## §5. Уравнения в полных дифференциалах

$M(x,y)\,dx + N(x,y)\,dy = 0$. (1.14) $du(x,y) = M(x,y)\,dx + N(x,y)\,dy \Rightarrow du(x,y)=0$, если $y(x)$ – решение (1.14)
то $du(x,y(x)) \equiv 0 \Rightarrow u(x,y(x)) = C$ (1.15). Чтобы левая часть (1.14) была полн. дифф. $\Longleftrightarrow$

$\dfrac{\partial M(x,y)}{\partial y} \equiv \dfrac{\partial N(x,y)}{\partial x}$ (1.16) $^*\Big[M(x,y)\,dx + N(x,y)\,dy = du = \dfrac{\partial u}{\partial x}dx + \dfrac{\partial u}{\partial y}dy \Rightarrow M = \dfrac{\partial u}{\partial x},\ N = \dfrac{\partial u}{\partial y},$

$\dfrac{\partial M}{\partial y} = \dfrac{\partial^2 u}{\partial x \partial y},\ \dfrac{\partial N}{\partial x} = \dfrac{\partial^2 u}{\partial x \partial y} \Longrightarrow \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}\Big]^*$ т.о. $\dfrac{\partial u}{\partial x} = M(x,y)$, $\dfrac{\partial u}{\partial y} = N(x,y)$, откуда

$u(x,y) = \int M(x,y)\,dx + C(y)$, далее дифф. $u(x,y)$, т.к. $\dfrac{\partial u}{\partial y} = N(x,y)$, получ. $\dfrac{\partial}{\partial y}\left(\int M(x,y)\,dx\right) + C'(y) = N(x,y)$

Пример 2. $x\,dx + y\,dy + (x^2+y^2)x^2\,dx = 0 \,\Big|\, * \ \mu = \dfrac{1}{x^2+y^2} \Rightarrow \dfrac{x\,dx + y\,dy}{x^2+y^2} + x^2\,dx = 0$

Пример 1. $(x+y+1)\,dx + (x-y^2+3)\,dy = 0$, $\dfrac{\partial(x+y+1)}{\partial y} = \dfrac{\partial(x-y^2+3)}{\partial x}$, $\dfrac{\partial u}{\partial x} = x+y+1$,

$u = \dfrac{x^2}{2} + yx + x + C(y)$, $\dfrac{\partial u}{\partial y} = x + C'(y)$, $C'(y) = -y^2+3$, $C(y) = -\dfrac{y^3}{3} + 3y + C_1$, $u = \dfrac{x^2}{2} + xy + x - \dfrac{y^3}{3} + 3y + C_1$

общий интеграл имеет вид $3x^2 + 6xy + 6x - 2y^3 + 18y = C_2$ (1.17)

Если $M(x,y)\,dx + N(x,y)\,dy = 0$ (1.14) м. подобрать ф-цию $\mu(x,y)$, после умн. на к-ю (1.14) превр. в полн. дифф. $du = \mu M(x,y)\,dx + \mu N(x,y)\,dy$.

Чтобы подобр. интегрирующий мн-ль: $\dfrac{\partial \mu M}{\partial y} = \dfrac{\partial \mu N}{\partial x}$ или $\dfrac{\partial \mu}{\partial y}M + \mu\dfrac{\partial M}{\partial y} = \dfrac{\partial \mu}{\partial x}N + \mu\dfrac{\partial N}{\partial x} \,\Big|\, \div\mu \Rightarrow$

$\dfrac{\partial \ln\mu}{\partial y}M - \dfrac{\partial \ln\mu}{\partial x}N = \dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}$ (1.18) Если $\mu = \mu(x)$, то $-\dfrac{d\ln\mu}{dx}N = \dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}$,

$\ln\mu = \int \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\,dx + \ln C$, $\mu = C e^{\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}dx}$ (1.19)