Dynamic incentive contracts for ESG investing*

Zhaojun Yang, Yuqian Zhang*

Department of Finance, Southern University of Science and Technology, Shenzhen, 518055, China.

Abstract

We develop a continuous-time model in which an ESG investor hires a manager to run a project and incentivizes the manager to fulfill ESG responsibilities. The effort and ESG investing of the manager are the manager's private information, and they determine the project cash flow and ESG performance. We derive the optimal contract and its implementation after introducing carbon credits following the cap-and-trade program in practice. We provide an example, comparative static analysis, and empirical implications. The results demonstrate that ESG investing enhances contract efficiency. The more significant the carbon emission reduction, or the less the cost of ESG investing, the higher the contract efficiency, the average q, the marginal q, and the optimal investment-capital ratios, implying that ESG investing mitigates inefficiencies arising from information asymmetry and enhances investment values. Our model predictions are partially verified by empirical facts.

Keywords: ESG investing, moral hazard, dynamic contracts, carbon credits, contract implementation.

JEL: D81, D82, E24, J41

1. Introduction

The impact of businesses on social welfare is of increasing concerns, whether positive or negative. Enterprises play a crucial role in green finance as they represent the demand side of financing in the market and provide the micro foundation for the development of the green

*The research reported in this paper was supported by the National Natural Science Foundation of China (Project No. 72031003).

*Corresponding author

Email addresses: yangzj@sustech.edu.cn (Zhaojun Yang), 12031196@mail.sustech.edu.cn (Yuqian Zhang)

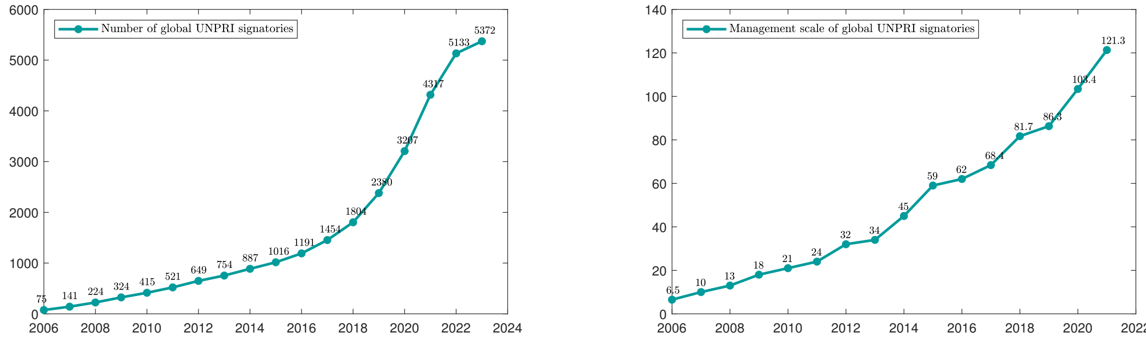
economy. Thus, it is essential to comprehensively strengthen the involvement of enterprises in the green transition. ESG, an acronym for environmental, social, and governance, represents a new evaluation standard that centers on corporations' investment philosophy, encompassing their comprehensive approach to green and sustainable development. The emergence of ESG concerns on a global scale in recent years has provided a foundational framework for corporate green transformation.

Some ESG investors of the firm are inherently concerned about these indicators (Engle et al., 2020). This reflects the fact that certain activities of the firm have external impacts, but their costs are not entirely borne by the firm. Corporate carbon emissions serve as typical examples of activities where the social costs are not completely borne by corporate decision-makers. The Intergovernmental Panel on Climate Change (IPCC) Special Report on Global Warming of 1.5°C[1] indicates that achieving net-zero global carbon emissions by the mid-century is crucial for limiting global warming to no more than 1.5°C, which in turn mitigates the adverse impacts of extreme weather events on human well-being. In light of this context, governments worldwide have established targets for carbon neutrality and implemented corresponding laws and regulations (Masood, 2021). Presently, over 190 countries have made commitments towards achieving carbon neutrality.

The Principles for Responsible Investment (UN PRI), supported by the United Nations, aim to assist investors in comprehending the influence of ESG factors on investment value. Their role is to assist signatory institutions in integrating these elements into their investment strategies, decision-making processes, and practicing active ownership. Its membership includes pension funds, insurance companies, sovereign wealth funds, development funds, investment managers, and service providers from various countries. According to Figure 1[2], more than 5,370 institutions have become signatories of the PRI, managing total assets exceeding $121 trillion. This data signifies the growing interest of investors in the ESG concept. ESG investments primarily reflect investors' non-monetary preferences (Starks, 2023). In traditional financial theory, there is a lack of description regarding non-monetary preferences related to ESG. Therefore, it is necessary to introduce stochastic processes that influence

[1] See, https://www.ipcc.ch/sr15/
[2] See, https://www.unpri.org/

non-monetary preferences when studying dynamic contracts from an ESG perspective.



(a) Number of global UNPRI signatories (b) Management scale of global UNPRI signatories

Figure 1. Cumulative number and management scale of global UNPRI signatories.

In the traditional corporate finance theory, company owners solely prioritize the financial performance while overlooking broader social indicators, such as ESG performance. Nevertheless, corporate ESG investing frequently entail substantial costs, the value of which is not readily apparent in project cash flows. In the absence of incentives, companies are prone to decline ESG investing to minimize costs and mitigate the impact on the project cash flow. Consequently, incentivizing companies to actively embrace ESG investment concepts is both significant and pressing. ESG investors who prioritize ESG metrics serve as sources of incentives. The decision-making of ESG investors regarding incentives holds significance in addressing this matter, as evidenced by the ISS Global Executive Compensation Analysis Database, which indicates a growth in the proportion of companies, from 3% in 2010 to over 30% in 2021, incorporating certain ESG metrics as executive key performance indicators (KPIs) (Cohen et al., 2023). The McKinsey Quarterly's November 14, 2019 report, "Five Ways ESG Creates Value" Outlines the ways companies are pursuing the value created by ESG strategies[3]. Therefore, it is necessary to theoretically solve the problems of how decisions are made, how incentives are transmitted, and how decisions affect the ESG behavior of enterprises. Subsequently, this provides scientific guidance for making and implementing

[3]See,https://www.mckinsey.com/capabilities/strategy-and-corporate-finance/our-insights/five-ways-that-esg-creates-value

specific decisions.

Our contributions. The main contributions of the paper are summarized as follows: First, we introduce ESG investing into the traditional contract design taking carbon emissions into account. Utilizing the classical principal-agent framework, we establish a continuous-time game model from ESG perspective. Solving the equilibrium, we delineate the optimal contract employing ordinary differential equations and elucidate ESG investor's incentive decisions to maximize the ESG performance of managers.

Second, we present the contract implementation by incorporating a portfolio comprising carbon credits and common securities. Introducing carbon credits into the portfolio grants the firm greater flexibility in its investment and financing policies. Additionally, we present a specific application example to offer tangible frameworks for policy formulation and implementation.

Lastly, we examine the impact of the conversion factor for carbon emissions and ESG costs on the manager's ESG behavior. This analysis also sheds light on how the conversion factor for carbon emissions and ESG costs influence firm value and investment opportunities. We demonstrate that ESG investing enhances contract efficiency. The more significant the carbon emission reduction, or the less the cost of ESG investing, the higher the contract efficiency, the average q, the marginal q, and the optimal investment-capital ratios, implying that ESG investing mitigates inefficiencies arising from information asymmetry and enhances investment values. Empirical facts partially verify our model predictions.

Relative literature. Holmstrom and Milgrom (1987) first introduce the continuous-time model to a principal-agent problem, and Holmstrom and Milgrom (1991) consider the effect on the distribution of work among tasks due to the nature of incentive contract. Schättler and Sung (1993) and Sung (1995) are the first to use the dynamic programming and martingale method of stochastic control theory to solve a moral hazard problem in a continuous-time model, where the dynamical system is often not Markovian. This is because a contract may depend on the entire historical output. Thus it cannot be solved directly by dynamic programming. Instead, taking a continuation value as a state variable and invoking a martingale method introduced by DeMarzo and Sannikov (2006) and Sannikov (2008), one can solve the sequential game problem.

Recently, Piskorski and Tchistyi (2010) address optimal mortgage design in a Markov regime-switching model. Schmid (2008), Rampini and Viswanathan (2010), and DeMarzo et al. (2012) introduce optimal investment into contract theory. Prat and Jovanovic (2014), DeMarzo and Sannikov (2016) and He et al. (2017) take unknown agent's quality and Bayesian learning into account in contract design. Williams (2011), and Williams (2015) apply the stochastic maximum principle approach for optimal security design. Zhu (2022) considers how anticipated disagreement between an investor and an entrepreneur affects optimal contracting and asset prices. Keppo et al. (2022) study a principal-agent problem under an investor-partner-manager structure.

ESG-related research has focused on exploring the relationship between corporate management characteristics and ESG activities. Some scholars have studied how corporate management affects ESG investing. Bénabou and Tirole (2010) demonstrate that ESG investing is a result of well-governed management decisions and is generated when managers act in their own interests. McCarthy et al. (2017) report a negative correlation between CEO confidence and firm ESG investing. Some studies specifically point out that ESG investing should be included in relevant performance indicators for corporate management. Maas (2018) finds that quantifiable and mandatory corporate social performance goals are an effective way to improve corporate social responsibility outcomes. Bonham and Riggs-Cragun (2022) point out that from the perspective of institutions and management, ESG indicators would be relied upon in executive compensation schemes if firm owners and their representatives are essentially concerned about ESG outcomes. Some scholars have studied the impact of ESG investing on corporate performance. Flammer et al. (2019) conclude that including corporate social responsibility variables in executive compensation often improves a firm's financial performance.

It can be seen that research on the relationship between corporate management characteristics and ESG activities currently focuses on empirical research. These empirical studies have discovered the mutual influence relationship between corporate ESG activities and management through statistical tests, but lack corresponding theoretical support. Theoretical research involving ESG activities mainly focuses on supply chain management. For example, Ma et al. (2017) designed optimal contracts for supply chains facing information asymmetry, where wholesale price contracts were developed as the basic model to gain a deeper understanding of the value of information sharing. However, theoretical research that simultaneously considers ESG, corporate governance, and firm operations is limited. Theoretical research that combines ESG and contract theory can provide theoretical support for the relationship between corporate management characteristics and ESG activities, reveal the underlying mechanisms, and fill the gap in this field.

Our paper is closely linked to the works of DeMarzo et al. (2012), which explore the optimal security design within principal-agent framework from a traditional perspective. In contrast, we develop a stochastic process model to capture carbon emissions in the firm's business activities. We conceptualize ESG investors as an incentive source, impacting managers' effort behavior and a firm's ESG activities. Subsequently, we build a continuous-time game model with ESG investing. In our model, the firm's ESG investing reduces carbon emissions. However, the high cost of ESG investing negatively affects the firm's cash flow. Simultaneously, moral hazard resulting from the agent's hidden-effort behavior can detrimentally impact social welfare. In combination, we examine incentive strategies offered by ESG investor and employ the martingale method to address the contract problem between the ESG investor and manager. To the best of our knowledge, we are the first to address incentive contracts in an A-K model taking ESG investing into account.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 considers optimal contract from the ESG perspective. Section 4 provides an implementation of the optimal contract. Section 5 provides numerical analysis to discuss how the model results are influenced by model parameters. Section 6 provides a model extension. Section 7 summarizes the relationship between the empirical results in the literature and our model predictions. Section 8 concludes. Proofs are relegated to the Appendix.

2. Model Setup

We consider an ESG investor[4], who hires a manager to run a project (or firm). The manager exerts hidden costly effort that increases project output; thus, there is a standard moral hazard problem. we assume the manager decides further whether the firm conducts ESG investing,[5] which linearly reduce carbon emissions arising from the firm's production activities.

We address a continuous-time infinite-horizon model. Denoting by K_t the firm's capital stock and by I_t its total investment rate, as is standard in capital accumulation models, we assume the firm's capital stock K_t evolves according to

$$\mathrm{d}K_t = F(I_t, K_t)\mathrm{d}t = (I_t - \delta K_t)\,\mathrm{d}t, \quad t \geq 0, \tag{1}$$

where δ represents the depreciation rate of the firm's capital stock.

According to Jermann (1998), investment incurs an adjusted cost. Following Hayashi (1982) and DeMarzo et al. (2012), we assume the adjustment cost is

$$G(I_t, K_t) = \frac{1}{2}\psi i_t^2 K_t,$$

where $\psi > 0$ is constant and $i_t = I_t/K_t$ called investment-to-capital ratio.

At any time interval $[t, t + \mathrm{d}t]$, the firm uses its capital stock K_t to generate cash flow equal to $K_t \mathrm{d}A_t$, where A_t is the cumulative productivity process. In contrast to the existing literature on contract theory, we assume ESG investing, measured by η, decreases the productivity by $\theta\eta_t$, where θ is the marginal cost of ESG investing, indicating the fixed marginal reduction of productivity due to ESG investing $\eta_t \in [0, \bar{\eta}]$ with $\bar{\eta} > 0$. The cumulative productivity process A_t is determined by the manager's effort a_t, ESG investing and random shocks; it is given by

$$\mathrm{d}A_t = (a_t - \theta\eta_t)\mathrm{d}t + \sigma \mathrm{d}Z_t$$

[4]In this paper, we use the terms "ESG investor", "principal" and "she" interchangeably, and use the terms "manager ", "agent" and "he" interchangeably.

[5]Thanks to Investopedia, ESG investing refers to a set of standards for a company's behavior used by socially conscious investors to screen potential investments.

where process $a \in \mathcal{E} = [0, \bar{a}]$ with $\bar{a} > 0$ being constant represents the unobserved effort of the agent, $\sigma > 0$ represents the productivity volatility and process Z is a Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ endowed with the information flow $\{\mathcal{F}_t\}_{0 \le t < \infty}\}$ satisfying the usual conditions. Accordingly, the project (cumulative) cash flow X is given by

$$\mathrm{d}X_t = K_t \mathrm{d}A_t - G(I_t, K_t)\mathrm{d}t = K_t \left[\left(a_t - \theta\eta_t - \frac{1}{2}\psi i_t^2 \right) \mathrm{d}t + \sigma \mathrm{d}Z_t \right], \ t \in [0, T], \tag{2}$$

where T is the project termination time. We say a process $X \in L^2$ if $\mathbb{E} \int_0^T X_t^2 \mathrm{d}t < \infty$.

The advantage of ESG investing is to reduce the carbon emissions. Specifically, we assume the carbon emissions process Y is given by

$$\mathrm{d}Y_t = K_t \left[(b - \eta_t)\, \mathrm{d}t + \phi \mathrm{d}B_t \right], \ t \in [0, T], \tag{3}$$

where B is another $\{\mathcal{F}_t\}_{0 \le t < \infty}$-adapted standard Brownian motion, independent of Z, defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, b indicates the expected growth rate of carbon emissions without ESG investing, and the constant ϕ represents the volatility of the carbon emissions.

As usual, the contract depends on the information available to participants. We assume both parties can observe the project cash flow, carbon emissions and capital stock. Letting $\mathcal{G}_t \equiv \sigma\{X_s, Y_s, K_s : 0 \le s \le t\}$, we denote by $\mathbb{G} \equiv \{\mathcal{G}_t\}_{t \ge 0}$ the augmented filtration generated by the cash flow X, carbon emissions Y and capital stock K, i.e. the information flow learned by participants who observe X, Y and K. Without loss of generality, we assume $\mathcal{F}_t = \sigma\{Z_s, B_s : 0 \le s \le t\}$ and denote $\mathbb{F} \equiv \{\mathcal{F}_t\}_{t \ge 0}$, which is the completion of the σ-algebra generated by Brownian motion Z and B. For all participants, cash flows X and carbon emissions Y are observable, but manager's effort a, ESG investing η, random shock Z and B are observable only for the manager. Briefly, process X, process Y and process K are $\mathbb{G}$-adapted, and a, η, Z and B are $\mathbb{F}$-adapted. We emphasize that the information flow of ESG investors is $\mathbb{G}$, while the manager's is $\mathbb{F}$. Thus, the manager know more than ESG investors. So the disparity in their access to information can lead to moral hazard. See Chapter 5 of Cvitanic and Zhang (2012) for a detailed discussion of information flows.

Suppose that the manager is necessary for operating the project, and the project is sufficiently profitable on average such that ESG investor never initiates contract termination.

The termination time T of the contract is endogenously determined by the manager. Upon termination, ESG investor receives the liquidation dollar value LK_T, while the manager gets exogenously given dollar value $RK_T > 0$ from an outside option.[6]

We assume the manager's ESG investing with his effort is private and unobservable. ESG investor offers him a contract that specifies the manager's compensation rate C per unit of time together with investment plan I. Therefore, The contract is defined by the pair (C, I), and the manager's problem is to choose a strategy (a, η, T) defined by effort level process a, ESG investing η, and contract termination time T. If the strategy (a, η, T) maximizes the manager's value under a given contract (C, I), then it is called incentive-compatible.

ESG investor's problem is how to design such a contract incentivizing the manager that the manager is willing to accept it and ESG investor's value is maximized while the manager takes an incentive-compatible strategy.

Following DeMarzo and Sannikov (2006) and DeMarzo et al. (2012), we assume both ESG investor and the manager are risk-neutral, but that manager's subjective discount rate γ is higher than the investor's, which is the risk-free rate r. As in Huang et al. (2023) and DeMarzo et al. (2012), the cost of effort has the linear form $g(a_t) \equiv \lambda K_t a_t, t \geq 0$, where the coefficient λ denotes the fixed marginal cost of effort and the cost has the same unit with the compensation. DeMarzo and Sannikov (2006) proves that in an optimal contract, the manager has zero dollars in his savings account all the time due to his higher subjective discount rate; thus, we do not consider savings in our model.

3. Optimal contracting

To develop economic intuition we first provide a benchmark model and consider the case in which all decisions are contractible. Accordingly, all incentives are unnecessary, leading us to derive the first-best strategies. After that, we turn to the optimal contract design under incomplete information and derive the main conclusions of the paper.

[6] This assumption is made for easy calculations, implying that, at the termination of the contract, the larger the size of the firm managed by the agent, the higher the external value he can receive. It is also reasonable because a larger firm indicates that the agent possesses a higher ability to manage a firm and thus a higher external option value.

3.1. Full information case

Thanks to the full information assumption, there is no information asymmetry and all financial decisions are contractible. Therefore, we only need to solve the following optimization problem:

$$\begin{aligned} &\sup_{C,I,a,\eta,T} \mathbb{E}\left\{\int_0^T e^{-rt}(\mathrm{d}X_t - \kappa \mathrm{d}Y_t) + e^{-rT}LK_T\right\}, \\ &\text{s.t. } \mathbb{E}\left\{\int_0^T e^{-\gamma t}[C_t - g(a_t)]\mathrm{d}t + e^{-\gamma T}RK_T\right\} \geq W_0 = w. \end{aligned}$$

where $\kappa > 0$ is an exogenous conversion factor from carbon emissions to their monetary value of ESG investor, and the exogenously given W_0 represents the minimum value obtained by the manager after entering the contract. Clearly, the larger the loss κ per unit of carbon emissions, the more ESG investor values the carbon emission reduction of the firm.

Solving it, we derive the first-best effort $a_t^{FB} = \bar{a}$ and the first-best ESG investing

$$\eta_t^{FB} = \begin{cases} 0, & \kappa < \theta, \\ \bar{\eta}, & \kappa \geqslant \theta, \end{cases} \qquad \text{for all } t \geq 0.$$

Referring to DeMarzo et al. (2012) and Hayashi (1982), we derive that the marginal value of capital (marginal q) equals the average value of capital (average or Tobin's q), both of which are given by

$$q^{FB} = \max_i \frac{\bar{a} - \kappa b + (\kappa - \theta)\eta^{FB} - \frac{1}{2}\psi i^2}{r + \delta - i}.$$

Therefore, we get the first-best investment-to-capital ratio

$$i^{FB} = \frac{\psi(r+\delta) - \sqrt{\psi^2(r+\delta)^2 - 2\psi(\bar{a} - \kappa b + (\kappa - \theta)\eta^{FB})}}{\psi},$$

and Tobin's q given by

$$q^{FB} = \psi(r+\delta) - \sqrt{\psi^2(r+\delta)^2 - 2\psi(\bar{a} - \kappa b + (\kappa - \theta)\eta^{FB})}, \tag{4}$$

which indicates that in the neoclassical model, the time-invariance of a firm's technology leads to constant optimal investment and contracting, independent of the firm's history or its cash flow volatility. By economic intuition, q^{FB} represents the value of the firm's cash flow (per unit of capital) prior to paying the agent's compensation. Investment is positive and $q^{FB} > 0$ when the firm is sufficiently productive and the investment has a positive net present value (NPV), i.e. $\bar{a} - \kappa b + (\kappa - \theta)\eta^{FB} > 0$.

3.2. The manager's problem under incomplete information

In general, the manager's decisions are not contractible, and the manager would deviate from the first-best strategy due to the costs of private effort and ESG investing. Therefore, ESG investors are required to provide incentives for the manger to work hard and enhance ESG investing. At least, the manager's compensation must increase as the project's output increases or carbon emissions reduce. To design an optimal contract, ESG investor must know how the manager responds to a contract offered by the investor. We answer such problems below.

Having been offered a given take-it-or-leave-it contract (C, I) by the principal, the agent needs to solve the following optimization problem:

$$\sup_{a,\eta,T} \mathbb{E}\left\{\int_0^T e^{-\gamma t}[C_t - g(a_t)]\mathrm{d}t + e^{-\gamma T} R K_T\right\}, \tag{5}$$

where control process a and η are $\mathbb{F}$-adapted and T is a $\mathbb{F}$-stopping time.

We define the manager's continuation value (or promised value) W_t at time t as the value of the manager's cash flow starting from current time $t \geq 0$ for a given participants' strategy profile $\{(C, I); (a, \eta, T)\}$, which is given by

$$W_t \equiv \mathbb{E}\left\{\int_t^T e^{-\gamma(s-t)}[C_t - g(a_t)]\mathrm{d}s + e^{-\gamma(T-t)} R K_T \middle| \mathcal{F}_t\right\}.$$

As in Sannikov (2008), we conclude from the martingale representation theorem the following lemma:

Lemma 3.1. *For a given strategy profile $\{(C, I); (a, \eta, T)\}$, there exists an $\mathbb{F}$-adapted processes pair (Σ^Z, Σ^B) such that the continuation value process W can be expressed as*

$$\mathrm{d}W_t = (\gamma W_t - C_t + \lambda K_t a_t)\,\mathrm{d}t + \Sigma_t^Z K_t \sigma \mathrm{d}Z_t + \Sigma_t^B K_t \phi \mathrm{d}B_t. \tag{6}$$

We assume that it is inefficient if the manager does not take his maximum effort $\bar{a}$ or ESG investing $\eta_t < \bar{\eta}$. Therefore, we call a contract (C, I) incentive-compatible if the solution of (5) satisfies $a_t = \bar{a}$ and $\eta_t = \bar{\eta}$ for all $0 \leq t \leq T$. Thanks to the Revelation Principle (Myerson, 1979), without loss of generality, we consider only the incentive-compatible contract in the following.

Intuitively, to make a contract incentive-compatible, ESG investor must provide the manager with sufficient incentives relative to the effort and ESG investing cost. Formally, we have

Proposition 3.2. *The contract (C, I) offered by ESG investor is incentive-compatible if and only if $\Sigma_t^Z \geqslant \lambda$ and $\Sigma_t^B \leq -\theta\Sigma_t^Z$, for $0 \leq t \leq T$. The manager's optimal exit time T is the first time of his continuation value W_t hitting the value RK_t from above, i.e. $T = \inf\{t \geq 0 : W_t = RK_t\}$.*

Prop. 3.2 says that ESG investor's offer (C, I) is incentive-compatible if and only if the sensitivity of the continuation value with respect to project output Σ_t^Z is greater than the marginal effort cost of the manager λ, and the sensitivity of the continuation value with respect to carbon emissions Σ_t^B is greater than the manager's sensitivity to the marginal cost of ESG responsibilities performance $\Sigma^Z\theta$, for any time $0 \leq t \leq T$. That is, the marginal benefit of the manager's action must be greater than his own marginal cost of the corresponding action.

Next, we turn to ESG investor's optimization problem to derive optimal contract design.

3.3. ESG investor's problem and optimal contract under incomplete information

There are numerous incentive-compatible contracts in general. Denote by $\mathcal{C}$ the set of all incentive-compatible contracts. As assumed before, the optimal contract, denoted by (C^*, I^*), must be incentive-compatible, i.e. $(C^*, I^*) \in \mathcal{C}$. Like Sannikov (2008), the optimal contract problem takes the continuation value W_t and capital stock K_t as state variables, and the control variables include $(C, I, \Sigma^Z, \Sigma^B)$. Specifically, ESG investor must solve the following stochastic dynamic programming:

$$
\begin{aligned}
&\sup_{\Sigma^Z;\Sigma^B;(C,I)\in\mathcal{C}} \mathbb{E}\left\{\int_0^T e^{-rt}(\mathrm{d}X_t-\kappa\mathrm{d}Y_t-C_t\mathrm{d}t)+e^{-rT}LK_T\middle|W_0=W;K_0=K\right\},\\
&\text{s.t.}\quad \mathrm{d}W_t=(\gamma W_t-C_t+\lambda K_t\bar{a})\,\mathrm{d}t+\Sigma_t^Z K_t\sigma\mathrm{d}Z_t+\Sigma_t^B K_t\phi\mathrm{d}B_t,\\
&\qquad\ \mathrm{d}K_t=F(I_t,K_t)\mathrm{d}t=(I_t-\delta K_t)\,\mathrm{d}t,\\
&\qquad\ \Sigma_t^Z\geqslant\lambda;\Sigma_t^B+\theta\Sigma_t^Z\leqslant 0;t\in[0,T],
\end{aligned}
\tag{7}
$$

where T is the first time of the continuation value W_t hitting the value RK_t according to Prop. 3.2. Therefore, ESG investor's value function $V(K_t,W_t)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$
\begin{aligned}
rV(K_t,W_t)=\sup_{\Sigma_t^Z\geq\lambda,-\Sigma_t^B\geq\theta\Sigma_t^Z,C_t,I_t}&\Big\{[(\bar{a}-\kappa b+(\kappa-\theta)\bar{\eta})K_t-G(I_t,K_t)-C_t]\\
&+F(I,K)V_K(K_t,W_t)+(\gamma W_t-C_t+\lambda K_t\bar{a})\,V_W(K_t,W_t)\\
&+\frac{1}{2}K_t^2((\Sigma_t^Z)^2\sigma^2+(\Sigma_t^B)^2\phi^2)V_{WW}(K_t,W_t)\Big\}.
\end{aligned}
\tag{8}
$$

for $0\leq t\leq T$, where the first item in curly brackets is the instantaneous expected utility of the principal, the second is the change in expectations due to the drift term of the capital stock, the third is expected change due to the drift, and the fourth is expected change due to the volatility.

An additional simplifying step is to exploit the scale invariance of the firm's technology to reduce ESG investor's problem to one dimension as in DeMarzo et al. (2012). For this aim, we define the scaled promised value $w_t=W_t/K_t$[7], the scaled investment $i_t=I_t/K_t$ and the scaled compensation rate $c_t=C_t/K_t$ per unit of capital. According to the Itô lemma, we conclude from (6) that the scaled promised value w_t evolves according to

$$
\mathrm{d}w_t=((\gamma-i_t+\delta)w_t-c_t+\lambda a_t)\,\mathrm{d}t+\Sigma_t^Z\sigma\mathrm{d}Z_t+\Sigma_t^B\phi\mathrm{d}B_t. \tag{9}
$$

[7] As in DeMarzo et al. (2012), we denote the scaled value of a variable by its corresponding lowercase letter.

Thanks to the scale invariance of the firm's technology, we write $V(K_t, W_t) = v(w_t)K_t$ and reduce the problem to one with a single state variable w_t. Specifically, we get from (8) and (9) that

$$(r+\delta)v(w_t) = \sup_{\Sigma_t^Z \geq \lambda, -\Sigma_t^B \geq \theta\Sigma_t^Z, c_t, i_t} \Bigg\{ \left[\bar{a} - \kappa b + (\kappa - \theta)\bar{\eta} - \frac{1}{2}\psi i_t^2 - c_t\right] + i_t v(w_t) \\ ((\gamma - i_t + \delta)w_t - c_t + \lambda\bar{a})\, v'(w_t) + \frac{1}{2}((\Sigma_t^Z)^2\sigma^2 + (\Sigma_t^B)^2\phi^2)v''(w_t) \Bigg\}. \tag{10}$$

Naturally, the firm is assumed to be so productive that termination/liquidation is inefficient throughout the text. We note that the instant payment at any time t is an admissible policy of ESG investor which costs her what her pays at most. Therefore, we have $v'(w_t) \geq -1$. Moreover, the payment has two opposite effects: One decreases the investor's value because it decreases the promised value, increasing the inefficient liquidation possibility; the other increases the value because the manager has a higher time discount rate than the investor, meaning that it is expensive for ESG investor to postpone payment to the manager. If the promised value is sufficiently small, the first effect dominates the second, inducing $v'(w_t) > 0$; otherwise, the opposite holds true. Formally, we show in Appendix **??** that the value function $v(\cdot)$ is concave, i.e. $v''(w_t) < 0$.

Therefore, we conclude from (10) that there is a threshold $\bar{w} > R$ such that $v'(\bar{w}) = -1$, the optimal compensation is given by

$$c_t^*(w_t) = \begin{cases} 0, & R < w_t < \bar{w}; \\ w_t - \bar{w}, & w_t \geq \bar{w}, \end{cases} \tag{11}$$

and the optimal investment-to-capital ratio controlled by ESG investor is given by

$$i_t^*(w_t) = \frac{v(w_t) - w_t v'(w_t)}{\psi}. \tag{12}$$

The optimal incentive factors are $\Sigma_t^{Z*}(w_t) = \lambda$ and $\Sigma_t^{B*}(w_t) = -\lambda\theta$.

The equation (11) shows that ESG investor pays the manager if and only if the promised value is greater than $\bar{w}$; the payment is equal to the difference between them. Therefore, the optimal promised value has a reflecting barrier at $\bar{w}$, because it behaves as a stochastic process given by (9) with $c_t = 0$, $a_t = \bar{a}$ and i_t given by (12) whenever it is less than $\bar{w}$ and is pushed down to be equal to $\bar{w}$ by the payment whenever it surpasses it.

As in many papers, e.g., DeMarzo and Sannikov (2006), Sannikov (2008), DeMarzo et al. (2012) and DeMarzo and Sannikov (2016) among others, the optimal incentive factor Σ^Z in contract implementation is set as the percentage of internal equity that the agent must hold. In our model, it only depends on agent's marginal effort cost λ. Intuitively, the sensitivity of the promised value to project output (equivalent to the marginal utility) must cover the agent's marginal effort cost. As argued by DeMarzo and Sannikov (2006), this incentive is costly, and thus it should be set as small as possible, leading to that $\Sigma^Z_t = \lambda$ for all time.

Optimal contract. Substituting (11) and (12) with $\Sigma^{Z*}_t(w_t) = \lambda$ and $\Sigma^{B*}_t(w_t) = \lambda\theta$ into the HJB equation (10), we obtain the following segmented second-order ordinary differential equation (ODE):

$$\begin{cases} (r+\delta)v(x) = \bar{a} - \kappa b + (\kappa - \theta)\bar{\eta} + \frac{(v(x) - xv'(x))^2}{2\psi} + ((\gamma+\delta)x + \lambda\bar{a})\, v'(x) \\ +\frac{1}{2}\lambda^2(\sigma^2 + \theta^2\phi^2)v''(x),\ x \in [R, \bar{w}), \\ v(x) = v(\bar{w}) - x + \bar{w},\ x \in [\bar{w}, \infty). \end{cases} \tag{13}$$

We require three boundary conditions to pin down a solution to this equation and the boundary $\bar{w}$. The first one arises because the agent must terminate the contract to hold the manager's scaled value R; thus, we have $v(R) = L$. The second boundary condition is the usual "smooth pasting" condition—the first derivatives must agree at the boundary, and so $v'(\bar{w}) = -1$. The final boundary condition is the "super contact" condition for the optimality of $\bar{w}$, which requires that the second derivatives match at the boundary. That is, $v''(\bar{w}-) = v''(\bar{w}+) = 0$.

To sum up, we get the following proposition.

Proposition 3.3. ***(Optimal contract)*** *Suppose that the project's cash flow is defined by (2), and the carbon emissions are defined by (3). The optimal compensation $c^*_t(w_t)$ is given by (11) that maximizes ESG investor's value while delivering the value $w_0 = w \geq R$ to the manager. The optimal investment-to-capital ratio that maximizes the investor's value is* *given by (12). If $w_t \in [R, \bar{w})$, the manager's continuation value w_t evolves according to*

$$\mathrm{d}w_t = \left[\left(\gamma - \frac{v(w_t) - w_t v'(w_t)}{\psi} + \delta\right) w_t + \lambda \bar{a}\right] \mathrm{d}t + \lambda \sigma \mathrm{d}Z_t - \lambda \theta \phi \mathrm{d}B_t,$$

with $\bar{w}$ being its reflecting barrier, and ESG investor's value is $v(w_t)$, where function $v(\cdot)$ is a solution of the following ODE:

$$\begin{aligned}(r+\delta)v(x) =& \bar{a} - \kappa b + (\kappa - \theta)\bar{\eta} + \frac{(v(x) - xv'(x))^2}{2\psi} \\ &+ ((\gamma+\delta)x + \lambda \bar{a})\, v'(x) + \frac{1}{2}\lambda^2(\sigma^2 + \theta^2\phi^2)v''(x),\end{aligned}$$

with boundary conditions $v(R) = L$, $v'(\bar{w}) = -1$, and $v''(\bar{w}-) = v''(\bar{w}+) = 0$. If $w_t \geq \bar{w}$, an immediate payment $w_t - \bar{w}$ is made by the investor to the manager; thus, we have

$$v(x) = v(\bar{w}) - x + \bar{w}.$$

We highlight that in contrast to the common contract theory, ESG investor must provide incentive for reducing carbon emissions. Specifically, to motivate the manager to take ESG investing $\bar{\eta}$, the compensation paid by investors to managers should exhibit a sensitivity to carbon emissions reduction $-\Sigma_t^B$ that is greater than the sensitivity to cash flows λ multiplied by the ESG cost θ, i.e., $-\Sigma_t^B \geq \lambda\theta$. The former reflects the marginal benefit from reducing carbon emissions while the latter reflects the marginal cost. The condition $\Sigma_t^B = -\lambda\theta$ ensures the minimum incentive cost for motivating the manager to take ESG investing $\bar{\eta}$ as argued by DeMarzo and Sannikov (2006).

4. Security implementation of the optimal contract

In this section we design common corporate securities (financial instruments) to implement the contract. If the cash flows allocated to the stakeholders perfectly match the claims of these securities, both the manager and venture capitalist voluntarily enter into the contract, and the manager's incentive compatibility constraint is imposed, then we call the optimal contract being implemented by these securities.

4.1. Security implementation

As in DeMarzo and Sannikov (2006), we present a scheme to implement the optimal contract by a portfolio of equity, credit line and carbon credits. The short-term cash flow (short-term financing) is realized through the credit line and carbon credits, and the long-term cash flow (long-term financing) through the issuance of equity. In Appendix **??**, we provide a proof of the optimal contract implementation discussed in this section.

First, we consider short-term financing. We introduce carbon credits following the cap-and-trade program in practice. The carbon credits are permits that allow the owner to emit a certain amount of carbon dioxide or other greenhouse gases. The carbon credits increase at the growth rate γ.[8] External regulators ration continuous carbon credits Π_t given by

$$\Pi_t = K_t(b - \bar{\eta}),$$

where b is defined in (3) and $\bar{\eta}$ represents the maximum ESG investing.[9]

The lower limit of the firm's carbon credits is 0. The carbon credits N_t evolve according to

$$\mathrm{d}N_t = \gamma N_t \mathrm{d}t - \mathrm{d}Y_t + \Pi_t \mathrm{d}t.$$

As for a credit line, suppose there is a liquid credit market that provides the firm with a credit line up to the limit of $\bar{M}$. The interest rate is γ on the credit line balance M_t. The manager decides to borrow and repay the funds on the credit line. The outstanding balance M_t on the credit line balance, the firm's capital stock and carbon credits N_t jointly determine the manager's continuation value in the following way:

$$W_t = RK_t + \lambda(\bar{M} - M_t) + \theta\lambda N_t, \tag{14}$$

where $\theta\lambda$ is the net present value per unit of carbon credits. If the outstanding balance M_t on the credit line exceeds $\bar{M}$ and carbon credits $N_t \leq 0$, the manager defaults and the

[8]This assumption is consistent with the increasingly strict carbon emission policy in the future.
[9]Thanks to Investopedia, companies that pollute are awarded credits that allow them to continue to pollute up to a certain limit.

project terminates. If the credit line is fully paid off ($M_t = 0$) and carbon credits $N_t > 0$, the firm's remaining cash flow is used to allocate dividends. From (14), we conclude that the credit limit $\bar{M}$ is determined by the dividend payment threshold $\bar{w}$ in the optimal contract. Specifically, we have

$$\bar{M} = K_t \left(\bar{w} - R\right) / \lambda. \tag{15}$$

We interpret $\bar{M} - M_t$ as the financial slack of a company, which represents the distance to bankruptcy. Carbon credits N_t serve as a flexible supplement to the company's financial slack. The company can sell surplus carbon credits in carbon markets to replenish its financial slack, and visa versa.

We turn to long-term financing and consider equity allocation. The investor grants a fraction λ of equity to the manager and the remaining $1 - \lambda$ of equity is held by herself. This allocation ensures the incentive compatibility: The share λ of equity eliminates the manager's motivation to save effort cost by reducing his workload since ESG investor can compensate the cost. We emphasize that prior to the dividend distribution, the investor obtains an extra yield $\mathrm{d}D_t$ given by

$$\mathrm{d}D_t = \left(K_t(\bar{a} - \theta\bar{\eta} - \frac{1}{2}\psi i_t^2) - \gamma K_t R/\lambda - \gamma \bar{M} \right) \mathrm{d}t, \tag{16}$$

where the first term of the right-hand side is the expected instantaneous cash flow received by the firm under the optimal contract, the second and last term reflect the maximum deduction due to the interest payments. As in DeMarzo et al. (2012), the claim on the extra yield plays the role of perpetual debt; differently, its coupon is not constant.

Thanks to (14) and (15), we divide the combinations of credit line and carbon credits into the following five different regions, as shown in Figure 2.

- **Region A:** It is a segment, an intersection of $N_t = 0$ and $0 \leq M_t < \bar{M}$, implying that carbon credits are exhausted but the balance M_t on the credit line has not exceeded the limit. The manager uses the credit line and carbon credits commonly, respectively.

- **Region B:** It is the intersection of $N_t > 0$ and $-\lambda(\bar{M} - M_t) \leq \theta\lambda N_t \leq (\bar{w} - R)K_t - \lambda(\bar{M} - M_t)$. It implies the manager can pay the credit line by selling some carbon credits: Selling carbon credits N_t can make the balance $M(t)$ on the credit line within the limit, but more is needed to fully pay the credit line off.

- **Region C:** That is, the intersection of $N_t \leq 0$ and $-\theta\lambda N_t \leq \lambda(\bar{M} - M_t) \leq (\bar{w} - R)K_t - \theta\lambda N_t$. It means that the manager must borrow from the credit line to buy the carbon emission right. For this aim, the remaining credit line is sufficient.

- **Region D:** It is the set defined by the inequality $\lambda(\bar{M} - M_t) + \theta\lambda N_t \geq (\bar{w} - R)K_t$ holds. It explains that the company has sufficient working capital. The manager should sell the firm's carbon credits to pay off the credit line and pay dividends as well.

- **Region E:** It is given by $\lambda(\bar{M} - M_t) + \theta\lambda N_t \leq 0$, meaning that the company suffers from severe working capital deficiency. Specifically, the credit line drawdowns have exceeded its limit even if all carbon credits were sold to pay it; thus, the project terminates.

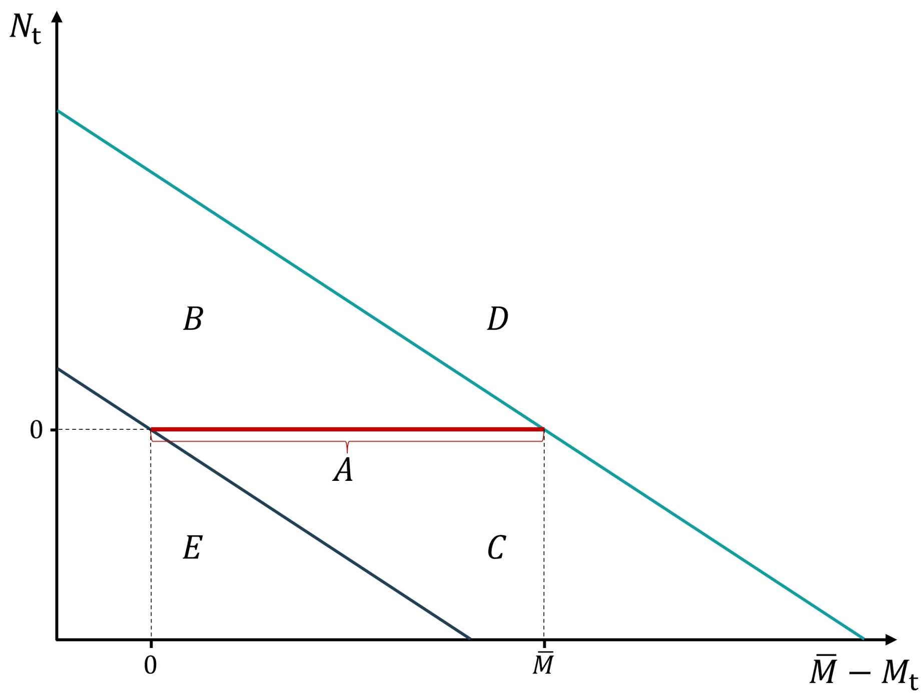


Figure 2. Five possible combinations of the credit line and carbon credits.

4.2. An example

The State Administration of Taxation (SAT) 2022 released "Guidelines on Tax and Fee Preferential Policies to Support Green Development"[10] saying that China has implemented 56 tax and fee preferential policies to support green development in four areas: supporting environmental protection, promoting energy conservation and environmental protection, encouraging the comprehensive utilization of resources, and promoting the development of low-carbon industries. Among them, the income from engaging in qualified environmental protection is regularly reduced or exempted from enterprise income tax, and the investment in acquiring specialized equipment for environmental protection is subject to a certain percentage of enterprise income tax credit. We refer to this policy and possible firm business scenarios in this subsection to provide an example of policy formulation and contract implementation.

Suppose a listed firm engages in production and operational activities; its cash flow primarily depends on the efforts of entrepreneurs hired by investors to manage these activities. Additionally, the firm's cash flow is subject to random factors as before. Entrepreneurial effort can generate an annual increase of $\bar{a} = 100$ million in cash flow per unit of capital stock, with the entrepreneur's marginal cost (equity share) being $\lambda = 60\%$. The firm's production and operation activities result in carbon emissions, which negatively impact the environment. We measure the extent of the carbon emissions' impact on the environment in dollars. The annual carbon emissions is $b = 20$ million per unit of capital stock. Entrepreneurs may choose to purchase green production equipment, which will reduce carbon emissions by $\eta = 60$ million annually per unit of capital stock. However, acquiring and using green production equipment incurs a certain cost, and the firm must carry out ongoing ESG expenditure of $\theta\eta = 36$ million annually per unit of capital stock.

Within the firm, investors need to incentivize managers through contracts to work as hard as possible to improve the firm's cash flow. In our model, we recommend that companies achieve optimal contracts in the form of the following internal equity, the credit line, and carbon credits issuance as shown in Table 1.

[10] See, http://www.chinatax.gov.cn/chinatax/n810341/n810825/c101434/c5175740/content.html

Table 1. Optimal security issuance for a typical scenario

Securities issuance	Symbol	Percentage/Amount
Internal equity	λ	60.00%
Net present value of carbon credits	$\theta\lambda$	0.36
Credit line limit	$\bar{M}$	2.09

The entrepreneur needs to hold 60.00% of the internal equity, which enables the entrepreneur to make up for the private cost of his efforts by the profit from the change of the firm's cash flow; thus, it motivates the manager to choose the highest level of effort. Net present value of carbon credits is 0.36. At the same time, the financial market provides the firm with a credit line of 2.09 billion, and the firm can borrow short-term money at any time within the limit, and the balance of short-term borrowing can be used as a ledger for investors to track the promised value of the entrepreneur. The company needs to regularly pay the minimum dividend distribution to external shareholders according to (16). If the firm's operation is good and the short-term loans are fully repaid, the firm's remaining cash flow can be used for dividend payment, and the manager can obtain the corresponding dividend according to the internal equity share of 60.00%. If the firm's operation is poor, the firm is unable to repay its debt, and the balance of the short-term borrowing exceeds the limit of the credit line provided by the financial market for the firm, the firm defaults.

5. Model interpretation and numerical analysis

We now turn to the model implications including the analysis of value functions, investment policies, Tobin's q and related corporate governance problems by numerical tests. We wonder how much contract efficiencies are under different situations, and how they are related to model parameters. We compare security designs with and without the ESG perspective. The latter can be recovered by exogenously letting $\kappa = 0$ and $\Sigma_t^B = 0$ in our model, which serves as the benchmark one. We denote the investor's value function from the traditional perspective by $j(w_t)$.

To conduct the numerical analysis, we select model parameters as typical as possible after referring to DeMarzo et al. (2012). We take the annualized baseline parameter values specified in Table 2 unless otherwise stated.

Table 2. Baseline parameter values

Model parameters	Symbol	Value
Risk-free rate	r	4.6%
The upper bound of agent's effort	$\bar{a}$	10%
Project cash flow volatility	σ	50%
The expected rate of carbon emissions	b	2%
Subjective discount rate of managers	γ	5%
Marginal effort cost	λ	60%
Upper bound of ESG investing	$\bar{\eta}$	6%
Marginal cost of ESG investing	θ	60%
Capital depreciation	δ	18%
Investment cost parameter	ψ	2
Conversion factor	κ	1
Outside option value of the manager	R	0
Liquidation value of the project	L	0

5.1. Scaled value function and contract efficiency

The information asymmetry between ESG investor and manager leads to the loss of the value of ESG investor on the one hand, and the loss of the total utility of the manager (the continuation value of the manager) on the other hand. Firstly, we define the contract efficiency of the model as follows:

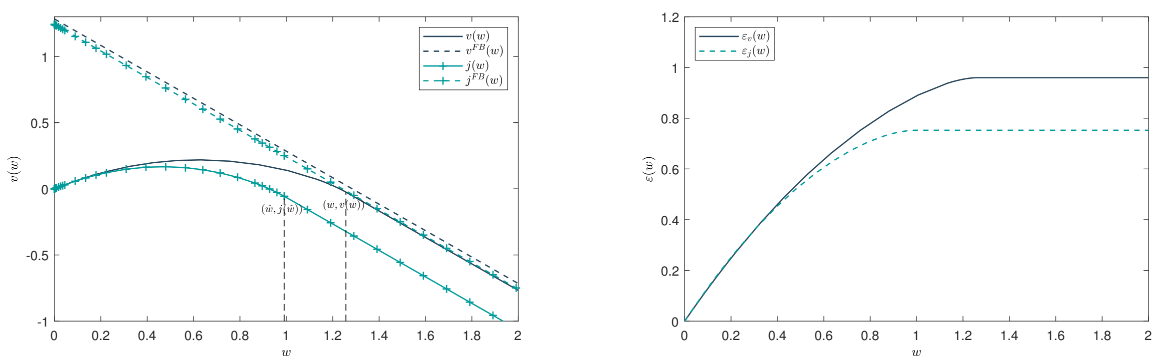
$$\varepsilon(w) \equiv \frac{V(W) + W}{v^{FB}(w)K + W} = \frac{v(w) + w}{v^{FB}(w) + w},$$

where the lowercase letters represent that the values are scaled, which are measured on a per unit of capital basis as $w = W/K$ and $v(w) = V(W)/K$.

Clearly, a higher contract efficiency indicates a less social welfare loss of a contract resulting from information asymmetry or incomplete information. Naturally, the contractual efficiency takes a value between 0 and 1, i.e. $\delta(w) \in [0, 1]$. The contract efficiency under full information is obviously equal to 1.

Figure 3(a) indicates that the scaled value function $v(w)$ from ESG perspective is higher than that $j(w)$ from traditional perspective in both full and incomplete information cases. It happens because the firm is allotted carbon credits which are valuable.

Figure 3(b) describes the contract efficiency in different models. The distinctions among them are similar to Figure 3(a). In particular, depending on the manager's promised value, the contract efficiency from ESG perspective exceed that from traditional perspective by over 20.73%. These conclusions are insightful for corporate governance, meaning that carbon credits are a powerful financial instrument to reduce the manager's moral hazard. The reason behind them is due to carbon credits, which make ESG investor easier to incentive the manager.

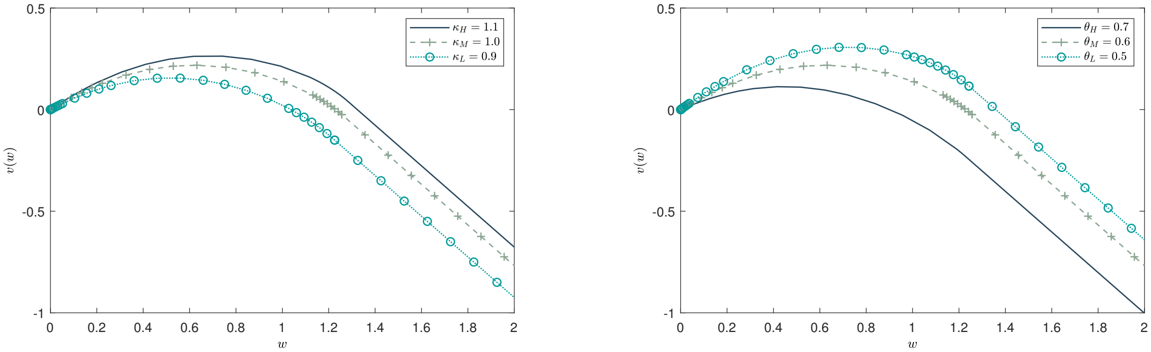


(a) $v(w)$, $v^{FB}(w)$, $j(w)$, and $j^{FB}(w)$

(b) $\varepsilon_v(w)$ and $\varepsilon_j(w)$

Figure 3. The figure plots (a) ESG investor's scaled value, and (b) the contract efficiency from ESG perspective and traditional perspective, respectively.

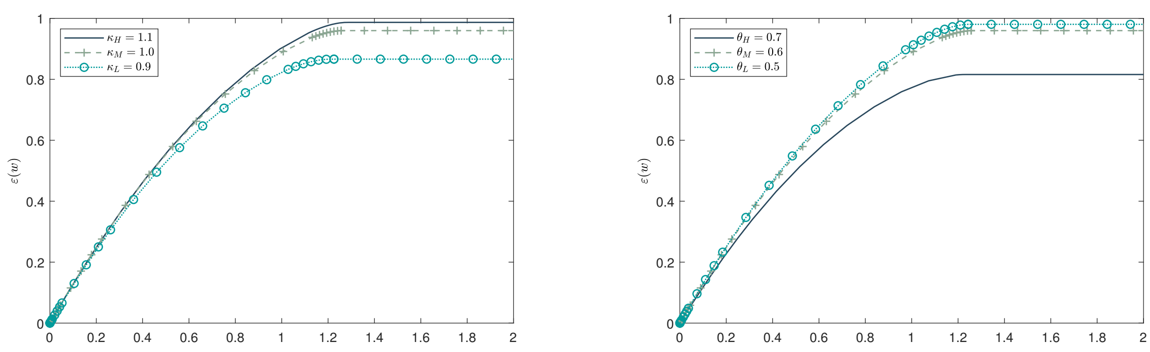
To evaluate a contract, equivalently, we analyze ESG investor's scaled value for a given promised value of the manager. We present ESG investor's scaled value under different situations and examine how it is related to model parameters. To be specific, this subsection focuses on ESG investor's scaled value. Figure 4(a) says that the higher the conversion factor κ of carbon emissions, the higher the ESG investor's scaled value. Figure 4(b) explains that the higher the marginal cost of ESG investing θ, the less ESG investor's scaled value. These are clear since the higher the marginal cost of ESG investing, the more ESG investor must pay to provide the manager with the given promised value.



(a) $v(w)$ vs. w with κ (b) $v(w)$ vs. w with θ

Figure 4. ESG investor's scaled value $v(w)$ versus the scaled promised value w with different (a) conversion factor for carbon emissions (ESG concerns) κ, and (b) marginal cost of ESG investing θ.

Last, we consider how the main model parameters impact on contract efficiency. Figure 5 shows numerical results. It says that the higher the conversion factor for carbon emissions, the higher the efficiency, and the higher the marginal cost of ESG investing, the lower the efficiency. These phenomena are consistent with what predicted before since they indicate that the more important the ESG investing, the higher the efficiency. In a word, ESG investing can improve corporate governance.



(a) $\varepsilon(w)$ vs. w with κ (b) $\varepsilon(w)$ vs. w with θ

Figure 5. The contract efficiency $\varepsilon(w)$ versus the scaled promised value w under different (a) ESG concerns κ, and (b) marginal cost of ESG investing θ.

5.2. Tobin's q with ESG investing

In a standard dynamic model of corporate investment, the q theory of investment proposed by Hayashi (1982) is important. There are two measures of Tobin's q, the average q and the marginal q. The average q indicates the average value of capital. The marginal q measures the marginal value of capital. A firm's optimal investment must make the marginal q equal to the marginal cost of capital. In the absence of fixed investment costs and financial market frictions with a homogeneous production technology, according to Hayashi (1982) and DeMarzo et al. (2012), marginal q equals average q.

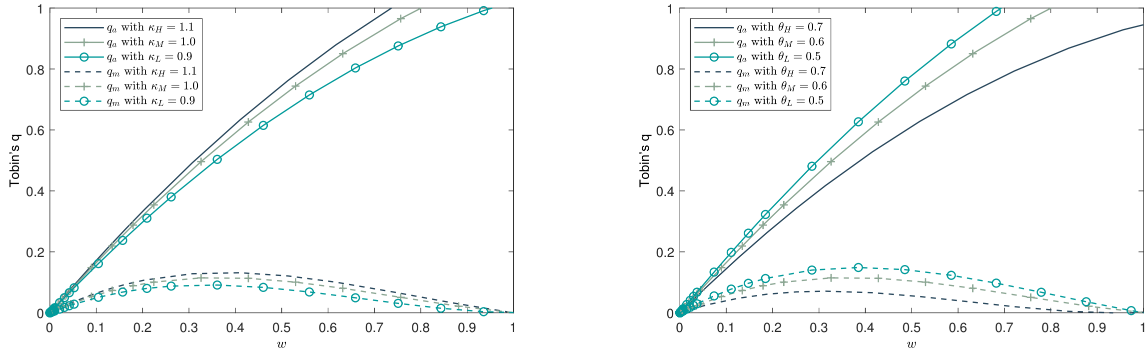
In our model, the average q, denoted by $q_a(w)$, and marginal q, denoted by $q_m(w)$ are calculated by

$$q_a(w) = \frac{V(K,W)+W}{K} = v(w)+w,\ q_m(w) = \frac{\partial(V(K,W)+W)}{\partial K} = v(w) - wv'(w), \quad (17)$$

respectively. Clearly, we have $v(w) < v^{FB}(w)$. Noting that $v'(w) \geq -1$ in our model, we thus have

$$q^{FB} > q_a(w) \geq q_m(w), \quad (18)$$

where the first inequality holds because the average and marginal qs are the same for the full information case. Figure 6 give numerical examples for the calculation of q. It is well known that marginal q and average q are forward-looking indicators for capturing future investment opportunities. The features shown in Figure 6 are consistent with the inequality (18). Figure 6(a) says that the higher the conversion factor for carbon emissions κ, the higher the average q and marginal q. Figure 6(b) explains that the higher the marginal cost of ESG investing θ, the less the average q and marginal q. These conclusions indicate that the higher the conversion factor for carbon emissions or the lower the marginal cost, the better the investment opportunities, consistent with the preceding predictions.

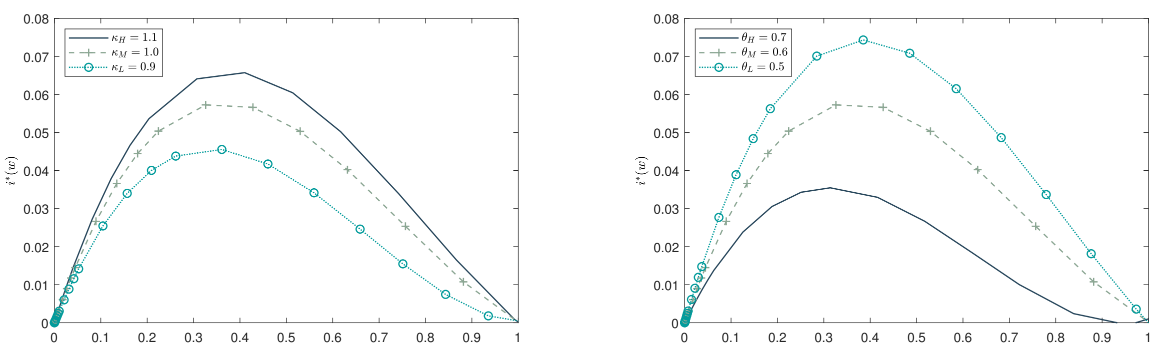


(a) q_a and q_m vs. w with κ

(b) q_a and q_m vs. w with θ

Figure 6. The average q (q_a) and the marginal q (q_m) versus the scaled promised value w under (a) different ESG concerns κ, and (b) marginal cost of ESG investing θ.

Finally, we consider how the main model parameters impact on the optimal investment-to-capital ratio. Figure 7 shows a similar story to Figure 6. It happens since a higher conversion factor for carbon emissions increases the value of the firm and the return on investment; however, a higher marginal cost of ESG investing will reduce the firm's cash flow and bring higher incentive costs.



(a) $i^*(w)$ vs. w with κ

(b) $i^*(w)$ vs. w with θ

Figure 7. The optimal investment-to-capital ratio $i^*(w)$ versus the scaled promised value w under different (a) ESG concerns κ, and (b) marginal cost of ESG investing θ.

5.3. Securities issuance

Table 3 shows the influence of different parameters on corporate securities issuance. First, a higher conversion factor for carbon emissions leads to an increase in the firm's short-term credit line (maximum financial slack). However, the opposite holds true for the marginal cost of ESG investing. As expected, the higher the payment threshold, the higher the credit line limit.

Table 3. Numerical analysis of security issuance

Parameters	Parameter values	Payment threshold	Credit line
	1.10	1.29	2.15
κ	1.00	1.26	2.09
	0.90	1.23	2.04
	0.70	1.23	2.06
θ	0.60	1.26	2.09
	0.50	1.27	2.12

6. An alternative model

This section considers an extension to the previous model, where ESG investing affects capital sustainability. To save space, the extension is written as short as possible.

ESG investing not only reflects the social responsibility and ethical level of enterprises, but also affects their long-term performance. Based on this observation, we extend the basic model to introduce a capital depreciation parameter affected by ESG investing. Specifically, in contrast to (1), the firm's capital stock K_t is given by the following modification:

$$\mathrm{d}K_t = F(I_t, K_t, \eta_t)\mathrm{d}t = (I_t - \delta^s(\eta_t)K_t)\,\mathrm{d}t, \quad t \geq 0,$$

where $\delta^s(\eta_t)$ represents the capital depreciation function affected by ESG investing.[11] First, consider the manager's problem. In response to a take-it-or-leave-it contract (C, I) offered by the ESG investor, the manager's control variates include effort level a, ESG investing η, and termination time T. We derive that the manager's scaled continuation value is given by

$$\mathrm{d}w_t = ((\gamma - i_t + \delta^s(\eta_t))w_t - c_t + \lambda a_t)\,\mathrm{d}t + \Sigma_t^Z \sigma \mathrm{d}Z_t + \Sigma_t^B \phi \mathrm{d}B_t. \tag{19}$$

[11]For ease of exposition, we use the superscript "s" to indicate that variable under the capital sustainability effect being considered.

Further, we derive the following proposition.

Proposition 6.1. *A contract (C, I) is incentive compatible if and only if the sensitivity process Σ^Z given by (19) is greater than the the effort cost coefficient λ, i.e. $\Sigma_t^Z \geq \lambda$. The optimal effort of the manager always arrives at its minimum, i.e. $a_t^s = \bar{a}$. The optimal ESG responsibility performance (i.e. ESG investing) of the manager for $0 \leq t \leq \tau$ is*

$$\eta_t^s = \zeta\left(-\frac{\Sigma_t^Z \theta + \Sigma_t^B}{w_t}\right), \tag{20}$$

where function $\zeta(\cdot)$ is the inverse function of the first-order derivative of $\delta^s(\cdot)$.

The ESG investor's value is similar to (7); we denote her scaled value by $v_s(w_t)$. After a similar computation, we derive the following HJB equation

$$\begin{aligned} rv_s(w_t) = \sup_{\Sigma_t^Z, \Sigma_t^B, c_t, i_t} \Bigg\{ & \left[\bar{a} - \kappa b + (\kappa - \theta)\eta_t^s - \frac{1}{2}\psi i_t^2 - c_t\right] + (i_t - \delta^s(\eta_t^s)) v_s(w_t) \\ & ((\gamma - i_t + \delta^s(\eta_t^s)) w_t - c_t + \lambda \bar{a})\, v_s'(w_t) + \frac{1}{2}((\Sigma_t^Z)^2 \sigma^2 + (\Sigma_t^B)^2 \phi^2) v_s''(w_t) \Bigg\}. \end{aligned}$$

To get a more explicit solution, we assume the linear relationship between capital depreciation and ESG investing as follows:

$$\delta^s(\eta_t) = \rho + \chi \eta_t,$$

where $\rho \in [0, 1]$ is an exogenous given parameter that represents capital depreciation without ESG investing, and χ is another positive exogenous given parameter representing the extra depreciation due to ESG investing. For example, to decrease carbon emission, the firm should buy new equipment instead of old one, leading to a higher depreciation.

Therefore, we conclude from (20) that

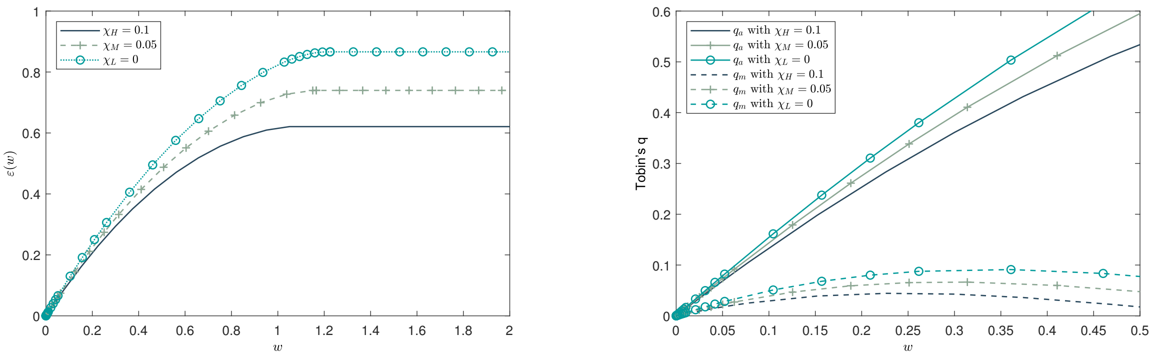
$$\eta_t^s = \begin{cases} \bar{\eta}, & \Sigma^B \leqslant -\Sigma^Z \theta - \chi w_t, \\ 0, & \Sigma^B > -\Sigma^Z \theta - \chi w_t. \end{cases}$$

The incentive compatible condition can be written as $\Sigma_t^Z \geq \lambda$ and $\Sigma^B \leqslant -\Sigma^Z \theta - \chi w_t$.

Through a familiar calculation process, we get the following ODE satisfied by ESG investor's scaled value function $v_s(\cdot)$:

$$\begin{cases} (r+\rho+\chi\bar{\eta})v_s(x) = \bar{a} - \kappa b + (\kappa-\theta)\bar{\eta} + \frac{(v_s(x)-xv_s'(x))^2}{2\psi} \\ +\left((\gamma+\rho+\chi\bar{\eta})x+\lambda\bar{a}\right)v_s'(x) + \frac{1}{2}(\lambda^2\sigma^2+(\lambda\theta+\chi x)^2\phi^2)v_s''(x),\ x\in[R,\bar{w}_s), \\ v_s(x) = v_s(\bar{w}_s) - x + \bar{w}_s,\ x\in[\bar{w}_s,\infty). \end{cases}$$

with three boundary conditions: (1) $v_s(R) = L$, (2) $v_s'(\bar{w}_s) = -1$, and (3) $v_s''(\bar{w}_s) = 0$.



(a) $\varepsilon(w)$ vs. w with χ (b) q_a and q_m vs. w with χ

Figure 8. The figure plots (a) the contract efficiency $\varepsilon(w)$, (b) the average q (q_a) and the marginal q (q_m) versus the scaled promised value w under different extra depreciation χ.

Figure 8 presents numerical examples illustrating how the contract efficiency $\varepsilon(w)$, average q (q_a), and marginal q (q_m) change with the extra depreciation χ. It says that the efficiencies and q values uniformly increase with the extra depreciation. Moreover, the efficiencies and q values relate to the manager's promised value in the same way predicted in the last section, implying the conclusions of our model are robust. As expected, if $\chi = 0$, we recover the model of Section 3. These findings indicate that a greater increase in capital depreciation caused by ESG investing corresponds to a larger loss in efficiency due to information asymmetry, along with inferior investment opportunities for the company.

7. Empirical implications

We now relate our theoretical research results to relevant empirical results. We address three different perspectives: investment and Tobin's q, firm value, and managerial compensation. The analysis shows that there are consistencies between our theoretical results and empirical results.

7.1. Investment and Tobin's q

Subsection 5.2 predicts that as investors' attention to ESG increases (i.e. the parameter κ or the significance of carbon emission reduction rises), the Tobin's q of the firm will also increase, both in terms of average q and marginal q. Conversely, there is an inverse relationship between a firm's ESG costs and its Tobin's q. Additionally, our model predicts that the investment-capital ratio is positively correlated with their attention to ESG, and negatively correlated with the firm's ESG costs.

Several literature sources have documented a positive correlation between a firm's ESG/CSR ratings and its financial performance or measures of firm value. Gao and Zhang (2015) used six of the seven KLD categories rated by Kinder, Lydenberg, and Domini (KLD) Research and Analytics, Inc (i.e. environment, product, labor relations, diversity, community, corporate governance, and humanities), and found a positive correlation between firm-level ESG/CSR scores and Tobin's q. Servaes and Tamayo (2013), Albuquerque et al. (2019), Liang and Renneboog (2017) and Ferrell et al. (2016) on the relationship between a firm's ESG characteristics and Tobin's q have also yielded similar conclusions. The conclusions of these empirical studies are consistent with our theoretical model predictions.

However, there are also different viewpoints in the literature regarding the relationship between a firm's ESG characteristics and Tobin's q. For example, Buchanan et al. (2018) found a negative significant coefficient estimate in the interaction between their ESG/CSR indicators, crisis indicators, and Tobin's q. Hsu et al. (2023) argue that a firm's environmental choices have no significant relationship with Tobin's q or long-term profitability. Their empirical results are contrary to the conclusions of our theoretical model. In addition, higher ESG/CSR performance (investing) reduces the cost of equity by skewing the investor base, see, e.g., Hong and Kacperczyk (2009) and El Ghoul et al. (2011). Our theoretical research did not involve these aspects.

7.2. Company values

Subsection 5.1 predicts that as investors' attention to ESG increases, the value of the firm will also increase. Conversely, there is an inverse relationship between a firm's ESG costs and its value. Additionally, a higher ESG attention can improve the contract efficiency, meaning that social welfare losses due to information asymmetry are reduced.

Across a lot of ESG literature, one of the most controversial questions is whether management choices related to corporate responsibility affect the firm performance and firm value, or further, whether the performance or valuation drives ESG choices. Recent financial research indicates that some investment groups are willing to exchange financial returns for improvements in ESG investing (see, e.g., Pástor et al., 2021; Riedl and Smeets, 2017; Barber et al., 2021; Krueger et al., 2020). Improving ESG investing has a significant positive impact on fundamental indicators such as firm performance, shareholder returns, and corporate profitability (see, e.g., Gyimah et al., 2021). Servaes and Tamayo (2013) shows that for companies with higher customer awareness, corporate social responsibility and corporate value are positively related. Ferrell et al. (2016) found a positive correlation between ESG/CSR scores and firm value, and extended their analysis to show that having higher ESG/CSR performance weakens the negative correlation between management stability and value. Their empirical analysis documents our predictions further.

7.3. Executive compensation

As shown in Table 3, our model reveals that an increase in investors' ESG attention will delay the payment of compensation to managers, and therefore reduce their compensation under the same conditions. Similarly, firm ESG costs can also delay the payment of managers' compensation.

One of the important topics in the ESG research field is about how a firm's ESG characteristics affect executive compensation. Jian and Lee (2015) found a negative correlation between ESG investing and the CEO compensation. Ferrell et al. (2016) also found a negative correlation between the measurement standard of CEO excess compensation and ESG investing. The empirical conclusions of these studies are consistent with our model implications.

8. Conclusion

Designing incentive contracts considering ESG strategy is a significant topic in financial contracting theory. In the existing research, investors focus on the financial performance of companies, neglecting ESG concerns. We are the first to explore incentive contract design related to ESG investing within a continuous-time principal-agent model. The firm's project operations generate cash flows, which are determined by the agent's private behavior subject to random shocks. We introduce dynamic carbon emissions that are determined by decision maker's ESG investing. To promote voluntary ESG investing, ESG investor exerts influence over their decisions through incentive policies.

In our model, project cash flows are impacted by the participants' behavior, focusing on the extensive research conducted in the security design literature regarding the influence of managers' private efforts. However, contract theory seldom considers carbon emissions, much less the firm's ESG investing as decision variables. The manager can shape the firm's ESG investing, indirectly impacting the average growth rate of project cash flows and the accumulation of carbon emissions resulting from project operations. We employ the martingale method to address this issue and determine the optimal solutions for workload recommendation, compensation payment, and incentive problems. We utilize a second-order ordinary differential equation to depict the contractual incentive relationship within the enterprise.

Green finance products provide new ideas for the implementation of the abstract optimal contract in practical settings. Consequently, we exploit a portfolio of equity, the credit line, and carbon credits to implement the optimal contract. The firm combines the credit line and carbon credits, functioning as "ledger", to monitor the continuation value of the manager. We present five potential scenarios illustrating the utilization of the credit line and carbon credits as combined sources of liquidity, thus facilitating increased flexibility in the firm's investment and financing policies. Furthermore, based on the latest government policies and potential firm operational scenarios, we provide a concrete application example to present a guide for policy formulation and implementation.

A key issue is how ESG-related factors influence companies' contract formation and investment prospects. Through a comparative analysis of the optimal contract and contract efficiency from both a traditional and ESG perspective, we discover that contract design with ESG concerns enhances contract efficiency. We observe a gradual increase in contract efficiency, average q, marginal q, and the optimal investment-to-capital ratio as the significance of ESG investing rises. This result implies that contract design considering ESG can mitigate inefficiencies arising from information asymmetry and improve investment opportunities. We arrive at the same conclusions as the costs of ESG investing decline. Furthermore, we extend our model to consider an extra depreciation due to ESG investing; we derive similar findings, implying that the results are robust. We relate our conclusions to empirical facts, which partially verify our model predictions.

References

Albuquerque, R., Koskinen, Y., Zhang, C., 2019. Corporate social responsibility and firm risk: Theory and empirical evidence. Management Science 65, 4451–4469.

Barber, B.M., Morse, A., Yasuda, A., 2021. Impact investing. Journal of Financial Economics 139, 162–185.

Bénabou, R., Tirole, J., 2010. Individual and corporate social responsibility. Economica 77, 1–19.

Bonham, J., Riggs-Cragun, A., 2022. An accounting framework for ESG reporting. Chicago Booth Research Paper .

Buchanan, B., Cao, C.X., Chen, C., 2018. Corporate social responsibility, firm value, and influential institutional ownership. Journal of Corporate Finance 52, 73–95.

Cohen, S., Kadach, I., Ormazabal, G., Reichelstein, S., 2023. Executive compensation tied to ESG performance: International evidence. Journal of Accounting Research 61, 805–853.

Cvitanic, J., Zhang, J., 2012. Contract theory in continuous-time models. Springer Science & Business Media.

DeMarzo, P.M., Fishman, M.J., He, Z., Wang, N., 2012. Dynamic agency and the q theory of investment. The Journal of Finance 67, 2295–2340.

DeMarzo, P.M., Sannikov, Y., 2006. Optimal security design and dynamic capital structure in a continuous-time agency model. The Journal of Finance 61, 2681–2724.

DeMarzo, P.M., Sannikov, Y., 2016. Learning, termination, and payout policy in dynamic incentive contracts. The Review of Economic Studies 84, 182–236.

El Ghoul, S., Guedhami, O., Kwok, C.C., Mishra, D.R., 2011. Does corporate social responsibility affect the cost of capital? Journal of Banking & Finance 35, 2388–2406.

Engle, R.F., Giglio, S., Kelly, B., Lee, H., Stroebel, J., 2020. Hedging climate change news. The Review of Financial Studies 33, 1184–1216.

Ferrell, A., Liang, H., Renneboog, L., 2016. Socially responsible firms. Journal of Financial Economics 122, 585–606.

Flammer, C., Hong, B., Minor, D., 2019. Corporate governance and the rise of integrating corporate social responsibility criteria in executive compensation: Effectiveness and implications for firm outcomes. Strategic Management Journal 40, 1097–1122.

Gao, L., Zhang, J.H., 2015. Firms' earnings smoothing, corporate social responsibility, and valuation. Journal of Corporate Finance 32, 108–127.

Gyimah, D., Kwansa, N.A., Kyiu, A.K., Sikochi, A.S., 2021. Multinationality and capital structure dynamics: A corporate governance explanation. International Review of Financial Analysis 76, 101758.

Hayashi, F., 1982. Tobin's marginal q and average q: A neoclassical interpretation. Econometrica: Journal of the Econometric Society , 213–224.

He, Z., Wei, B., Yu, J., Gao, F., 2017. Optimal long-term contracting with learning. The Review of Financial Studies 30, 2006–2065.

Holmstrom, B., Milgrom, P., 1987. Aggregation and linearity in the provision of intertemporal incentives. Econometrica: Journal of the Econometric Society , 303–328.

Holmstrom, B., Milgrom, P., 1991. Multitask principal–agent analyses: Incentive contracts, asset ownership, and job design. The Journal of Law, Economics, and Organization 7, 24–52.

Hong, H., Kacperczyk, M., 2009. The price of sin: The effects of social norms on markets. Journal of Financial Economics 93, 15–36.

Hsu, P.H., Liang, H., Matos, P., 2023. Leviathan Inc. and corporate environmental engagement. Management Science 69, 7719–7758.

Huang, Y., Ju, N., Xing, H., 2023. Performance evaluation, managerial hedging, and contract termination. Management Science 69, 4953–4971.

Jermann, U.J., 1998. Asset pricing in production economies. Journal of Monetary Economics 41, 257–275.

Jian, M., Lee, K.W., 2015. Ceo compensation and corporate social responsibility. Journal of Multinational Financial Management 29, 46–65.

Keppo, J., Touzi, N., Zuo, R., 2022. Dynamic contracting in asset management under investor-partner-manager relationship. Available at SSRN: https://ssrn.com/abstract=3761499 .

Krueger, P., Sautner, Z., Starks, L.T., 2020. The importance of climate risks for institutional investors. The Review of Financial Studies 33, 1067–1111.

Liang, H., Renneboog, L., 2017. Corporate donations and shareholder value. Oxford Review of Economic Policy 33, 278–316.

Ma, P., Shang, J., Wang, H., 2017. Enhancing corporate social responsibility: Contract design under information asymmetry. Omega 67, 19–30.

Maas, K., 2018. Do corporate social performance targets in executive compensation contribute to corporate social performance? Journal of Business Ethics 148, 573–585.

Masood, E., 2021. Net-zero carbon pledges must be meaningful to avert climate disaster. Nature 592.

McCarthy, S., Oliver, B., Song, S., 2017. Corporate social responsibility and CEO confidence. Journal of Banking & Finance 75, 280–291.

Myerson, R.B., 1979. Incentive compatibility and the bargaining problem. Econometrica: Journal of the Econometric Society , 61–73.

Pástor, L., Stambaugh, R.F., Taylor, L.A., 2021. Sustainable investing in equilibrium. Journal of Financial Economics 142, 550–571.

Piskorski, T., Tchistyi, A., 2010. Optimal mortgage design. The Review of Financial Studies 23, 3098–3140.

Prat, J., Jovanovic, B., 2014. Dynamic contracts when the agent's quality is unknown. Theoretical Economics 9, 865–914.

Rampini, A.A., Viswanathan, S., 2010. Collateral, risk management, and the distribution of debt capacity. The Journal of Finance 65, 2293–2322.

Riedl, A., Smeets, P., 2017. Why do investors hold socially responsible mutual funds? The Journal of Finance 72, 2505–2550.

Sannikov, Y., 2008. A continuous-time version of the principal-agent problem. The Review of Economic Studies 75, 957–984.

Schättler, H., Sung, J., 1993. The first-order approach to the continuous-time principal–agent problem with exponential utility. Journal of Economic Theory 61, 331–371.

Schmid, L., 2008. A quantitative dynamic agency model of financing constraints. Available at SSRN 1437845 .

Servaes, H., Tamayo, A., 2013. The impact of corporate social responsibility on firm value: The role of customer awareness. Management Science 59, 1045–1061.

Starks, L.T., 2023. Presidential Address: Sustainable Finance and ESG Issues—Value versus values. The Journal of Finance 78, 1837–1872.

Sung, J., 1995. Linearity with project selection and controllable diffusion rate in continuous-time principal-agent problems. The RAND Journal of Economics , 720–743.

Williams, N., 2011. Persistent private information. Econometrica 79, 1233–1275.

Williams, N., 2015. A solvable continuous time dynamic principal–agent model. Journal of Economic Theory 159, 989–1015.

Zhu, J.Y., 2022. Anticipating disagreement in dynamic contracting. Review of Finance 26, 1241–1265.

Appendices

Proof of the concavity of the scaled value function. Relative to DeMarzo et al. (2012)'s proof, we provide a similar but a bit more formal proof as follows. Differentiating (13), we obtain

$$\begin{aligned}(r+\delta)v'(x) = &-\frac{1}{\psi}xv''(x)[v(x)-xv'(x)] + [(\gamma+\delta)x+\lambda\bar{a}]\,v''(x)\\ &+(\gamma+\delta)v'(x)+\frac{1}{2}\lambda^2(\sigma^2+\theta^2\phi^2)v'''(x).\end{aligned} \tag{21}$$

Noting that $v'(\bar{w}) = -1$ and $v''(\bar{w}) = 0$, we get

$$-(r+\delta) = -(\gamma+\delta)+\frac{1}{2}\lambda^2(\sigma^2+\theta^2\phi^2)v'''(\bar{w}). \tag{22}$$

We immediately have $v'''(\bar{w}) > 0$ since $\gamma > r$. Therefore, there exists $\epsilon > 0$, such that $v''(y) < 0$ for any $y \in (\bar{w}-\epsilon, \bar{w})$. Denoting $h(x) \equiv v(x) - xv'(x)$, thanks to (13), we express $h(x)$ as

$$(r+\delta)h(x) = \bar{a} - \kappa b + (\kappa-\theta)\bar{\eta} + \frac{h^2(x)}{2\psi} + ((\gamma-r)x+\lambda\bar{a})\,v'(x) + \frac{1}{2}\lambda^2(\sigma^2+\theta^2\phi^2)v''(x).$$

Suppose that there exists some $\tilde{w} < \bar{w}$ such that $v''(\tilde{w}) = 0$, and $\tilde{w}$ is just the largest one within such zero points. Thus we have $v''(y) < 0$ for all $\tilde{w} < y < \bar{w}$. Therefore, we conclude that

$$(r+\delta)h(\tilde{w})=\bar{a}-\kappa b+(\kappa-\theta)\bar{\eta}+\frac{h^2(\tilde{w})}{2\psi}+((\gamma-r)\tilde{w}+\lambda\bar{a})\,v'(\tilde{w}). \tag{23}$$

We get from (4) that q^{FB} is the smaller root of the quadratic equation of variable z: $Q(z)\equiv z^2/(2\psi)-(r+\delta)z+\bar{a}-\kappa b+(\kappa-\theta)\bar{\eta}=0$. Noting that $h(x)=v(x)-xv'(x)=q_m(x)<q^{FB}$ for all promised values x from (17) and (18), we derive that $Q(h(x))>0$ for all promised values x including $\tilde{w}$. Thus, we derive $v'(\widetilde{w})<0$ from (23), which leads to $v'''(\tilde{w})>0$ from (21) after noting that $v''(\tilde{w})=0$. This is a contradiction since the chosen value $\widetilde{w}$ satisfies $v''(\widetilde{w})=0$ and $v''(y)<0$ for all $\tilde{w}<y<\bar{w}$. Therefore, $v(\cdot)$ is strictly concave over the interval $(R,\bar{w})$. □

Proof of the contract implementation. The dynamic evolution of the firm's carbon credits N_t satisfies

$$\mathrm{d}N_t=\gamma N_t\mathrm{d}t-\mathrm{d}Y_t+\Pi_t\mathrm{d}t. \tag{24}$$

The firm's credit line balance M_t satisfies the following stochastic differential equation:

$$\mathrm{d}M_t=\gamma M_t\mathrm{d}t+\mathrm{d}D_t+Div_t\mathrm{d}t-\mathrm{d}X_t, \tag{25}$$

where Div_t denotes the dividend paid. We stress that Div_t is the remaining cash flow after having paid the credit line and the extra yield D to ESG investor.

According to (14) with (15) and (16), we get the promised value of the agent:

$$\begin{aligned}
\mathrm{d}W_t=&-\lambda\mathrm{d}M_t+\lambda\theta\mathrm{d}N_t\\
=&-\lambda\left(\gamma M_t\mathrm{d}t+\mathrm{d}D_t+Div_t\mathrm{d}t-\mathrm{d}X_t\right)+\lambda\theta\left(\gamma N_t\mathrm{d}t-\mathrm{d}Y_t+\Pi_t\mathrm{d}t\right)\\
=&\left(\gamma W_t-\lambda Div_t\right)\mathrm{d}t+\lambda(\mathrm{d}X_t-\bar{a}K_t\mathrm{d}t+\theta\bar{\eta}K_t\mathrm{d}t+\frac{1}{2}\psi i_t^2K_t\mathrm{d}t)\\
&-\lambda\theta(\mathrm{d}Y_t-bK_t\mathrm{d}t+\bar{\eta}K_t\mathrm{d}t)\\
=&\left(\gamma W_t-\lambda Div_t\right)\mathrm{d}t+\lambda\sigma K_t\mathrm{d}Z_t-\lambda\theta\phi K_t\mathrm{d}B_t,
\end{aligned} \tag{26}$$

where we take $\lambda Div_t=C_t-\lambda\bar{a}K_t$. $\lambda Div_t=C_t-\lambda\bar{a}K_t$ implies that the dividend received by the agent who owns the portion of internal equity corresponds to his compensation net of the cost of effort. □

Proof of Prop. 3.2. For the problem (5), the first order condition of the drift term of Equation (9) states that the optimal effort behavior a_t^* of the manager can be expressed as

$$a_t^* = \begin{cases} \bar{a}, & \Sigma^Z \geqslant \lambda, \\ 0, & \Sigma^Z < \lambda. \end{cases} \tag{27}$$

Similarly, the first-order condition indicates the manager's optimal ESG investing η_t^* can be expressed as

$$\eta_t^* = \begin{cases} \bar{\eta}, & -\Sigma^B \geqslant \Sigma^Z \theta, \\ 0, & -\Sigma^B < \Sigma^Z \theta. \end{cases} \tag{28}$$

Last, the manager has a perpetual American exit option; thus, he must terminate the contract immediately to get the outside option value RK_T once his promised value gets less than RK_T. □