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Учреждение образования
«Брестский государственный технический университет»
Кафедра ИИТ

Аттестационная работа
по дисциплине: **Математическая статистика**
Тема: Выборочный метод. Элементы теории корреляции

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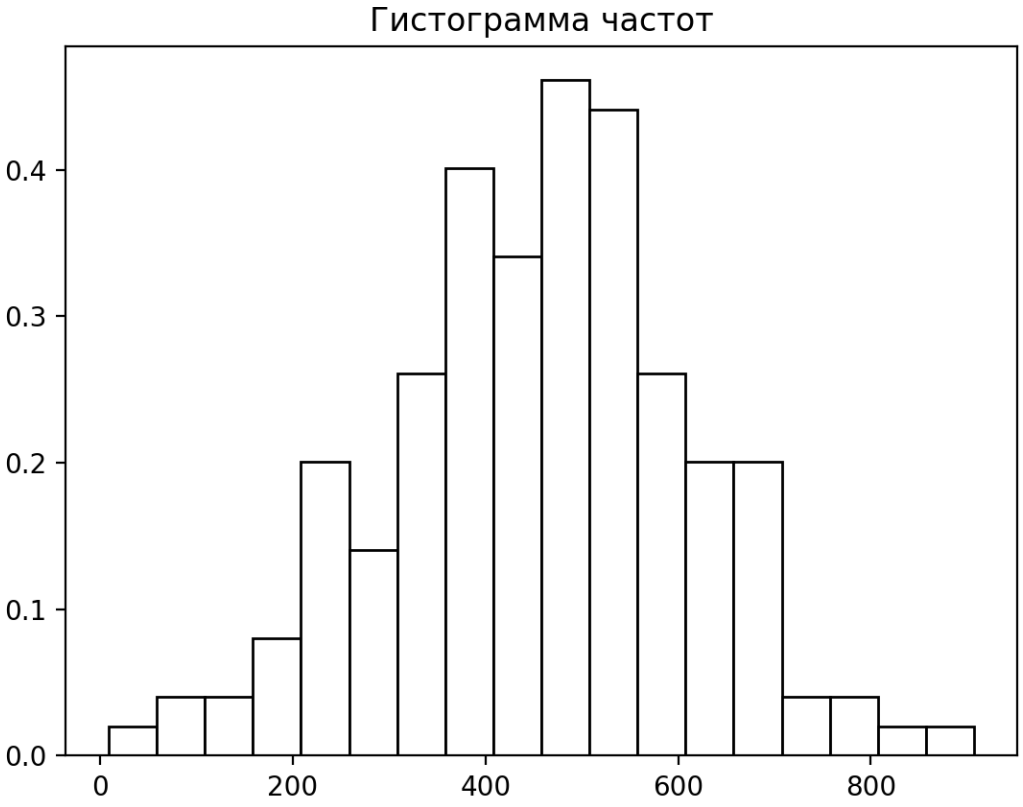
Брест 2021

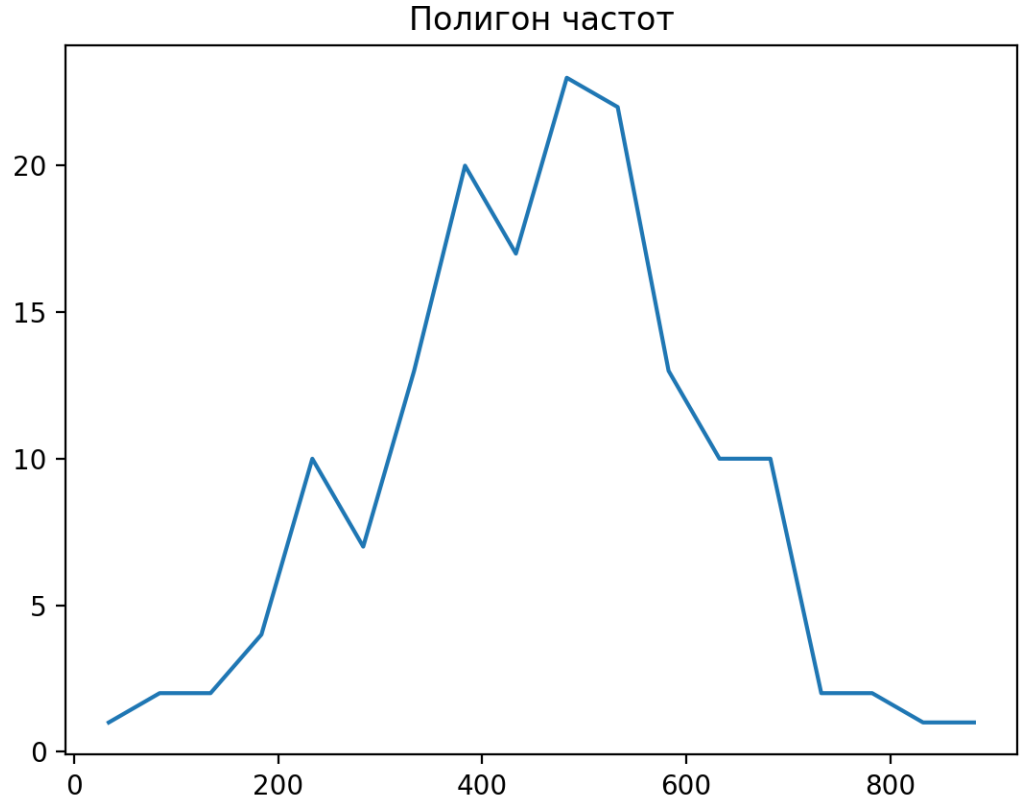
Задание 1 Изначальная выборка:

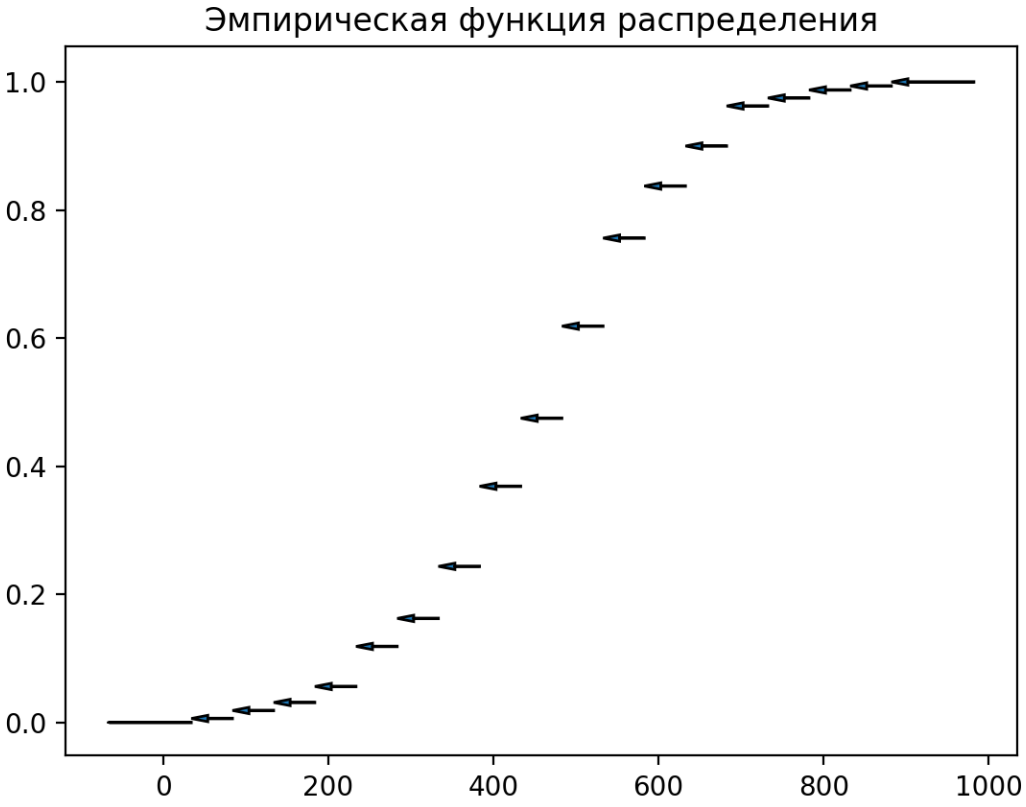
596.33 383.19 647.44 477.37 792.69 482.05 400.08 381.28 313.42 900.00 463.59 488.34 471.48 475.06 414.22 321.03 534.87 667.84 472.06 593.00 583.34 454.50 397.68 553.65 684.05 796.19 715.45 688.49 324.96 661.44 559.07 698.96 382.79 568.19 627.06 552.12 338.17 243.64 599.94 548.57 532.31 209.61 611.21 320.79 717.78 567.37 403.42 705.66 624.11 471.04 354.48 429.41 388.35 670.01 438.85 621.10 611.86 662.92 362.38 358.76 398.89 273.35 469.37 434.85 494.38 188.17 477.30 216.81 190.81 408.04 9.00 240.83 585.51 705.23 564.10 558.02 400.77 428.32 405.27 221.73 496.03 372.93 428.31 457.70 518.59 508.23 342.10 314.49 417.41 461.55 517.25 584.51 435.63 484.29 808.20 506.08 359.76 489.47 235.19 393.88 542.45 656.96 518.78 537.45 465.59 551.23 329.04 127.73 498.11 469.92 268.03 74.54 491.10 289.69 354.53 533.77 552.07 231.61 517.18 435.38 59.80 225.38 468.84 528.83 161.00 259.54 151.32 260.25 544.46 374.75 317.44 374.20 595.16 516.74 290.70 455.10 392.82 532.09 269.01 324.98 552.81 414.07 388.27 632.44 670.02 249.29 642.66 641.95 318.12 482.56 532.18 489.71 445.34 224.29 594.73 425.48 437.70 185.61 543.90 455.33

$$h = \frac{x_{\max} - x_{\min}}{1 + 3.322 \cdot \ln n} = 48.889$$

$[x_i..x_{i+1}]$		n_i	W_i	W_i/h_i	$F'(x)$	x_i
9	58.889	1	0.006	0.020	0.006	33.945
58.889	108.778	2	0.013	0.040	0.019	83.834
108.778	158.667	2	0.013	0.040	0.031	133.722
158.667	208.556	4	0.025	0.080	0.056	183.612
208.556	258.445	10	0.062	0.200	0.119	233.500
258.445	308.334	7	0.044	0.140	0.162	283.389
308.334	358.223	13	0.081	0.261	0.244	333.279
358.223	408.112	20	0.125	0.401	0.369	383.168
408.112	458.001	17	0.106	0.341	0.475	433.057
458.001	507.890	23	0.144	0.461	0.619	482.946
507.890	557.779	22	0.138	0.441	0.756	532.835
557.779	607.668	13	0.081	0.261	0.837	582.724
607.668	657.557	10	0.062	0.200	0.900	632.612
657.557	707.446	10	0.062	0.200	0.962	682.502
707.446	757.335	2	0.013	0.040	0.975	732.390
757.335	807.224	2	0.013	0.040	0.987	782.280
807.224	857.113	1	0.006	0.020	0.994	832.168
857.113	907.002	1	0.006	0.020	1.000	882.058


Гистограмма частот
0.4
0.3
0.2
0.1
0.0
0
200
400
600
800


Полигон частот
20
15
10
5
0
0
200
400
600
800



$$\bar{x_s} = 457.3773$$
$$\bar{D_s} = 24017.6150$$
$$\sigma = 154.9761$$
$$S^2 = 24168.6691$$
$$S = 155.4627$$

Задание 2 Доверительный интервал для мат ожидания:

$$\bar{x_s} - t\frac{s}{\sqrt{n}} < \bar{x_{tr}} < \bar{x_s} + t\frac{s}{\sqrt{n}}$$
$$t_{0.025;159} = 1.975$$
$$\bar{x_{tr}} \in (433.1038; 481.6509)$$

Доверительный интервал среднего квадратического отклонения генеральной

совокупности:

$$s \cdot q_1 < \sigma_{tr} < s \cdot q_2$$

$$q_1 = \sqrt{\frac{n-1}{\chi_{0.025;159}}}$$

$$q_2 = \sqrt{\frac{n-1}{\chi_{0.975;159}}}$$

$$\sigma_{tr} \in (142.0230; 170.7810)$$

$$(\alpha - 3\sigma; \alpha + 3\sigma) \approx (\bar{x_s} - 3s; \bar{x_s} + 3s) \approx (-9.0108; 923.7656)$$

Выборочные данные удовлетворяют правилу 3σ нормального распределения.

$$p_i = P(x_{i-1} < X < xi) = \Phi(\frac{x_i - \bar{x}}{\sigma}) - \Phi(\frac{x_{i-1} - \bar{x}}{\sigma})$$

$[x_i..x_{i+1}]$		Частоты n_i	Выравнивающие частоты $n' = np_i$
9	58.889	1	0.505
58.889	108.778	2	1.149
108.778	158.667	2	2.355
158.667	208.556	4	4.356
208.556	258.445	10	7.272
258.445	308.334	7	10.954
308.334	358.223	13	14.889
358.223	408.112	20	18.261
408.112	458.001	17	20.211
458.001	507.890	23	20.185
507.890	557.779	22	18.191
557.779	607.668	13	14.793
607.668	657.557	10	10.856
657.557	707.446	10	7.188
707.446	757.335	2	4.295
757.335	807.224	2	2.316
807.224	857.113	1	1.127
857.113	907.002	1	0.495

Частоты n_i	Выравнивающие частоты $n' = np_i$	$(n'_i - n_i)/n'_i$
5	4.009	0.245
4	4.356	0.029
10	7.272	1.024
7	10.954	1.427
13	14.889	0.240
20	18.261	0.166
17	20.211	0.510
23	20.185	0.392
22	18.191	0.797
13	14.793	0.217
10	10.856	0.067
10	7.188	1.100
6	8.232	0.605

$\chi^2_{watch} = \sum(n'_i - n_i)/n'_i = 6.8199$ Для уровня значимости $\alpha = 0.05$ и $k = 10$ соответствует значение $\chi^2_{crit} = 18.3070$.

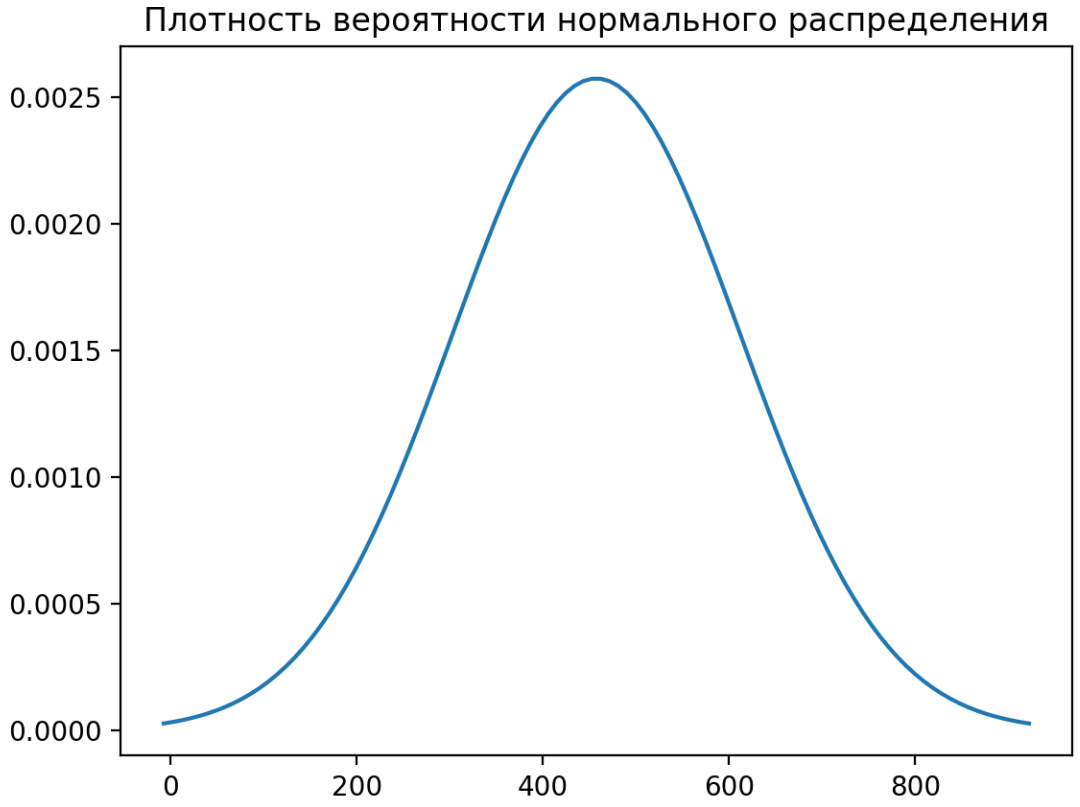
Частоты	Эмпирическая функция распределения $F'(x)$	Теоретическая функция распределения $F(x)$	Разности $\lvert F'(x) - F(x)\rvert$
1	0.006	0.005	0.001
2	0.019	0.012	0.007
2	0.031	0.027	0.004
4	0.056	0.054	0.002
10	0.119	0.100	0.019
7	0.162	0.168	0.006
13	0.244	0.261	0.017
20	0.369	0.375	0.007
17	0.475	0.502	0.027
23	0.619	0.628	0.009
22	0.756	0.741	0.015
13	0.837	0.834	0.004
10	0.900	0.902	0.002
10	0.962	0.947	0.016
2	0.975	0.974	0.001
2	0.987	0.988	0.001
1	0.994	0.995	0.001
1	1.000	0.998	0.002

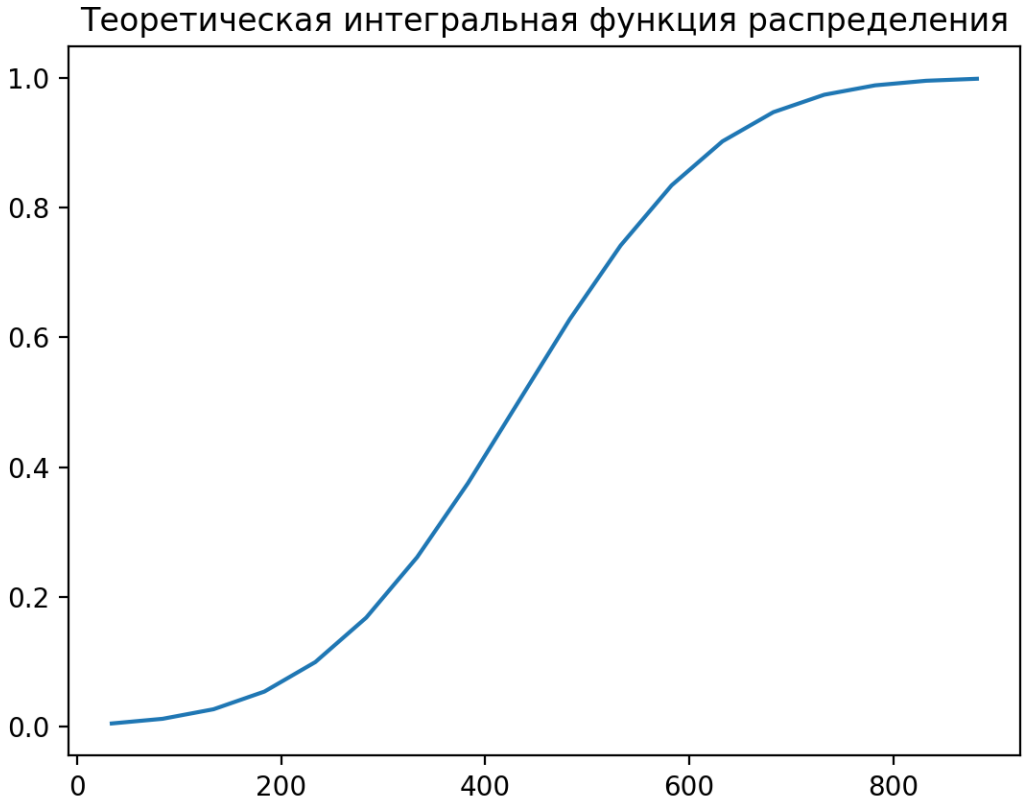
$$\lambda_{watch} = \sqrt{n} \cdot \max_x |F'(x) - F(x)| = 0.3365$$
$$\lambda_{crit} = 1.36$$

Так как:

$$\chi^2_{watch} < \chi^2_{crit}$$
$$\lambda_{watch} < \lambda_{crit}$$

Нет оснований отвергать гипотезу о нормальном распределении.





Задание 3 Для начала нужно заполнить таблицы:

	Y	18.500	19.700	20.900	22.100	23.300	24.500	25.700	26.900
X	v / u	-3.500	-2.500	-1.500	-0.500	0.500	1.500	2.500	3.500
12.500	-2.500	4	8	6					
20	-1.500		7	14	8				
27.500	-0.500				15	13	7		
35	0.500				9	18	9	6	
42.500	1.500						9	5	1
50	2.500							6	3

n_i	$n_i u_i$	$\sum n_{ij} v_j$	$n_i u_i^2$	$u_i \sum n_{ij} v_j$
18	-45	-43	112.500	107.500
29	-43.500	-42.500	65.250	63.750
35	-17.500	9.500	8.750	-4.750
42	21	33.000	10.500	16.500
15	22.500	29.500	33.750	44.250
9	22.500	25.500	56.250	63.750

m_i	$m_i v_i$	$\sum n_{ij} u_i$	$m_j v_j^2$	$v_j \sum n_{ij} u_i$
-3.500	-14.000	-10	49.000	35.000
-2.500	-37.500	-30.500	93.750	76.250
-1.500	-30.000	-36	45.000	54.000
-0.500	-16.000	-15	8.000	7.500
0.500	15.500	2.500	7.750	1.250
1.500	37.500	14.500	56.250	21.750
2.500	42.500	25.500	106.250	63.750
3.500	14.000	9	49.000	31.500

Выборочные средние X и Y:

$$\bar{u} = \frac{\sum n_i u_i}{n} = -0.2702$$
$$\bar{v} = \frac{\sum m_j v_j}{n} = 0.0810$$
$$\bar{x} = 29.2229$$
$$\bar{y} = 22.7972$$

Дисперсии признаков X и Y:

$$\sigma_u = \sqrt{\bar{u^2} - (\bar{u})^2 = 1.3660}$$
$$\sigma_v = \sqrt{\bar{v^2} - (\bar{v})^2 = 1.6725}$$
$$\sigma_x = \sigma_u h_x = 10.2455$$
$$\sigma_y = \sigma_v h_y = 2.0070$$

Таблицы условных средних:

X	12.500	20	27.500	35	42.500	50
$\bar{x_y}$	19.833	20.941	23.026	23.643	25.060	26.100

Y	18.500	19.700	20.900	22.100	23.300	24.500	25.700	26.900
$\bar{y_x}$	12.500	16	17.750	27.734	31.855	35.600	42.500	48.125

Коэффициент корреляции признаков X и Y совпадает с коэффициентом корреляции условных вариант:

$$r = \frac{1}{\sigma_u \sigma_v}\left(\frac{\sum n_{ij} u_i v_j}{n} - \bar{u}\bar{v}\right) = 0.9328$$

Следовательно коэффициент детерминации $r^2 = 0.8701$. Значит 87% рассеивания зависимой переменной объясняется линейной регрессией Y на X.

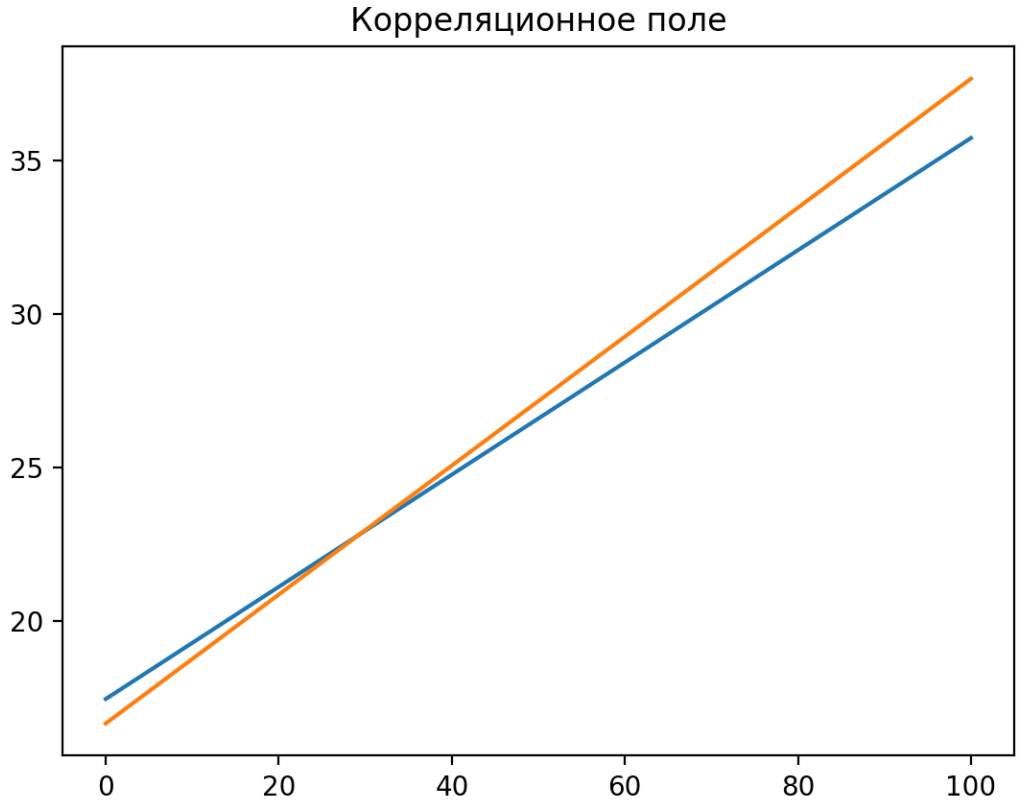
$$\sigma_{\bar{x}}^2 = \sqrt{\frac{1}{n}\sum(\bar{x}_i - \bar{x})^2 n_i} = 9.0427$$
$$\sigma_{\bar{y}} = \sqrt{\frac{1}{n}\sum(\bar{y}_i - \bar{y})^2 n_i} = 1.7722$$
$$\eta_{Y/X} = \frac{\sigma_{\bar{y}}}{\sigma_y} = 0.88299$$
$$\eta_{Y/X} = \frac{\sigma_{\bar{x}}}{\sigma_x} = 0.88260$$

Эмпирическая линейная регрессия Y на X:

$$\bar{y}_x - \bar{y} = b_1(x - \bar{x})$$
$$b = r\frac{\sigma_y}{\sigma_x} = 0.1827$$
$$y - 22.7972 = 0.1827(x - 29.2229)$$
$$y = 0.1827x + 17.4586$$

Эмпирическая линейная регрессия X на Y:

$$\bar{x}_y - \bar{x} = a(y - \bar{y})$$
$$a = r\frac{\sigma_x}{\sigma_y} = 4.7617$$
$$x - 29.2229 = 4.7617(y - 22.7972)$$



95% доверительный интервал для коэффициентов регрессии. Если уравнение имеет вид $\bar{y}_x = b_0 + b_1 x$, то $b_0 \in (-0.1754; 0.5409)$, а $b_1 \in (17.4456; 17.8154)$

Для оценки значимости выборочного коэффициента вычислим:

$$t_{watch} = r\sqrt{\frac{n-2}{1-r^2}} = 31.2775$$
$$t_{crit} = 1.98$$
$$t_{watch} > t_{crit}$$

Нулевую гипотезу отвергаем. Коэффициент корреляции значимо отличается от нуля.

data.py

```
import numpy as np

initArray = np.sort(
    np.array(
        [
            596.33, 383.19, 647.44, 477.37, 792.69, 482.05, 400.08,
            381.28, 313.42, 900.00, 463.59, 488.34, 471.48, 475.06,
            414.22, 321.03, 534.87, 667.84, 472.06, 593.00, 583.34,
            454.50, 397.68, 553.65, 684.05, 796.19, 715.45, 688.49,
            324.96, 661.44, 559.07, 698.96, 382.79, 568.19, 627.06,
            552.12, 338.17, 243.64, 599.94, 548.57, 532.31, 209.61,
            611.21, 320.79, 717.78, 567.37, 403.42, 705.66, 624.11,
            471.04, 354.48, 429.41, 388.35, 670.01, 438.85, 621.10,
            611.86, 662.92, 362.38, 358.76, 398.89, 273.35, 469.37,
            434.85, 494.38, 188.17, 477.30, 216.81, 190.81, 408.04,
```

```
            9.00, 240.83, 585.51, 705.23, 564.10, 558.02, 400.77,
            428.32, 405.27, 221.73, 496.03, 372.93, 428.31, 457.70,
            518.59, 508.23, 342.10, 314.49, 417.41, 461.55, 517.25,
            584.51, 435.63, 484.29, 808.20, 506.08, 359.76, 489.47,
            235.19, 393.88, 542.45, 656.96, 518.78, 537.45, 465.59,
            551.23, 329.04, 127.73, 498.11, 469.92, 268.03, 74.54,
            491.10, 289.69, 354.53, 533.77, 552.07, 231.61, 517.18,
            435.38, 59.80, 225.38, 468.84, 528.83, 161.00, 259.54,
            151.32, 260.25, 544.46, 374.75, 317.44, 374.20, 595.16,
            516.74, 290.70, 455.10, 392.82, 532.09, 269.01, 324.98,
            552.81, 414.07, 388.27, 632.44, 670.02, 249.29, 642.66,
            641.95, 318.12, 482.56, 532.18, 489.71, 445.34, 224.29,
            594.73, 425.48, 437.70, 185.61, 543.90, 455.33
        ]
    )
)
```

task.py

```
from scipy import *
from scipy import stats

from data import initArray
import matplotlib.pyplot as plt
import numpy as np
from latextable import *


def task():
    n = initArray.size
    step = (initArray.max() - initArray.min()) / (1 + 3.322 * np.log(initArray.size))
    step = np.round(step, 3)
    intervals = create_intervals(step)
    intervalMiddles = create_interval_middles(intervals)
    frequencies = create_frequencies(intervals)
    relativeFrequencies = np.array([x / initArray.size for x in frequencies])
    frequenciesOfDensities = np.array([x / step for x in frequencies])
    distributionFunction = create_distribution_function(relativeFrequencies)
    Exini = np.sum(np.array([intervalMiddles[i] * frequencies[i] for i in range(intervalMiddles.size)]))
    Exi2ni = np.sum(np.array([(intervalMiddles[i] ** 2) * frequencies[i] for i in range(intervalMiddles.size)]))
    En = np.sum(frequencies)
    X = Exini / En
    D = (Exi2ni / En) - (X ** 2)
    sigma = D ** 0.5
    S2 = En / (En - 1) * D
    S = S2 ** 0.5

    a = 0.05

    t = 1.975
```

```
32      alpha = (X - t * S / (n ** 0.5), X + t * S / (n ** 0.5))
33      print("Доверительный интервал alpha:", alpha)
34      delta = (np.sqrt((n - 1) / stats.chi2.ppf((1 - a)/2, df=n)) * S,
    np.sqrt((n - 1) / stats.chi2.ppf(a / 2, df=n)) * S)
35      print("Доверительный интервал delta:", delta)
36
37      laplassianAplphas = np.array([(intervals[i] - X) / sigma for i in
     range(intervals.size)])
38      laplassians = np.array(
39          [laplas_integral(laplassianAplphas[i]) - laplas_integral(
    laplassianAplphas[i - 1])
40           for i in range(1, intervals.size)])
41
42      npi = np.array([x * n for x in laplassians])
43      merge_config = create_merge_config(5, frequencies)
44      mergedFrequencies = merge_limits(merge_config, frequencies)
45      mergedNpi = merge_limits(merge_config, npi)
46
47      niNpiDeltas = np.array([mergedNpi[i] - mergedFrequencies[i] for i
     in range(mergedNpi.size)])
48      niNpiSquaredDeltas = np.array([(mergedNpi[i] - mergedFrequencies[
    i]) ** 2 for i in range(mergedNpi.size)])
49      niNpiSquaredDividedDeltas = np.array([niNpiSquaredDeltas[i] /
    mergedNpi[i] for i in range(mergedNpi.size)])
50
51      xSquaredWatched = np.sum(niNpiSquaredDividedDeltas)
52      k = niNpiDeltas.size - 2 - 1
53
54      print("k =", k)
55      X2Crit = stats.chi2.ppf(1 - a, df=k)
56      print("X^2Watched =", xSquaredWatched)
57      print("X^2Crit =", X2Crit, xSquaredWatched < X2Crit)
58
59      hypotheticalFunction = np.array([laplas_integral(
    laplassianAplphas[i]) for i in range(1, laplassianAplphas.size)])
60      hypotheticalDistributionFunctionDifferences = np.array(
61          [np.abs(distributionFunction[i] - hypotheticalFunction[i])
62           for i in range(distributionFunction.size)])
63      maxDifference = np.max(
    hypotheticalDistributionFunctionDifferences)
64      print(hypotheticalFunction)
65      print(distributionFunction)
66
67      lambdaWatched = (n ** 0.5) * maxDifference
68      print("lambdaWatched =", lambdaWatched)
69      lambdaCrit = 1.36
70      print("lambdaCrit =", lambdaCrit, lambdaWatched < lambdaCrit)
71
72      print(X - 3*S, X + 3 * S)
73
74      def print_values1():
75          print(f"Xв = {X}")
76          print(f"Dв = {D}")
77          print(f"Dв ^ 0.5 = {D ** 0.5}")
78          print(f"S ^ 2 = {S2}")
79          print(f"S = {S}")
```

```
80
81      def draw_graphs1():
82          plt.figure(dpi=200)
83          plt.bar(intervalMiddles, frequenciesOfDensities, width=step,
    edgecolor="black", color='white')
84          plt.title('Гистограмма частот')
85          plt.savefig("gr11.png")
86
87          plt.figure(dpi=200)
88          plt.plot(intervalMiddles, frequencies)
89          plt.title('Полигон частот')
90          plt.savefig("gr12.png")
91
92          plt.figure(dpi=200)
93          x = np.insert(intervalMiddles, 0, [intervalMiddles[0] - 100])
94          x = np.append(x, [intervalMiddles[-1] + 100])
95          y = np.insert(distributionFunction, 0, [0])
96          plt.title('Эмпирическая функция распределения')
97          for i in range(1, y.size):
98              plt.arrow(x[i + 1], y[i], x[i] - x[i + 1] + 20, 0,
    head_width=0.01, head_length=20)
99          plt.arrow(x[1], y[0], x[0] - x[1], 0)
100         plt.savefig("gr13.png")
101
102     def draw_graphs2():
103         plt.figure(dpi=200)
104         plt.title('Плотность вероятности нормального распределения')
105         x = np.linspace(X - 3 * sigma, X + 3 * sigma, 100)
106         plt.plot(x, stats.norm.pdf(x, X, sigma))
107         plt.savefig("gr21.png")
108
109         plt.figure(dpi=200)
110         plt.title('Теоретическая интегральная функция распределения')
111         plt.plot(intervalMiddles, hypotheticalFunction)
112         plt.savefig("gr22.png")
113
114     def print_tables1():
115         table = Texttable()
116         table.set_cols_align(["l", "l", "l", "l", "l", "l", "l"])
117         table.header(['start', 'end', 'ni', 'Wi', 'Wi / h', 'F\'(x)',
     'xi'])
118         for i in range(intervalMiddles.size):
119             table.add_row(
120                 [intervals[i], intervals[i + 1], frequencies[i],
    relativeFrequencies[i],
121                  frequenciesOfDensities[i], distributionFunction[i],
    intervalMiddles[i]])
122         print(draw_latex(table))
123
124     def print_tables2():
125         table = Texttable()
126         table.set_cols_align(["l", "l", "l", "l"])
127         table.header(['start', 'end', 'ni', 'n\' = npi'])
128         for i in range(intervalMiddles.size):
129             table.add_row(
130                 [intervals[i], intervals[i + 1], frequencies[i], npi[
```

```
    i]])
131         print(draw_latex(table))
132
133         table = Texttable()
134         table.set_cols_align(["l", "l", "l"])
135         table.header(['Частоты', 'Выравнивающие частоты', '(n\'i - ni
    ) / n\'i'])
136         for i in range(mergedFrequencies.size):
137             table.add_row(
138                 [mergedFrequencies[i], mergedNpi[i],
    niNpiSquaredDividedDeltas[i]])
139         print(draw_latex(table))
140
141         table = Texttable()
142         table.set_cols_align(["l", "l", "l", "l"])
143         table.header(
144             ['Частоты', 'Эмпирическая функция распределения', 'Теорет
    ическая функция распределения', 'Разности'])
145         for i in range(intervalMiddles.size):
146             table.add_row(
147                 [frequencies[i], distributionFunction[i],
    hypotheticalFunction[i],
148                  hypotheticalDistributionFunctionDifferences[i]])
149         print(draw_latex(table))
150
151     print_values1()
152     print_tables1()
153     draw_graphs1()
154     draw_graphs2()
155     print_tables2()
156
157
158 def create_merge_config(count: int, array: np.ndarray) -> []:
159     left = 0
160     right = 0
161     acc = 0
162     for i in range(array.size):
163         acc += array[i]
164         if acc >= count:
165             left = i + 1
166             break
167     acc = 0
168     for i in reversed(range(array.size)):
169         acc += array[i]
170         if acc >= count:
171             right = i
172             break
173     return [left, right]
174
175
176 def merge_limits(config: [], arr: np.ndarray):
177     arr = np.split(arr, config)
178     return np.array([np.sum(arr[0]), *arr[1], np.sum(arr[2])])
179
180
181 def laplas_integral(x: float) -> float:
```

```
182     return stats.norm.cdf(x)
183
184
185 def gaussian(x: float) -> float:
186     return 1 / np.sqrt(2 * np.pi) * np.exp(-x ** 2 / 2)
187
188
189 def create_frequencies(intervals: np.ndarray) -> np.ndarray:
190     frequencies = np.zeros(intervals.size - 1)
191     for i in range(intervals.size - 1):
192         if i == intervals.size - 1:
193             ni = np.count_nonzero((initArray >= intervals[i]) & (
    initArray <= intervals[i + 1]))
194         else:
195             ni = np.count_nonzero((initArray >= intervals[i]) & (
    initArray < intervals[i + 1]))
196         frequencies[i] = ni
197     return frequencies
198
199
200 def create_intervals(step: float) -> np.ndarray:
201     intervals = np.array([initArray.min()])
202     while intervals.max() < initArray.max():
203         intervals = np.append(intervals, intervals.max() + step)
204     return intervals
205
206
207 def create_interval_middles(intervals: np.ndarray) -> np.ndarray:
208     intervalMiddles = np.zeros(intervals.size - 1)
209     for i in range(intervals.size - 1):
210         intervalMiddles[i] = (intervals[i] + intervals[i + 1]) / 2
211     return intervalMiddles
212
213
214 def create_distribution_function(relative_frequencies: np.ndarray) ->
    np.ndarray:
215     distributionFunction = np.zeros(relative_frequencies.size)
216     distributionFunction[0] = relative_frequencies[0]
217     for i in range(1, relative_frequencies.size):
218         distributionFunction[i] = distributionFunction[i - 1] +
    relative_frequencies[i]
219     return distributionFunction
220
221
222 if __name__ == "__main__":
223     task()
```

task3.py

```
1 import matplotlib.pyplot as plt
2 import numpy as np
3 from latextable import draw_latex
4 from scipy import stats
5 from texttable import Texttable
6
7 matrix = np.asarray([
```

```
8       [4, 8, 6, 0, 0, 0, 0, 0],
9       [0, 7, 14, 8, 0, 0, 0, 0],
10      [0, 0, 0, 15, 13, 7, 0, 0],
11      [0, 0, 0, 9, 18, 9, 6, 0],
12      [0, 0, 0, 0, 0, 9, 5, 1],
13      [0, 0, 0, 0, 0, 0, 6, 3],
14  ])
15
16  stepX = 7.5
17  stepY = 1.2
18
19
20  def task():
21      n = np.sum(matrix)
22      xAxis = create_axis(12.5, stepX, 6)
23      yAxis = create_axis(18.5, stepY, 8)
24      Ui = np.array((xAxis - np.average(xAxis)) / stepX)
25      Vj = np.array((yAxis - np.average(yAxis)) / stepY)
26      Ni = np.array(matrix.sum(1)).flatten()
27      Mj = np.array(matrix.sum(0)).flatten()
28      NiUi = np.array(Ni * Ui)
29      MjVj = np.array(Mj * Vj)
30
31      sumNijVj = np.zeros(matrix.shape[0])
32      for i in range(matrix.shape[0]):
33          for j in range(matrix.shape[1]):
34              sumNijVj[i] += Vj[j] * matrix.item((i, j))
35
36      sumNijUi = np.zeros(matrix.shape[1])
37      for i in range(matrix.shape[0]):
38          for j in range(matrix.shape[1]):
39              sumNijUi[j] += Ui[i] * matrix.item((i, j))
40
41      MjVjpow2 = np.array(Mj * (Vj ** 2))
42      NiUipow2 = np.array(Ni * (Ui ** 2))
43
44      UiSumNijVj = np.array(sumNijVj * Ui)
45      ViSumNijUi = np.array(sumNijUi * Vj)
46
47      barU = NiUi.sum() / n
48      barV = MjVj.sum() / n
49
50      barX = np.average(xAxis) + stepX * barU
51      barY = np.average(yAxis) + stepY * barV
52
53      sigmaU = (NiUipow2.sum() / n - barU ** 2) ** 0.5
54      sigmaV = (MjVjpow2.sum() / n - barV ** 2) ** 0.5
55
56      sigmaX = sigmaU * stepX
57      sigmaY = sigmaV * stepY
58
59      rSquare = 1 / (sigmaU * sigmaV) * (UiSumNijVj.sum() / n - barU *
    barV)
60      r = rSquare ** 0.5
61
62      barYs = np.array([(matrix[i:i + 1] * yAxis).sum() / matrix[i:i +
```

```
    1].sum() for i in range(matrix.shape[0])])
63      barXs = np.array(
64          [(matrix[:, i:i + 1].flatten() * xAxis).sum() / matrix[:, i:i
        + 1].sum() for i in range(matrix.shape[1])])
65
66      sigmaBarX = ((1 / n) * np.sum(Mj * (barXs - barX) ** 2)) ** 0.5
67      sigmaBarY = ((1 / n) * np.sum(Ni * (barYs - barY) ** 2)) ** 0.5
68
69      NuyYtoX = sigmaBarY / sigmaY
70      NuyXtoY = sigmaBarX / sigmaX
71
72      XtoYRegressionA = r * sigmaX / sigmaY
73      YtoXRegressionB = r * sigmaY / sigmaX
74
75      B0 = YtoXRegressionB
76      B1 = barY - barX * YtoXRegressionB
77      a = 0.05
78      t = 1.98
79      sigmaYX = (n * sigmaY ** 2 * (1 - rSquare) / (n - 2)) ** 0.5
80      bigSquareRoot = (1 / sigmaX) * ((sigmaX ** 2 + barX ** 2) / n) **
        0.5
81      bigSquareRoot2 = 1 / (sigmaX * n ** 0.5)
82      B0Interval = (B0 - t * sigmaYX * bigSquareRoot, B0 + t * sigmaYX
       * bigSquareRoot)
83      B1Interval = (B1 - t * sigmaYX * bigSquareRoot2, B1 + t * sigmaYX
        * bigSquareRoot)
84
85      TWatch = r * ((n - 2) / (1 - rSquare)) ** 0.5
86
87      TCrit = 1.984
88
89      print('barU', barU)
90      print('barV', barV)
91      print('barX', barX)
92      print('barY', barY)
93      print('sigmaU', sigmaU)
94      print('sigmaV', sigmaV)
95      print('sigmaX', sigmaX)
96      print('sigmaY', sigmaY)
97      print('r', r)
98      print('rSquare', rSquare)
99      print('sigmaBarX', sigmaBarX)
100     print('sigmaBarY', sigmaBarY)
101     print('NuyYtoX', NuyYtoX)
102     print('NuyXtoY', NuyXtoY)
103     print('sigmaBarX', sigmaBarX)
104     print('sigmaBarY', sigmaBarY)
105     print('XtoYRegressionA', XtoYRegressionA)
106     print('YtoXRegressionB', YtoXRegressionB)
107     print('B0', B0)
108     print('B1', B1)
109     print('t', t)
110     print('sigmaYX', sigmaYX)
111     print('bigSquareRoot', bigSquareRoot)
112     print('bigSquareRoot2', bigSquareRoot2)
113     print('B0Interval', B0Interval)
```

```
114     print('B1Interval', B1Interval)
115     print('Twatch', TWatch)
116     print('TCrit', TCrit, TWatch < TCrit)
117 
118     def draw_correlation_field():
119         plt.figure(dpi=200)
120         plt.title('Корреляционное поле')
121 
122         x = np.linspace(0, 100, 2)
123 
124         y1 = (x - barX) * YtoXRegressionB + barY
125         plt.plot(x, y1)
126 
127         y2 = (x - barX) / XtoYRegressionA + barY
128         plt.plot(x, y2)
129 
130         plt.savefig("gr31.png")
131 
132     def print_tables():
133         table = Texttable()
134         table.set_cols_align(["l"] * (matrix.shape[1] + 2))
135         table.add_row(['~', '~', *yAxis])
136         table.add_row(['~', '~', *Vj])
137         for i in range(Ui.size):
138             table.add_row(
139                 [xAxis[i], Ui[i], *matrix[i]])
140         print(draw_latex(table))
141 
142         table = Texttable()
143         table.set_cols_align(["l"] * 5)
144         table.add_row(['n_{i}', 'n_{i}u_{i}', 'sum', 'a', 'a'])
145         for i in range(Ui.size):
146             table.add_row(
147                 [Ni[i], NiUi[i], sumNijVj[i], NiUipow2[i], UiSumNijVj
    [i]]
148             )
149         print(draw_latex(table))
150 
151         table = Texttable()
152         table.set_cols_align(["l"] * 5)
153         table.add_row(['m_{i}', 'm_{i}v_{i}', 'sum', 'a', 'a'])
154         for i in range(Vj.size):
155             table.add_row(
156                 [Vj[i], MjVj[i], sumNijUi[i], MjVjpow2[i], ViSumNijUi
    [i]]
157             )
158         print(draw_latex(table))
159 
160         table = Texttable()
161         table.set_cols_align(["l"] * (xAxis.size + 1))
162         table.add_row(['$X$', *xAxis])
163         table.add_row(['$\\bar{x_y}$', *barYs])
164         print(draw_latex(table))
165 
166         table = Texttable()
167         table.set_cols_align(["l"] * (yAxis.size + 1))
```

```
168         table.add_row(['$Y$', *yAxis])
169         table.add_row(['$\\bar{y_x}$', *barXs])
170         print(draw_latex(table))
171 
172     print_tables()
173 
174 def create_axis(start: float, step: float, count: int) -> np.ndarray:
175     result = np.zeros(count)
176     for i in range(count):
177         result[i] = start + step * i
178     return result
179 
180 if __name__ == "__main__":
181     task()
```