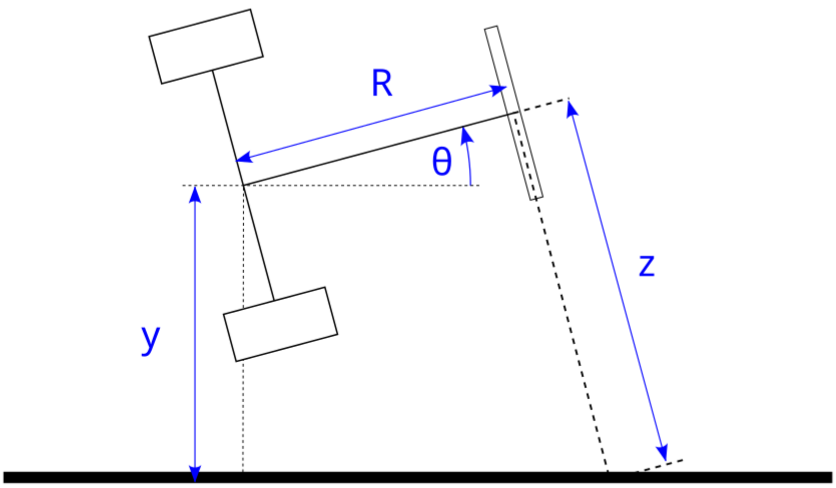
I implemented LQR method and Kalman filter on a line follower.

The robot is composed of two wheels and a row of sensors at the front. The distance between sensors and the center of two wheels is R. The robot is moving forward at a constant speed v. The angular velocity ω is the input of the system to make sure that the robot follows the line. The sensor can measure the distance away from the line, and the measured value z is the output of the system. y is the distance between the line and the center of two wheels. θ is the angle between the moving directions and the line.



It can be seen that $\dot{y} = v \sin\theta$ and $z = R \tan\theta + y \sec\theta$

I design the state to be $\begin{bmatrix} y \\ \theta \end{bmatrix}$, the state space model can be linearized at

$y = \theta = 0$. Therefore,

$$\frac{d}{dt}\begin{bmatrix} y \\ \theta \end{bmatrix} = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\omega$$

$$z = \begin{bmatrix} 1 & R \end{bmatrix}\begin{bmatrix} y \\ \theta \end{bmatrix}$$

This system is controllable and observable.

Because the sensor value is discrete, the variance of output is

$$\frac{1}{12} * (the\ interval\ between\ sensors)^2$$

Based on simulation, this method works great on following a straight line.

Interestingly, it works on following a curve under certain conditions.

When calculating the LQR gain, the weighted matrix QW for the state which is a 2*2 matrix. The weight matrix RW for input is float value. If the weight for θ is smaller or RW is smaller, the robot will rotate more aggressively to follow the line, and therefore it can follow a sharper curve. On the other hand. The optimal weight for y in QW depends on R. If R decrease, the weight for y needs to be increased. Also, the upper bound for v is determined by the width of sensors. When v increases, it required wider range of distribution of sensors so that robot can correct the direction in time. The feedback frequency could be as low as 100Hz under some conditions.