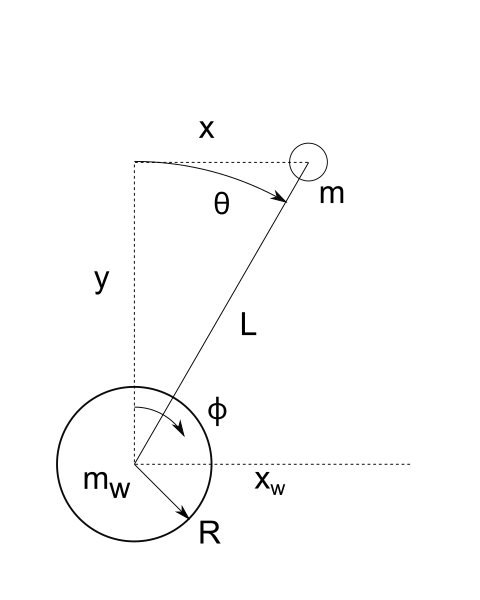
Inverted pendulum on wheel



x\_w: horizontal position of the center of wheel relative to a defined origin

x: horizontal position of the center of pendulum relative to a defined origin

y: vertical position of the center of pendulum relative to a defined origin

phi: clockwise rotational angle of the wheel from +y axis

theta: clockwise rotational angle of the pendulum from +y axis

tau: clockwise torque applied to wheel from pendulum

m: mass of the pendulum part

m\_w: mass of the wheel part

R: radius of the wheel

L: length between center of pendulum and center of wheel

I: inertia of the pendulum

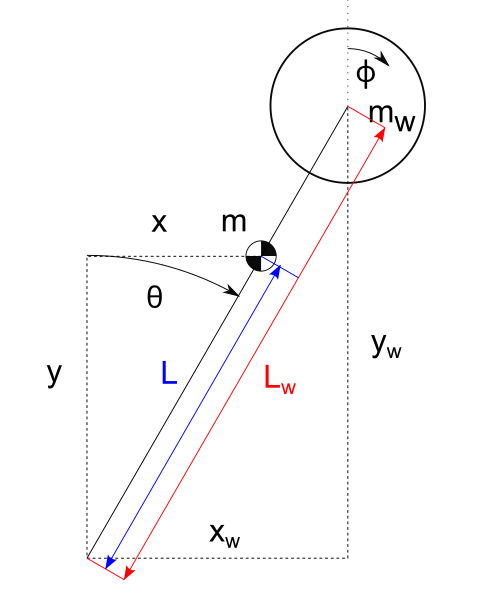
I\_w: intertia of wheel

Br: friction between wheel and floor

Bm: friction between wheel and pendulum

linearization at

Reaction wheel on inverted pendulum



x\_w: horizontal position of the center of wheel relative to a defined origin

y\_w: vertical position of the center of wheel relative to a defined origin

x: horizontal position of the center of pendulum relative to a defined origin

y: vertical position of the center of pendulum relative to a defined origin

phi: clockwise rotational angle of the wheel from +y axis

theta: clockwise rotational angle of the pendulum from +y axis

tau: clockwise torque applied to wheel from pendulum

m: mass of the pendulum part

m\_w: mass of the wheel part

L: length between center of pendulum and pivot point

L\_w: length between center of wheel and pivot point

I: inertia of the pendulum

I\_w: intertia of wheel

Ba: friction between pendulum and floor

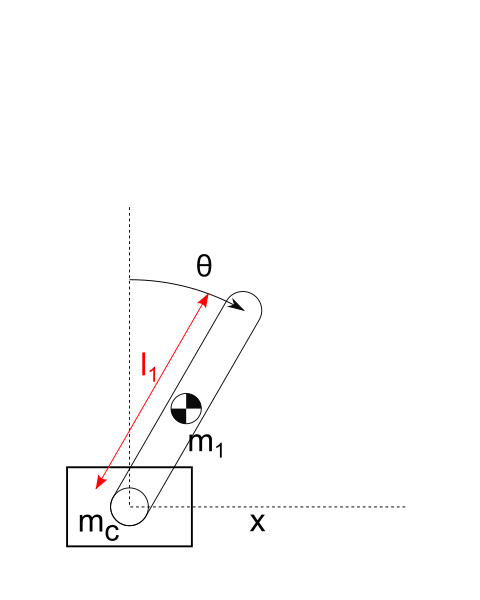
Bm: friction between wheel and pendulum

linearization at

Let

Let , then such that

Inverted pendulum on cart

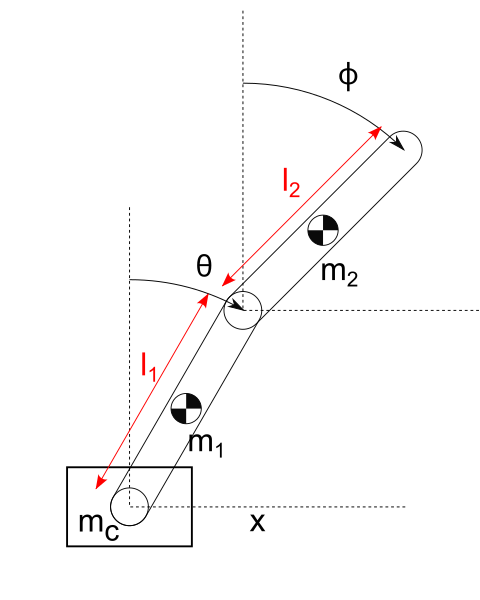


Cart has mass m\_c

Pendulum1 mass m\_1, length l1, moment of inertia I1, angle from y to x (CW) is theta

Set point is

Double Inverted pendulum on cart



Cart has mass m\_c

Pendulum1 mass m\_1, length , MoI , angle from y to x (CW) is theta

Pendulum2 mass m\_2, length , MoI angle from y to x (CW) is phi

X part

Therefore,

Theta part

Therefore,

Phi part

Therefore,

Combine all three equations

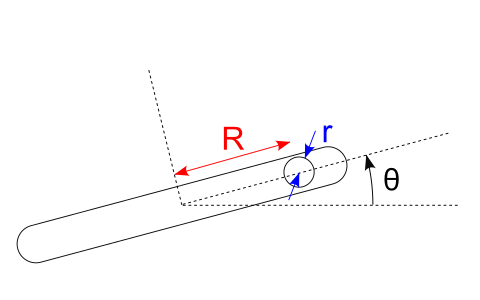
If , then

If , then

They reduce to the single pendulum case

Set point is

Ball rolling on a rotated linear slide (assume no slip)



theta: counterclockwise rotational angle of the linear slide from +x axis

R: distance between mass and origin

r: radius of the ball

phi: clockwise rotated angle of the ball

tau: counterclockwise torque applied to linear slide

m: mass of the pendulum part

I: inertia of the linear slide

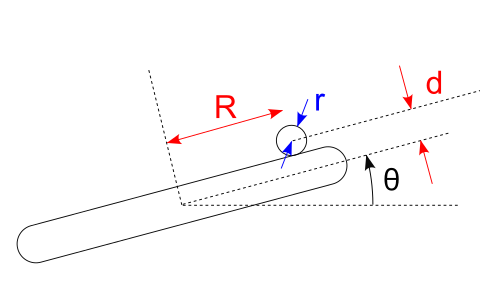
b: friction at R direction

Note that

Let

Let , then such that

Ball rolling on a rotated linear slide with offset (assume no slip)



theta: counterclockwise rotational angle of the linear slide from +x axis

phi: counterclockwise rotated angle of the ball

R: distance between mass and origin

tau: counterclockwise torque applied to linear slide

m: mass of the pendulum part

r: radius of the ball

d: offset between center of ball and origin

I: inertia of the linear slide

b: friction at R direction

Let

If

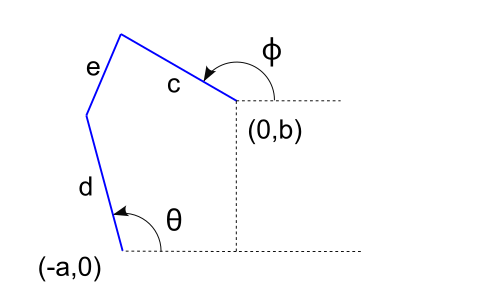
if

At the same time, if

On the other hand, if

if

Ball rolling on a platform inverse kinetic



Motor angle is theta from x axis

Platform angle is phi from x axis

Motor at (-a,0)

Platform pivot at (0,b)

Platform hinge is c away from pivot

Motor arm length is d

Fourth linkage length is e, which is the distance between and

Therefore,

The other way around

If we want to simplify the equation

Let

such that

Angular velocity and quaternion

let

then

let

then

body ref:

extra, if , then

fixed ref:

Gyroscope zxz

wheel

Wheel moment of inertia is

Center of mass radius is R

Wheel mass is m

P=mgR

Center of mass at

Let

Gyroscope xyz

wheel

Wheel moment of inertia is

Center of mass radius is R

Wheel mass is m

P=mgR

Center of mass at

Let

Control moment gyroscope

Robot forward is , it can tilt along x axis for ,

The wheel is fixed on the robot, spinning along z axis for ,

Wheel rotation axis can be tilted along y axis for by applying torque

For the wheel, the rotation is

Wheel angular velocity

Wheel moment of inertia is

(2 Wheel mass \* wheel height^2+ Body mass \* body height^2) is I\_b

g\*(2 Wheel mass \* wheel height+ Body mass \* body height) is p

Let

Linearization at

Control theory

Plant without noise: input=u, output=y

Plant with noise: input=(u,wd,wn), output=y,

in this case, wd has length of x

LQG feedback: input=(u,y,r), output=u

QN is the covariance of wd, RN is the covariance of wn

Control with desired point

Let is the reference value, is a scaling coefficient

If , then

Discrete Kalman filter

Xk is the real state at time k

uk is the input at time k

yk is the real measurement at time k

Xhatk is the estimated state at time k

From initial state

Prediction based on previous :

Update:

To minimize the trace of , the kalman gain is derived as following:

Therefore