac{\partial u}{\partial t}\Big|_{i,k} = \frac{u_{i,k+1} - u_{i,k}}{\tau}$$

τ:时间步长

h:空间步长

$$\frac{u_{i,k+1} - u_{i,k}}{\tau} = \lambda \frac{u_{i-1,k} - 2u_{i,k} + u_{i+1,k}}{h^2} + q|_{i,k}$$

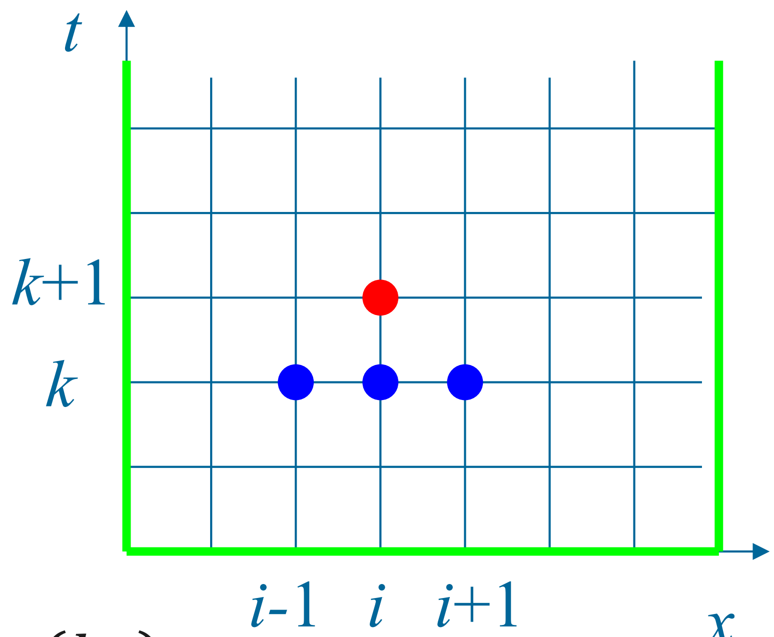
$$\begin{cases} \boxed{u_{i,k+1} = \dfrac{\lambda\tau}{h^2} u_{i+1,k} + \left(1 - 2\dfrac{\lambda\tau}{h^2}\right) u_{i,k} + \dfrac{\lambda\tau}{h^2} u_{i-1,k} + \tau q|_{i,k}} \\ u_{i,0} = \varphi(ih) \\ u_{0,k} = \boldsymbol{g}_1(k\tau) \\ u_{N,k} = \boldsymbol{g}_2(k\tau) \end{cases}$$

$$N = \frac{l}{h},$$

$$M = \frac{T}{\tau}$$

稳定条件 $$\frac{\tau\lambda}{h^2} \le \frac{1}{2}$$

计算步骤：



1. 设定λ, l, h, τ, NT
2. 计算位置坐标的节点数N
3. 计算初值：$u_{i,0} = \varphi(ih)$

   计算边界值：$u_{0,k} = g_1(k\tau), u_{N,k} = g_2(k\tau)$
4. 用有限差分法计算$u_{i,k+1}$

边界条件差分格式：

第一类边界条件

$$\begin{cases} u(0,t) = g_1(t) \\ u(l,t) = g_2(t) \end{cases} \Longrightarrow \begin{cases} u_{0,k} = g_1(k\tau) \\ u_{N,k} = g_2(k\tau) \end{cases}$$

第二类边界条件

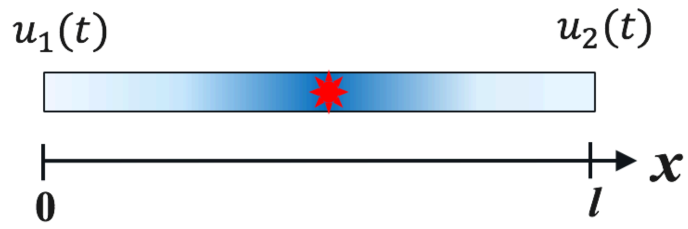
$$\begin{cases} \dfrac{\partial u(0,t)}{\partial x} = g_1(t) \\ \dfrac{\partial u(l,t)}{\partial x} = g_2(t) \end{cases} \Longrightarrow \begin{cases} \dfrac{u_{1,k} - u_{0,k}}{h} = g_1(k\tau) \\ \dfrac{u_{N,k} - u_{N-1,k}}{h} = g_2(k\tau) \end{cases}$$

第三类边界条件

$$\begin{cases} \dfrac{\partial u(0,t)}{\partial x} - \lambda_1(t)u(0,t) = g_1(t) \\ \dfrac{\partial u(l,t)}{\partial x} - -\lambda_2(t)u(l,t) = g_2(t) \end{cases} \Longrightarrow \begin{cases} \dfrac{u_{1,k} - u_{0,k}}{h} - \lambda_1(k\tau)u_{0,k} = g_1(k\tau) \\ \dfrac{u_{N,k} - u_{N-1,k}}{h} - \lambda_2(k\tau)u_{N,k} = g_2(k\tau) \end{cases}$$

举例：

$$\begin{cases}\dfrac{\partial u(x,t)}{\partial t} = \dfrac{\partial^2 u}{\partial^2 x} + 5.0e^{-2.0(x-5.0)^2} \\ u(0,x) = 0.1 \cdot x(10.0 - x) \\ u(t,0) = 0.0 \\ u(t,10) = 0.0 \end{cases}$$



计算过程：

1. 设定λ, l, h, τ, NT

$$\tau = 0.05; h = 0.5;$$
$$l = 10.0;\ \lambda = 1.0;$$
$$NT = 1000$$

$$\frac{\lambda\tau}{h^2} = \frac{1}{5} \leq \frac{1}{2}$$

2. 计算位置坐标的节点数N

$$N = \frac{l}{h} = 20$$

3. 计算初值：$u_{i,0} = \varphi(ih)$

计算边值：$u_{0,k} = g_1(k\tau), u_{N,k} = g_2(k\tau)$

$$u(\mathrm{x}, 0) = 0.1x\,(10.0 - x)$$

$$u_{i,0} = 0.1 \cdot ih(10.0 - ih)$$

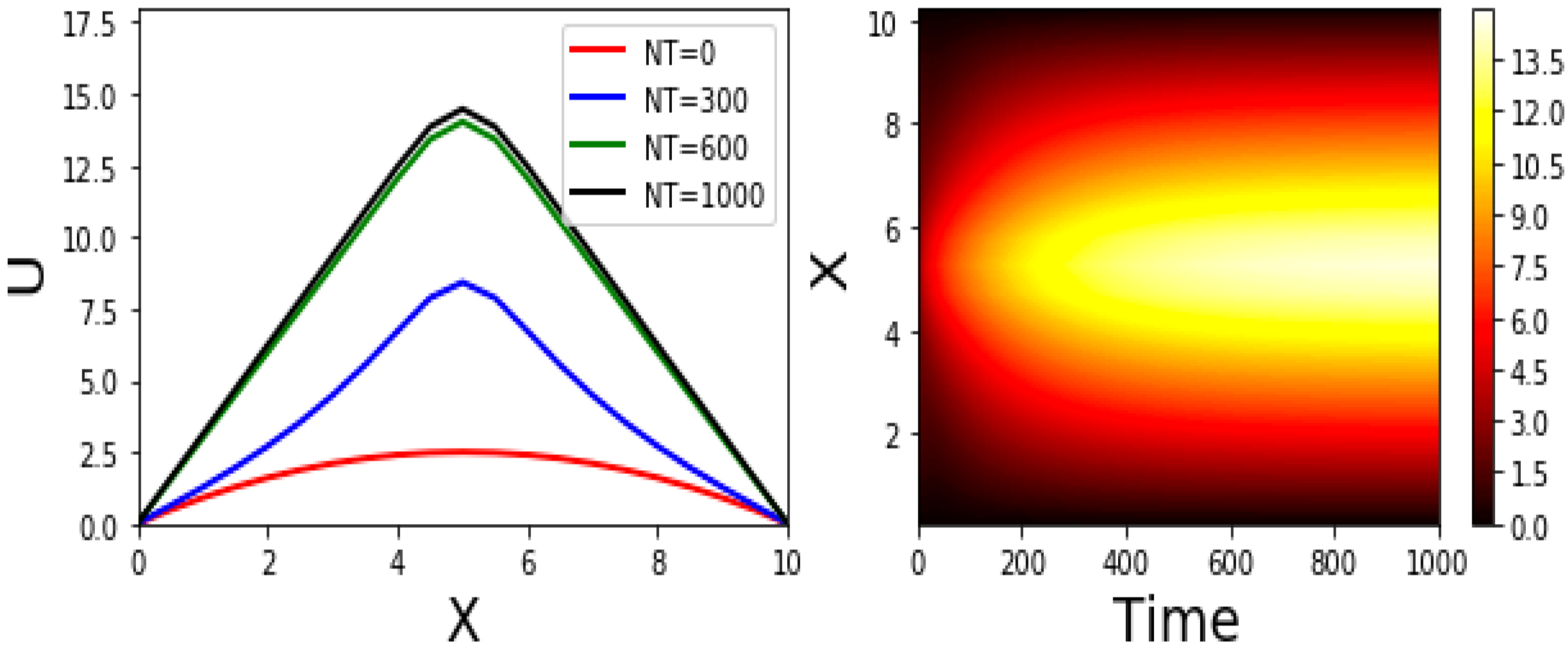
$$u(0, \mathrm{t}) = u(10, \mathrm{t}) = 0)$$

$$u_{0,k} = u_{\mathrm{N},k} = 0$$

4. 用有限差分法计算$u_{i,k+1}$

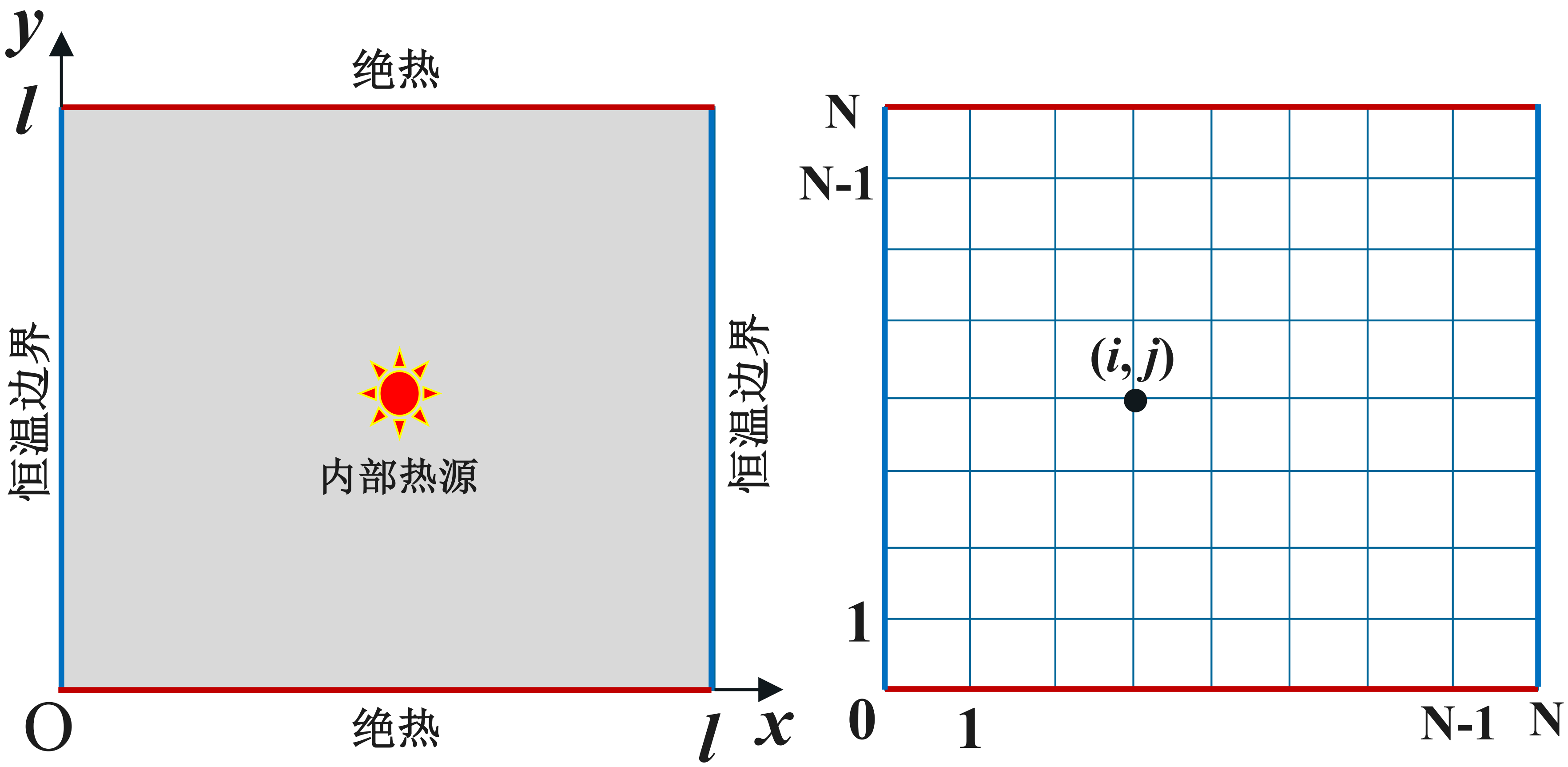
$$u_{i,k+1} = \frac{\lambda\tau}{h^2} u_{i+1,k} + \left(1 - 2\frac{\lambda\tau}{h^2}\right) u_{i,k} + \frac{\lambda\tau}{h^2} u_{i-1,k} + \tau e^{-2.0(ih-5.0)^2}$$

计算结果：



17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
U
0
2
4
6
8
10
X
NT=0
NT=300
NT=600
NT=1000
10
8
6
4
2
X
0
200
400
600
800
1000
Time
13.5
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0.0

二维热传导方程的差分解法



$$\frac{\partial u}{\partial t} = \lambda\left(\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y}\right) + q(t, x, y)$$

二维热传导方程

建立差分格式：

$$\frac{\partial u(x,y,t)}{\partial t}\Big|_{i,k} = \frac{u_{i,j,k+1} - u_{i,j,k}}{\tau}$$

$$\frac{\partial^2 u}{\partial^2 x}\Big|_{i,\mathbf{j},k} = \frac{u_{i-1,\mathbf{j},k} - 2u_{i,\boldsymbol{j},k} + u_{i+1,\mathbf{j},k}}{h^2}$$

$$\frac{\partial^2 u}{\partial^2 y}\Big|_{i,j,k} = \frac{u_{i,\mathrm{j}-1,k} - 2u_{i,\mathrm{j},k} + u_{i,\mathrm{j}+1,k}}{h^2}$$

$$\frac{u_{i,j,k+1} - u_{i,j,k}}{\tau} = \lambda\frac{u_{i-1,j,k} - 2u_{i,j,k} + u_{i+1,j,k}}{h^2} + \lambda\frac{u_{i,j-1,k} - 2u_{i,j,k} + u_{i,j+1,k}}{h^2} + q|_{i,j,k}$$

整理得递推公式：

$$u_{i,j,k+1} = \left(1 - \frac{4\tau\lambda}{h^2}\right)u_{i,j,k} + \frac{\tau\lambda}{h^2}\left(u_{i-1,j,k} + u_{i,j-1,k} + u_{i+1,j,k} + u_{i,j+1,k}\right) + \left. q\right|_{i,j,k}$$

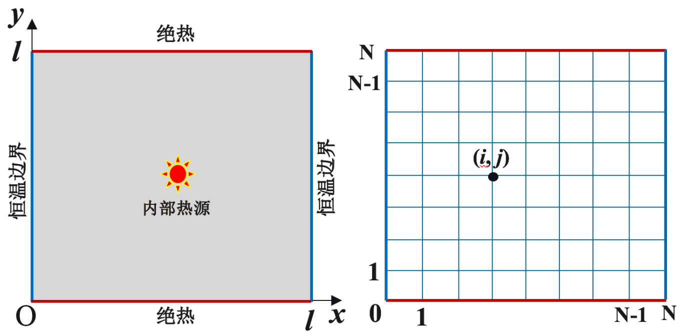
初始条件：

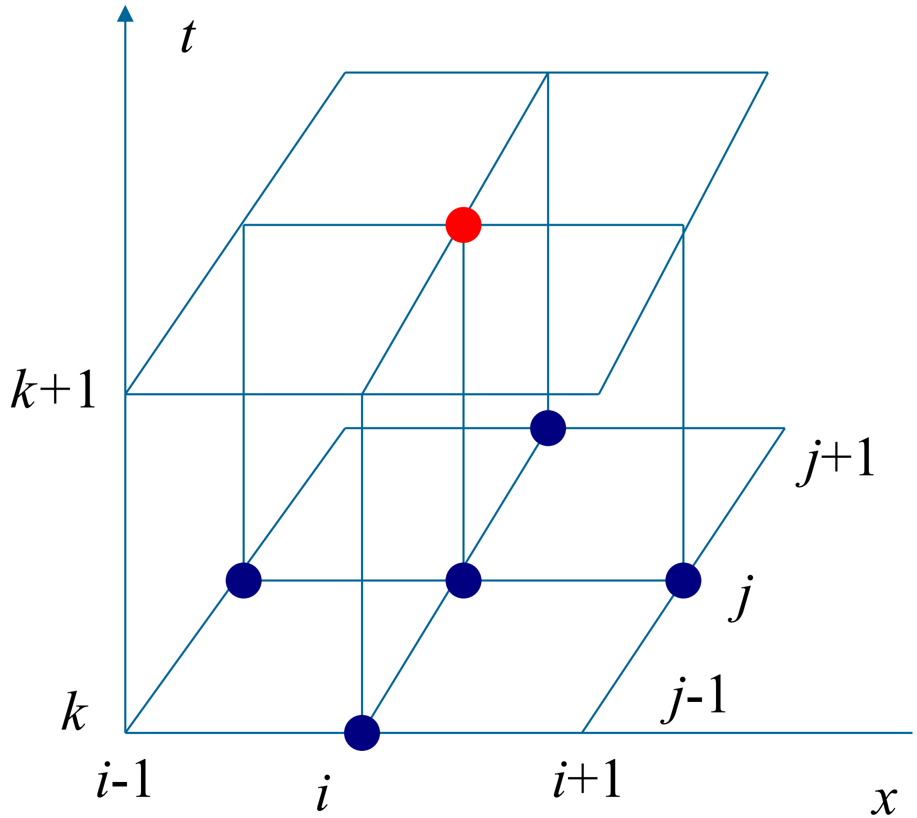
$$u_{i,j,0} = 0$$

边界条件：

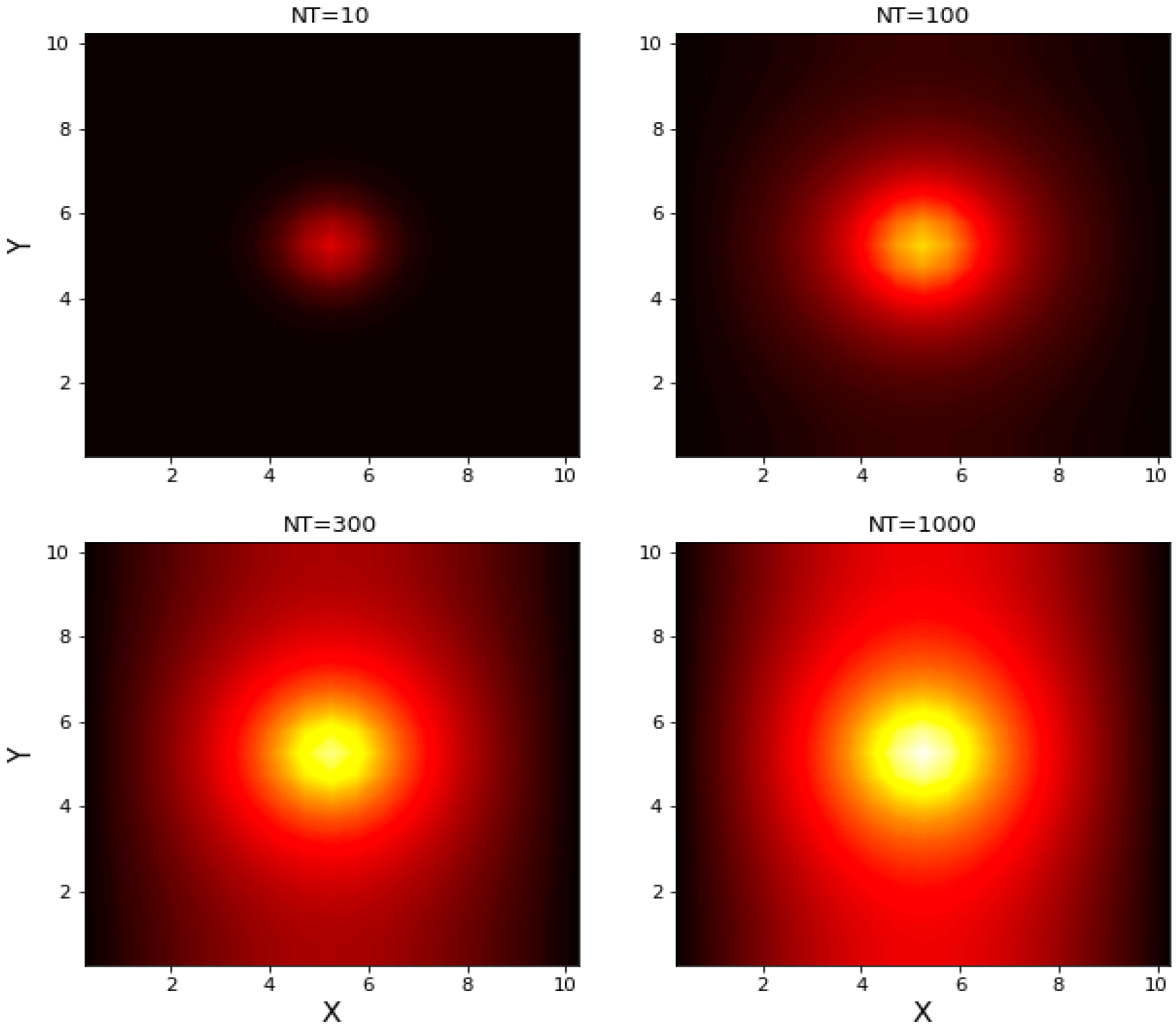
$$u_{0,j,k} = 0;\ u_{N,j,k} = 0$$

$$u_{i,0,k} = u_{i,1,k};\ u_{i,N,k} = u_{i,N-1,k}$$





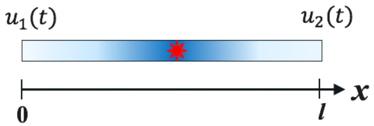
计算结果：

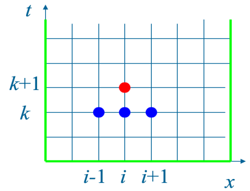


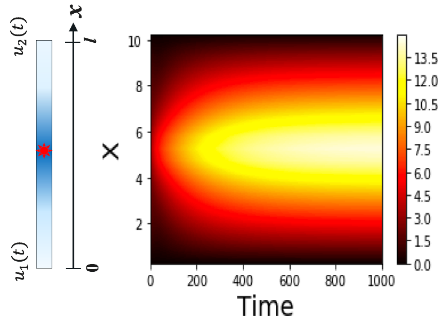
热传导方程：

$$\frac{\partial u}{\partial t}=\lambda\left(\frac{\partial^2 u}{\partial^2 x}+\frac{\partial^2 u}{\partial^2 y}+\frac{\partial^2 u}{\partial^2 z}\right)+q(t,x,y,z)$$

一维热传导：







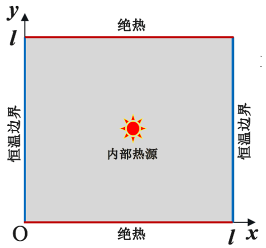
$$\begin{cases}\dfrac{\partial u(x,t)}{\partial t}=\lambda\dfrac{\partial^2 u}{\partial^2 x}+q(t,x) \\ u(0,x)=\varphi\,(x) & \text{初始条件} \\ u(t,0)=g_1(t) & \text{边界条件} \\ u(t,l)=g_2(t) \end{cases}$$

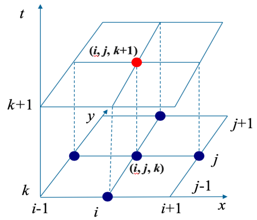
$$\frac{u_{i,k+1}-u_{i,k}}{\tau}=\lambda\frac{u_{i-1,k}-2u_{i,k}+u_{i+1,k}}{h^2}+\left.q\right|_{i,k}$$

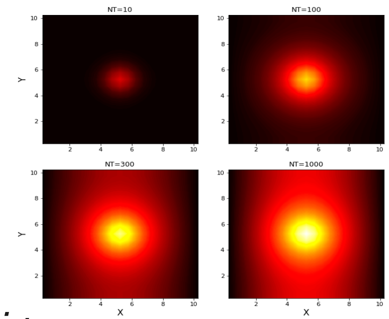
$$u_{i,k+1}=\frac{\lambda\tau}{h^2}u_{i+1,k}+\left(1-2\frac{\lambda\tau}{h^2}\right)u_{i,k}+\frac{\lambda\tau}{h^2}u_{i-1,k}+\tau\left.q\right|_{i,k}$$

$$\frac{\tau\lambda}{h^2}\leq\frac{1}{2}$$

二维热传导：







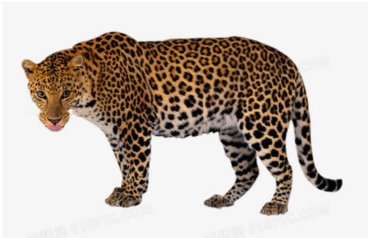
$$\frac{\partial u}{\partial t}=\lambda\left(\frac{\partial^2 u}{\partial^2 x}+\frac{\partial^2 u}{\partial^2 y}\right)+q(t,x,y)$$

$$\frac{u_{i,j,k+1}-u_{i,j,k}}{\tau}=\lambda\frac{u_{i-1,j,k}-2u_{i,j,k}+u_{i+1,j,k}}{h^2}+\lambda\frac{u_{i,j-1,k}-2u_{i,j,k}+u_{i,j+1,k}}{h^2}+\left.q\right|_{i,j,k}$$

$$u_{i,j,k+1}=\left(1-\frac{4\tau\lambda}{h^2}\right)u_{i,j,k}+\frac{\tau\lambda}{h^2}\left(u_{i-1,j,k}+u_{i,j-1,k}+u_{i+1,j,k}+u_{i,j+1,k}\right)+\left.q\right|_{i,j,k}$$

$$\frac{\lambda\tau}{h^2}\leq\frac{1}{4}$$

反应扩散问题

Gray-Scott model：

#在分子V的催化下，U生成V。

#分子V可自发生成P。

化学反应：

$$U + 2V \rightarrow 3V$$
$$V \rightarrow P$$

反应扩散方程：

$$\frac{\partial U}{\partial t} = D_u \nabla^2 U - UV^2 + F(1 - U)$$
$$\frac{\partial V}{\partial t} = D_v \nabla^2 V + UV^2 - (F + r)V$$

D_u， D_v：扩散系数；

F：分子U的供应速率，以及
分子U,V,P的消除速率；

r：V→P的反应速率。

$$\frac{\partial U}{\partial t} = D_u \nabla^2 U - UV^2 + F(1-U)$$
$$\frac{\partial V}{\partial t} = D_v \nabla^2 V - UV^2 - (F+r)V$$

建立差分方程：

$$\frac{u_{i,j,k+1}-u_{i,j,k}}{\tau} = D_u\left(\frac{u_{i-1,j,k}-2u_{i,j,k}+u_{i+1,j,k}}{h^2} + \frac{u_{i,j-1,k}-2u_{i,j,k}+u_{i,j+1,k}}{h^2}\right) - u_{i,j,k}v_{i,j,k}^2 + f(1-u_{i,j,k})$$

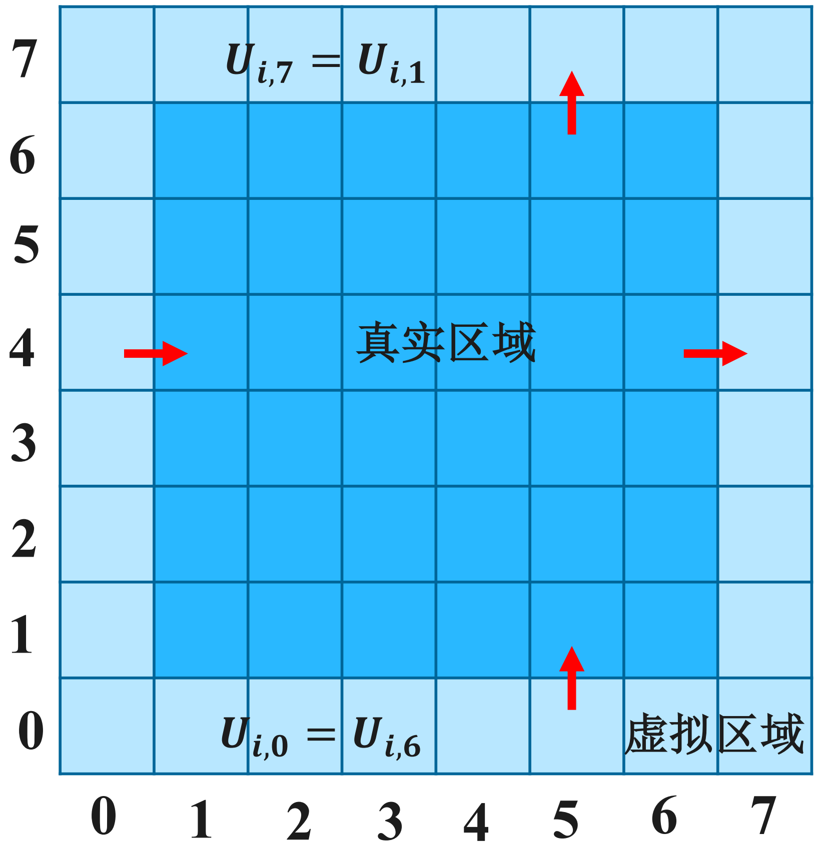
$$\frac{v_{i,j,k+1}-v_{i,j,k}}{\tau} = D_v\left(\frac{v_{i-1,j,k}-2v_{i,j,k}+v_{i+1,j,k}}{h^2} + \frac{v_{i,j-1,k}-2v_{i,j,k}+v_{i,j+1,k}}{h^2}\right) + u_{i,j,k}v_{i,j,k}^2 - (f+r)v_{i,j,k}$$

有限差分格式：

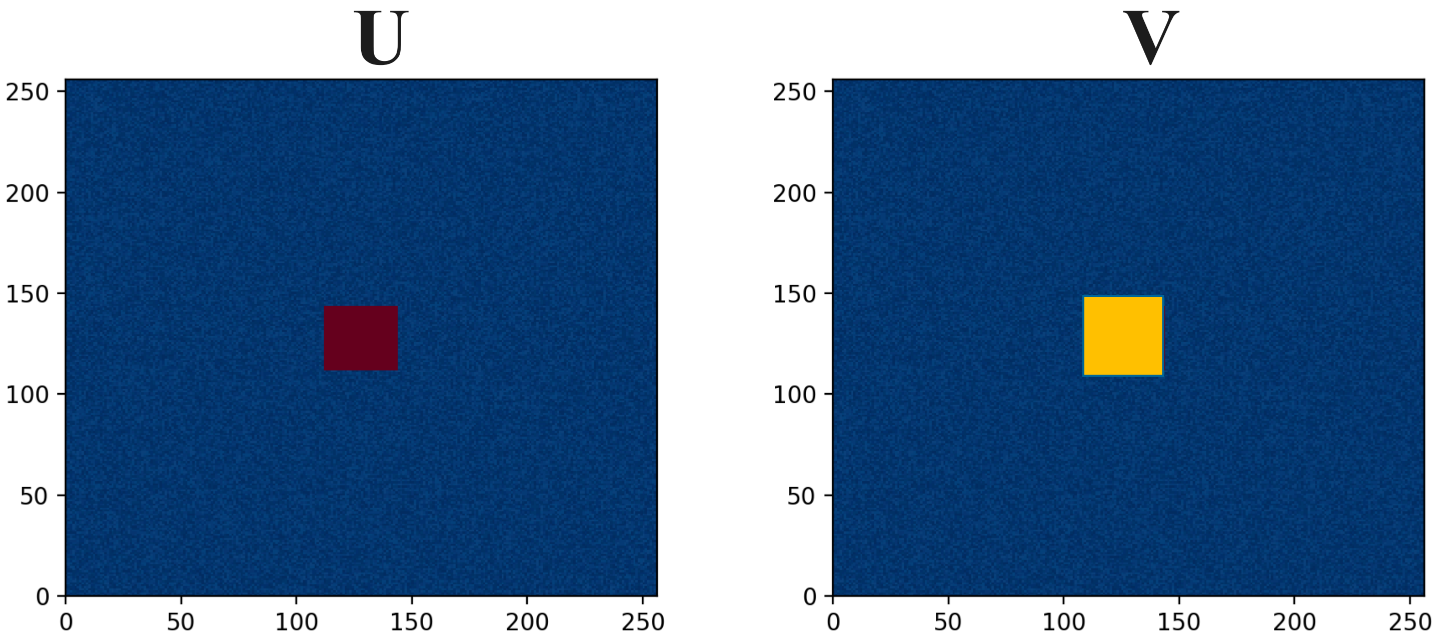
$$u_{i,j,k+1} = \left(1-\frac{4\tau D_u}{h^2}\right)u_{i,j,k} + \frac{\tau D_u}{h^2}(u_{i-1,j,k}+u_{i,j-1,k}+u_{i+1,j,k}+u_{i,j+1,k}) - u_{i,j,k}v_{i,j,k}^2 + f(1-u_{i,j,k})$$

$$v_{i,j,k+1} = \left(1-\frac{4\tau D_v}{h^2}\right)v_{i,j,k} + \frac{\tau D_v}{h^2}(v_{i-1,j,k}+v_{i,j-1,k}+v_{i+1,j,k}+v_{i,j+1,k}) + u_{i,j,k}v_{i,j,k}^2 - (f+r)v_{i,j,k}$$

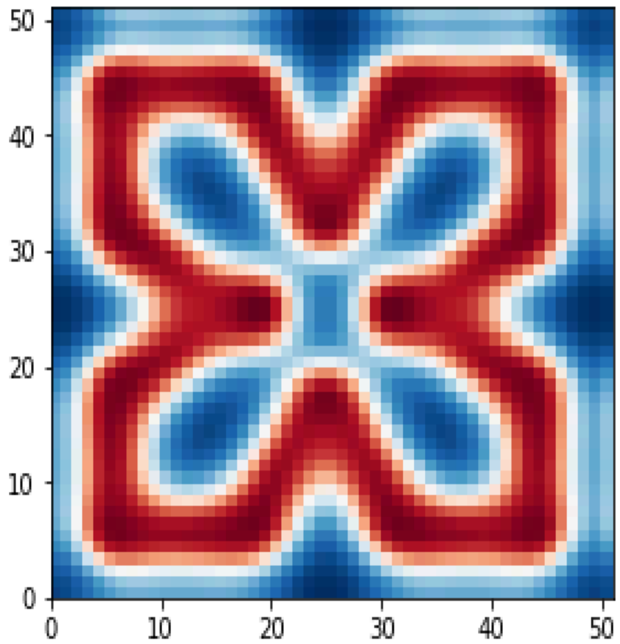
周期性边界条件

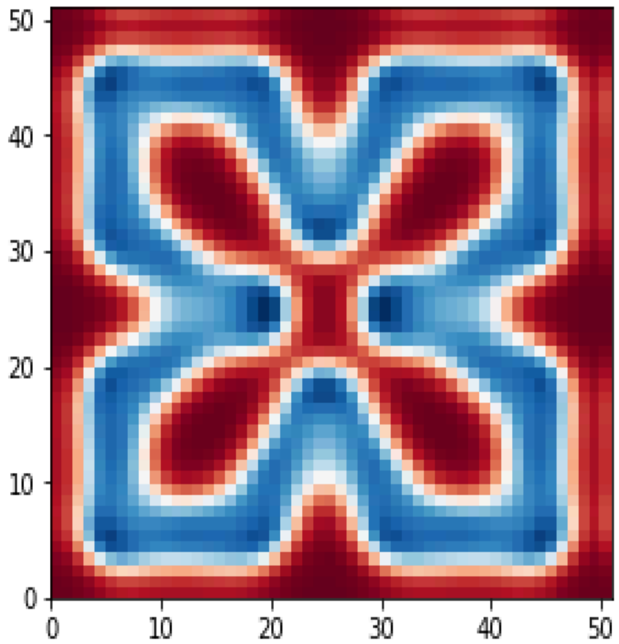


初始条件



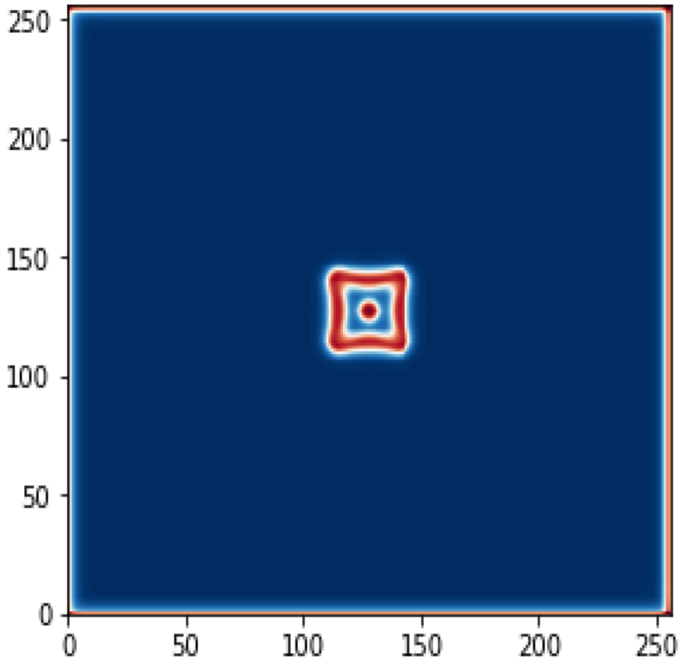
50×50二维网格斑图（3000步）

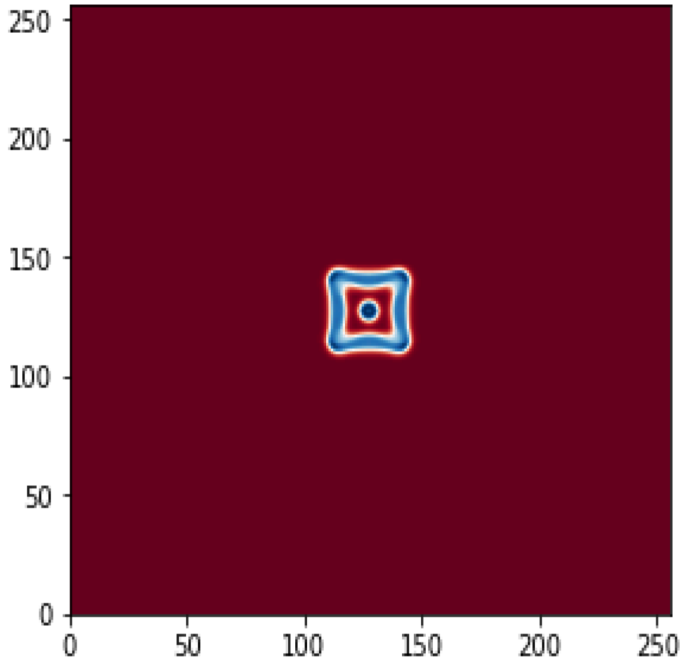




$D_u, D_v, f, r = 0.16, 0.08, 0.060, 0.062$

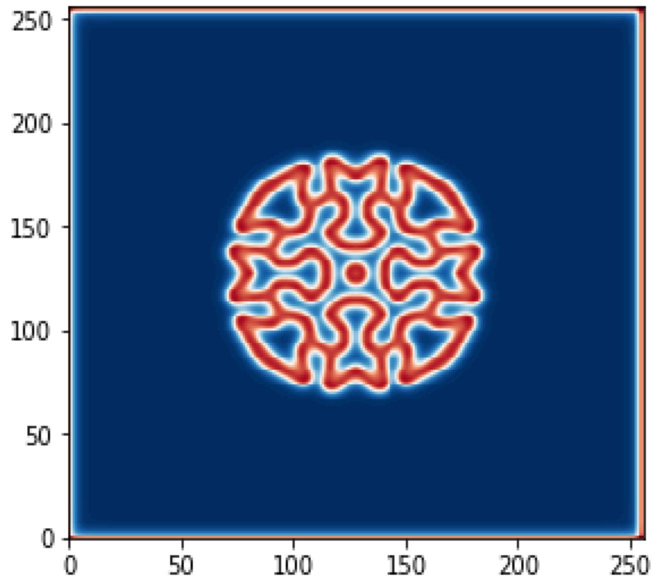
256×256二维网格斑图（300步）

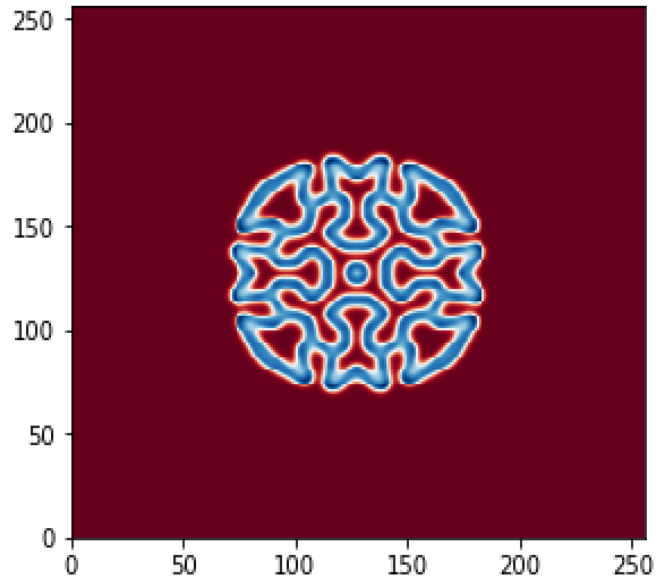




$$D_u, D_v, f, r = \ 0.16, 0.08, 0.060, 0.062$$

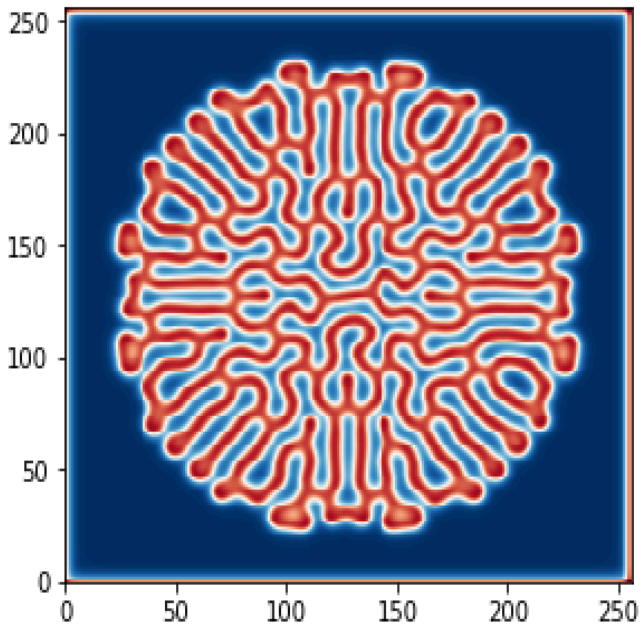
256×256二维网格斑图（3000步）

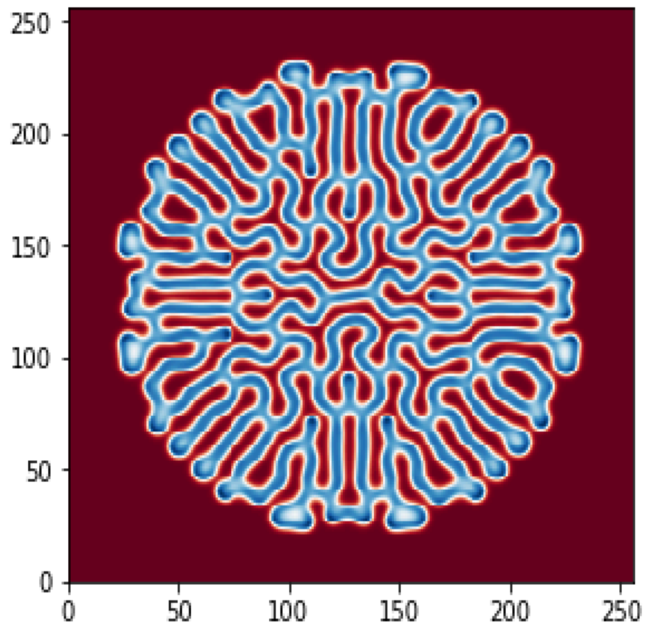




$D_u, D_v, f, r = \ 0.16, 0.08, 0.060, 0.062$

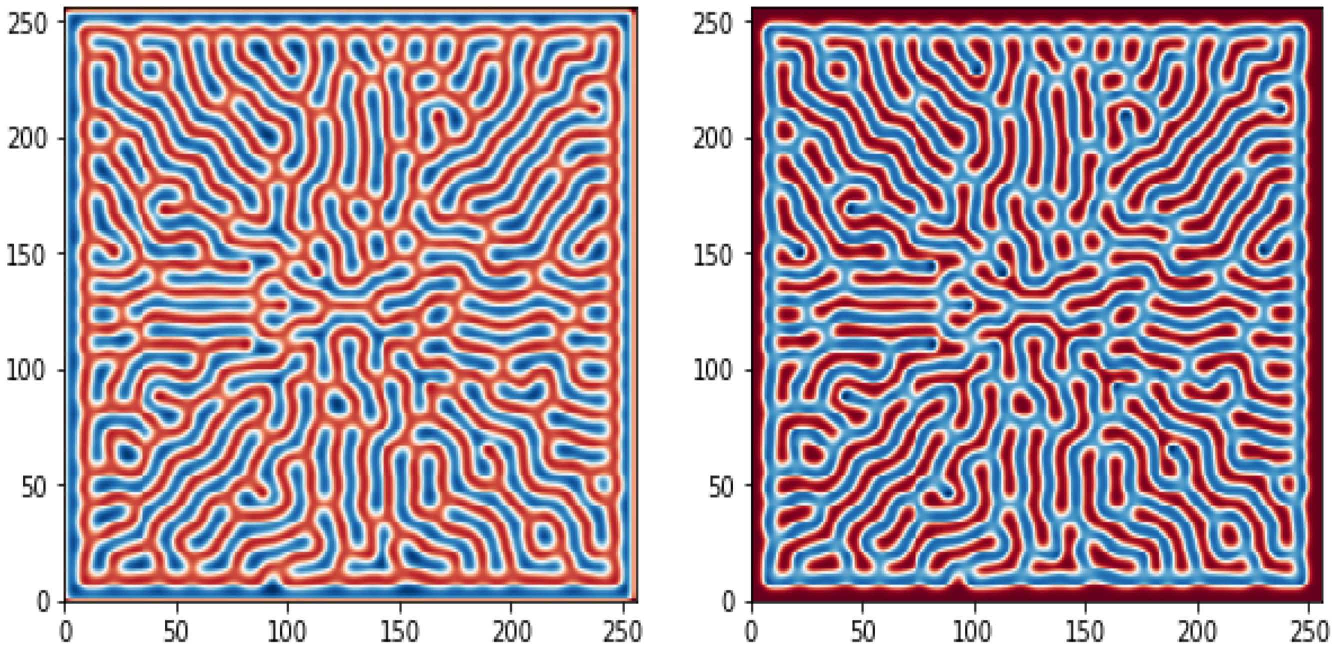
256×256二维网格斑图（10000步）





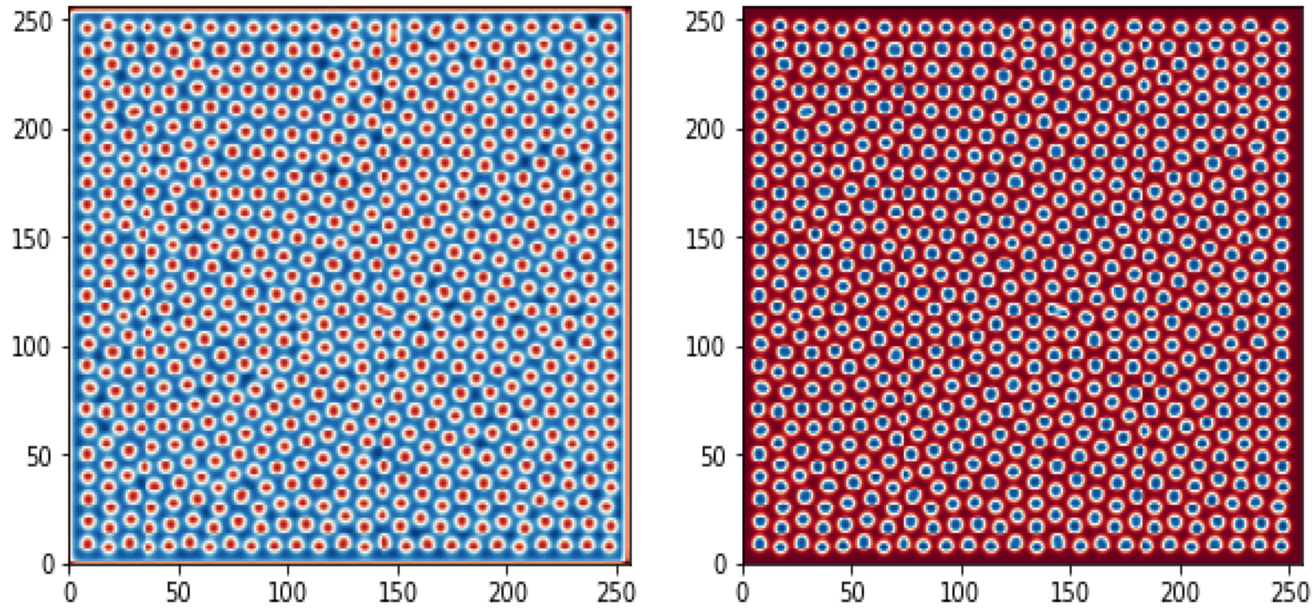
$D_u, D_v, f, r = \ 0.16, 0.08, 0.060, 0.062$

256×256二维网格斑图（30000步）



$D_u, D_v, f, r = \ 0.16, 0.08, 0.060, 0.062$

256×256二维网格斑图（30000步）



$$D_u, D_v, f, r = \ 0.14, 0.06, 0.035, 0.065$$

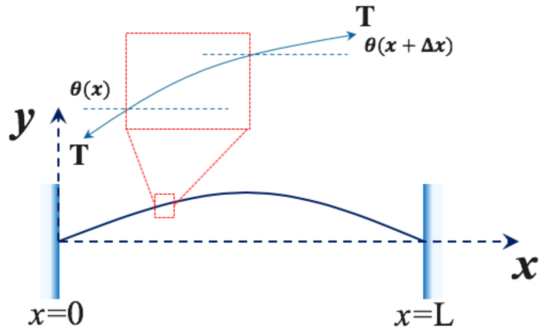
波动方程

——偏微分方程的数值解法（双曲型）

弦线的横振动方程

$$\underline{\rho(x)}\frac{\partial^2 y}{\partial t^2}=\underline{T}\frac{\partial^2 y}{\partial x^2}+\underline{P(x,t)}$$

线密度　　张力　　外力



$$F_y=T\sin\theta(x+\Delta x)-T\sin\theta(x)\approx T\frac{\partial^2 y}{\partial^2 x}$$

对均匀弦线，无外力的自由振动情况：

$$\frac{\partial^2 y}{\partial t^2}=v^2\frac{\partial^2 y}{\partial x^2}$$

其中 $v=\sqrt{\dfrac{T}{\rho}}$ 为波速

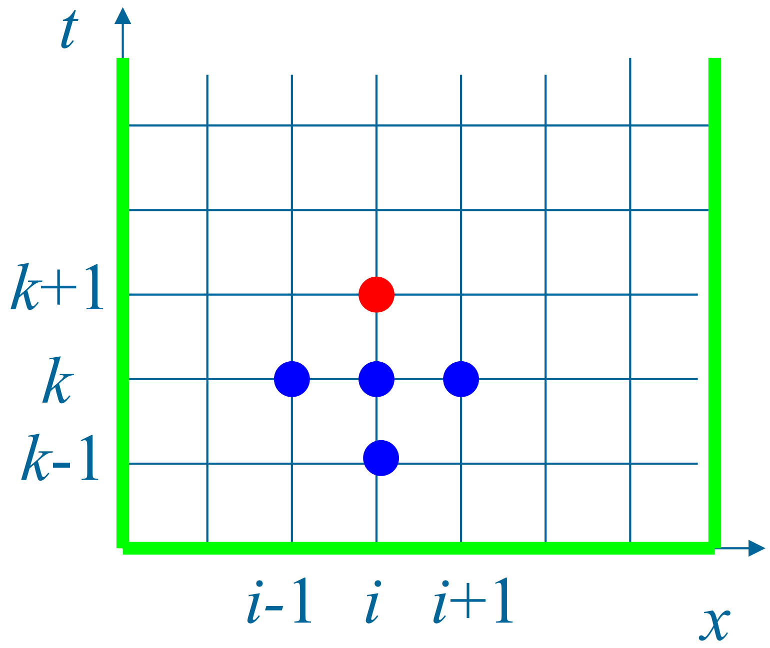
一维波动方程

问题转化为求如下定解问题

$$
\begin{cases}
\dfrac{\partial^2 y}{\partial t^2} = v^2 \dfrac{\partial^2 y}{\partial x^2} \quad 0 < x < l;\ 0 < t < T \\[2em]
y(x,0) = \varphi(x) \\
\dfrac{\partial y(x,0)}{\partial t} = \psi(x) & \text{初始条件} \\[2em]
y(0,t) = g_1(t) \\
y(l,t) = g_2(t) & \text{边界条件}
\end{cases}
$$

思路：用差分代替微分

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$



建立差分格式：

$$\frac{\partial^2 y}{\partial x^2} = \frac{y_{i+1,k} - 2y_{i,k} + y_{i-1,k}}{h^2}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{y_{i,k+1} - 2y_{i,k} + y_{i,k-1}}{\tau^2}$$

差分格式： $y_{i,k+1} = 2(1 - (\frac{\tau v}{h})^2) y_{i,k} + (\frac{\tau v}{h})^2 (y_{i+1,k} + y_{i-1,k}) - y_{i,k-1}$

收敛条件：$\frac{\tau v}{h} \leq 1$

初始条件差分格式：

$$y(x,0)=\varphi(x)$$

$$\frac{\partial y(x,0)}{\partial t}=\psi(x)$$

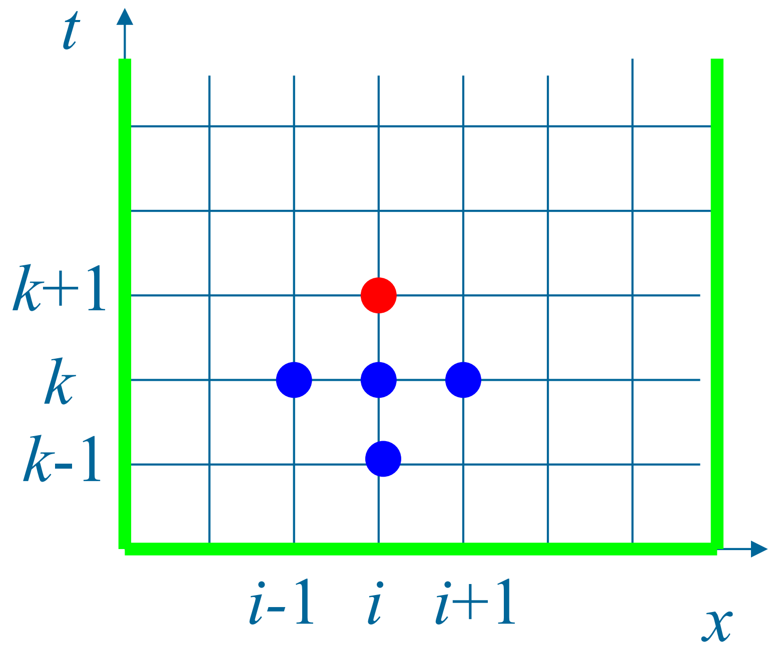
向前差分：

$$\frac{\partial y_{i,0}}{\partial t}=\frac{y_{i,1}-y_{i,0}}{\tau}\quad i=0,1,\cdots,N$$

初始条件向前差分格式 $y_{i,1}=y_{i,0}+\tau\psi(ih)$

一维波动方程定解问题的差分格式

$$
\begin{cases}
y_{i,k+1} = 2\left(1-\left(\dfrac{\tau v}{h}\right)^2\right)y_{i,k} + \left(\dfrac{\tau v}{h}\right)^2\left(y_{i+1,k}+y_{i-1,k}\right) - y_{i,k-1} \\
\qquad i = 1,2,\cdots,N-1; \quad k = 1,2,\cdots,M-1 \\
y_{i,0} = \phi(ih) \quad i = 0,1,\cdots,N \\
y_{i,1} = \phi(ih) + \tau\psi(ih) \quad i = 0,1,\cdots,N \\
y_{0,k} = g_1(k\tau) \quad k = 0,1,\cdots,M \\
y_{N,k} = g_2(k\tau) \quad k = 0,1,\cdots,M
\end{cases}
$$



计算步骤：

1.给定v,l,h,τ,T

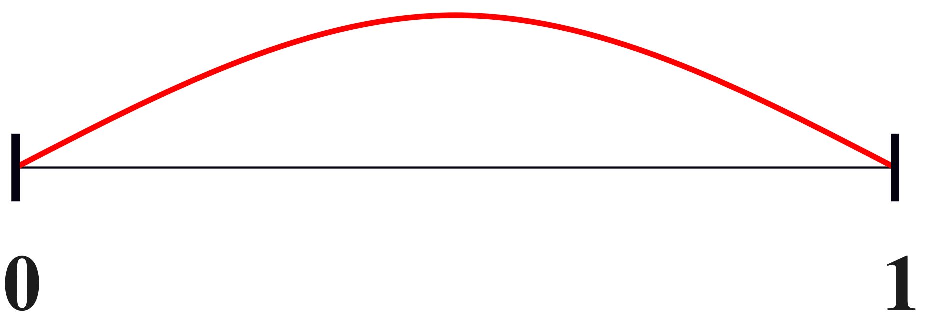
2. 计算$N=l/h,M=T/\tau$

3. 计算初值和边值

4. 用差分格式计算$y_{i,k+1}$

举例：

$$\begin{cases} \dfrac{\partial^2 y}{\partial t^2} = \dfrac{\partial^2 y}{\partial x^2} \quad 0 < x < 1 \quad 0 < t \\ y(x,0) = \sin \pi x \quad \dfrac{\partial y(x,0)}{\partial t} = 0 \qquad 0 \le x \le 1 \\ y(0,t) = y(1,t) = 0 \quad 0 < t \end{cases}$$



取 $\frac{v\tau}{h}=1$ $h=0.2$ $\tau=h/v=0.2$

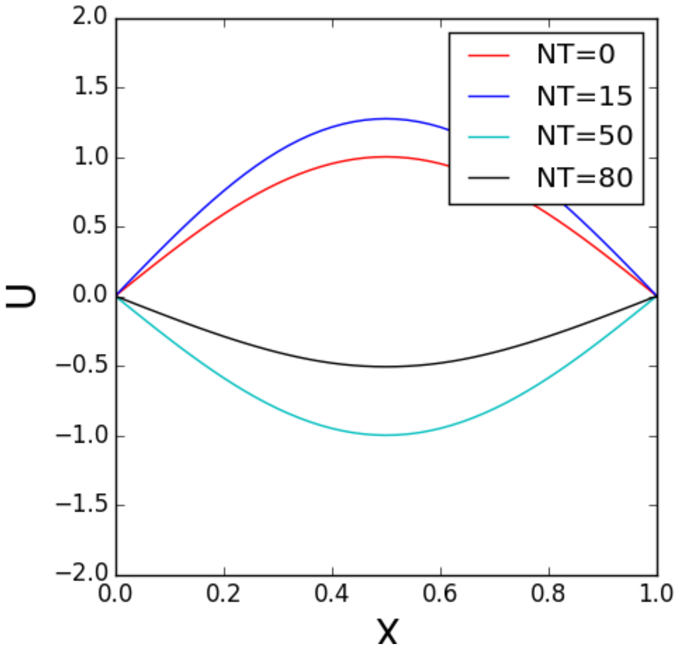
取 $M=100$ $N=1/h=5$

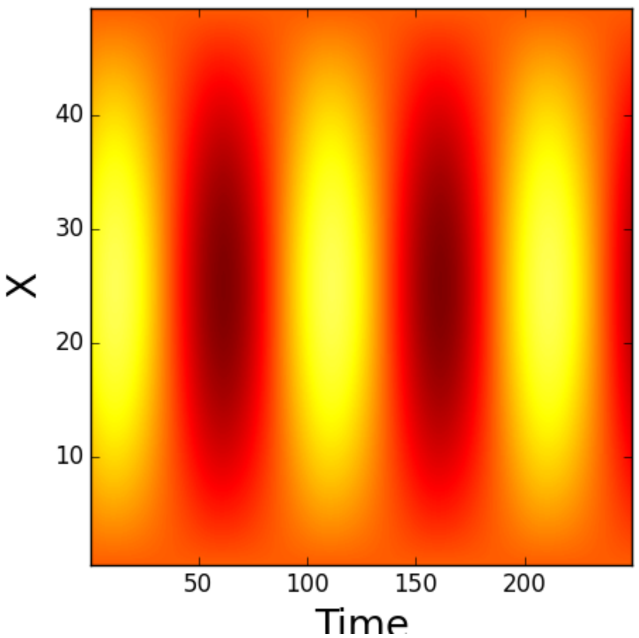
建立差分格式：

$$\begin{cases} y_{i,k+1}=y_{i+1,k}+y_{i-1,k}-y_{i,k-1} & i=1,2,3,4 \quad k=1,2,\ldots \\ y_{i,0}=\sin ih\pi \quad i=1,2,3,5 & \\ y_{i,1}=\sin ih\pi & i=1,2,3,4 \\ y_{0,k}=y_{1,k}=0 \quad k=1,2,\ldots & \end{cases}$$

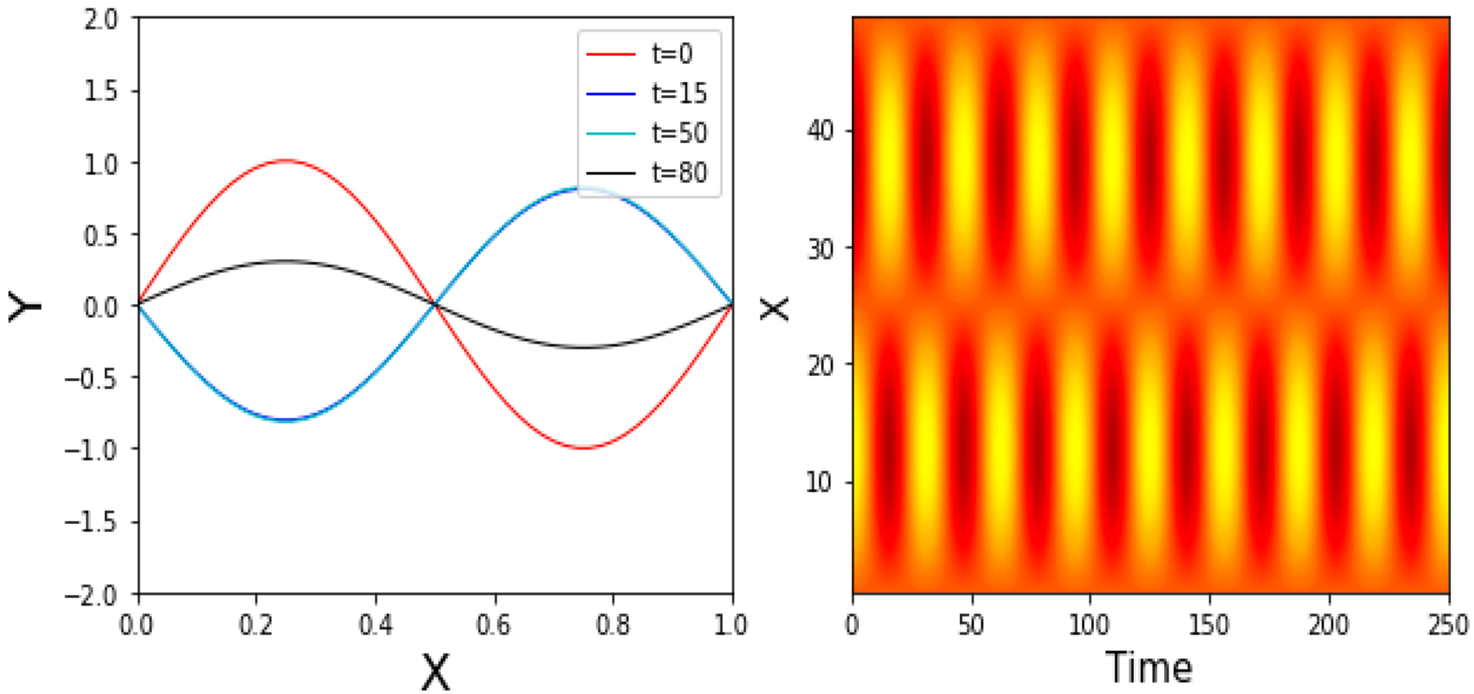
计算结果：

初始条件为$(x,0)=\sin(\pi x)$





初始条件为$(x, 0) = \sin(2\pi x)$



解析解: $y(x,t) = \sum_0^\infty B_n \sin(\frac{\pi(n+1)}{L}x)\cos(\frac{2\pi n}{L}t)$

1. 二维膜振动：

$$
\begin{cases}
\dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} \qquad 0 < x < \pi; 0 < y < \pi \qquad 0 < t \\
u(x, y, 0) = 3\sin 2x \sin y; 0 \le x \le \pi; 0 \le y \le \pi \\
\dfrac{\partial u(x, y, 0)}{\partial t} = 0; 0 \le x \le \pi; 0 \le y \le \pi \\
u(0, y, t) = u(\pi, y, t) = 0 \qquad 0 < t \\
u(x, 0, t) = u(x, \pi, t) = 0 \qquad 0 < t
\end{cases}
$$

$$\frac{\partial^2 u(x,y,z)}{\partial t^2} = \frac{u(x,y,t+\Delta t) - 2u(x,y,t) + u(x,y,t-\Delta t)}{(\Delta t)^2}$$

$$\frac{\partial^2 u(x,y,z)}{\partial x^2} = \frac{u(x+\Delta x,y,t) - 2u(x,y,t) + u(x-\Delta x,y,t-\Delta t)}{(\Delta x)^2}$$

$$\frac{\partial^2 u(x,y,z)}{\partial y^2} = \frac{u(x,y+\Delta y,t) - 2u(x,y,t) + u(x,y-\Delta y,t)}{(\Delta y)^2}$$

$$u_{i,j}^{k+1} = 2u_{i,j}^{k} - u_{i,j}^{k-1} + (\frac{c\Delta t}{\Delta x})^2 \left[u_{i+1,j}^{k} + u_{i-1,j}^{k} - 4u_{i,j}^{k} + u_{i,j+1}^{k} + u_{i,j-1}^{k} \right]$$

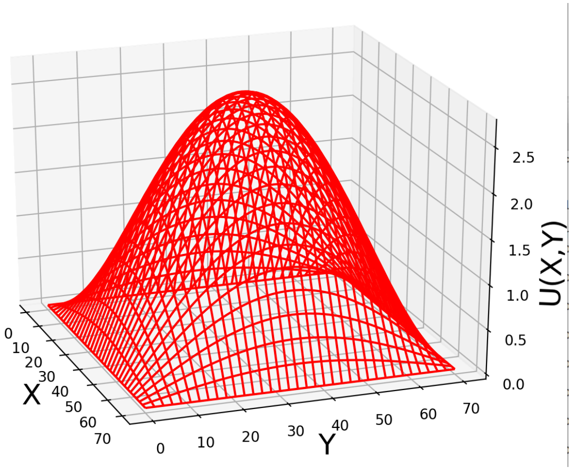
初始条件： $u_{i,j}^{0} = 3*\sin(2*\mathrm{i}\Delta \mathrm{x})\sin(\mathrm{j}\Delta \mathrm{x})$ $u(x,y,0) = \sin 2x \sin y$

$u_{i,j}^{1} = 3*\sin(2*\mathrm{i}\Delta \mathrm{x})\sin(\mathrm{j}\Delta \mathrm{x})$ $\frac{\partial u(x,y,0)}{\partial t} = 0$

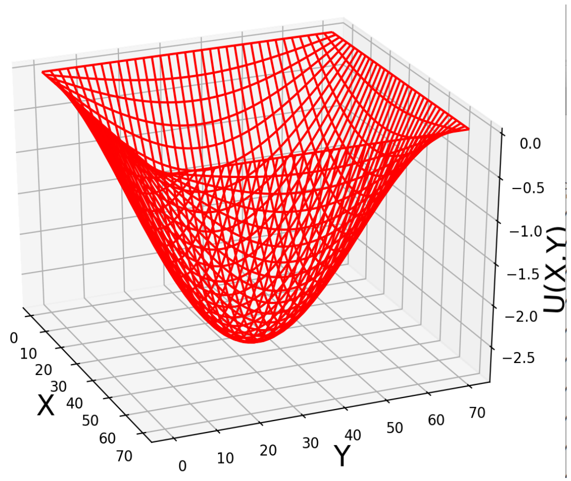
边界条件： $u_{0,j}^{k} = u_{\mathrm{N},j}^{k} = u_{\mathrm{i},0}^{k} = u_{\mathrm{i,N}}^{k} = 0$

初始条件为$u(x,0) = \sin(x)\sin(y)$

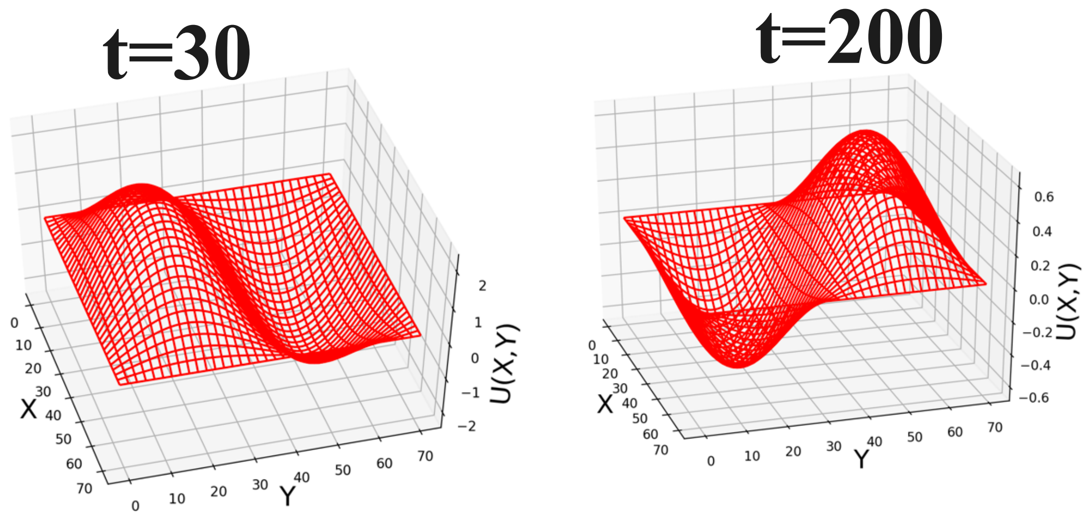
t=30



t=200

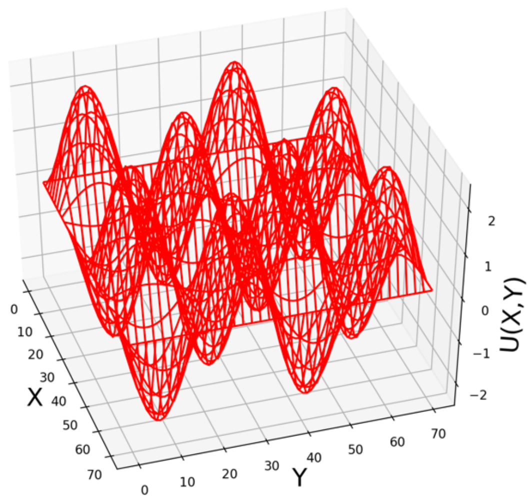


$$u(x, 0) = \sin(2x)\sin(y)$$

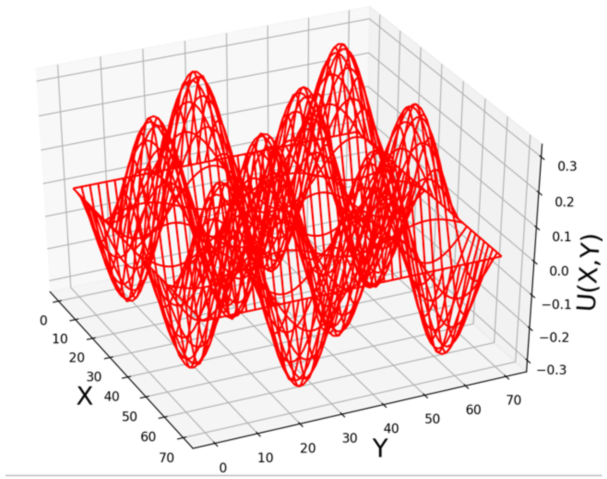


$$u(x,0) = \sin(\mathbf{4}x)\sin(\mathbf{4}y)$$

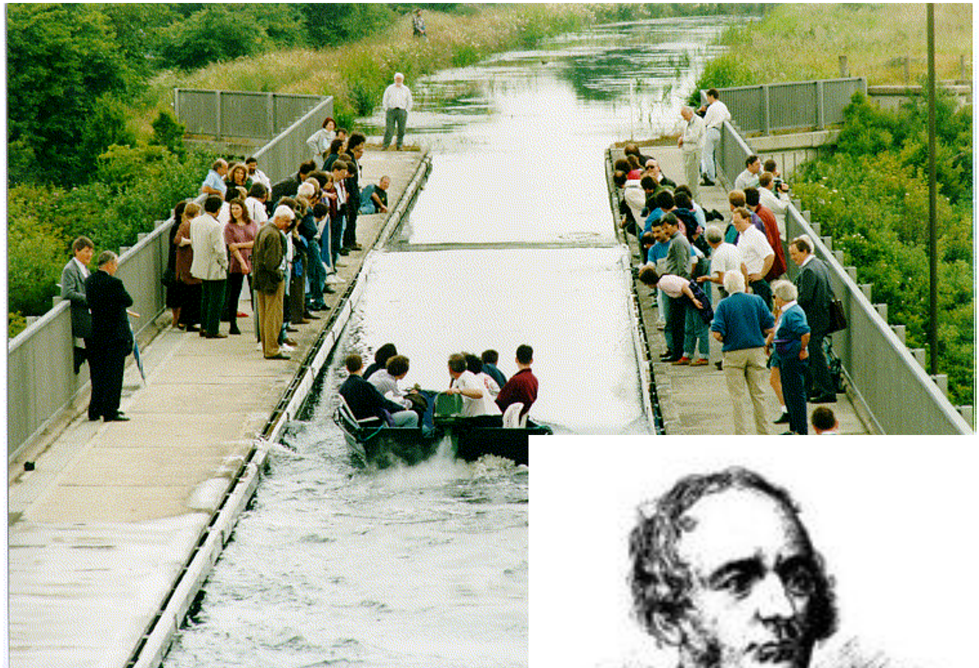
t=30



t=100

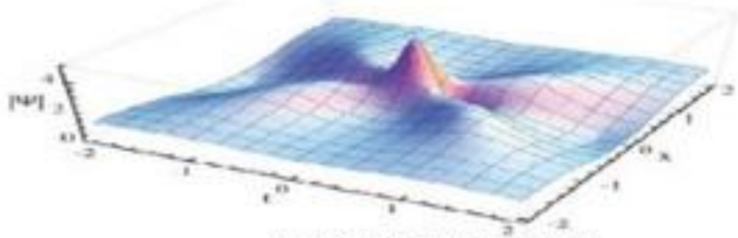


2. KdeV方程：孤子传播



(a) 罗素1834年第一次发现孤子

(b) 河中的孤子



(c) 计算机模拟产生的孤子

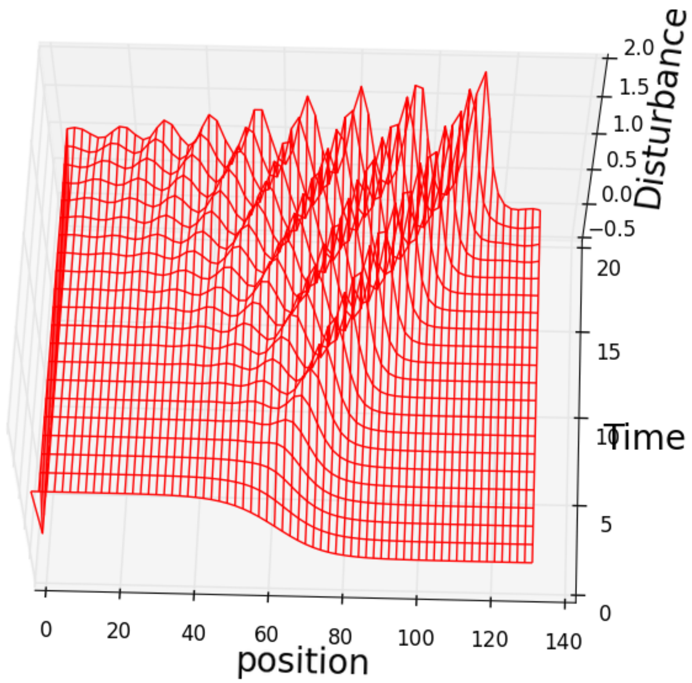
$$\frac{\partial u(x,t)}{\partial t}+\varepsilon\cdot u(x,t)\cdot\frac{\partial u(x,t)}{\partial x}+\alpha\frac{\partial^3 u(x,t)}{\partial x^3}=0 \qquad u(\mathrm{x},\mathrm{t}=0)=\frac{1}{2}\left[1-\tanh\left(\frac{x-25}{5}\right)\right]$$

中心差分： $\frac{\partial u}{\partial t}=\frac{u_{i,k+1}-u_{i,k-1}}{2\Delta t}$； $\frac{\partial u}{\partial x}=\frac{u_{i+1,k}-u_{i-1,k}}{2\Delta x}$； $\frac{\partial^3 u}{\partial x^3}=\frac{u_{i+2,k}+2u_{i\text{-}1,k}\text{-}2u_{i+1,k}-u_{i-2,k}}{2\Delta x^3}$.

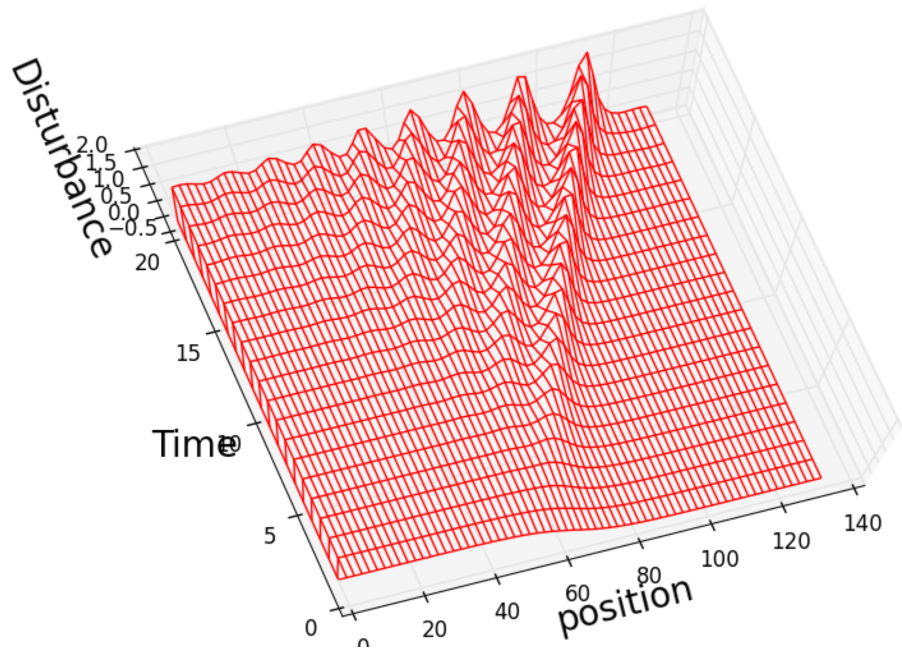
差分方程：
$$\begin{aligned}u_{i,k+1}=u_{i,k-1}&-\frac{\varepsilon}{3}\frac{\Delta t}{\Delta x}\left[u_{i+1,k}+u_{i,k}+u_{i-1,k}\right]\left[u_{i+1,k}-u_{i-1,k}\right]\\&-\alpha\frac{\Delta t}{\Delta x^3}\left[u_{i+2,k}+2u_{i-1,k}-2u_{i+1,k}-u_{i-2,k}\right]\end{aligned}$$

$$\begin{aligned}u_{i,2}=u_{i,1}&-\frac{\varepsilon}{6}\frac{\Delta t}{\Delta x}\left[u_{i+1,1}+u_{i,1}+u_{i-1,1}\right]\left[u_{i+1,1}-u_{i-1,1}\right]\\&-\alpha\frac{\Delta t}{2\Delta x^3}\left[u_{i+2,1}+2u_{i-1,1}-2u_{i+1,1}-u_{i-2,1}\right]\end{aligned}$$

一些参数取值： $\varepsilon=0.2,\ \alpha=0.1$， $\Delta x=0.4$, $\Delta t=0.1$



Disturbance
2.0
1.5
1.0
0.5
0.0
−0.5
20
15
10
5
0
Time
0
20
40
60
80
100
120
140
position



Disturbance
2.0
1.5
1.0
0.5
0.0
−0.5
20
15
10
5
0
Time
0
20
40
60
80
100
120
140
position

课堂练习：含时一维薛定谔方程：

$$i\frac{\partial\psi(x,t)}{\partial t}=-\frac{1}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}+V(x)\psi(x,t)$$

其中：$V(x)=\begin{cases}\infty, & x<0, or\ x>15,\\ 0, & 0\le x\le 15.\end{cases}$

初始条件：$\psi(x,t=0)=\exp\left[-\frac{1}{2}\left(\frac{x-5}{\sigma_0}\right)^2\right]e^{ik_0x}$

一些参数取值：$\sigma_0=0.5, \Delta x=0.02, k_0=17\pi, \Delta t=\frac{1}{2}\Delta x^2, 2m=1$； $\mathrm{hbar}=1$

要求：计算几率密度的时间演化

提示：

$$\psi(x,t) = R(x,t) + i \cdot I(x,t)$$

$$\frac{\partial R(x,t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2 I(x,t)}{\partial x^2} + V(x)I(x,t)$$

$$\frac{\partial I(x,t)}{\partial t} = +\frac{1}{2m}\frac{\partial^2 R(x,t)}{\partial x^2} + V(x)R(x)$$

几率密度： $\rho(t) = R^2(t) + I^2(t)$

作业： 自拟题目

自行选择或设计一个普通物理，理论力学，或其他课程中涉及到的物理问题并用所学数值方法求解。要求该物理问题需要用到数值求解偏微分方程。

要求：

1）所选择或设计的物理问题需要用到数值求解偏微分方程方法；
2）写出计算用到的主要公式；
3）写出计算程序代码 (python)；
4）将计算结果用图形表示出来。
5）有一定原创性(勿从其他“计算物理”教材中照抄，可从普通物理，理论力学等物理课程教材中的题目选择或适当改进。)