Lagrangian mechanics

For an object with mass , position and velocity

is the position relative to point P, and it can be written as a function of parameters. However, P depends on the trajectory, but the velocity of P is constrained such that there is no slipping

Assuming parameters are

Then, velocity

The constraints of point P are the following:

are function of

Therefore,

Because is a function of ,

Therefore,

For an object with moment of inertia , rotation

and angular velocity

And . Therefore,

Necessary terms are , and

Noted that also

If , then

Wheel on the ground in ZXY rotation

-z faces up, +x faces forward, move forward

Mass , moment of inertia

Constraints are

Let

Translational kinetic energy

Rotational kinetic energy

Let

Wheel on the ground in ZXZ rotation

+z faces up, +x faces forward, when set, wheel is lay flat on the ground. When , the wheel stays up

Wheel radius

Mass , moment of inertia

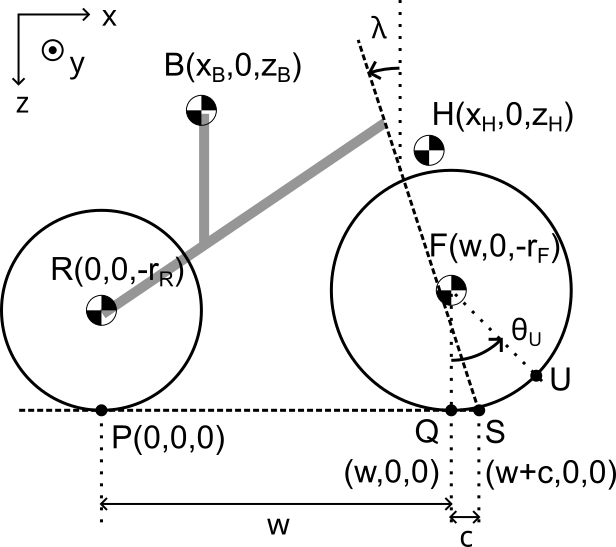
In conclusion,

Conversion between ZXY and ZXZ

For ZXY,

For ZXZ,

Bike



All the parameters:

To make sure that the front wheel is contact with ground, 2 constrains are required:

The first constraint means that point U is contact with ground, but U is just a point on front wheel, so the second constraint makes sure that U is the lowest point of front wheel.

Based on definition:

where

For the first constraint, let

For the second constraint, let

Combine the two constraints, let , because ,

At set point

Therefore,

then

This equation below is used in potential energy

Tires don’t slip.

Because , we only need to constrain the xy component of

Such that

At set point

Because , therefore

Others

Therefore,

In conclusion, the only free parameters are . If we set to constant, there are only two parameters and .

Translational kinetic energy

Because , also the only parameter depending on and the derivative might not be zero is , therefore

Because any parameter derivative of is zero,

Rotational kinetic energy

Cheat sheet of derivative

Translational kinetic energy

For RB, , for RS,

For SH, , for SF,

Therefore

If Y=z or , . Therefore,

For point R

In conclusion,

For point B

For B,

In conclusion,

For point H and F

for RS,

For SH, , for SF,

In conclusion, for H

In conclusion, for F

Rotational kinetic energy

For point R

For point B

For point H

For point F

Gravitational potential energy:

Because gravity is facing toward direction instead of direction

Combine everything

From literature

B4:

B6:

B7:

B1:

B2:

B3:

B2+B3:

The parameters provided in the article are

For point R

can be model as an annulus,

The inner radius of annulus

For point B

For point H

The principal moment of inertia can be modeled as a solid cylinder,

The equivalent height is 41cm which is a little bit short.

For point F

can be model as an annulus,

The inner radius of annulus